A SunCam Online Continuing Education Course

Electromagnetic Theory

Electric & Magnetic Theory & Application

COURSE 1
Part I: Fundamentals
Part II: Electric & Magnetic Phenomena
Part III: Electrostatics
Part IV: Electrokinetics

COURSE 2
Part V: Magnetostatics
Part VI: Magnetokinetics
Part VII: Electromagnetic Compatibility and Interference

by

John A Camara, BS, MS, PE, TF

Copyright© 2025 John A Camara

Nomenclature[1]

a	acceleration	m/s^2
a	unit vector	-
A, **A**	area	m^2
B, **B**	magnetic flux density	T (Wb/m^2)
c	speed of light, 2.9979×10^8	m/s
C	capacitance	F
d	distance	m
D, **D**	electric flux density	C/m^2
e^-	charge of an electron, -1.6022×10^{-19}	C
E, **E**	electric field strength	V/m
F, **F**	force	N
G	conductance	S (A/V)
G	universal gravitational constant, 6.672×10^{-11}	$\mathbf{N \cdot m^2 / kg^2}$
h	Planck's constant, 6.6256×10^{-34}	$\mathbf{J \cdot s}$
H, **H**	magnetic field strength	A/m
i	variable current	A
I, **I**	constant or rms current	A
J, **J**	current density	A/m^2
k	electrostatic proportionality constant, 8.987×10^9	$\mathbf{N \cdot m^2 / C^2}$
l, **l**	length	m
ℓ	azimuthal quantum number	-
L	inductance	H
m	mass	kg
m	spatial quantum number	-
m_s, *s*	spin quantum number	-
M, **M**	magnetization	A/m
M	mutual induction	H
n	principle quantum number	-
N	number of turns	-
p	pole strength	Wb (N/T)
p^+	charge on a proton, 1.6022×10^{-19}	C
P	polarization	C/m^2

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

 Copyright© 2025 John A Camara

q, Q	charge	C
r, $\mathbf{r}$	radius	m
R	ratio	-
R	resistance	Ω
$\mathcal{R}$ or $\mathfrak{R}$	reluctance	A/Wb
s, $\mathbf{s}$	surface area	m^2
S	elastance	1/F
t	thickness	m
t	time	s
u	energy density	J/m^3
U	energy	J
υ or v	variable voltage	V_{volume}
V	constant or rms voltage	V
v, $\mathbf{v}$	velocity*	m/s
V_{volume}	volume	m^3

*Velocity is often used in tests where it would be more technically correct to us “speed”. Velocity is a vector with speed as its magnitude.

 Copyright© 2025 John A Camara

Symbols

Γ	reciprocal inductance	1/H
ε	permittivity	F/m $\left(C^2 / N \cdot m^2\right)$
ε_0	permittivity of free-space, 8.854×10^{-12}	F/m $\left(C^2 / N \cdot m^2\right)$
θ	angle	degrees
Λ	flux linkage	Wb
μ	mobility	$m^2 / V \cdot s$
μ	permeability	H/m $\left(N / A^2\right)$
μ_0	free-space permeability, 1.2566×10^{-6}	H/m $\left(N / A^2\right)$
ρ	charge density	C/m^3
σ	conductivity	$S \cdot m$
ϕ, Φ	magnetic flux	Wb
χ	susceptibility	-
ψ, Ψ	electric flux	C
ω	angular velocity	rad/s

Subscripts

1–2	From 1 to 2 Or effect on 2 caused by the field on 1	-
ave	average	-
c	conduction	-
ci	cast iron	-
d	displacement drift	-
e	electric	-
l	line	-
m	magnetic	-
m	mean	-
r	relative	-

Copyright© 2025 John A Camara

TABLE OF CONTENTS

 Copyright© 2025 John A Camara

NOTE

The content, figure, table, equation, and example numbering is a continuation from Course 1.

List of Figures

List of Tables

List of Equations

 Copyright© 2025 John A Camara

List of Examples

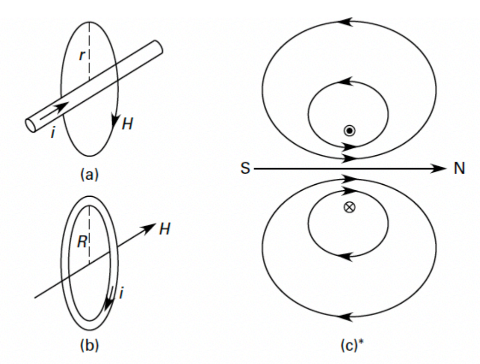
COURSE INTRODUCTION

The theoretical information is primarily from the author's books, Refs. [A] and [B]. The NESC Ref. [C] and NEC Ref. [D] though not covered in this course are useful sources for electrical engineers. Information useful in many aspects of electric engineering may be found in [E] and [F]. Reference [G] has detailed descriptions of analysis techniques. Reference [H] covers many terms in EE with excellent definitions and explanations. References [I] and [J] provide indepth information on magnetics, though one should use the latest versions or similar references. The appendices cover information useful in many engineering tasks with App. H focused on this course and provides a side by side comparison of electric and magnetic equations.

PART 5: MAGNETOSTATICS

Magnetic Poles

Mobile charges set in motion by an electric field carry their own electric fields with them. The subsequent three-dimensional field can be resolved into longitudinal and transverse components, referenced to the direction of motion. The transverse fields produce magnetic forces between moving charges. This force also acts between the conductors in which the charges move. It is this force that constitutes the basis for motor and generator theory in electric power engineering. Moving charges represented by the current, *i*, and the magnetic field established by their motion are shown in Fig. 10(a). Another completely valid view of the same situation is shown in Fig. 10(b). If the charges move with a uniform motion, the field is referred to as *magnetostatic*. If the charges move nonuniformly, the field is referred to as *magnetokinetic*.



*The symbol "⊗" indicates the current flows into the page. The symbol "⦿" indicates flow out of the page. That is, the dot indicates the head of an arrow and the "⊗" represents the tail. Assume positive current flow. This correlates with the *right-hand rule* that states that placing the thumb in the direction of the current flow and curling the fingers inward gives the direction of the magenetic field.

Figure 10: Magnetic Fields of Moving Charges

 Copyright© 2025 John A Camara

A magnetic field can exist only with two equal and opposite poles, referred to as the *north pole* and the *south pole*.[2] The combination of the north and south poles linked together is referred to as a *dipole*. The magnetic *dipole moment*, *m*, depends on the pole strength, *p*, and the distance between the poles.

Equation 42: Magnetic Dipole Moment

$$m = pd$$

The magnetic moment is a mathematical construct. Because the poles cannot be separated for measurement, the distance, *d*, is not measurable. Nevertheless, the magnetic moment is an important and fundamental quantity, and it can be measured. Magnetic moments are measured in terms of *Bohr magnetons*. In ferromagnetic material, a Bohr magneton is the moment produced by one unpaired electron, which is equivalent to 9.24×10^{-27} $A \cdot m^2$.

Figure 10(c) represents a loop of current creating a magnetic field. As the diameter of the current loop becomes infinitely small, the field approaches that of a magnetic dipole. A magnetic dipole field is shown in Fig. 11.

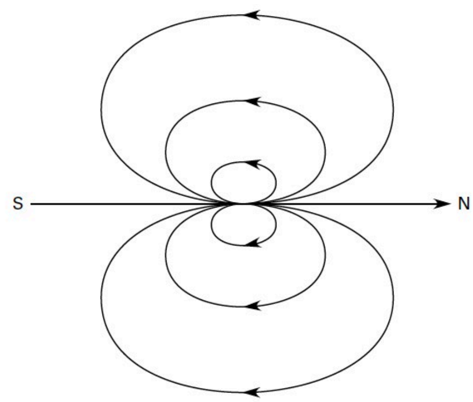


Figure 11: Magnetic Dipole Field

The infinitely small loop of current can be viewed as a single electron moving in an orbit. Hence, the magnetic moments of individual atoms and the usefulness of the Bohr magneton.[3]

For ease of presentation, and because in magnetic materials many groups of atoms tend to align themselves into domains, individual electrons or loops of current are not usually shown. Instead

[2] An electric charge can exist as a single charged object. Magnetic poles cannot exist as single north or south poles. (Mathematically there is no reason for the nonexistence of such a magnetic monopole, but none has yet been found.)

[3] It was Ampère who suggested over a hundred years ago that the magnetism observed in materials was caused by "molecular currents."

they are more often shown as in Fig. 12, which illustrates the standard convention that lines of *magnetic flux* are directed outward from the north pole (that is, the *magnetic source*), and inward at the south pole (that is, the *magnetic sink*).

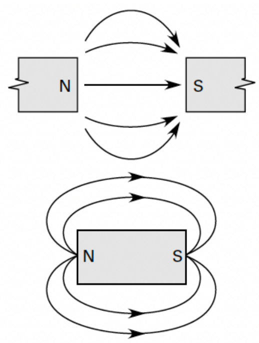


Figure 12: Typical Magnetic Field

Biot-Savart Law

The Biot-Savart law is basic to magnetostatics and magnetokinetics just as Coulomb's law is basic to electrostatics and electrokinetics. The law, in conjunction with Ampère's law, relates the force $\mathbf{F}_{1\text{-}2}$ between two current elements as in Eq. 43.

Equation 43: Biot-Savart Law

$$\mathbf{F}_{1-2} = \frac{\mu}{4\pi}\left(\frac{\left(I_2 d\mathbf{l}_2\right)\times\left(I_1 d\mathbf{l}_1\right)\times\mathbf{r}_{1-2}}{r_{1-2}^2}\right)$$

The term μ is the magnetic permeability and $\mathbf{r}_{1\text{-}2}$ is a unit vector from the point represented by differential current element $d\mathbf{l}_1$ to $d\mathbf{l}_2$.[4] Both current elements are in the magnetic field of the other. For parallel currents, Eq. 43 becomes

Equation 44: Biot Savart for Parallel Currents

$$F_{1-2} = \frac{\mu}{4\pi r^2} I_1 dI_1 I_2 dI_2$$

[4] The magnetic permeability, μ, can only be used where one is operating over a linear region of the *BH* curve. If not, the magnetic permeability of free space, μ_0, should be used in its place. For hard ferromagnetic materials, **B** and **M** are directly related and μ need not be used.

 Copyright© 2025 John A Camara

Because differential elements of current cannot be isolated, Eq. 43 and Eq. 44 are of use only when integrated over the entire path of the currents I_1 and I_2. The situation described by Eq. 43 and Eq. 44 is shown in Fig. 13.

Figure 13: Magnetic Forces Between Current Elements

The magnetic force between two moving charges is given by Eq. 45. A specific case is shown in Fig. 13.

Equation 45: Magnetic Force between Moving Charges

$$\mathbf{F}_{1-2} = \frac{\mu}{4\pi}\left(\frac{Q_1 Q_2}{r_{1-2}^2}\right)\mathbf{v}_2 \times \left(\mathbf{v}_1 \times \mathbf{r}_{1-2}\right)$$

If the charges are moving parallel to one another, the maximum magnetic force is

Equation 46: Magnetic Force between Parallel Charges

$$F = \frac{\mu}{4\pi r^2} Q_1 Q_2 \mathrm{v}^2$$

Comparison of Magnetic and Electric Force

The maximum coulombic force between moving charges in free space can be deduced from Eq. 3 to be

Equation 47: Maximum Coulombic Force

$$F_e = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$

The maximum magnetic force between moving charges in free space can be deduced from Eq. 45 to be

Equation 48: Maximum Magnetic Force

$$F = \frac{\mu_0}{4\pi r^2} Q_1 Q_2 \mathrm{v}^2$$

 Copyright© 2025 John A Camara

Comparing the maximum magnetic force F_m to the maximum electric force F_e gives

Equation 49: Ratio of Maximum Magnetic Force to Maximum Electric Force

$$\frac{F_m}{F_e} = \varepsilon_0 \mu_0 \mathrm{v}^2$$

Consider Eq. 49. In electrical engineering practice, the velocities of electrons are much less than the velocity of light, therefore the magnetic force is much weaker than the electric force. Additionally, the speed of light squared is fundamentally related to magnetic permeability and the electric permittivity as follows.

Equation 50: Speed of Light Relationship

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Finally, the magnetic force can be viewed as the electric force multiplied by the factor $(\mathrm{v}/c)^2$. That is, the magnetic force is a result of charges in relative motion

Equation 51: Magnetic Force versus Electric Force

$$F_m = F_e \left(\frac{\mathrm{v}}{c}\right)^2$$

The magnetic and electric forces on two reference charges moving in parallel are shown in Fig. 14.

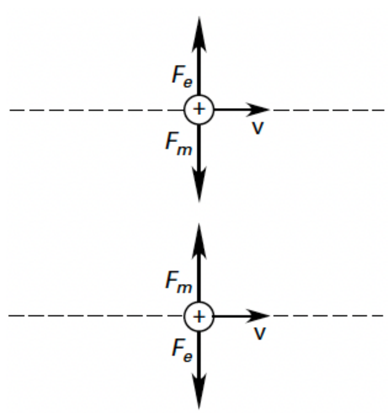


Figure 14: Magnetic and Electric Forces on Moving Charges

Magnetic Superposition

Magnetic forces occur as a result of the interaction of a pair of current-carrying conductors or the equivalent. One sets up the magnetic field and the other exhibits a force reaction to this same magnetic field. In free space and most nonmagnetic materials, the magnitude of the magnetic field is proportional to the total current that sets up the field as long as the permeability of the medium is constant. However, this is not true for iron and other ferromagnetic materials. Therefore, in general, the principle of linear superposition does not apply to current-carrying conductors in the vicinity of ferromagnetic materials.

Magnetic Fields

A magnetic field is a region in which a moving electric charge exerts a measurable force on another moving charge. It is a vector force field because a moving test charge placed in the field will experience a force in a given direction and with a specific magnitude. The direction of the field is along the flux lines, which are perpendicular to the current (that is, the moving charges creating the magnetic field). These lines, ideally, represent the direction of the force exerted on an infinitely small, isolated positive test charge moving with speed v. The actual direction is represented by the unit vector **a**. Equation 52 represents *the strength of the B-field*, or the *magnetic flux density*, in a substance with permeability μ at a distance r from current element $d\mathbf{l}_1$ that gives the direction of one turn, *N*, of the total current, *I*.[5] The denominator, $4\pi r^2$, represents the surface area of a sphere centered on the current element d**l** whose surface just touches the point in space where the **B**-field is being evaluated. Specific configurations may be evaluated with Eq. 52 by changing **B** to *d***B** and *l* to *d***l** and integrating.

Equation 52: The B Field

$$\mathbf{B} = \frac{\mu I l}{4\pi r^2}\mathbf{a}$$

B should not be called the magnetic field strength. Magnetic field strength is designated by H.[6] Although **H**, which will be defined later, is directly related to the current *I* (as **E** is directly related to the charge), the fundamental field is **B**, as **B** enters into the force relations (as **E** enters into the

[5] The **B** field has two sources. One is the current, and the other is the magnetization, **M**.

[6] For historical reasons, the concept of the **B** field in terms of magnetic flux density developed first, prior to the concept of a field or field strength **H**. The names are opposites of those used when discussing electric quantities. Nevertheless, **B** is analogous to **E**, and **H** is analogous to **D**.

force relations in electrokinetics). **B** is analogous with **E**, and **H** is analogous with **D**. This is true though a review of the units in each case would seem to indicate otherwise.

The amount of *magnetic flux* in a magnetic field is symbolized by Φ or by the commonly used ϕ, both measured in webers (Wb). The magnetic flux density, **B**, is measured in teslas (T). One T is equivalent to 1 Wb/m^2. **B** is also known as the *magnetic induction*, the *intensity of magnetization*, and the *dipole moment per unit volume*.[7] **B** is given in terms of the flux in Eq. 53 and Eq. 54.

Equation 53: Magnitude of B

$$B = |\mathbf{B}| = \frac{\phi}{A}$$

Equation 54: The B Field in Terms of Flux

$$\mathbf{B} = \frac{\phi}{A}\mathbf{a}$$

Permeability and Susceptibility

The behavior and magnitude of magnetic forces between moving charges depend on the environment in which the moving charges are located. The largest magnetic forces occur in ferromagnetic materials. Another way of viewing this phenomenon is to say that magnetic flux does not pass equally through all materials. In fact, diamagnetic material repels magnets.

The *permeability of free space* (free space is a vacuum), μ_0, represents the magnetic properties of free space. However, it is better regarded as a constant relating the units in mechanical and electromagnetic systems. 1.2566×10^{-6} $N/A^2 = 4\pi \times 10^{-7}$ N/A^2 (H/m) is the value for μ_0 in the SI system. The product of the permeability of free space and the *relative permeability* of a medium gives the *permeability* of the medium.

Equation 55: Permeability

$$\mu = \mu_0 \mu_r$$

Permeability is often defined in terms of the magnetic flux density and the magnetic field strength necessary to create it, as in Eq. 56.

---

[7] The word "induction" stems from an earlier era. When an unmagnetized piece of iron was brought near a magnet, magnetic poles were said to be "induced" in the iron

 Copyright© 2025 John A Camara

Equation 56: Permeability using B and H

$$\mu = \frac{B}{H}$$

When **B** and **H** are not parallel, the permeability, μ, is a tensor. In general, when losses occur within a material, the permeability becomes a complex number and curves that show the variations in the real and imaginary terms are called the *magnetic spectrum* or *permeability spectrum* of the material.

The permeability can also be expressed as follows.

Equation 57: Permeability using Susceptibility

$$\mu = \mu_0 \left(1 + \chi_m\right)$$

The term χ_m is a dimensionless quantity called the *magnetic susceptibility*. Magnetic susceptibility is a measure of the alignment in a magnetic material. It is also the ratio of the magnetization, *M*, to the external applied magnetic field, *H*.

Equation 58: Magnetic Susceptibility

$$\chi = \frac{M}{H}$$

For diamagnetic and paramagnetic materials, the susceptibility is essentially constant. For ferromagnetic materials, the susceptibility is nonlinear with respect to the applied field, as shown in Fig. 2 of Course 1.

Magnetic Flux

The magnetic flux, $\boldsymbol{\Psi} m$, or more correctly, Φ, is defined as the surface integral of the normal component of the magnetic flux density, **B**, over an area, *A*, and is given by Eq. 59.

Equation 59: Magnetic Flux

$$\Phi = \Psi_m = \iint \mathbf{B} \cdot d\mathbf{A}$$

 Copyright© 2025 John A Camara

It is also useful to define the flux, ϕ, as the magnetic lines of force in a region.[8] In this case, the flux is given by Eq. 60, with N representing the number of complete turns of closely spaced conductors of length l carrying current I. The term NI in Eq. 60 is sometimes referred to as the *magnetomotive force* (mmf or preferably F_m).

Equation 60: Magnetic Flux

$$\phi = \mu N I l = BA$$

The orientation of the magnetic field lines and magnetic flux lines depends on the properties of the material, specifically the permeability. By convention, the flux lines exit the north pole as given by the right-hand rule.[9] The magnetic field lines always form closed loops. They do not terminate on anything but themselves (see Fig. 10 and Fig. 11). Since no magnetic charges exist, the magnetic flux on a closed sphere (or any closed surface) is equal to zero.

Magnetic Field Strength

The *magnetic field strength*, **H**, in units of A/m, is derived from the magnetic flux density.

Equation 61: magnetic Field Strength

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B}$$

The magnetic field strength is a vector point function whose curl is the current density. Because it is proportional to the magnetic flux density in regions where no magnetized substances exist, and because the cgs system units define **H** in terms of lines of flux per unit area, **H** is referred to as the magnetic flux density in texts on magnetic theory. This explains its correlation with the electric flux density, **D**.[10] When using the SI system of units, **H** should be referred to only as the magnetic field strength.

---

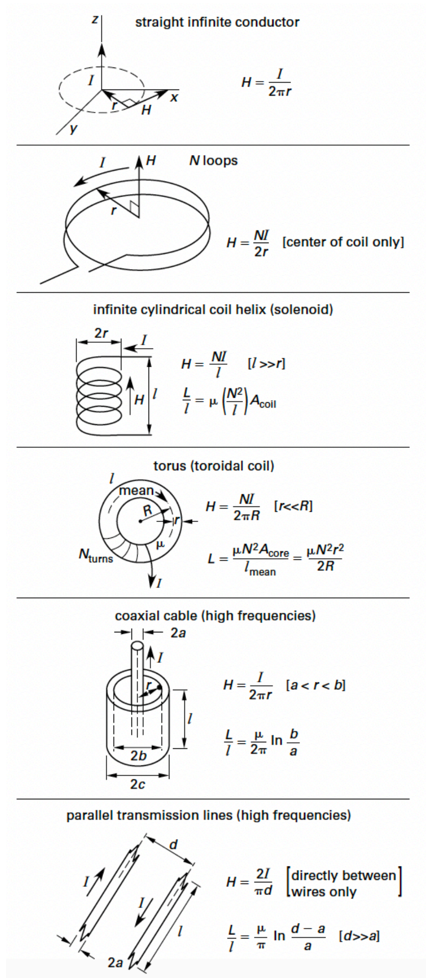
[8] The symbol for the flux, **Ψ**m, does not necessarily have to change. It is shown here as commonly used, ϕ. The standard symbol for magnetic flux is Φ. The additional symbols are used to alert the reader to the range of symbols that may be encountered.

[9] In the case of straight wire, the thumb indicates the current direction and the fingers curl in the direction of the magnetic field. For a coil of wire, the fingers curl in the direction of the positive current flow and the thumb points in the direction of the magnetic field direction. The thumb points toward the north pole.

[10] There are other reasons that **H** is analogous to **D**. For instance, **D** is used to avoid direct reference to polarization charges and the polarization, **P**. It is then easier to work with **E** and **D**. **H** is used to avoid direct reference to the magnetization, **M**. It is then easier to work with **B** and **H**.

The magnetic field and inductance for various configurations are given in Table 8.

Table 8: Magnetic Field and Inductance for Various Configurations



Example 8

A long wire with a radius of 0.001 m carries 30 A of current. What are the magnetic field strength and the magnetic flux density 0.01 m from the surface of the wire? (This is roughly equivalent to asking what are the magnetic field strength and flux density at the inside surface of a conduit that contains a 10 AWG copper wire which is carrying the maximum current allowed by the National Electrical Code.)

Solution

From Table 8, the magnetic field strength is

$$H = \frac{I}{2\pi r} = \frac{30\ \text{A}}{2\pi\left(0.001\ \text{m} + 0.01\ \text{m}\right)} = 434.06\ \frac{\text{A}}{\text{m}}$$

From Eq. 56, using the permeability of free space, the flux density is

$$B = \mu H = \left(4\pi \times 10^{-7}\ \frac{\text{Wb}}{\text{A}\cdot\text{m}}\right)\left(434.06\ \frac{\text{A}}{\text{m}}\right) = 5.45 \times 10^{-4}\ \text{T}$$

Gauss' Law for Magnetic Flux

A law similar to Gauss' law for electric flux can be written for magnetic flux. Such a law states that the amount of flux passing through any closed surface is equal to zero.

Equation 62: Gauss' Law for Magnetic Flux

$$\Phi = \psi_m = \iint \mathbf{B} \cdot d\mathbf{A} = 0$$

The physical significance of Eq. 62 is that magnetic flux lines form continuous closed loops. Additionally, this indicates that the divergence of any magnetic field is zero. In other words, no magnetic charge exists. The law refers to magnetic flux rather than magnetostatics as it is also applicable to time-varying magnetic fields (that is, magnetokinetics).

Inductance and Reciprocal Inductance

When charges are in motion, a current is said to be flowing and a magnetic field is established. When the current varies with time, the generated magnetic field opposes the current change and in doing so "stores" energy in the magnetic field. *Inductance*, L, is the property of a system of conductors and circuits that permits this storage.

If a time-varying current is applied to a conductor, usually in the form of a coil of wire, the magnetic field builds up, producing a potential difference across the ends of the conductor. The amount of potential, v, is proportional to the magnitude of the rate of change of current. The

 Copyright© 2025 John A Camara

proportionality constant is called the *inductance*, L, and is measured in henries, H. This is shown in Eq. 63.[11]

Equation 63: Inductance

$$v = L\frac{di}{dt}$$

The *reciprocal inductance*, Γ, a seldom used term, is the reciprocal of the inductance.

Equation 64 Reciprocal Inductance

$$\Gamma = \frac{1}{L}$$

The type of inductance just described is also called *self-inductance*. It can also be described in terms of *flux linkage*, Λ (measured in Wb). Flux linkage is defined as the lines of flux that link the entire magnetic circuit. For example, in Fig. 15 the "linked flux" is the flux through the center of the solenoid that links the current flows of all the wire turns (the small flux loops around each turn of wire are ignored). Using this definition, the inductance would be given by Eq. 65.

Equation 65: Self Inductance

$$L = \frac{\Lambda}{I} = \frac{N\phi}{I}$$

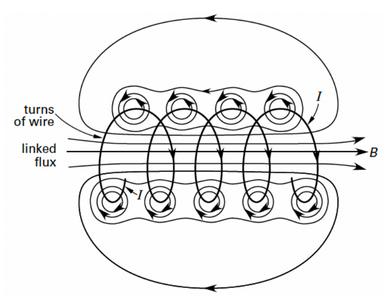


Figure 15: Magnetic Flux Links in a Solenoid

Mutual inductance, M, vital to transformer theory, is the term used to describe the fact that a changing current in one circuit induces an electromotive force in another. It is given by Eq. 66, where Λ represents the flux links in one winding and I is the current in the other winding.

[11] This equation technically belongs in Part 6 of this chapter. It is often stated as the definition of inductance and is used here for that reason.

Course **Equation 66: Mutual Inductance**

$$M = \frac{\partial \Lambda}{\partial I}$$

Inductors

An *inductor* is a device that stores magnetic energy. It typically consists of coils of wire with a core material in the center, which can be air. A common type of inductor consists of N turns of wire with radius r and an iron core center with permeability μ. This type of inductor is called a *solenoid*, and its inductance is determined by Eq. 67.

Equation 67: Solenoid Inductance

$$L = \mu \frac{N^2 A_{\text{coil}}}{l}$$

The formula for the inductance of other configuration is given in Table 8.

The total energy, U (in joules), in a magnetic field of an inductor with inductance L and current I is

Equation 68: Energy in an Inductor

$$U = \tfrac{1}{2} L I^2$$

For an inductor in series, the total inductance is

Equation 69: Inductors in Series

$$L_{\text{total}} = L_1 + L_2 + L_3 + \cdots + L_n$$

For inductors in parallel, the reciprocal of the total inductance is

Equation 70: Inductors in Parallel

$$\frac{1}{L_{\text{total}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}$$

Energy Density in a Magnetic Field

The average energy (in joules) in a magnetic field is

 Copyright© 2025 John A Camara

Equation 71: Energy in Magnetic Field

$$U_{ave} = \tfrac{1}{2}\phi N I$$

The average energy density at any point in this magnetic field is

Equation 72: Energy Density in Magnetic Field

$$u_{ave} = \frac{U_{ave}}{V_{volume}} = \tfrac{1}{2} BH = \tfrac{1}{2}\mu H^2 = \tfrac{1}{2}\frac{B^2}{\mu}$$

The average energy stored in inductors that are mutually coupled is given by Eq. 73. Here M is the mutual inductance (with units of henries) and I_1 and I_2 represent the currents in the two inductors.

Equation 73: Energy in Mutually Coupled Inductors

$$U_{ave} = MI_1I_2$$

PART 6: MAGNETOKINETICS

Speed and Direction of Charge Carriers

A positive free charge moving with velocity, **v**, in a magnetic field experiences a force given by Eq. 74.

Equation 74: Magnetic Field Force

$$\mathbf{F} = Q\mathbf{v} \times \mathbf{B}$$

The magnetic force **can change a charge's direction but not its kinetic energy**. This is in contrast to the force the charge experiences in an electric field, which does work on the charge and therefore changes its kinetic energy.[12]

If there are many charges flowing through a conductor in a magnetic field, that is, if a current-carrying conductor exists in an external magnetic field, the differential force equation is

[12] Consider that if a charge is not in motion with respect to a magnetic field, it cannot "see" the magnetic field and, therefore, experiences no change in kinetic energy. A charge stationary with respect to an electric field can "see" the field and, therefore, experiences a force that changes its kinetic energy.

Course **Equation 75: Differential Force**

$$d\mathbf{F} = (dq)(\mathbf{v}\times\mathbf{B}) = (Idt)(\mathbf{v}\times\mathbf{B}) = I(d\mathbf{l}\times\mathbf{B})$$

In Eq. 75, $d\mathbf{l}$ is a unit length in the direction of conventional (i.e., positive) current flow *I*. If the conductor is straight and the magnetic field is constant along the length of the conductor, a version of *Ampère's law* results.

Equation 76: Ampere's Law, General Magnetic

$$F = NIBl\sin\theta$$

N represents the number of turns of current-carrying conductor in the magnetic field and θ is the angle between the conductor (i.e., the current element $d\mathbf{l}$) and the magnetic field, **B**. If these are at right angles, which is often the case, the force is represented by

Equation 77: Ampere's Law, Right-Angle Magnetic

$$F = NIBl$$

The situations in Eq. 75 through Eq. 77 are illustrated in Fig. 16.

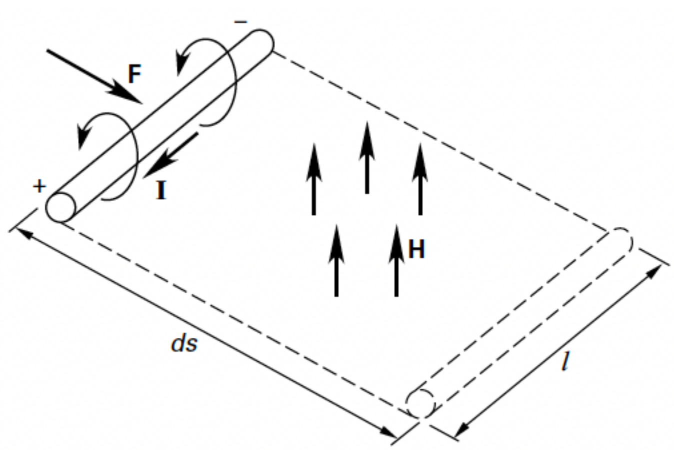


Figure 16: Conductor in a Magnetic Field

Care must be taken to properly interpret directions in the figure. If an external magnetic force is applied in the direction shown, the conductor has a velocity in the same direction. Applying Eq. 74 then gives the direction of force on the charge carriers within the conductor—in this case, in the direction of the current shown. This is the basis of generator theory. If instead a current is applied to a loop of wire, of which the conductor represents one section, the velocity in Eq.74 is in the direction of the charge carriers, that is, the direction of the current flow shown. In this case, the force given by Eq. 74 is in a direction opposite to the force arrow shown. This is the basis of motor theory.

If a magnetic field, **B**, is uniform within a region and a charge has an initial velocity perpendicular to this field, the resulting path of the particle is a circle of radius *r*. The magnitude of the force is determined by Eq. 74. Such a force would be directed toward the center of the circle. The centripetal acceleration is of a magnitude $\omega^2 r = \mathrm{v}^2/r$. Applying Newton's second law gives the following for the radius

Equation 78: Force in Uniform Magnetic Field

$$|Q|\mathrm{v}B = m\frac{\mathrm{v}^2}{r}$$

Rearranging Eq. 78 gives

Equation 79: Radius of Charge in Uniform Magnetic Field

$$r = \frac{m\mathrm{v}}{|Q|B}$$

When both electric and magnetic fields are in a given region at the same time, the force is given by the *Lorentz force equation*, Eq. 80.

Equation 80: Lorentz Force Equation

$$\mathbf{F} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Electromagnetic Oscillations and Waves

A time-varying electric field produces time-varying currents that generate magnetic fields. If the electromagnetic variations are restricted to a small region, such as within a circuit, no spatial variations of the field occur and electrical oscillations are produced. If the electromagnetic variations are generated over large areas of space, compared to the wavelength of the oscillation, electromagnetic waves are generated that propagate through space with a velocity given by

Equation 81: Propagation Velocity

$$\mathrm{v} = \frac{1}{\sqrt{\varepsilon\mu}}$$

 Copyright© 2025 John A Camara

When a conduction current generates the electromagnetic field, the magnetic field strength, **H**, is proportional to the electric field strength, **E**, and an induction field is set up, keeping the energy within the circuit. If the electromagnetic field is generated by a displacement current, the magnetic field strength, **H**, is directly proportional to the rate of change of the electric field strength, $d\mathbf{E}/dt$. A radiation field is thereby created, and energy can leave the system (i.e., be radiated into space).

Electric and Magnetic Energy Conversion

Applying the law of conservation of energy to a lossless, energy-storing system in which charges move in directed nonuniform motion means the sum of the instantaneous values of electric and magnetic energy must remain constant. Changes in the electric energy necessarily mean changes in the magnetic energy as given in Eq. 82.

Equation 82: Electric to Magnetic Conversion

$$du_e = -du_m$$

Voltage and the Magnetic Circuit

An *electric potential* or *voltage* is the work done on a unit charge to bring it from some specified reference point to another point. The symbol *V* is generally used for constant voltages and *v* is used for time-varying systems. The unit is the *volt* with 1 V = 1 J/C. By convention, current flows from the positive terminal on a voltage source to the negative outside the source and from negative to positive inside the source. Also by convention, the current flows from the positive terminal to the negative terminal through a resistor.[13] This situation is illustrated in Fig. 17 along with an analogous magnetic circuit.

[13] These directions are based on conventional current flow and are arbitrary. As long as one remains consistent when choosing reference directions for both voltage and current, the correct results will be obtained.

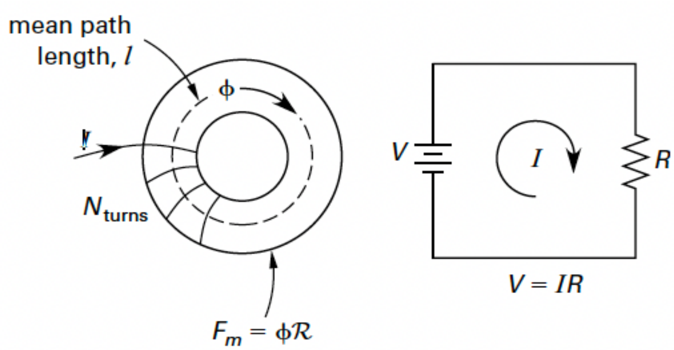


Figure 17 Magnetic-Electric Circuit Analogy

In a magnetic circuit, the flux density is proportional to the *magnetomotive force* (mmf), F_m, given in Eq. 83.

Equation 83: Magnetomotive Force

$$F_m = IN$$

The magnetomotive force is in units of amp-turns. Turns, like revolutions or radians, are for clarification only and can be disregarded in equation manipulations.

The *reluctance*, $\Re$, measured in A/Wb, is analogous to electrical resistance and is given by

Equation 84: Reluctance

$$\Re = \frac{1}{\mu A}$$

The permeability depends on the flux density. Therefore, iterations between Eq. 84 and Eq. 83 may be needed if either the reluctance or flux density is an unknown.

The magnetic equation that correlates with Ohm's law in electric circuits is

Equation 85: Ohm's Law, Magnetic

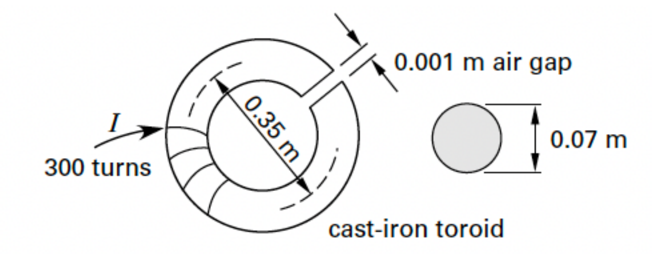
$$F_m = Hl = \phi\Re$$

 Copyright© 2025 John A Camara

Example 9

A 0.001 m slice is taken from a cast-iron toroidal coil (μ_r = 2000) with a 0.35 m mean toroidal diameter and a 0.07 m core diameter. A steady unknown current flows through 300 turns of wire wrapped around the coil, producing a constant flux density in the core and air gap.

What current is required to establish a flux of 10^{-2} Wb across the air gap?



Solution

The mean path length through the cast iron is

$$I_{ci} = \pi(0.35\text{ m}) - 0.001\text{ m} = 1.0986\text{ m}$$

The cross-sectional area of the flux path is

$$A = 2\pi r^2 = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.07\text{ m})^2 = 0.003848\text{ m}^2$$

The total reluctance is the sum of the reluctances of the cast iron and air paths. From Eq. 84,

$$\begin{aligned}\Re &= \frac{l}{\mu A} = \frac{1}{\mu_0 A}\Sigma\frac{1}{\mu_r}\\ &= \frac{1}{\left(4\pi\times10^{-7}\ \frac{\text{Wb}}{\text{A}\cdot\text{m}}\right)(0.003848\text{ m}^2)}\left(\frac{1.0986\text{ m}}{2000}+\frac{0.001\text{ m}}{1}\right)\\ &= 3.204\times10^5\ \text{A/Wb}\end{aligned}$$

The flux, ϕ, is given as 10^{-2} Wb. From Eq. 83, the magnetomotive force is

 Copyright© 2025 John A Camara

$$F_m = IN = I(300)$$

Using Eqs. 83 and 85, the current is

$$I = \frac{\phi\Re}{N} = \frac{\left(10^{-2}\ \text{Wb}\right)\left(3.203\times10^{5}\ \frac{\text{A}}{\text{Wb}}\right)}{300\ \text{turns}} = 10.68\ \text{A}$$

Magnetic Field-Induced Voltage

When a conductor "cuts" lines of magnetic flux at a given rate, a voltage called the *induced electromotance*, or *electromotive force* (emf) is produced in the conductor, as illustrated in Fig. 16. The magnitude of such an *electromagnetic induction* is given in Eq. 86, which is the mathematical statement of *Faraday's law*. Faraday's law states that the induced voltage is proportional to the time rate of change of the magnetic flux linked with the circuit. The minus sign is mandated by *Lenz's law*, which states that the current flow in a conductor caused by an induced voltage moves in a direction that creates a magnetic flux that opposes the magnetic flux inducing the voltage. More succinctly, the direction of the induced emf is such as to oppose any change in current. This is a consequence of the law of conservation of energy. (The minus sign indicates this opposing nature of the induced voltage. It is not shown in all equations. Instead, the polarity of the voltage in the circuit indicates the opposition.)

Equation 86: Electromagnetic Induction—Faraday's Law

$$V = -N\frac{d\phi}{dt}$$

Because the magnetic flux linkage is given by $\Lambda = N\phi$, and the induced voltage is the time rate of change of these linkages, the induced voltage can be written in terms of the velocity of the conductors.

Equation 87: Electromagnetic Induction—Conductor Speed

$$V = N\frac{d\phi}{dt} = NBl\frac{ds}{dt} = NBlv$$

In both Eq. 86 and Eq. 87, N is the number of conductors, l is the length of the conductor in the magnetic field, and ds is the differential distance traveled by the conductor through the field (see

Fig. 16). Equation 86 is the flux-changing method of generating a voltage. Equation 87 is the flux-cutting method of generating a voltage.

Eddy Currents

Eddy currents, also known as *Foucault currents*, occur as a result of induced voltages in a conducting medium by a varying magnetic field. Because the currents so induced cannot leave the conducting medium, they "deflect" off the edges to form loops, or circulating currents. Such currents result in Joule heating of the conducting medium and can result in considerable losses if not adequately addressed in the design of the system.

Displacement Current Magnetic Effect

The displacement current explained in earlier occurs only in dielectrics and insulators. The conduction electrons in these materials are tightly bound and cannot move freely through the material. Nevertheless, the displacement current creates a magnetic field intensity, **H**, proportional to the time rate of change of the electric field intensity, **E**, producing the displacement current. (By contrast, in a conduction current, **H** and **E** are in time phase.) Because **H** is proportional to $d\mathbf{E}/dt$, and **E** is proportional to $d\mathbf{H}/dt$, it is possible to generate electromagnetic waves in nonconducting material.

Magnetic Hysteresis

Plotting the values of the magnetic flux density, **B**, for various values of the magnetic field strength, or magnetizing force, **H**, for a ferromagnetic material reveals a nonlinear relationship. This is illustrated in Fig. 18. The phenomenon causing values of **B** to lag behind **H** so that the fields differ in magnitude is called *hysteresis*. The loop traces a complete cycle of **BH** values and is termed the *hysteresis loop*.

 Copyright© 2025 John A Camara

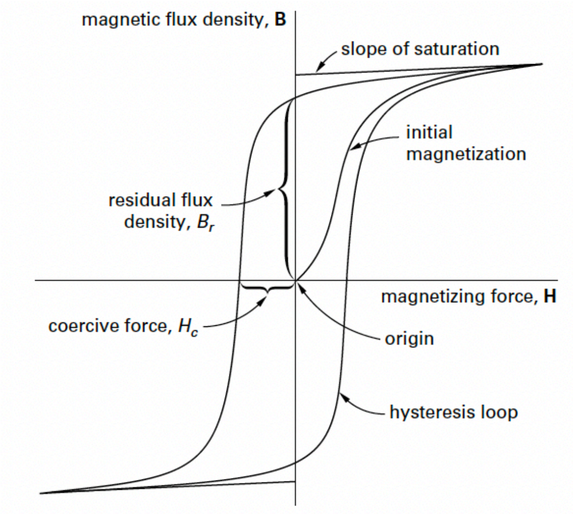


Figure 18: Magnetic Hysteresis

PART 7: ELECTROMAGNETIC COMPATIBILITY AND INTERFERENCE

Electromagnetic Compatibility and Interference: Overview

Electromagnetic compatibility (EMC) is the ability of electronic circuits or systems to operate in the expected electromagnetic environment at the designed efficiency levels.

Electromagnetic interference (EMI) is an electromagnetic disturbance that causes the performance of a device, piece of equipment, or system to degrade. If the electromagnetic disturbance comes from an external source, it is called *radiated EMI*. When it comes from an internal source, it is called *conducted EMI*.

Conducted EMI and radiated EMI can each be divided into two types of interference: *continuous wave* (CW) and *transient*. A continuous wave is generally sinusoidal, and its oscillations repeat under steady-state conditions. A transient wave is a pulse, a damped oscillation, or any other temporary phenomenon resulting in a deviation from steady-state conditions.

A circuit, device, or system is *electromagnetically compatible* if it complies with the EMI standards for another device that it will operate in or near (i.e., it is not expected to disturb the operation of the other device).

EMC also encompasses EMI, which is the discipline of controlling the unintentional generation of electromagnetic energy. The term *radio frequency interference* (RFI) was used prior to the term EMI given that radiated interference was generally associated with the spectrum of radio waves.

Standards for protecting against interference in electronic systems were first developed and controlled by the military. The Federal Communications Commission (FCC) has since started regulating digital products.

Sources of interference include lightning, relays, motors, fluorescent lights, transmitting radio antenna, digital computers, and any circuit that pulses.[14] Noise from these sources, whether conducted or radiated, can prevent the proper operation of circuits. Additionally, noise decreases the signal strength within the available bandwidth, resulting in potential errors. The throughput of a communication schema is measured by the spectral efficiency, which is the information rate that can be transmitted over a given bandwidth minus the bandwidth used by error correcting codes. Spectral efficiency is measured in bits/s/Hz. Thus, the spectral efficiency is the net bit rate (bit/s) per hertz of bandwidth.

Equation 88: Spectral Efficiency

$$\eta_s = \frac{R}{\mathrm{BW}}$$

A system is electromagnetically compatible if three general criteria are met:

- The system does not interfere with other systems.
- The system is not susceptible to interference from other systems.
- The system does not interfere with itself.

Crosstalk is one example of internal interference. The four basic EMI problems are shown in Fig. 19.

[14] While not often thought of as a source of EMI, the pulses can be the 1 and 0 signals used inside computers.

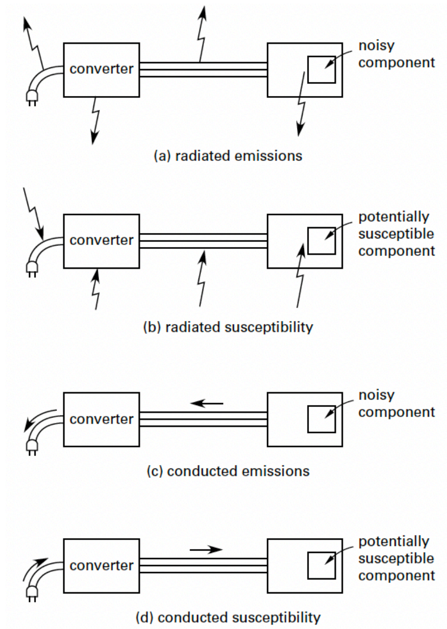


Figure 19: Basic EMI Problems

The relative size of a component determines whether it will be an EMC/EMI concern. For nonconductive media (i.e., lossless media), the wavelength, λ, and frequency, f, are related to the speed of wave propagation, v, which is usually close to the speed of light, c.

Equation 89: Wavelength of EMI/EMC

$$\lambda = \frac{\mathrm{v}}{f} \cong \frac{c}{f}$$

If a given electronic circuit, potentially susceptible component board (PCB) trace, component, or wire is small compared to the wavelength of concern, it is not expected to suffer interference from the wavelength.[15] The following equation is derived from Eq. 89, where the k-factor (i.e., wavelength factor) is a unitless term used to compare the largest dimension, D_L, to the wavelength of concern.

[15] Essentially, the wavelength can't "see" the item in question. This is the reason one can use lumped parameters such as resistors, inductors, and capacitors to represent a circuit of components connected by perfectly conducting wires. The lumped parameters also simplify the mathematics involved. If the wavelength is near that of the system, a *distributed parameter* model is used, for example, on long power transmission lines. The variable k is often used in wave equations.

Course **Equation 90: Wavelength Factor**

$$k = \frac{D_L}{\lambda} = \frac{D_L f}{\text{v}}$$

A structure or circuit is *electrically small* if its largest dimension, D_L, is much smaller than the wavelength, that is, $D_L \ll \lambda$ or $k \ll 1$. Although no set criteria exist, a rule of thumb is that the term electrically small indicates $D_L < 0.1\lambda$.

Equation 90 can be used if the velocity of propagation is known. If not, the velocity can be calculated from Eq. 91.

Equation 91: Velocity of Propagation

$$\text{v} = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

Relative permittivity and permeability vary widely. A quicker way to determine the wavelength of a given signal is to remember the following: in free space (including air), one wavelength is 1 m for a frequency of 300 MHz. Thus, as shown in Eq. 92, the frequency can be determined from this ratio. The subscripts used for the variables for frequency and wavelength indicate that as the known value of the frequency increases, the unknown value of the wavelength decreases.

Equation 92: Unknown Wavelength

$$\frac{\lambda_{\text{unknown}}}{1\ \text{m}} = \frac{300\ \text{MHz}}{f_{\text{known}}}$$

Example 10

A telephone system used for sending and receiving faxes has a download rate of 112 kbit/s. The filtering on the network limits the bandwidth to 3000 Hz.

What is the spectral efficiency of this setup?

Solution

The spectral efficiency is given by Eq. 88.

 Copyright© 2025 John A Camara

$$\eta_s = \frac{R}{BW} = \frac{(112\text{ kbit})\left(10^3\ \frac{\text{bit}}{\text{kbit}}\right)\left(1\ \frac{\text{bit}}{\text{s}}\right)}{3000\text{ Hz}} = 37.3 = 37.3\text{ bit}\cdot\text{s/Hz}$$

The result is a unitless quantity consistent with normal efficiency, but it is shown in this manner for clarification.

Electromagnetic Compatibility and Interference: Requirements

The requirements imposed on a design come from the product manufacturer and, of course, the various government agencies tasked with regulating the use of radio frequency devices. These rules and regulations are detailed in the *Code of Federal Regulations* (CFR), Title 47, Part 15 (47CFR15). The purpose of 47CFR15 is to control the emissions from radio frequency devices. This is done by separating digital devices into classes (by radiative distance) and limiting the electric fields allowed to be radiated (in µV/m) or conducted (in µV).[16]

If the interference is intentional, it is called *jamming* instead of interference. Numerous methods of jamming exist, the simplest being to send noise at the target frequency, thus lowering the signal-to-noise ratio (SNR). A method of overcoming jamming is called *spread-spectrum modulation* in which the carrier frequency constantly changes, making it more difficult to jam.

The radio spectrum for radio and all other frequencies is controlled by the FCC for nonfederal use, and by the National Telecommunications and Information Administration (NTIA) for federal use. The spectrum allocation chart is much too detailed to show here, but it is available online. The takeaway from the chart is the complexity of the allocations and, for EMI purposes, the frequencies that may be in use and of concern in electronic design.

Electromagnetic Compatibility and Interference: Electrical Components

Any electrical component can contribute to EMI. Conductors, that is, wires and PCB lands, are important in this respect. In the conducted emission range of 150 kHz–30 MHz and, more importantly, in the radiated emission range of 30 MHz–40 GHz, the electrical response of wires is

[16] The Greek letter mu, μ, in this case is NOT the efficiency. It is an abbreviation for 10^{-6}, which is a common usage. The CFR rules also use dBμV/m and dBμV.

 Copyright© 2025 John A Camara

not ideal.[17] Specifically of concern is the inductance of wires, both internal and external, due to nearby wiring. The nonideal behavior of electrical components over these frequency bands impacts the EMC design process. The inductance, capacitance, resistance, and frequency behavior of distributed versus lumped components is covered in Refs. [A] and [B]. Bode plots and filtering, essential to EMC and EMI, are also covered in of Refs. [A] and [B].

Wires can also experience crosstalk, which is the unintended transfer of electromagnetic energy between wires or PCB lands. Crosstalk occurs when both the source of the emission and the receptor are within the same system, and thus is an example of a system interfering with itself. Crosstalk can only occur when there are three or more conductors in a system since adding a third conductor provides the possibility of creating interference. Near-field and far-field effects on electronics and electrical components are addressed in the FCC regulations.

Electromagnetic Compatibility and Interference: Shielding

A *shield* is a device that limits propagated waves to a finite space. It can be a room or a metal braid around a cable. Shields can prevent external electromagnetic fields from the internal dimensions (the finite space) of the circuit. Shields can also retain the field within the circuit space to prevent the energy from radiating. To be effective, shields must be attached to a zero potential point.[18]

The shielding effectiveness is the ratio of the incident field to the transmitted field. While a positive quantity, shielding effectiveness measures the reduction in intensity of the field across the shield. For electric and magnetic fields, the shielding effectiveness, η, is measured in decibels.

Equation 93: Shield Effectiveness

$$\eta_{\text{shield}} = 20\log\frac{|\mathbf{E}_i|}{|\mathbf{E}_t|}$$

[17] Conducted emission measurements are made with a circuit called a line impedance stabilization network (LISN), which is used to remove differences in AC power supplied to the components being tested. The radiated emission measurements are made in an electromagnetic isolated room (e.g., a Faraday cage).

[18] A rule of thumb for the connection (i.e., grounding) of wire overbraids used as shielding is to ground the shield at the equipment to be protected. That is, if the source is being protected, ground the shield to the source zero potential. If the load is to be protected, ground the shield to the load zero potential. Exercise caution, however, as numerous cases exist where grounding at both ends is warranted—such as secure communication links. Guidance can be found in the Department of Defense Military Handbook publication *Grounding, Bonding, and Shielding for Electronic Equipments and Facilities* (MIL-HDBK-419A) and various IEEE documents.

Course **Equation 94: Shield Effectiveness**

$$\eta_{\text{shield}} = 20\log\frac{|\mathbf{H}_i|}{|\mathbf{H}_t|}$$

The incident, transmitted, and reflected conditions are shown in Fig. 20. If the shield is larger than the skin depth, the multiple reflections and transmission can be ignored, and the initial reflection on the left and transmission on the right are the most impactful components. However, all these components can be measured and either graphed or placed in tables (usually in decibels). The shielding effectiveness is the function of three components: the *reflection loss*, R_{dB}, the *absorption loss*, A_{dB}, and multiple reflections, M_{dB}. The shielding effectiveness is determined by adding all three of these components.

Equation 95: Shield Effectiveness Components

$$\eta_{\text{shield}} = R_{\text{dB}} + A_{\text{dB}} + M_{\text{dB}}$$

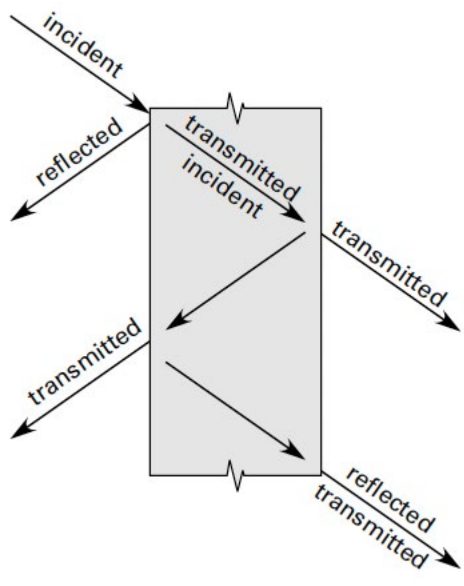


Figure 20: Shield Reflections and Transmissions

SUMMARY

This course is meant to provide an overview of the items of concern. I hope the task is accomplished. All the best.

REFERENCES

A. Camara, John A. *Electrical Engineering Reference Manual.* Belmont, CA: PPI, 2009.
B. Camara, John A. *PE Power Reference Manual*. Belmont, CA: PPI (Kaplan), 2021.
C. Marne, David J., and John A. Palmer. *National Electrical Safety Code*® (NESC®) *2023 Handbook*. New York: McGraw Hill, 2023.
D. Earley, Mark, ed. *NFPA 70, National Electrical Code Handbook*. Quincy, Massachusetts: NFPA, 2020.

NOTE

Electrical refers to something related to electricity while “electric” refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code.

E. IEEE 315-1975. *Graphic Symbols for Electrical and Electronics Diagrams.* New York: IEEE, approved 1975, reaffirmed 1993.
F. IEEE 280-2021. *IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering.* New York: IEEE.
G. Grainger, John J., and William Stevenson, Jr. *Power System Analysis*. New York, McGraw Hill, 1994.
H. Parker, Sybil P., editor in chief. *McGraw-Hill Dictionary of Scientific and Technical Terms,* 5th ed. New York, McGraw-Hill, 1994.
I. Edminister, Joseph A. *Schaum's Outline of Electromagnetics*, 2nd ed. New York, McGraw-Hill, 1995.
J. Plonus, Martin A. *Applied Electromagnetics*. New York, McGraw-Hill, 1978.

 Copyright© 2025 John A Camara

Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s \cdot V)$
C	$A \cdot s$	J/V	$(N \cdot m)/V$	$V \cdot F$
F	C/V	C^2/J	s/Ω	$(A \cdot s)/V$
F/m	$C/(V \cdot m)$	$C^2/(J \cdot m)$	$C^2/(N \cdot m^2)$	$s/(\Omega \cdot m)$
H	W/A	$(V \cdot s)/A$	$\Omega \cdot s$	$(T \cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	$\text{radians}/(2\pi \cdot s)$
J	$N \cdot m$	$V \cdot C$	$W \cdot s$	$(kg \cdot m^2)/s^2$
m^2/s^2	J/kg	$(N \cdot m)/kg$	$(V \cdot C)/kg$	$(C \cdot m^2)/(A \cdot s^3)$
N	J/m	$(V \cdot C)/m$	$(W \cdot C)/(A \cdot m)$	$(kg \cdot m)/s^2$
N/A^2	$Wb/(N \cdot m^2)$	$(V \cdot s)/(N \cdot m^2)$	T/N	$1/(A \cdot m)$
Pa	N/m^2	J/m^3	$(W \cdot s)/m^3$	$kg/(m \cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg \cdot m^2)/(A^2 \cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2 \cdot s^3)/(kg \cdot m^2)$
T	Wb/m^2	$N/(A \cdot m)$	$(N \cdot s)/(C \cdot m)$	$kg/(A \cdot s^2)$
V	J/C	W/A	C/F	$(kg \cdot m^2)/(A \cdot s^3)$
V/m	N/C	$W/(A \cdot m)$	$J/(A \cdot m \cdot s)$	$(kg \cdot m)/(A \cdot s^3)$
W	J/s	$V \cdot A$	V^2/Ω	$(kg \cdot m^2)/s^3$
Wb	$V \cdot s$	$H \cdot A$	T/m^2	$(kg \cdot m^2)/(A \cdot s^2)$

 Copyright© 2025 John A Camara

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022\times10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	2.09×10^{7} ft	6.370×10^{6} m
Earth-Moon separation[2]	⊕☾	1.26×10^{9} ft	3.84×10^{8} m
Earth-Sun separtion[2]	⊕⊙	4.89×10^{11} ft	1.49×10^{11} m
Moon radius[2]	☾	5.71×10^{6} ft	1.74×10^{6} m
Sun radius[2]	⊙	2.28×10^{9} ft	6.96×10^{8} m
first Bohr radius	a_0	1.736×10^{-10} ft	5.292×10^{-11} m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{27} kg or 10^{-3} kg mol^{-1} / N_A
Earth[2]	⊕	4.11×10^{23} slugs	6.00×10^{24} kg
Earth [customary U.S.][2]	⊕	1.32×10^{25} lbm	-
Moon[2]	☾	1.623×10^{23} lbm	7.36×10^{22} kg
Sun[2]	⊙	4.387×10^{30} lbm	1.99×10^{30} kg
electron rest mass	m_e	2.008×10^{-30} lbm	9.109×10^{-31} kg
neutron rest mass	m_n	3.693×10^{-27} lbm	1.675×10^{-27} kg
proton rest mass	m_p	3.688×10^{-27} lbm	1.672×10^{-27} kg
Pressure			

 Copyright© 2025 John A Camara

atmospheric		**14.696 (14.7) lbf/in^2**	**1.0133×10^5 Pa**
Temperature			
standard		32°F (492°R)	0°C (273 K)
absolute zero		-459.67°F (0°R)	-273.16°C (0 K)
Velocity[3]			
Earth escape		**3.67×10^4 ft/sec**	**1.12×10^4 m/s**
light (vacuum)	c, c_0	**9.84×10^8 ft/sec**	**2.9979 (3.00) $\times 10^8$ m/s**
sound [air, STP]	a	**1090 ft/sec**	**331 m/s**
sound [air, 70°F, (20°C), 1 atm]		**1130 ft/sec**	**344 ft/s**
Volume			
Volume: molal ideal gas (STP)[4]		**359 ft^3 / lbmol**	**22.41 m^3/kmol**

Table 1 Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit “lbmol” is an actual unit, not a misspelling.

 Copyright© 2025 John A Camara

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	∝B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/□R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} $N \cdot m^2 / kg^2$
nuclear magneton	∝N		5.050×10^{-27} J/T
permeability of a vacuum	∝0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ɛ0		8.854×10^{-12} $C^2 / N \cdot m^2$ (F/m)
Planck's constant	h		6.6256×10^{-34} $J \cdot s$
Planck's constant: h/2π	□		1.0546×10^{-34} $J \cdot s$
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm-□R	287 J/kg·K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m$^2 \cdot$K^4
triple point, water		32.02□F, 0.0888 psia	0.01109□C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol-□R 1.986 BTU/lbmol-□R	8314 J/kmol·K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

 Copyright© 2025 John A Camara

Appendix D: Mathematical Constants

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Appendix E: The Greek Alphabet

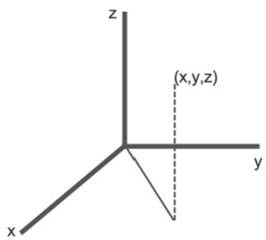
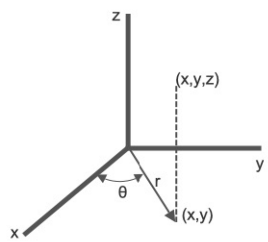
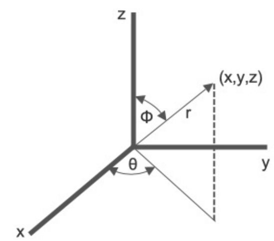
Α	α	alpha	Ν	υ	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	o	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

Appendix F: SI Prefixes

symbol	prefix	value
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}
da	deka	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

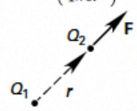
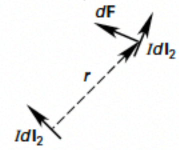
 Copyright© 2025 John A Camara

Appendix G: Coordinate Systems & Related Operations

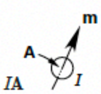
Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\theta + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\phi + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\theta$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \theta & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\phi & \frac{1}{r}\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$

 Copyright© 2025 John A Camara

Appendix H: Comparison of Electric & Magnetic Equations

equation description	electric version	magnetic version	remarks
experimental force law	Coulomb's law $\mathbf{F} = \left(\frac{Q_1 Q_2}{4\pi\epsilon r^2}\right)\mathbf{r}$  	force between two current elements $d\mathbf{F} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_2 d\mathbf{l}_2 \times (I_1 d\mathbf{l}_1 \times r)}{r^2}\right)$  	The term Idl in the magnetic column is the equivalent of a "magnetic charge" q_m. The I or the dl can be the vector. The r is a unit vector pointing from 1 to 2.
field definitions from force law	$\mathbf{F} = Q\mathbf{E}$	$d\mathbf{F} = \mathbf{I} \times \mathbf{B}\, dl$ current element $d\mathbf{F} = \mathbf{J} \times \mathbf{B}\, dV$ distributed current element $d\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$ moving charge	The V used in this row represents volume, not voltage. The v is the velocity.
general force law	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ $d\mathbf{F} = (\rho\mathbf{E} + \mathbf{J} \times \mathbf{B})\, dV$ where $dQ = \rho\, dV$		The V in this row represents the volume, not voltage. The v is the velocity.
definition of scalar and vector potential	$E = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$	A is the magnetic vector potential.
Poisson's equation for the potential function	$\nabla^2 V = -\frac{\rho}{\epsilon}$	$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$	From a knowledge of the charge distribution, the potential can be found and then the E and B fields determined.
Gauss's law enclosing charge and Ampère's law enclosing current	$\iint \mathbf{D} \cdot d\mathbf{A} =$ $\iiint \rho\, dV = Q$ $\nabla \cdot \mathbf{D} = \rho$	$\oint H \cdot dl = I$ $\nabla \times \mathbf{H} = \mathbf{J}$	The V in this row represents volume.
constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	$\mathbf{B} = \mu\mathbf{H}$ $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	The second set of equations is always valid. The first set assumes the medium is linear and isotropic.
definitions of relative permittivity and permeability	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_0 =$ 8.854×10^{-12} F/m	$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ H/m	

Copyright© 2025 John A Camara

equation description	electric version	magnetic version	remarks
capacitance and inductance of a field cell	$\epsilon_0 = \frac{C}{l}$	$\mu_0 = \frac{L}{l}$	Field cells are a construct designed to represent free space in terms of a parallel plate capacitor and an inductor. This capacitance and inductance exist regardless of the presence of an electric or magnetic field.
capacitance and inductance	$C = \frac{Q}{V}$	$L = \frac{\Lambda}{I}$	Λ is the flux linkage.
energy density of a field	$U = \frac{1}{2}\epsilon E^2$	$U = \frac{1}{2}\mu H^2$	Both energy and momentum are carried by a field.
energy stored by capacitance and inductance	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	
electromotive and magnetomotive force with sources present	$\oint \mathcal{E} \cdot d\mathbf{l} = \mathcal{E} = V$	$\oint \mathbf{H} \cdot d\mathbf{l} = NI = F_m = V_m$	The $\mathcal{E}$ is the emf, not the permittivity. Without sources present, both line integrals are equal to zero.
dipole moments	$\mathbf{p} = q\mathbf{d}$ 	$\mathbf{m} = I\mathbf{A}$ 	
dipole torque	$\mathbf{T} = \mathbf{p} \times \mathbf{E}$	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$	This torque occurs due to the dipole being immersed in an external **E** or **B** field.
dipole potential energy	$W = -\mathbf{p} \cdot \mathbf{E}$	$W = -\mathbf{m} \cdot \mathbf{B}$	

 Copyright© 2025 John A Camara