A SunCam Online Continuing Education Course

Wind Power Primer

History / Theory / Application

by

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Nomenclature[1]

A	area**	m^2
AEP	Annual Energy Production	MWh/year
ASD	Adjustable Speed Device	-
CF	Capacity Factor	%
d	diameter	m
f	frequency	Hz
GS	Generator Size	MW
h, H	height	m
n	speed	rpm
P	power	W or kW
p	poles	-
P/A	wind power density	W/m^2
r	radius	m
v, v	velocity*	m/s
VFD	Variable Frequency Device	-
VSD	Variable Speed Device	-
V_{volume}	volume	m^3

*Velocity is often used in tests where it would be more technically correct to us "speed". Velocity is a vector with speed as its magnitude.
**Area in terms of wind is defined as the area swept by the blades.

Symbols

ρ	air density	kg/m^3
ω	angular velocity	rad/s
E	energy	kW□h

Subscripts

0,o	initial or first value, original or origin, reference value	-
avg	average	-
d	decrease	-

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

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COURSE INTRODUCTION

The theoretical information is primarily from the author's books, Refs. [A] and [B]. The NESC Ref. [C] and NEC Ref. [D] though not covered in this course are useful sources for electrical engineers. Information useful in many aspects of electric engineering may be found in [E] and [F]. Reference [G] has detailed descriptions of analysis techniques. Reference [H] covers many terms in EE with excellent definitions and explanations. References [I] and [J] provide indepth information on magnetics, though one should use the latest versions or similar references. Reference [K] provides indepth coverage of generation and transmission fundamentals. Reference [L] contains useful atmospheric information. The appendices cover information useful in many engineering tasks with App. H provides a side by side comparison of electric and magnetic equations.

HISTORY & OVERVIEW

Wind was a major source of power even before modern times. It was used to pump water on farms, grind grain, and travel the world in sailing ships. The peak time for use of farm windmills was back in the 1930s and 1940s [Ref K]. They were used for power as well in rural area until expansion of the electric grid by the Rural Electrification Act of 1936. The oil crisis in 1973 sparked interest in large wind turbines, which expanded to distributed systems in the 1980s and to our current day.

Advantages include a) renewability, b) useful in many areas of the world, and c) the system does not require water. Disadvantages include a) wind is variable and thus not reliable, b) it's a low density energy source, c) high initial costs, d) requires storage and/or backup power systems, and e) windy areas tend to be located distance from load centers (cities), f) frequency stabilization with wind changes and large load changes, g) gearbox failures and maintenance difficulties, and h) though admittedly subjective, wind turbines scar the landscape over wide areas, even off-shore (having likely negative impact on marine life and fisheries).

Small turbines utilized for households, small businesses, and telecommunications are generally produced ≤100 kW.[2]

At the time of this writing, wind power generates approximately 10% of the nation's electricity.

[2] The average house uses 30 kW/day according to the US Energy Information Agency.

WIND FUNDAMENTALS

Wind Power

Wind power increases as the cube of the wind speed.

Equation 1: Wind Power

$$\frac{P}{A} = 0.5\rho v^3$$

$$P = 0.5A\rho v^3$$

The terms are *P* for power, *A* for area, ρ for air density, and v for the air speed. The *P/A* is known as the *wind power density*. Using the average wind speed normally underestimates the power, which is conservative. Of note, wind speed varies with height, so the height of the wind generator must also be known and accounted for when using data to determine the wind speed.

Since air density depends on the barometric pressure and temperature, the wind power decreases with elevation above sea level.[3]

Equation 2: Power Decrease per Elevation

$$P_{\text{decrease}} = \frac{10\%}{1000\text{ m}}$$

The density of air at sea level is 1.225 kg/m³ per the International Standard Atmosphere (ISA). Pure, dry air has a density of 1.293 kg/m^3 at a temperature of 273 K and a pressure of 101.325 kPa.[4]

Example 1

An on-shore blade for a wind turbine is 50 m long.[5] Using a density of 1.225 kg/m³ and a wind speed of 10 mph according to a local weather report.

How much power can be expected from the turbine?

---

[3] This may seem at odds with information presented later, specifically that wind speed increases with height, which it does. But at the air density decrease at higher elevations results in the power decrease.

[4] The temperature 273 K is the freezing point of water while 101.325 kPa is 1 standard atmosphere. See Ref [L].

[5] Off-shore turbine blades can be twice that length.

Solution

The applicable equation follows.

$$\frac{P}{A} = 0.5\rho v^3$$
$$P = 0.5A\rho v^3$$

Now, convert to the units necessary in the equation.

First, convert the 10 mph to m/s.

$$v = \left(10\,\frac{\text{miles}}{\text{hour}}\right)\left(\frac{1{,}609\ \text{m}}{1\ \text{mile}}\right)\left(\frac{1\ \text{hour}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$
$$= 4.469\ \frac{\text{m}}{\text{s}} \quad \left(4.5\ \frac{\text{m}}{\text{s}}\right)$$

Now, obtain the area using the diameter (length) of the blade.

$$A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}d^2$$
$$= \frac{\pi}{4}(50\ \text{m})^2$$
$$= 1963.495\ \text{m}^2 \quad \left(2000\ \text{m}^2\right)$$

Use the given information regarding density and substitute.

$$\frac{P}{A} = 0.5\rho v^3$$
$$P = 0.5A\rho v^3$$
$$= 0.5\left(2000\ \text{m}^2\right)\left(1.225\ \text{kg/m}\ \right)\left(4.5\ \frac{\text{m}}{\text{s}}\right)^3$$
$$= 111628\ \text{W} \quad (112\ \text{kW})$$

Wind Shear

Wind speed changes with height and is called *wind shear*. The wind speed at a given height can be estimated using a known wind speed and the following.

Equation 3: Wind Shear

$$\frac{v}{v_0} = \left(\frac{H}{H_0} \right)^{\alpha}$$

The v_0 is the known wind speed at height H_0 , the reference value. The exponent, alpha, is the wind shear exponent determined from experiments considering stable atmospheric conditions. The value is

Equation 4: Wind Shear Exponent

$$\alpha = \frac{1}{7} = 0.14$$

The result means the wind speed doubles from 10 m to 50 m, a good thumb rule. The world meteorological standard for wind speed uses the 10 m height as its standard.

The actual value for the exponent varies somewhat by location as does the wind, even for closely related sites. Meaning, wind power is very site specific. Wind maps for the U.S. for land and off shore at varying heights are available from the Department of Energy's National Renewable Energy Laboratory (NREL). An example is shown in Fig. 1.

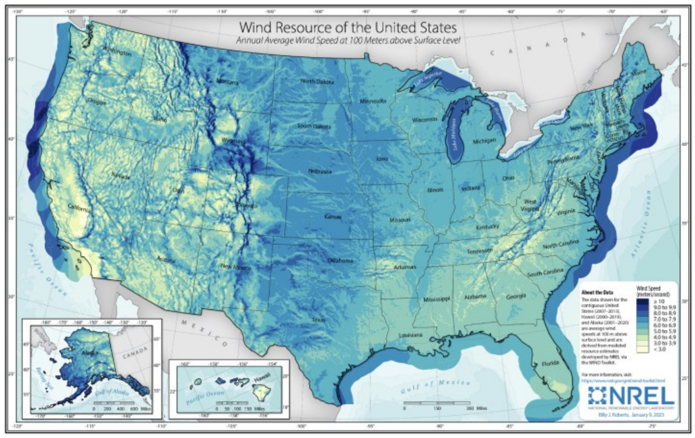

Figure 1: Wind Map at 100 m above Surface Level

NREL specifies wind power by class as shown in Table 1.

Table 1: Wind Power Class at 10 m and 50 m

Wind Power Class*	10 m (33 ft)		50 m (164 ft)	
	Wind Power Density (W/m^2)	Speed(b) m/s (mph)	Wind Power Density (W/m^2)	Speed(b) m/s (mph)
1	0	0	0	0
2	100	4.4 (9.8)	200	5.6 (12.5)
3	150	5.1 (11.5)	300	6.4 (14.3)
4	200	5.6 (12.5)	400	7.0 (15.7)
5	250	6.0 (13.4)	500	7.5 (16.8)
6	300	6.4 (14.3)	600	8.0 (17.9)
7	400	7.0 (15.7)	800	8.8 (19.7)
	1000	9.4 (21.1)	2000	11.9 (26.6)

WIND TURBINE CONSTRUCTION

Classification of wind turbines is by a) axis (horizontal or vertical), b) innovative or unusual types (Savonius; Giromill, Darrieus, et al) and c) interaction of blades with the wind (drag or lift). See Fig. 2 for examples.

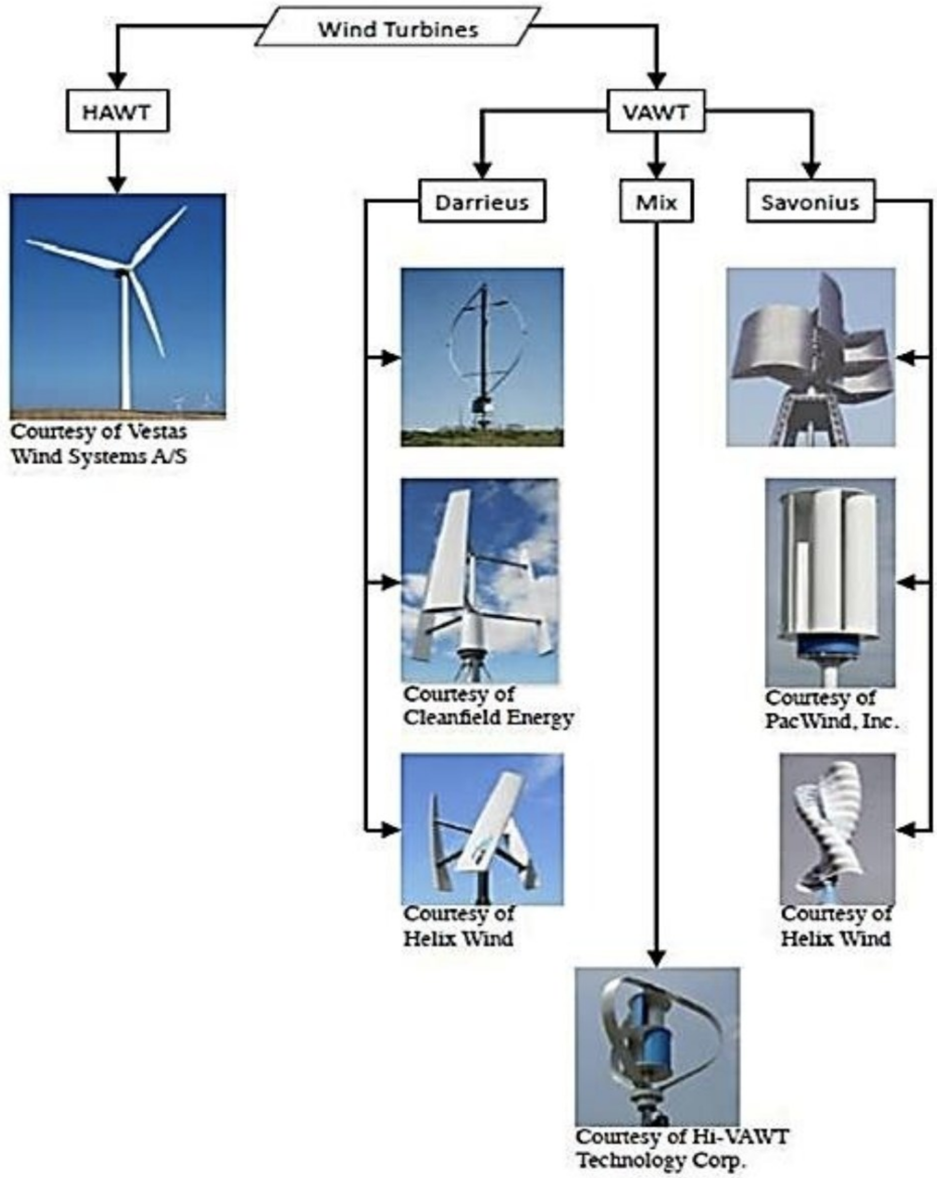

Figure 2: Horizontal and Vertical Axis Wind Turbines
[Source: *https://www.researchgate.net*]

Both the Horizontal Axis Wind Turbine (HAWT) and the Vertical Axis Wind Turbine (VAWT) have similar sub-systems, except that the VAWTS do not have a yaw system, as they are not sensitive to wind direction. The various subsystems are shown in Fig. 3.[6]

[6] The electrical engineer is going to be focused on the generator with an AC output, the storage of the energy as DC generally in batteries, and the conversion to AC at a constant frequency for the electrical grid.

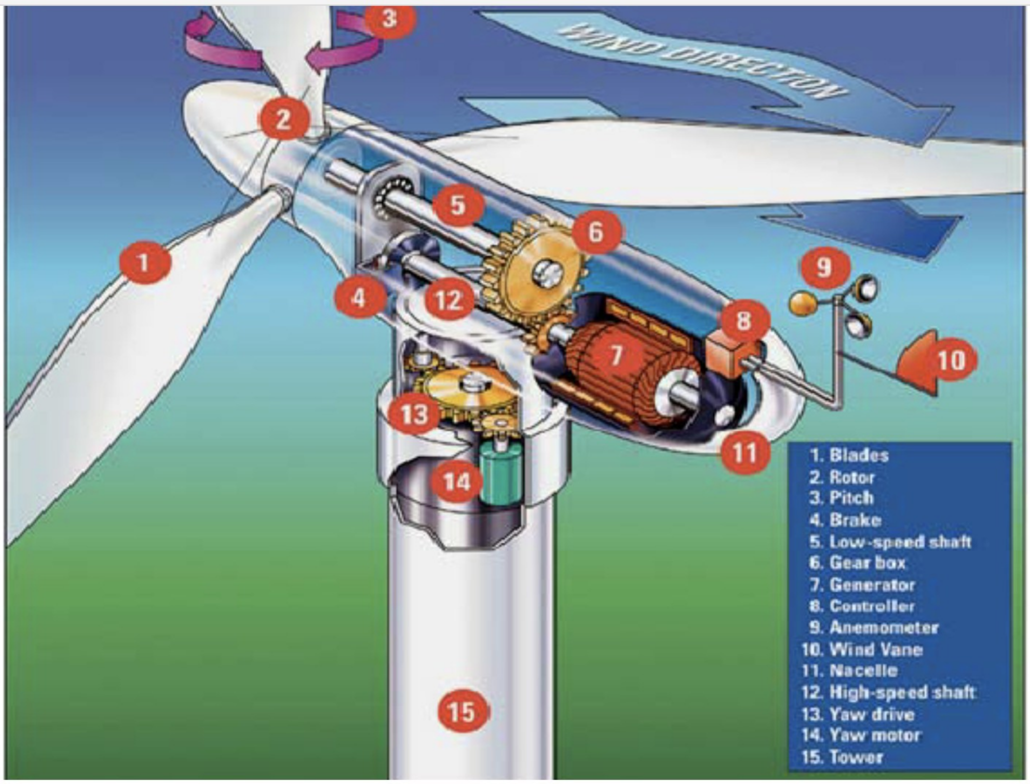

Figure 3: Wind Turbine Cut-Away

[Source: *https://www.researchgate.net*]

Drag device designs are where the wind pushes against the blade forcing the rotor to turn. Such devices are limited in efficiency since the blade speed cannot be greater than the wind speed. The theoretical efficiency is 15% maximum and thus they are not used commercially for the generation of electricity.

Lift devices use airfoils for the blades similar to propellers or airplane wings. Such designs have wing tip speeds up to 7 times the wind speed.

The output of a wind turbine is given by the rotational kinetic energy it imparts to the rotor and is generally output as electrical energy. The output can be from a synchronous or induction generator connected *directly to the grid*; or from a variable frequency alternator (permanent magnet alternator)[7] or a direct current generator connected *indirectly via an inverter to the grid*.

[7] Very much like your car's alternator.

ELECTRICAL POWER RATINGS

The *power coefficient* is defined as the power output compared to the power available in the wind. Based on calculations using conservation of energy and momentum, the theoretical maximum is some 59%.

The *average power* is the total energy production over a given time period, often quarterly or annually.

Equation 5: Average Power

$$P_{avg} = \frac{E}{t}$$

Which brings one to the *capacity factor*: the ratio of the actual use to the available capacity, that is, the *power rating*.

Equation 6: Capacity Factor

$$CF = \frac{P_{\text{used}}}{P_{\text{rated}}}$$

Example 2

A given wind turbine produces an annual energy production of 5000 MW-h and is rated for 2 MW.

What is the average power during the year? What is the unit's capacity factor?

Solution

The annual average power is calculated from the following. Use 8760 h as the hours within a year.

$$\begin{aligned} P_{avg} &= \frac{E}{t} \\ &= \frac{5000\ \text{MW}\cdot\text{h}}{8760\ \text{h}} \\ &= 0.57\ \text{MW} \end{aligned}$$

Since the capacity factor is the actual use compared to the rating of the turbine, the following is applicable.

$$
\begin{aligned}
CF &= \frac{P_{\text{actual}}}{P_{\text{available}}} \\
&= \frac{0.57\ \text{MW}}{2\ \text{MW}} \\
&= 0.28 \quad (28\%)
\end{aligned}
$$

ENERGY PERFORMANCE

The performance in terms of average annual energy production is generally estimated through three different methods.

- Generator Size (Rated Power)
- Rotor Sweep Area using Wind Map Values
- Manufacturer's Curve of Energy vs. Average Wind Speed

A good first order approximation using the generator size is given by the following where *AEP* is the Annual Energy Production, *CF* is the Capacity Factor, *GS* is the rated power [Generator Size] of the wind turbine (previously shown as $P_{\text{available}}$), and 8670 is the hours in a year.

Equation 7: Annual Energy Production—Generator Size

$$AEP = CF \cdot GS \cdot 8760 \quad [\text{kWh/year or MWh/year}]$$

Using the Rotor Sweep area and a Wind Map gives the following estimate. *AEP* is the Annual Energy Production, *CF* is the Capacity Factor, *GS* is the rated power [Generator Size] of the wind turbine (previously shown as $P_{\text{available}}$), A_r is the Area of the Rotor [Rotor/Blade Sweep], W_m is the power/area taken from a Wind Map, and 8670 is the hours in a year.

Equation 8: Annual Energy Production—Rotor Size

$$AEP = CF \cdot Ar \cdot W_m \cdot 8760 \quad [\text{W/year}]$$

The manufacturer may provide a curve similar to Fig. 4. Once can then determine the average wind speed for the area of turbine usage and use the figure to get the power or energy production (depending upon the y-axis utilized).

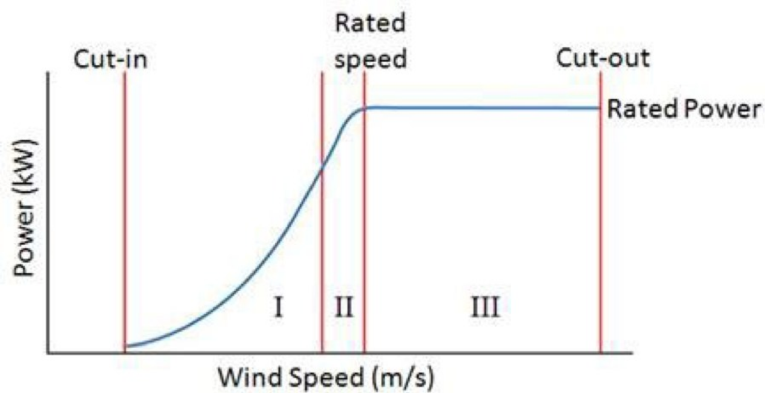

Figure 4: Energy Production—Average Wind Speed

POWER CONVERSION

As previously mentioned, so of the major challenges for the electrical engineer will be the design of a wind power system able to maintain (or respond to) changes in wind speed or large changes in load while maintaining 60 Hz supplied to the grid. See Fig. 5. Here the ac output of the wind turbine generator is converted to dc and stored in a battery where it can be inverted to supply ac to the transformer and ultimately to the grid at 60 Hz. This can be accomplished using a Variable Frequency device.

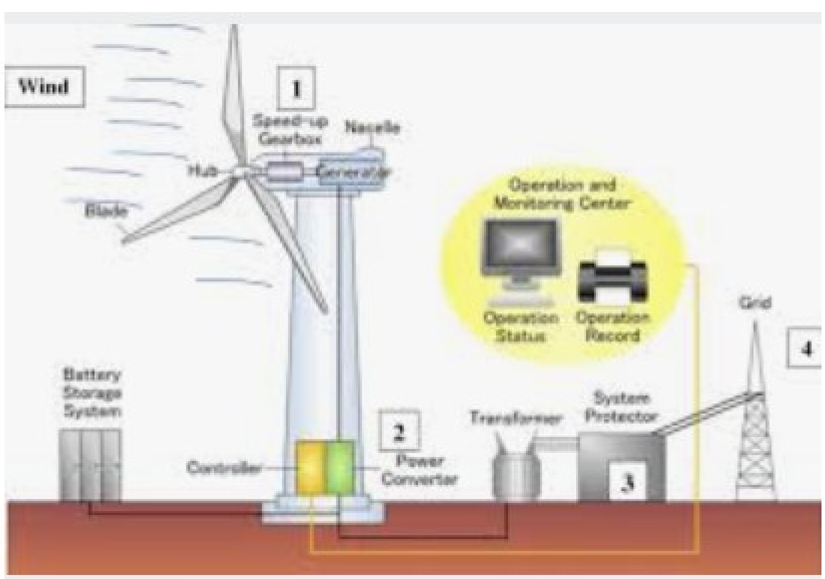

Figure 5: Energy Storage & Conversion / Inversion

Variable Frequency Device

A variable speed drive (VSD) is an electrical device whose speed varies across a considerable range as a function of the load. It is sometimes called an adjustable speed drive (ASD). When the frequency is controlled, the device is electronic and is called a variable frequency drive (VFD). Such drives change the frequency in order to change the speed of the rotational field, as shown by

Equation 9: Frequency vs. Poles

$$np = 120f$$

A variable speed drive (VSD) is an electrical device whose speed varies across a considerable range as a function of the load. It is sometimes called an adjustable speed drive (ASD). When the frequency is controlled, the device is electronic and is called a variable frequency drive (VFD). Such drives change the frequency in order to change the speed of the rotational field, as shown by

Since the number of poles, p, is generally fixed,[8] changing the frequency changes the speed of the rotational field, n, which varies the overall speed of the motor. A VFD converts AC input power to a controllable output which varies in frequency and produces a wave that closely approximates a sine wave.

A simplified electrical schematic of a VFD is shown in Fig. 6.

[8] While generally true especially once designed, on a wind turbine the number of poles designed into a generator may be increased to allow for slower rotation of the blades and rotor.

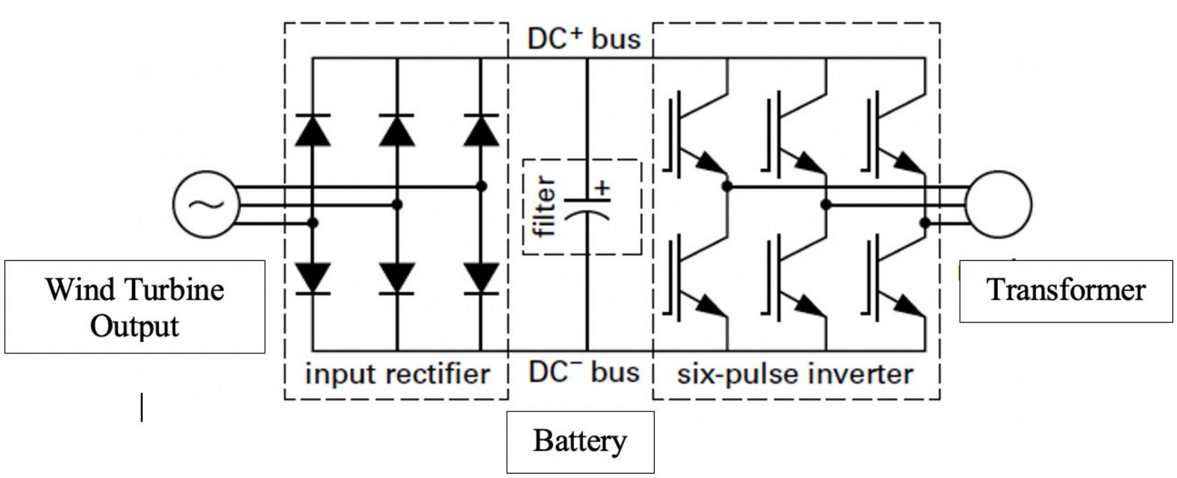

Figure 6: VFD Functional Schematic

The AC power source (wind turbine) is applied to an input rectifier section that changes the signal from AC to DC.[9] The VFD shown is a six-pulse device, since it generates three pulses on the positive portion of the cycle, and three on the negative portion. This creates a rippled DC signal in the DC bus section. Additional pulses can be added to smooth the DC signal.

Even without additional pulses to smooth the signal, the DC bus section contains filtering components such as inductors, DC links, chokes, and other items to minimize ripples in the signal. However, the DC bus section's major components are capacitors [or a battery], only one of which is shown in Fig. 6. The capacitors in the DC bus section store the energy from the pulses, maintaining the DC voltage level and delivering the energy to the inverter section.

The inverter section is controlled by circuitry that controls the firing of the individual transistors. Insulated gate bipolar transistors (IGBTs) are shown. They are common in such circuits because of their high switching speeds and low power consumption in the switching mode. IGBTs use pulse-width modulation (PWM), a common method for regulating power supplies, to create an easily modified square wave whose frequency and overall value can be changed. The modifications produce a variable sine wave. VFDs have several benefits compared to other VSDs. They reduce energy consumption, since the speed can be varied to provide only the flow required for a given application. Because the frequency at start-up is low (sometimes called a soft start), VFDs also produce lower levels of mechanical stress during start-up than other types of VSDs. Some designs use an active front end, which controls the firing of transistors. This helps minimize harmonics.

[9] Diodes are shown for simplicity, but silicon-controlled rectifiers (SCRs) or transistors can be used for more precise control.

SUMMARY

This short course is meant to provide an overview of the items of concern to the electrical engineer involved in the design of components for a wind turbine. I hope the task is accomplished. All the best.

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NOTE

Electrical refers to something related to electricity while "electric" refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code.

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Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s\cdot V)$
C	$A\cdot s$	J/V	$(N\cdot m)/V$	$V\cdot F$
F	C/V	C^2/J	s/Ω	$(A\cdot s)/V$
F/m	$C/(V\cdot m)$	$C^2/(J\cdot m)$	$C^2/(N\cdot m^2)$	$s/(\Omega\cdot m)$
H	W/A	$(V\cdot s)/A$	$\Omega\cdot s$	$(T\cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	$\text{radians}/(2\pi\cdot s)$
J	$N\cdot m$	$V\cdot C$	$W\cdot s$	$(kg\cdot m^2)/s^2$
m^2/s^2	J/kg	$(N\cdot m)/kg$	$(V\cdot C)/kg$	$(C\cdot m^2)/(A\cdot s^3)$
N	J/m	$(V\cdot C)/m$	$(W\cdot C)/(A\cdot m)$	$(kg\cdot m)/s^2$
N/A^2	$Wb/(N\cdot m^2)$	$(V\cdot s)/(N\cdot m^2)$	T/N	$1/(A\cdot m)$
Pa	N/m^2	J/m^3	$(W\cdot s)/m^3$	$kg/(m\cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg\cdot m^2)/(A^2\cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2\cdot s^3)/(kg\cdot m^2)$
T	Wb/m^2	$N/(A\cdot m)$	$(N\cdot s)/(C\cdot m)$	$kg/(A\cdot s^2)$
V	J/C	W/A	C/F	$(kg\cdot m^2)/(A\cdot s^3)$
V/m	N/C	$W/(A\cdot m)$	$J/(A\cdot m\cdot s)$	$(kg\cdot m)/(A\cdot s^3)$
W	J/s	$V\cdot A$	V^2/Ω	$(kg\cdot m^2)/s^3$
Wb	$V\cdot s$	$H\cdot A$	T/m^2	$(kg\cdot m^2)/(A\cdot s^2)$

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022 \times 10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	2.09×10^{7} ft	6.370×10^{6} m
Earth-Moon separation[2]	⊕☾	1.26×10^{9} ft	3.84×10^{8} m
Earth-Sun separtion[2]	⊕⊙	4.89×10^{11} ft	1.49×10^{11} m
Moon radius[2]	☾	5.71×10^{6} ft	1.74×10^{6} m
Sun radius[2]	⊙	2.28×10^{9} ft	6.96×10^{8} m
first Bohr radius	a_0	1.736×10^{-10} ft	5.292×10^{-11} m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{27} kg or 10^{-3} kg mol^{-1} / N_A
Earth[2]	⊕	4.11×10^{23} slugs	6.00×10^{24} kg
Earth [customary U.S.][2]	⊕	1.32×10^{25} lbm	-
Moon[2]	☾	1.623×10^{23} lbm	7.36×10^{22} kg
Sun[2]	⊙	4.387×10^{30} lbm	1.99×10^{30} kg
electron rest mass	m_e	2.008×10^{-30} lbm	9.109×10^{-31} kg
neutron rest mass	m_n	3.693×10^{-27} lbm	1.675×10^{-27} kg
proton rest mass	m_p	3.688×10^{-27} lbm	1.672×10^{-27} kg
Pressure			

atmospheric		14.696 (14.7) lbf/in^2	1.0133□10^5 Pa
Temperature			
standard		32˚F (492˚R)	0˚C (273 K)
absolute zero		–459.67˚F (0˚R)	–273.16˚C (0 K)
Velocity[3]			
Earth escape		3.67□10^4 ft/sec	1.12□10^4 m/s
light (vacuum)	c, c_0	9.84□10^8 ft/sec	2.9979 (3.00) □10^8 m/s
sound [air, STP]	a	1090 ft/sec	331 m/s
sound [air, 70°F, (20°C), 1 atm]		1130 ft/sec	344 ft/s
Volume			
Volume: molal ideal gas (STP)[4]		359 ft^3 / lbmol	22.41 m^3/kmol

Table 1 Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit “lbmol” is an actual unit, not a misspelling.

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	μ_B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/ R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N·m^2 / kg^2
nuclear magneton	μ_N		5.050×10^{-27} J/T
permeability of a vacuum	μ_0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N·m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J·s
Planck's constant: h/2π			1.0546×10^{-34} J·s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm- R	287 J/kg·K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2·K^4
triple point, water		32.02 F, 0.0888 psia	0.01109 C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol- R 1.986 BTU/lbmol- R	8314 J/kmol·K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

Appendix D: Mathematical Constants

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

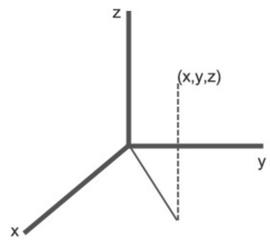
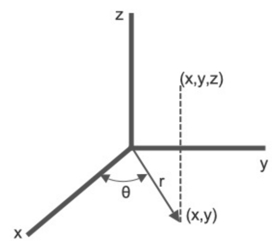
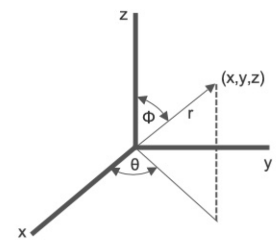
Appendix E: The Greek Alphabet

Α	α	alpha	Ν	ν	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	o	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

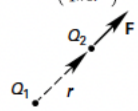
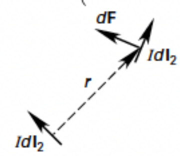
Appendix F: SI Prefixes

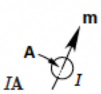
symbol	prefix	value
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}
da	deka	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

Appendix G: Coordinate Systems & Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\boldsymbol{\phi} & \frac{1}{r}\boldsymbol{\theta} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$

Appendix H: Comparison of Electric & Magnetic Equations

equation description	electric version	magnetic version	remarks
experimental force law	Coulomb's law $\mathbf{F} = \left(\frac{Q_1 Q_2}{4\pi\epsilon r^2}\right)\mathbf{r}$  	force between two current elements $d\mathbf{F} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_2 d\mathbf{l}_2 \times (I_1 d\mathbf{l}_1 \times r)}{r^2}\right)$  	The term Idl in the magnetic column is the equivalent of a "magnetic charge" q_m. The I or the dl can be the vector. The r is a unit vector pointing from 1 to 2.
field definitions from force law	$\mathbf{F} = Q\mathbf{E}$	$d\mathbf{F} = \mathbf{I}\times\mathbf{B}\,dl$ current element $d\mathbf{F} = \mathbf{J}\times\mathbf{B}\,dV$ distributed current element $d\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$ moving charge	The V used in this row represents volume, not voltage. The v is the velocity.
general force law	$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ $d\mathbf{F} = (\rho\mathbf{E} + \mathbf{J}\times\mathbf{B})\,dV$ where $dQ = \rho\,dV$		The V in this row represents the volume, not voltage. The v is the velocity.
definition of scalar and vector potential	$E = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$	**A** is the magnetic vector potential.
Poisson's equation for the potential function	$\nabla^2 V = -\frac{\rho}{\epsilon}$	$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$	From a knowledge of the charge distribution, the potential can be found and then the **E** and **B** fields determined.
Gauss's law enclosing charge and Ampère's law enclosing current	$\iint \mathbf{D}\cdot d\mathbf{A} =$ $\iiint \rho\,dV = Q$ $\nabla\cdot\mathbf{D} = \rho$	$\oint H\cdot d\mathbf{l} = I$ $\nabla\times\mathbf{H} = \mathbf{J}$	The V in this row represents volume.
constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	$\mathbf{B} = \mu\mathbf{H}$ $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	The second set of equations is always valid. The first set assumes the medium is linear and isotropic.
definitions of relative permittivity and permeability	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_0 =$ 8.854×10^{-12} F/m	$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ H/m	

equation description	electric version	magnetic version	remarks
capacitance and inductance of a field cell	$\epsilon_0 = \frac{C}{l}$	$\mu_0 = \frac{L}{l}$	Field cells are a construct designed to represent free space in terms of a parallel plate capacitor and an inductor. This capacitance and inductance exist regardless of the presence of an electric or magnetic field.
capacitance and inductance	$C = \frac{Q}{V}$	$L = \frac{\Lambda}{I}$	Λ is the flux linkage.
energy density of a field	$U = \frac{1}{2}\epsilon E^2$	$U = \frac{1}{2}\mu H^2$	Both energy and momentum are carried by a field.
energy stored by capacitance and inductance	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	
electromotive and magnetomotive force with sources present	$\oint \mathcal{E} \cdot d\mathbf{l} = \mathcal{E} = V$	$\oint \mathbf{H} \cdot d\mathbf{l} = NI = F_m = V_m$	The $\mathcal{E}$ is the emf, not the permittivity. Without sources present, both line integrals are equal to zero.
dipole moments	$\mathbf{p} = q\mathbf{d}$ 	$\mathbf{m} = I\mathbf{A}$ 	
dipole torque	$\mathbf{T} = \mathbf{p} \times \mathbf{E}$	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$	This torque occurs due to the dipole being immersed in an external **E** or **B** field.
dipole potential energy	$W = -\mathbf{p} \cdot \mathbf{E}$	$W = -\mathbf{m} \cdot \mathbf{B}$	

