A SunCam online continuing education course

Introduction to Culvert Hydraulics

by

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Introduction:
This course presents an overview of the hydraulics of culvert flow. Analysis of the flow through a culvert can be very complicated and a variety of assumptions often need to be made to properly model the situation. For practical purposes, this is generally accomplished through the use of software packages or culvert design charts, commonly called nomographs. This course presents some practical guidelines for modeling culvert flow and also presents some of the theoretical background underpinning these guidelines.

When you complete this course you should be able to analyze a wide variety of culverts under a range of flow conditions.

For this course the primary reference is The USDOT's "Hydraulic Design of Highway Culverts". However, there are a number of regional guides to culvert analysis and design. For example, the New York State Department of Transportation has published a pamphlet entitled "Culvert Design: An Overview of the NYS Highway Design Manual" which includes guidelines for use in New York State.

Table of Contents:

A typical concrete box culvert inlet is shown in the photograph below. This course will describe how to design and analyze this (and many other) types of structure.

The photograph below shows a large, circular concrete culvert with block wingwalls.

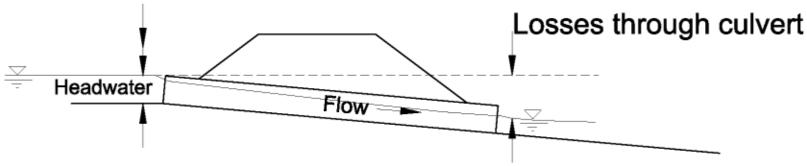
The photograph below shows the same structure under construction. The culvert is easier to see in this view.

Inlet Control vs. Outlet Control:

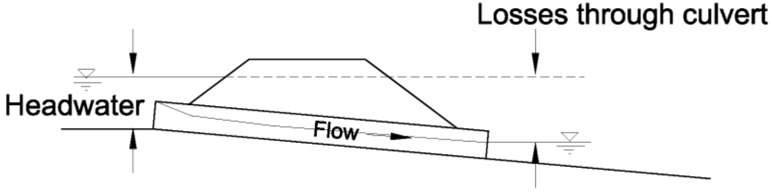
One reason that the analysis of culvert flow can be extremely complex is because, in many cases, the flow is non-uniform – meaning that there are regions within the culvert of both gradually varying and rapidly varying flow. (Gradually varying flow is simply, as the name implies, flow characterized by gradual variations in flow depth and velocity. These variations are generally caused by gentle changes in channel slope and/or geometry. Rapidly varying flow, by contrast, involves situations where the water surface undergoes abrupt changes over short distances, often caused by sharp expansions or contractions or by abrupt changes in the channel slope). Often hydraulic jumps (places where the flow suddenly changes from super-critical to sub-critical) occur within the culvert barrel. Therefore, an exact analysis of flow within the culvert requires backwater and drawdown calculations, energy and momentum balance equations, and a complete hydraulic model study. This is a cumbersome approach. Therefore, the Federal Highway Administration (FHWA) has developed a systematic, simplified approach to culvert analysis based on the various types of flow and on the location of the control section. (The FHWA considers the control section the location where there is a unique relationship between the flow rate and the upstream water surface).

The location of the control section determines which of the two basic types of flow will occur within the culvert. These flow types are: Inlet Control and Outlet Control and shown schematically in the following figures and they are described more fully below:

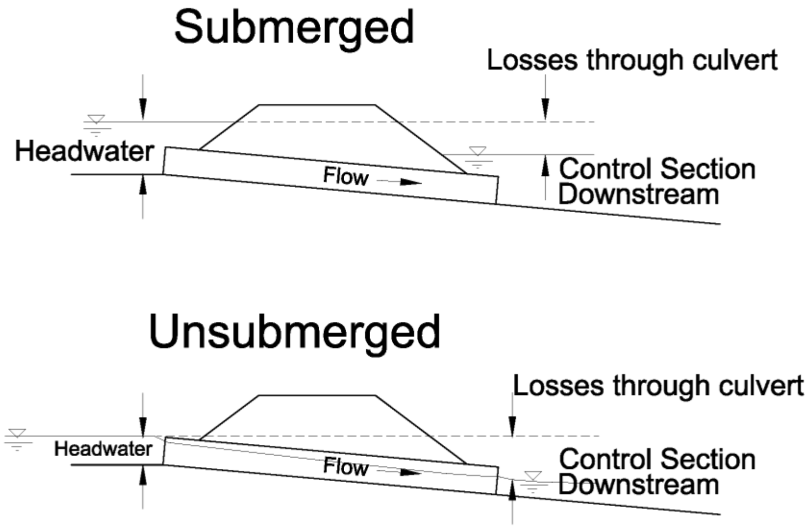
Projecting End-Unsubmerged



Projecting End-Submerged



Culvert: Inlet Control Flow Conditions



Culvert: Outlet Control Flow Conditions

Inlet Control: Inlet control occurs when the culvert barrel is capable of conveying more flow than the inlet will accept. In this type of flow the control section is located just inside the entrance of the culvert. Critical flow occurs at or near this critical section and the flow regime immediately downstream of this section will be sub-critical. Hydraulic characteristics downstream of the inlet control section do not affect the culvert flow capacity. Therefore, the major flow controls are upstream water surface elevation and the inlet geometry, which includes the inlet shape, inlet cross-sectional area, and inlet configuration.
The factors that affect the flow under inlet control are listed below:

1. Headwater Depth: This is measured from the invert of the inlet control section to the surface of the upstream pool.
2. Inlet area: This is the cross-sectional area of the culvert face. Generally, the inlet face area is equivalent of the barrel area. However, for tapered inlets (such s with flared end sections) the face area is enlarged and the control section is at the throat of the taper.
3. Inlet configuration: This describes the entrance type such as, for example: thin edge projecting, mitered, square edges in a headwall, tapered, or one of several

other configurations. A number of inlet configurations are illustrated in this course.

4. Inlet shape: This is generally the same shape as the culvert barrel. Typical culvert shapes are circular, rectangular, and elliptical, although they can take a number of shapes. In fact, sometimes in older culverts, the engineer will encounter a long culvert that changes shape along its length. If the inlet face is a different size or shape than the culvert barrel, then there is the possibility that there will be an additional control section within the culvert.
5. Barrel slope: This actually has a very small effect on the flow under inlet control conditions.

Outlet Control: Outlet control occurs when the culvert barrel is not capable of conveying as much flow as the inlet opening will accept. In this case, the control section is located at the barrel exit or even further downstream. Under outlet control conditions, the flow within the culvert barrel can be either sub-critical or pressure flow. (Pressure flow occurs when the water is flowing due to pressure differences. Obviously, fluid flows from a higher pressure toward a lower pressure). When analyzing a culvert flowing under outlet control conditions, all of the parameters listed above for inlet control (upstream water surface elevation and the inlet geometry) must be considered in addition to the barrel roughness, barrel length, barrel slope, and tailwater depth. The barrel slope is the primary factor influencing whether a culvert will operate under inlet or outlet control conditions.
Some of these parameters are defined below:

1. Barrel roughness: This is a function of the culvert material. Typical culvert materials are concrete, corrugated metal, and plastic. The roughness is modeled using a resistance coefficient (such as the Manning's "n' value). Typical n values for culverts range between 0.012 and 0.024. The table below shows some typical n values for various culvert materials. (a more detailed table is included later in the course):

Culvert Material	Manning's "n' value
Concrete	0.013
Plastic	0.012
Ductile iron	0.016
Corrugated metal	0.022
Clay	0.014

2. Barrel area: Obviously, a larger barrel will carry more flow than a smaller barrel (given the same slope, roughness, etc.)

3. Barrel shape: Typical barrel shapes are box, arch, and circle.
4. Barrel length: This is the total length of the culvert from the inlet to the outlet.
5. Barrel slope: This is the actual slope of the culvert barrel and is often (but not always) the same as the slope of the stream bed.
6. Tailwater elevation: This is the downstream water surface elevation. The tailwater elevation can be calculated by using a normal depth calculation, or from backwater calculations, or it can observed in the field.

Before moving on to a detailed discussion of the analysis of culvert flow under inlet and outlet control, a brief review of critical, sub-critical, and super-critical flow regimes might be useful. To determine the flow regime (critical, sub-critical, or supercritical), it is necessary to calculate the Froude number. The Froude Number is a measurement of bulk flow characteristics such as waves, flow/depth interactions at cross sections or between boulders, etc. It is defined as the ratio of inertial forces divided by gravitational forces.

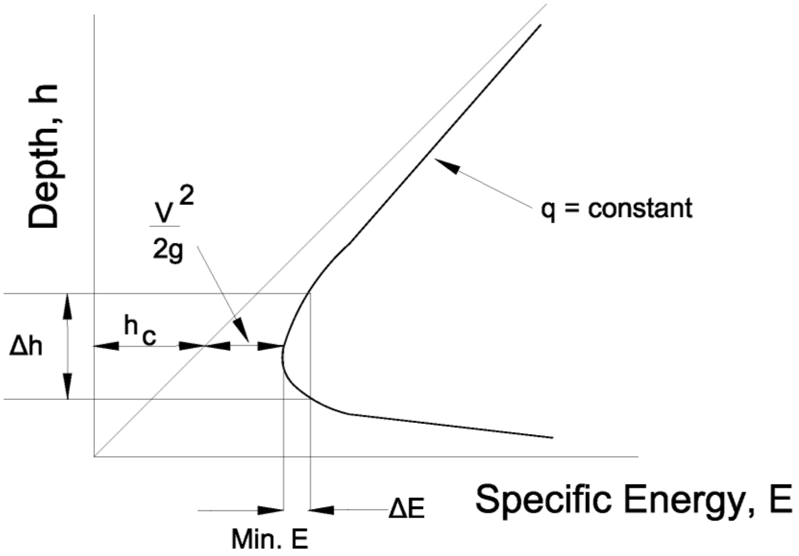
The Froude Number is calculated using the following formula:
$Fr=V/(gD)^{1/2}$, where:
Fr is the Froude Number
V is the velocity in feet per second
g is the acceleration due to gravity (32 feet per second squared)
D is the hydraulic depth (cross sectional area divided by flow top width)

If the Froude Number is greater than 1, the flow is supercritical.
If the Froude Number is equal to 1, the flow is critical.
If the Froude Number is less than 1, the flow is subcritical.

If the Froude Number is 1(i.e. critical; flow) the resulting depth is known as critical depth. Depths near critical depth are inherently unstable because small changes in surface energy can lead to large changes in the local flow depth.

The specific energy is calculated using the following equation:
$E = h + (V^2 / 2g)$, where:
E is the specific energy
v is the velocity
h is the flow depth
g is the acceleration due to gravity (32 feet per second squared)

The solution to this equation is a parabola as shown in the specific energy curve below. Note that for any value of specific energy on the x axis there are two corresponding depths on the y axis. The lower depth is the supercritical value and the larger depth represents the sub-critical flow depth. (The one exception to the two solution rule is at the minimum energy value on the curve, which corresponds to critical depth).



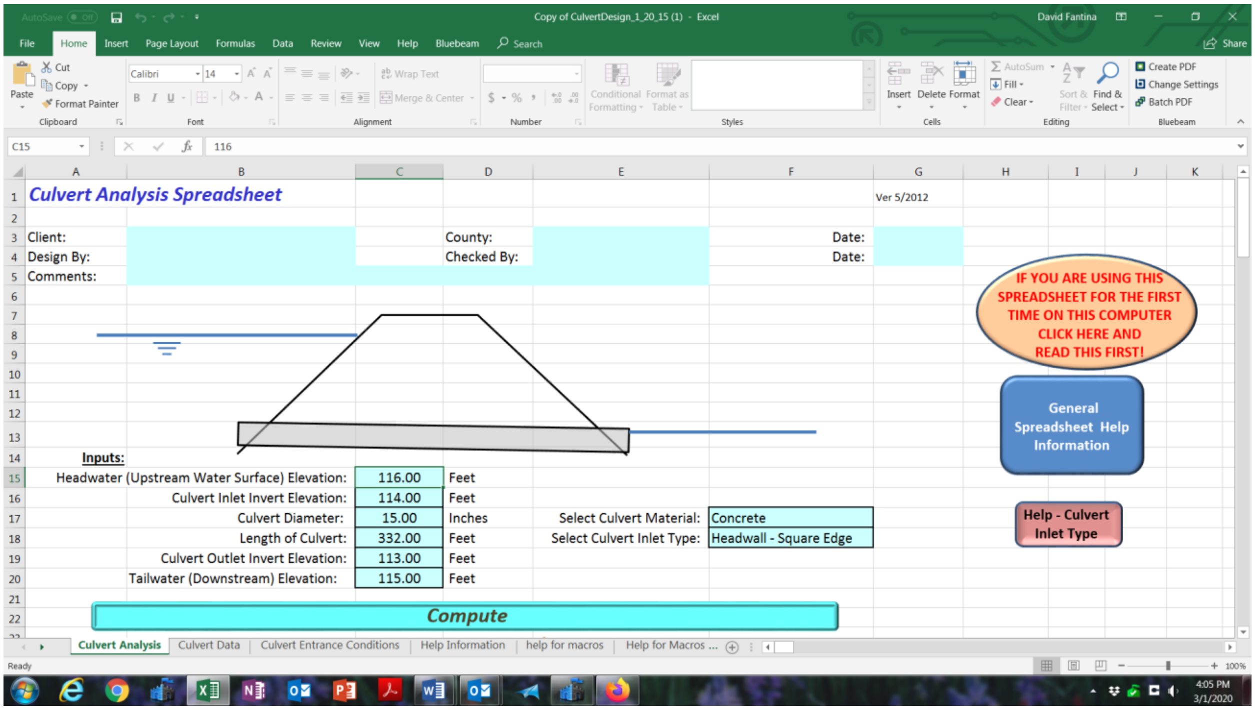
Specific Energy Curve for Open Channel Flow

The denominator of the above equation represents the speed of a small wave on the surface relative to the speed of the water and is called the wave celerity. When Fr = 1 and the flow is critical, the celerity equals the velocity. Therefore, any disturbance to the water surface will remain stationary under this condition. If Fr < 1 (sub-critical flow), the flow is controlled from a downstream point and disturbances are transmitted upstream. This leads to backwater effects. If Fr > 1 (super-critical flow) the flow is controlled upstream and disturbances are transmitted downstream.

Inlet Control Analysis:

For culverts with inlet control, the design engineer can make use of a Excel spreadsheets or other computer software or he or she can refer to a number of culvert design charts. A screen shot from a typical culvert design program (available from the New Jersey Department Of Transportation) is shown below. Note that this particular screenshot shows a significant amount of input data including:

1. Upstream & downstream inverts.
2. Culvert length & diameter. (This is a circular culvert. Otherwise, it would show a height and width instead of a diameter value).
3. Culvert material & inlet type.
4. Tailwater depth.



Culvert Analysis Spreadsheet

Ver 5/2012

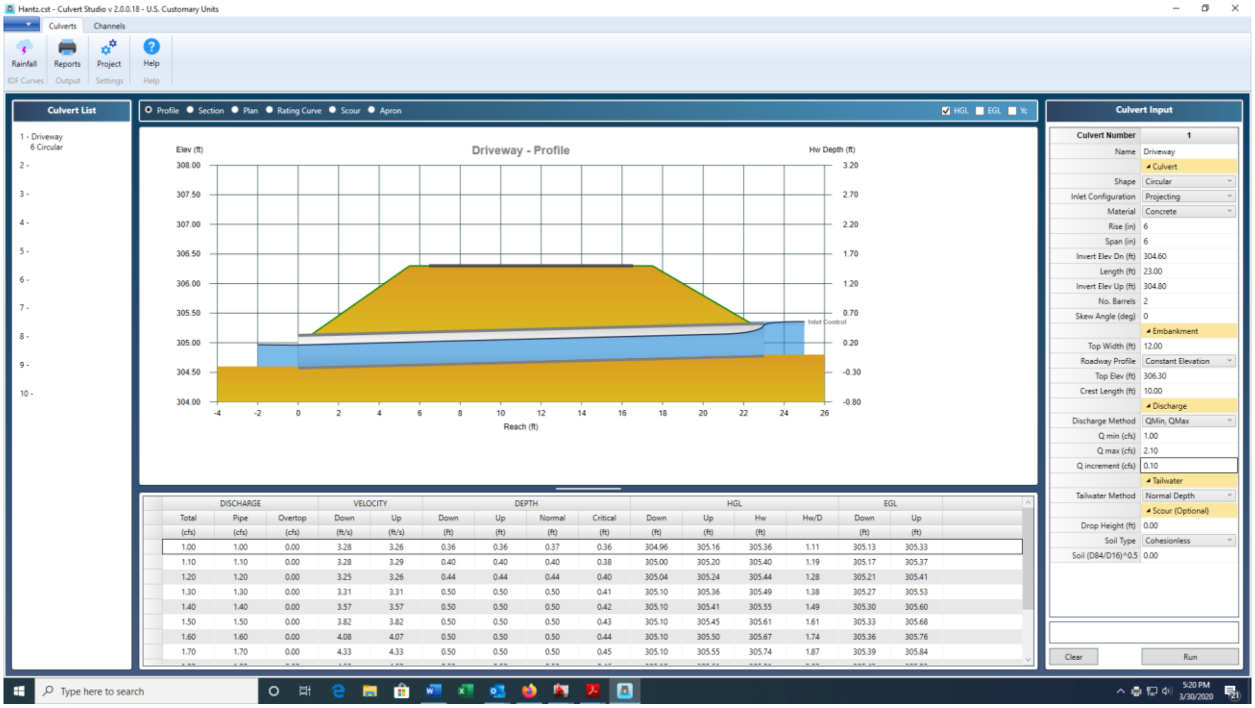
Client:
County:
Date:
Design By:
Checked By:
Date:
Comments:

Inputs:

Headwater (Upstream Water Surface) Elevation:	116.00	Feet
Culvert Inlet Invert Elevation:	114.00	Feet
Culvert Diameter:	15.00	Inches
Length of Culvert:	332.00	Feet
Culvert Outlet Invert Elevation:	113.00	Feet
Tailwater (Downstream) Elevation:	115.00	Feet

Select Culvert Material: Concrete
Select Culvert Inlet Type: Headwall - Square Edge

There are several commercially-available culvert design & analysis software packages. The screen shot below is taken from a program called Culvert Studio. Like the screen shot above, this one shows the basic input data for the culvert (including size & shape, number of barrels, length, slope, etc.) and also a schematic of the flow conditions through the culvert. The Culvert Studio package allows the user to determine exactly what data is presented in the final report.



DISCHARGE			VELOCITY		DEPTH				HGL				EGL	
Total (cfs)	Pipe (cfs)	Overtop (cfs)	Down (ft/s)	Up (ft/s)	Down (ft)	Up (ft)	Normal (ft)	Critical (ft)	Down (ft)	Up (ft)	Hw (ft)	Hw/D	Down (ft)	Up (ft)
1.00	1.00	0.00	3.28	3.26	0.36	0.36	0.37	0.36	304.96	305.16	305.36	1.11	305.13	305.33
1.10	1.10	0.00	3.28	3.29	0.40	0.40	0.40	0.38	305.00	305.20	305.40	1.19	305.17	305.37
1.20	1.20	0.00	3.25	3.26	0.44	0.44	0.44	0.40	305.04	305.24	305.44	1.28	305.21	305.41
1.30	1.30	0.00	3.31	3.31	0.50	0.50	0.50	0.41	305.10	305.36	305.49	1.38	305.27	305.53
1.40	1.40	0.00	3.57	3.57	0.50	0.50	0.50	0.42	305.10	305.41	305.55	1.49	305.30	305.60
1.50	1.50	0.00	3.82	3.82	0.50	0.50	0.50	0.43	305.10	305.45	305.61	1.61	305.33	305.68
1.60	1.60	0.00	4.08	4.07	0.50	0.50	0.50	0.44	305.10	305.50	305.67	1.74	305.36	305.76
1.70	1.70	0.00	4.33	4.33	0.50	0.50	0.50	0.45	305.10	305.55	305.74	1.87	305.39	305.84

Culvert Number	1
Name	Driveway
	Culvert
Shape	Circular
Inlet Configuration	Projecting
Material	Concrete
Rise (in)	6
Span (in)	6
Invert Elev Dn (ft)	304.60
Length (ft)	23.00
Invert Elev Up (ft)	304.80
No. Barrels	2
Skew Angle (deg)	0
	Embankment
Top Width (ft)	12.00
Roadway Profile	Constant Elevation
Top Elev (ft)	306.30
Crest Length (ft)	10.00
	Discharge
Discharge Method	QMin, QMax
Q min (cfs)	1.00
Q max (cfs)	2.10
Q increment (cfs)	0.10
	Tailwater
Tailwater Method	Normal Depth
	Scour (Optional)
Drop Height (ft)	0.00
Soil Type	Cohesionless
Soil (D84/D16)^0.5	0.00

However, in order to understand the mathematics behind the software we will take a look at some of the underlying equations on which the software or charts are constructed. The two basic conditions of inlet control depend upon whether or not the inlet of the culvert is submerged by the upstream headwater. If the inlet is submerged, the inlet will perform as an orifice and the following submerged inlet control equation governs: (Note that this equation should be used when $Q/AD^{0.5} = 4.0$).

Equation 1:

$HW_i/D=C(K_UQ/AD^{0.5})^2 +Y+K_SS$ (Terms for all of these equations are defined on the next page).

On the other hand, if the inlet of the culvert is not submerged, either equation 2 or equation 3 should be used. (Generally, these equations are good up to about a value of $Q/AD^{0.5}$ =3.5. Equation 3 is easier to apply in most cases.)

Equation 2:

$HW_i/D=(H_C/D)+K(K_uQ/AD^{0.5})^M+ K_SS$

Equation 3:
$HW_i/D=K(K_uQ/AD^{0.5})^M$

In Equations (1), (2), and (3), above, the parameters are defined as follows:
HW_i is defined as the headwater depth above the inlet control section invert in feet
D is the interior height of the culvert barrel in feet
H_c is specific head at critical depth in feet.
Q is the discharge in cubic feet per second
A is the cross sectional area of the culvert barrel
S is the slope of the culvert barrel in feet per foot
K, M, c, & Y are constants taken from the following table
Ku is a unit conversion (which is equal to 1.0 if standard English units are used).
Ks is a slope correction constant, which is equal to +0.7 for mitered inlets and -0.5 for other inlet configurations.

Table of Inlet Control Constants:

Culvert Shape & Material	Inlet Configuration	Equat. Form	Unsubmerged K	Unsubmerged M	Submerged c	Submerged Y
Circular Concrete	Square Edge with Headwall	1	0.0098	2.0	0.0398	0.67
Circular Concrete	Groove End with Headwall	1	0.0018	2.0	0.0292	0.74
Circular Concrete	Groove End Projecting	1	0.0045	2.0	0.0317	0.69
Circular CM	Headwall	1	0.0078	2.0	0.0379	0.69
Circular CM	Mitered to Slope	1	0.0210	1.33	0.0463	0.75
Circular CM	Projecting	1	0.0340	1.50	0.0553	0.54
Circular	Beveled Ring, 45° bevels	1	0.0018	2.50	0.0300	0.74

Circular	Beveled Ring, 33.7° bevels	1	0.0018	2.50	0.0243	0.83
Rectangular Box Culvert	30 to 75° wingwall flares	1	0.026	1.0	0.0347	0.81
Rectangular Box Culvert	90° and 15° wingwall flares	1	0.061	0.75	0.0400	0.80
Rectangular Box Culvert	0° Wingwall	1	0.061	0.75	0.0423	0.82
Rectangular Box Culvert	45° Wingwall, flare d=0.43D	2	0.510	0.667	0.0309	0.80
Rectangular Box Culvert	18° to 33.7° Wingwall flare d=0.83D	2	0.486	0.667	0.0249	0.83
Rectangular Box Culvert	90° Headwall with ¾" chamfers	2	0.515	0.667	0.0375	0.79
Rectangular Box Culvert	90° Headwall with 45° bevels	2	0,495	0.667	0.0314	0.82
Rectangular Box Culvert	90° Headwall with 33.7° bevels	2	0.486	0.667	0.0252	0.865
Rectangular Box Culvert	¾" chamfers, 45° skewed headwall	2	0.545	0.667	0.04505	0.73
Rectangular Box Culvert	¾" chamfers, 30° skewed headwall	2	0.533	0.667	0.0425	0.705
Rectangular	¾" chamfers,	2	0.522	0.667	0.0402	0.68

Box Culvert	15° headwall					
Rectangular Box Culvert	45° bevels, 10°-45° skewed headwall	2	0.498	0.667	0.0327	0.75
Circular	Smooth tapered inlet throat	2	0.534	0.555	0.0196	0.90
Circular	Rough tapered inlet throat	2	0.519	0.64	0.0210	0.90
Elliptical Face	Tapered inlet, beveled edges	2	0.536	0.622	0.0368	0.83
Elliptical Face	Tapered inlet, square edges	2	0.5035	0.719	0.0478	0.80
Elliptical Face	Tapered inlet, thin edge projecting	2	0.547	0.80	0.0598	0.75
Rectangular Concrete	Tapered inlet throat	2	0.475	0.667	0.0179	0.97
Rectangular Concrete	Side tapered, less favorable edges	2	0.56	0.667	0.0446	0.85
Rectangular Concrete	Side tapered, more favorable edges	2	0.56	0.667	0.0378	0.87
Rectangular Concrete	Slope tapered, less favorable edges	2	0.50	0.667	0.0446	0.65
Rectangular Concrete	Slope tapered, more favorable edges	2	0.50	0.667	0.0378	0.71

Boxes CM	90° Headwall	1	0.0083	2.0	0.0379	0.69
Boxes CM	Thick wall projecting	1	0.0145	1.75	0.0419	0.64
Boxes CM	Thin wall projecting	1	0.0340	1.5	0.0496	0.57
Horizontal Ellipse Concrete	Square edge with headwall	1	0.0100	2.0	0.0398	0.67
Horizontal Ellipse Concrete	Groove end with headwall	1	0.0018	2.5	0.0292	0.74
Horizontal Ellipse Concrete	Groove end projecting	1	0.0045	2.0	0.0317	0.69
Vertical Ellipse Concrete	Square edge with headwall	1	0.0100	2.0	0.0398	0.67
Vertical Ellipse Concrete	Groove end with headwall	1	0.0018	2.5	0.0292	0.74
Vertical Ellipse Concrete	Groove end projecting	1	0.0095	2.0	0.0317	0.69
Pipe Arch 18” corner radius CM	90° headwall	1	0.0083	2.0	0.0379	0.69
Pipe Arch 18” corner radius CM	Mitered to slope	1	0.0300	1.0	0.0463	0.75
Pipe Arch 18” corner radius CM	Projecting	1	0.0340	1.5	0.0496	0.57
Pipe Arch 18” corner radius CM	Projecting	1	0.0300	1.5	0.0496	0.57
Pipe Arch	No bevels	1	0.0088	2.0	0.0368	0.68

18” corner radius CM						
Pipe Arch 18” corner radius CM	33.7° Bevels	1	0.0030	2.0	0.0269	0.77
Pipe Arch 31” corner radius CM	Projecting	1	0.0300	1.5	0.0496	0.57
Pipe Arch 31” corner radius CM	No bevels	1	0.0088	2.0	0.0368	0.68
Pipe Arch 31” corner radius CM	33.7° Bevels	1	0.0030	2.0	0.0269	0.77
Arch CM	90° headwall	1	0.0083	2.0	0.0379	0.69
Arch CM	Mitered to slope	1	0.0300	1.0	0.0473	0.75
Arch CM	Thin wall projecting	1	0.0340	1.5	0.0496	0.57

The somewhat home-made looking pipe entrance shown below carries a small stream under a cemetery path in New Jersey. In order to determine the constants for an entrance like this, the engineer has to use some reasoning in order to negotiate the table above. The pipe is a concrete and the “headwall” consisting of the wooden structure shown in the photograph. The values for

"Circular Concrete, Square Edge with Headwall" (the first row in the table above) might be appropriate here.

The photograph below shows a different entrance configuration. This pipe carries a small drainage ditch under a rural road in the Great Swamp National Wildlife Refuge in New Jersey. The values for "Circular Concrete, Groove End Projecting" would appear to be appropriate in this case.

Some examples will illustrate how to use these equations:

Inlet Control Example A: Consider the culvert shown in the photograph below. This structure is a rectangular box culvert with a 15° headwall. The culvert is 5 feet wide and 3 feet high. The peak10 year design flow into the culvert has been determined to be 85 CFS. Find the headwater depth. (Based on the photograph, we will assume that the inlet is not submerged).

Solution:

From the table above we can find the following constants:

K is 0.061

M is 0.75

$HW_i/D=(0.061)(1.0X85/(15)(3)^{0.5})^{0.75} = 0.148$

Since the height of the culvert (D) is known to be 3 feet, the headwater depth can be determined as: HWi=0.148 X 3 = 0.45 feet. (Note that this validates the assumption that the culvert is not submerged).

Inlet Control Example B: Consider the culvert shown in the photograph below. This structure is an elliptical concrete culvert without a headwall. The culvert has a 53" span and a 34" rise. The peak10 year design flow into the culvert has been determined to be 60 CFS. Based on the photograph, we will assume that the inlet is not submerged. The cross-sectional area of this culvert is approximately equivalent to a 42" round culvert or 9.6 SF. Find the headwater depth.

Solution:

From the table above we can find the following constants: (Use the entry for "horizontal ellipse concrete groove end projecting"):

K is 0.0018

M is 2.5

$HW_i/D=(0.0018)(1.0X60/(9.6)(34/12)^{0.5})^{2.5} = 0.0478$

Since the height of the culvert (D) is known to be 34 inches or2.83 feet, the headwater depth can be determined as: HWi=0.04784 X 2.83 = 0.135feet. (Note that, once again, this validates the assumption that the culvert is not submerged).

Outlet Control Analysis:

Under Outlet Control conditions with full barrel flow, the total energy loss within the culvert is made up of the following:

- The entrance loss (H_e).
- The friction loss though the culvert barrel (H_f).
- The exit loss (H_o).

In some, but not all culvert barrels the energy loss can also include the following:

- Bend losses (H_b).
- Junction losses (H_j).
- Grate losses (H_j).

Therefore, the total loss within the culvert barrel (H_L) can be calculated as:

$H_L = H_e + H_f + H_o + H_b + H_j + H_j$

Obviously, not all culverts have bends, junctions, or grates so the last three terms in the energy loss equation above only come into play if and when applicable. Because they are not found in "typical" culverts they will not be included further in this analysis.

Calculating each of these individual losses correctly is essential when modelling culvert flow. The following equations can be used in this regard:

$H_e = K_e (V^2/2g)$

$H_f = (K_u n^2 L/R^{1.33})(V^2/2g)$

$H_o = V^2/2g$

In the above equations, the following parameters are used:

K_e is the entrance loss coefficient (see the table below).

V is the barrel velocity, which can be determined by dividing the flow though the barrel by its cross-sectional area (if the culvert is flowing full).

g is the acceleration due to gravity (32 ft/sec^2)

K_u is a constant which is equal to 29 in standard English units.

n is the manning's roughness coefficient.

L is length of the culvert barrel in feet.

R is the hydraulic radius.

A table of entrance losses is included below.

Pipe Culverts	----
Type of Structure & Characteristics of Entrance	**Entrance Loss Coefficient**
Concrete Pipe Projecting from Fill with no Headwall	-----

Socket end of pipe	0.5
Square cut end of pipe	0.2
Concrete Pipe with Headwall or Headwall & Wingwalls	-----
Socket end of pipe	0.2
Square cut end of pipe	0.5
Rounded entrance with rounding radius = 1/12 of pipe diameter	0.2
Concrete Pipe	-----
Mitered to conform to fill slope	0.7
End section conformed to fill slope	0.5
Beveled edges, 33.7 to 45 degree bevels	0.2
Side slope tapered inlet	0.2
Corrugated Metal Pipe or Pipe-Arch	-----
Projected from fill with no headwall	0.9
Headwall or headwall and wingwalls square edge	0.5
Mitered to conform to fill slope	0.7
End section conformed to fill slope	0.5
Beveled edges, 33.7 to 45 degree bevels	0.2
Side slope tapered inlet	0.2
Reinforced Concrete Box Culverts	----
Type of Structure & Characteristics of Entrance	**Entrance Loss Coefficient**
Headwall parallel to embankment with no wingwalls	-----
Square edge on three edges	0.5
Three edges rounded to radius of 1/12 of barrel dimension	0.2
Wingwalls at 30 to 75 degrees to Barrel	-----
Square edge at crown	0.4
Top corner rounded to 1/12 of barrel dimension	0.2
Wingwalls at 10 to 25 degrees to Barrel	-----
Square edge at crown	0.5

Wingwalls parallel (extension of sides)	----
Square edge at crown	0.7
Side or slope tapered inlet	0.2

The photograph below shows the entrance to an elliptical concrete culvert with a headwall.

Under outlet control conditions the hydraulic resistance of the culvert barrel must be calculated using a friction loss equation. There are several different equations for calculating the hydraulic resistance. Two that will be discussed here are the Darcy equation and the Manning equation.

The Darcy equation is generally expressed in the following form:

$H_f=f(L/D)(V^2/2g)$, where

H_f is the friction head loss

f is known as the Darcy resistance factor

L is the culvert length

D is the culvert diameter

V is the mean velocity

g is the acceleration due to gravity

In order to determine the Darcy resistance factor, one refers to a chart known as the Moody Diagram, which relates this factor to the Reynolds number and relative roughness.

A version of the Moody Diagram is presented below.

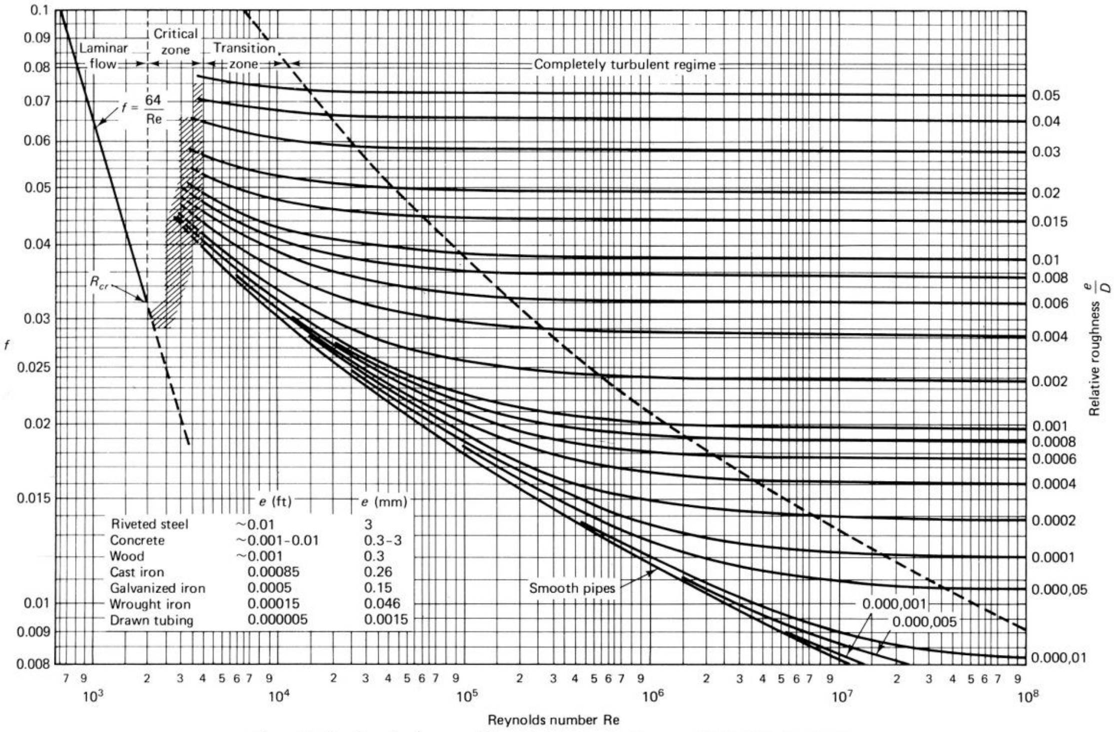


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

Even a brief glance at the Moody Diagram shown above reveals that it is not the easiest chart to interpret! However, it is an invaluable tool in many respects.

On the left axis of the diagram is the friction factor (f), and on the right axis is the relative roughness (*e*/D). The bottom axis (or X axis) is the Reynolds Number.

The relative roughness is based on the properties of the pipe. A very smooth pipe is represented by the bottom line on the diagram. In order to read the diagram, first calculate the Reynolds Number and plot that value on the x axis. Then follow that line up vertically until it intercepts the pipe with the correct relative roughness. From that point, draw a line horizontally to the left to determine the friction factor.

One of the main benefits of the Moody Diagram is that it allows the user to determine the state of the flow. It might be useful here to briefly review the various flow states that can occur in culverts (and in natural channels).

The basic types of flow that can develop are as follows:

1. Laminar flow is present when the water travels smoothly or in regular paths without abrupt changes in velocity or pressure.
2. Transitional flow is, as the name implies, an in-between state of flow somewhat between laminar and turbulent.
3. Turbulent flow is characterized by chaotic changes in pressure and flow velocity. This type of flow occurs if the viscous forces are weak in comparison with the inertial forces.

The various flow types can be determine by referring to the Moody Diagram and by calculating the Reynolds Number, which was referred to above. The Reynolds number (R) is calculated by the following formula:

$R=VL/v$, where

V is the velocity of the flow in feet per second,

L is a parameter known as the characteristic length, which (in culvert flow) is considered to be equal to the hydraulic radius of the culvert. The hydraulic radius is defined as the ratio of the cross-sectional area of the culvert to the wetted perimeter. (Note that in a circular culvert flowing full, this can be simplified to R=Pipe diameter/4).

v is the kinematic viscosity in feet^2/second. (The kinematic viscosity is somewhat dependent on temperature: for water at 68 degrees Fahrenheit, $v=1.08\text{X}10^{-5}$).

The following fairly simple example will illustrate the use of the Moody Diagram.

Moody Diagram Example:

A 24” diameter smooth, plastic culvert is flowing full with a velocity of 8 feet per second at a temperature of 68 degrees. What is the Reynold’s Number and what is the flow type within the culvert?

Solution:

First, calculate the Reynolds Number:

$R=VL/v$,
In this case, V is given as 8 ft/second and L can be calculated as the pipe diameter divided by four (i.e. 24” or 2’/4 = 0.5ft).
v is $1.08X10^{-5}$ (see above)
$R=(8)(0.5)/\ 1.08X10^{-5} = 3.7037X10^{5}$
Looking at the Moody Diagram with this value of R and a smooth pipe, one can see that the flow regime is laminar.

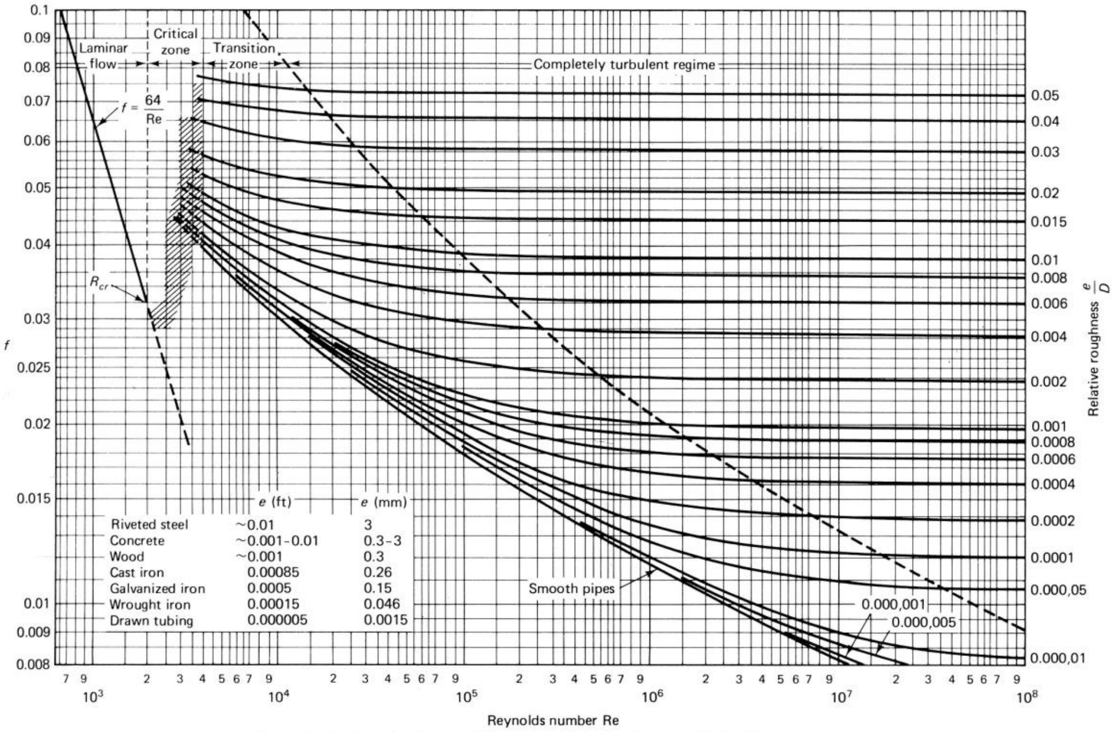


Figure 7.13 Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

While the Darcy equation is theoretically correct, the Manning Equation, shown below is an empirical equation that is more commonly employed:
$V=(1.486/n)R^{2/3}S^{\frac{1}{2}}$, where
R is the hydraulic radius, which is defined above under the discussion of the Darcy equation and the Moody Diagram.
S is the slope of the culvert in feet per foot.
n is the roughness coefficient (see below).
This form of the Manning equation calculates the velocity in the culvert. In order to determine the flow rate, this velocity is multiplied by the flow area.

The Manning's n value can be based on either hydraulic test results or on theoretically calculated resistance values. However, in practical terms a table such as the one below is generally used:

Culvert Material	**Description**	**Minimum n**	**Normal n**	**Maximum n**
Brass	Smooth	0.009	0.010	0.013
Steel	Lockbar & welded	0.010	0.012	0.014
Steel	Riveted & spiral	0.013	0.016	0.019
Cast Iron	Coated	0.010	0.013	0.014
Cast Iron	Uncoated	0.011	0.024	0.016
Wrought Iron	-----	0.012	0.015	0.017
Corrugated metal	Subdrain	0.017	0.019	0.021
Corrugated metal	Storm drain	0.021	0.024	0.030
Plastic	PVC	0.009	0.012	0.016
Concrete	Straight & free of debris	0.010	0.011	0.013
Concrete	With bends, connections, & some debris	0.011	0.013	0.014
Brick	Lined with cement mortar	0.012	0.015	0.017

Manning's Equation Example:
A 36" corrugated metal storm drain culvert flowing full has a slope of 1%. Using the Manning's equation, what is the velocity of the flow through the structure?
Solution:
Based on the table above, use an n value of 0.019. Remember that the hydraulic radius of a circular pipe flowing full is the diameter divided by 4. Therefore R = 36" or 3' divided by 4 = 0.75.
$V=(1.486/0.019).75^{2/3}0.01^{½} = 6.5$ FPS.

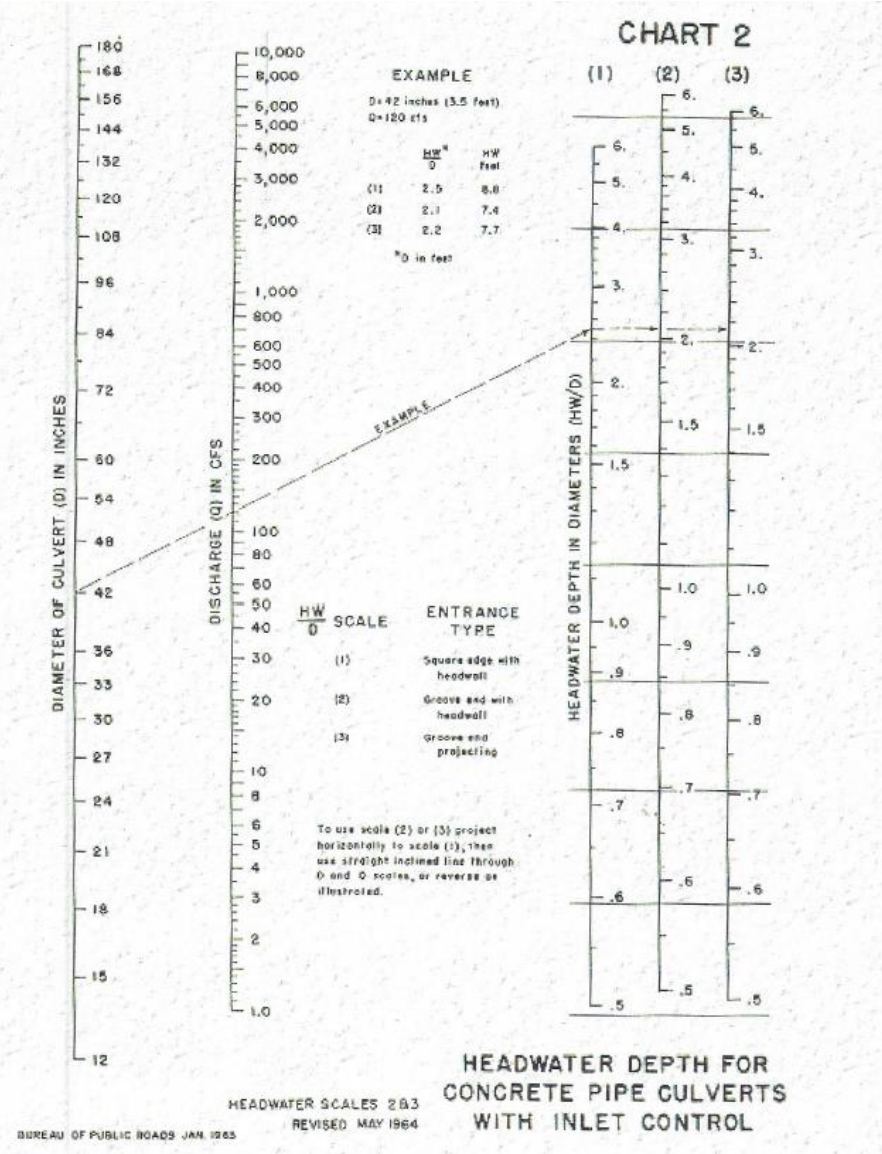
Interestingly, the Darcy f value and the Manning n value are related by the following equation: $n=0.0926R^{1/6}f^{1/2}$, where R is, again, the hydraulic radius of the culvert.

For example, if the Manning's n value has been determined to be 0.013 on a 24" pipe flowing full, the equation above can be rearranged to find a Darcy's f value of 0.0248.

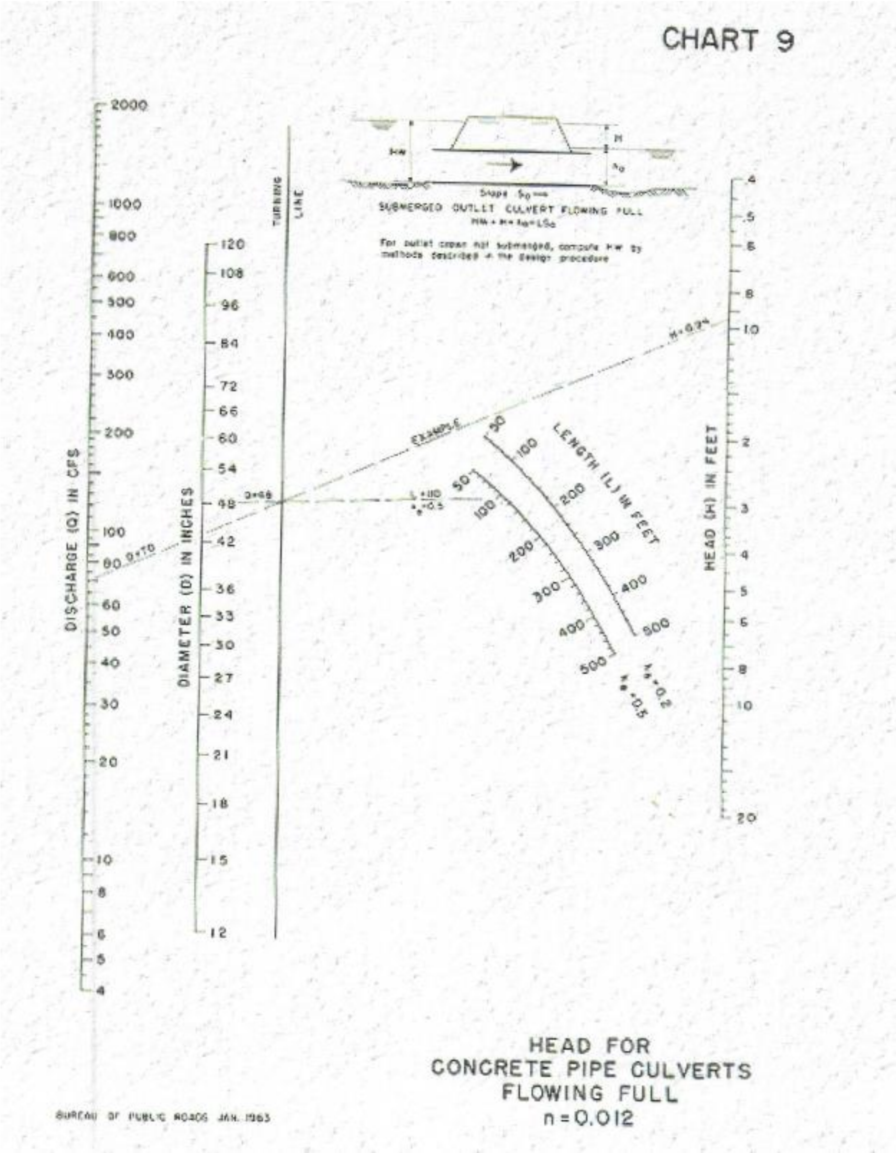
Another fairly standard-looking culvert with headwall and wingwalls is shown in the photograph below. Note that it can be seen that this structure may be prone to becoming clogged to some extent with vegetation. The design engineer should always take this into account when designing (or analyzing) a culvert. One way to account for this is to modify the loss coefficient at the entrance of the structure.

Culvert Analysis Using Nomographs:

Before moving on, it should be mentioned that a simple, tried and true way of analyzing culverts is by using standard nomographs which have been published by Federal Highway Department. Long before computers were in general use, engineers were making use of these graphs to design and analyze a wide variety of culverts and they are still just as useful today. A variety of nomographs are published in the reference noted in the introduction to this course. The two nomographs shown below, however, are taken from an older publication entitled “Hydraulic Charts for the Selection of Highway Culverts”, prepared by the US. Department of Transportation in December 1965. This older publication is used to be consistent with the remainder of this course (the newer publication shows the charts in metric units).
The first nomograph (Chart 2) is shown below and is to used for circular concrete pipes culverts glowing with inlet control.



The second nomograph (Chart 9) is also to be used for a circular concrete culvert, this time flowing under outlet control conditions.



The procedure for using the nomographs is quite simple and each of the nomographs shown above has an illustrative example embedded in it. Specifically, the procedure can be described as follows:

1. Inlet control nomograph: To determine the headwater depth (HW), given the culvert size and flow (Q), do the following:

- Use a straight edge to connect the culvert size and the flow and extend the line to the right to intersect the HW/D line. (For the example shown in Chart 2, above, the Q value is 120 CFS and the culvert diameter is 42” or 3.5 feet. Connecting these points yields a HW/D value of 2.5 (for chart (1) which represents a culvert with the entrance being a square edge with a headwall). Therefore, HW is calculated as 2.5 X 3.5' = 8.75 feet deep.

- If the culvert geometry is different than square edge with headwall and scale (2) or (3) has to be used, simply draw a horizontal line from scale (1).
- The nomograph can be used in a similar way to determine the capacity of a culvert given the diameter and available headwater depth.
- Finally, the nomograph can be used to determine the required culvert size given the Q and HW by using a trial and error approach.

2. Outlet control nomograph: To determine the required head (H) for a given culvert size and length and discharge (Q) do the following:

- Connect the culvert size with the culvert length using a straight-edge. (In the example shown on Chart 9, above, the culvert diameter is 48" and the length is 110 feet).
- The given Q on Chart 9 is 70 CFS. Connect this Q value with the location on the "Turning Line" where the line connecting the 48" culvert and 100' length intersects.
- Then follow this line to the right until it connects the H line. In the example shown on the nomograph, the H value is 0.94 feet. (Note that the head used in the outlet control nomographs is different than the headwater shown in the inlet control nomographs. This difference is shown graphically on the nomographs, themselves).

Because it is usually not obvious as to whether a culvert will be flowing under inlet or outlet control conditions it is a good idea to use a nomograph for each condition and pick the more conservative answer

Analysis of Culverts with an Adverse Slope:

The inlet and outlet analyses discussed in this course assume that the culvert in question has a positive slope. However, in many real-life examples, culverts are found to have a zero slope (horizontal) or even an adverse slope. This sometimes occurs when the entrance side of the culvert settles more than the exit. However, the analyses discussed above can also be applied to culverts without a positive slope. The following adjustments should be made:

Inlet Control:

- Mitered inlets: For a culvert with a horizontal slope, 0.014 should be subtracted from the HW/D value obtained from the nomograph. For a culvert with an adverse slope, 0.014 should be added to the HW/D value obtained from the nomograph and an additional 0.7S should be added. (S, in this case, is the adverse slope in feet per foot. e.g. An adverse slope of 2.5% would be represented as 0.025).

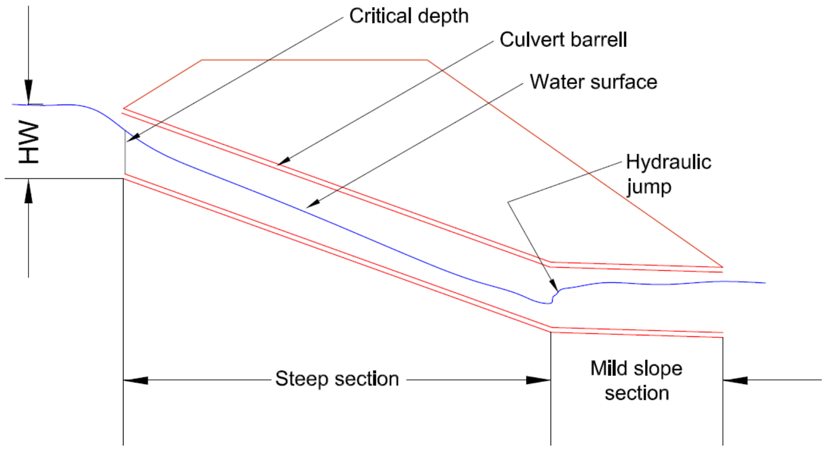
- Inlets that are not mitered: For a culvert with a horizontal slope, 0.01 should be subtracted from the HW/D value obtained from the nomograph. For a culvert with an adverse slope, 0.01 should be added to the HW/D value obtained from the nomograph and an additional 0.5S should be added.

Outlet Control:

The outlet control solution does not need to be modified for a horizontal culvert or a culvert with an adverse slope.

Broken-Back Culverts:

When designing a culvert in a steep area, the engineer must be aware of the potential for excessive velocities at the downstream end of the barrel. There are several ways to mitigate the erosive potential of this velocity and conduit outlet protection in the form of rip-rap or a preformed scour hole is often employed. However, the engineer can also consider using a break in the culvert barrel slope. Such a culvert (shown schematically below) is known as a broken-back culvert. The example shown below is only one of many different configurations that can be used. Depending on the topography the engineer, can employ more than one “break” in the culvert and can use a mild sloping section upstream of the sleep section.



Schematic Broken Back Culvert

In order to reduce the velocity at the downstream face of the culvert, the intent of the broken-back configuration is to force a hydraulic jump within the culvert barrel (which will convert the flow regime to a high-velocity supercritical flow to a lower velocity subcritical flow). There must be sufficient tailwater depth to ensure that a hydraulic jump will actually occur within the barrel. The engineer should generally make use of roughness baffles, or other energy dissipating devices within the culvert barrel to ensure that this is the case.

The following steps should be used when designing a broken -back culvert:

1. Establish a flow-line profile.
2. Size the culvert.
3. Begin to calculate a supercritical profile.
4. Complete the profile calculations.
5. Consider hydraulic jump calculations.

Of course there are a number of commercially-available software programs that will adequately analyze a broken-back culvert.

Storage Routing Calculations:

There is often a significant amount of storage volume provided upstream of a highway culvert. This storage volume can reduce the peak rate of flow through the culvert and can reduce the required capacity of the culvert. A detailed discussion of storage-routing calculations is beyond the scope of this course but the engineer can make use of any of a variety of software packages to accurately model this situation. (The culvert will essentially act as a controlled outlet from a ”detention basin” which is the area formed by the backflow upstream of the culvert. Therefore, the engineer simply needs to input the various storage values and the rating curve for the culvert at various elevations to model the situation. The culvert entrance can be considered to act as an orifice in this case). The photograph on the following page shows a low area that is subject to inundation in large storm events that has only a relatively small culvert (shown in the center of the photograph below the roadway guardrail) as an outlet. As mentioned above, this is the type of situation that may require a storage-routing analysis in order to properly model the real-life flow conditions. This situation is basically modeled the same way (and using the same software) as an engineer would route a storm through a detention basin.

Roadway Overtopping:

Culverts are generally designed so that the entire design storm is conveyed through the culvert barrel. However, in large storms events, or if the culvert is undersized (or for a variety of other reasons including, but not limited to (i) the culvert becoming partially or completely blocked with debris or (ii) backwater effects downstream of the culvert) a portion of the flow will bypass the culvert and overtop the roadway. Flow over the road will approximate flow over a broad-crested weir. Recall that the standard weir flow equation is: $Q=cLH^{1.5}$, where:
Q is the flow over the weir in CFS.
c is the broad-crested weir coefficient (generally 2.6 is used).
L is the weir "length" or, in this case, the length of the roadway that is overtopped.
H is the head in feet.

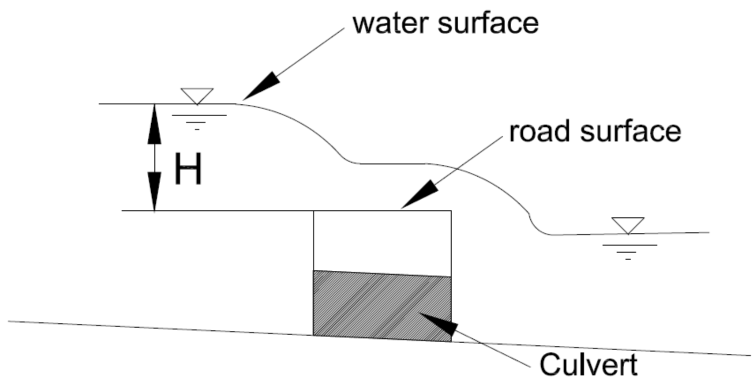
However, this form of the equation can be refined to model flow across a roadway. The refined equation is: Q= $C_dLHW_r^{1.5}$, where:

Q is again the flow over the roadway in CFS

C_d is overtopping discharge coefficient which is a function of headwater depth, length of the roadway to be overtopped, and the submergence factor. C_d=KtCr, and both of these factors are based on the charts shown below.

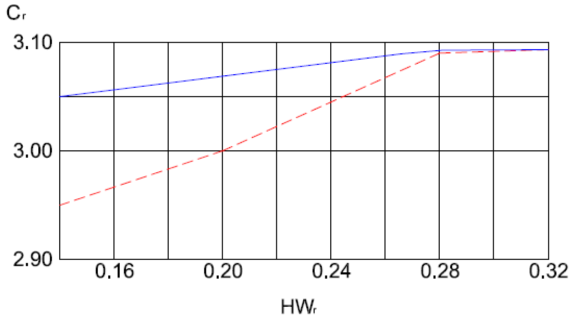
L is again the roadway crest length in feet.

HW_r is the upstream depth (measured from the roadway crest to the water surface upstream of the weir drawdown. Note that it is important to measure the head sufficiently upstream of the weir to get a valid measurement – see the diagram below).

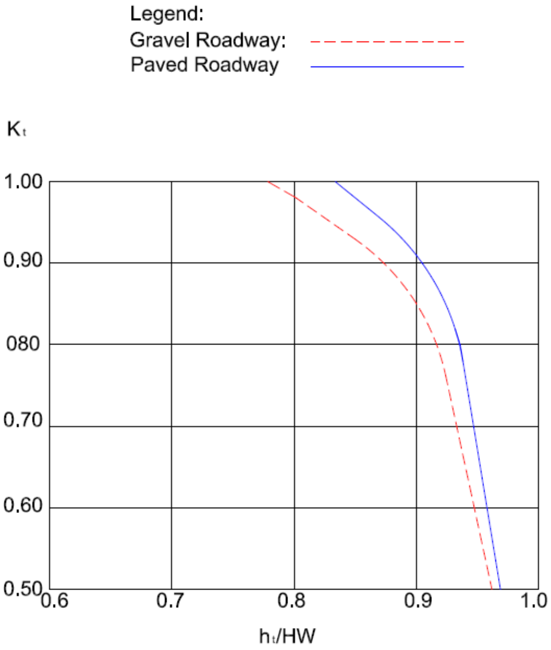


Schematic Road Overtopping Diagram

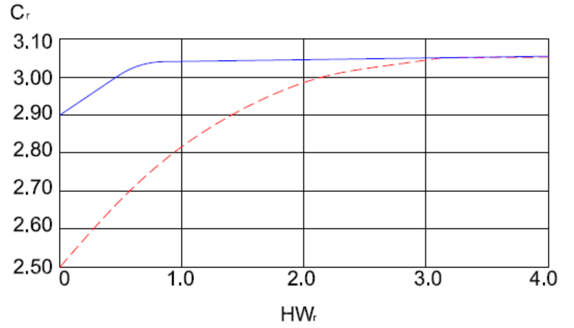
The following charts should be used to determine the overtopping discharge coefficient described above:



Discharge Coefficient for $HW_r/L_r>0.15$



Submergence Factor



Discharge Coefficient for $HW_r/L_r\leq 0.15$

The photograph below shows a circular, concrete pipe and a stone headwall. Note that there is not much freeboard available above the head and there is a possibility that the road will be overtopped.

Roadway Overtopping Example:

Consider the following situation: A 52 foot long, 36" circular concrete culvert travels under a paved roadway. During a major flood event the headwater depth over the road is two feet and a 25 foot length of road will be inundated. The tailwater is 21" (or 1.75') above the crown of the pipe. What is the flow over the roadway during the peak of the flooding?

Solution:

Once again, the governing equation is: $Q = C_d L HW_r^{1.5}$

First determine the value of HW_r/L: In this case it is 2/25 = 0.08, which is less than 0.15. Therefore the lower chart for C_r is used, above. Reading up from the bottom to intersect with the paved road line yields a C_r value of 3.03.

The factor h_t/HW_r is calculated as 1.75'/2' =0.875. Reading the chart on the right above, yields a submergence factor of 0.96 for this value and a paved road.

Therefore: Cd=(0.96)(3.03) = 2.91

The flow is calculated as:

$Q = (2.91)(25)(2)^{1.5}$ = 206 CFS over the roadway.

There are a number of simplifications inherent in this approach. For one thing it does not take into account the effects of a sloping roadway. In this case, the engineer has a few choices as to how to proceed. He or she can do wither of the following:

1. Take an "average" elevation of the roadway surface and use that as the crest elevation of the weir.
2. Break the roadway surface into multiple smaller weir units.

Either of these methods have advantages and disadvantages and it is up to the engineer to decide which to use in a particular circumstance.

A concrete box culvert is shown in the photograph below. This structure would probably be less subject to overtopping during a large storm event than the culvert shown in the preceding photo.

Twin circular concrete culverts are shown in the photograph below. Twin culverts can be analyzed in a very similar manner to the procedures outlined herein. Unless there is a reason to assume otherwise, it is best to assume that 50% of the flow is carried by each structure. Note that the pathway above these culverts is somewhat sloping and, if it is overtopped, the engineer will have to take that into consideration.

Additional Considerations:

Obviously, it is vitally important to properly design the hydraulics of a culvert so that it will carry the design flow under whatever roadway or path is being analyzed. However, there are a number of other factors that should be kept in mind when designing a culvert. Some of these include the following:

1. Some culverts are constructed with horizontal or vertical bends. These structures require special hydraulic considerations.
2. The engineer must be cognizant of the structural stability of the culvert especially if it is designed to carry highway loads.

3. In some cases a culvert may need to be equipped with a protective barrier either to prevent children from entering the culvert or else to prevent the culvert becoming clogged during flood events. The photograph below shows a culvert in a public park with a cage barrier:

4. The cost of the culvert installation must always be kept in mind. In some cases, of course, a particular culvert design must be used regardless of cost. However, the engineer should always strive to ensure that the most cost-effective design is being employed. A related consideration is the accessibility of the materials used in the design. The pandemic of a few years ago, showed that the supply chain can sometimes break down and make certain materials impossible to get or prohibitively expensive.
5. The on-going maintenance requirements of the culvert should be considered. In this regard some pertinent questions the engineer can ask are:
    - What type of maintenance will this particular structure require?
    - Is there adequate access to maintain the culvert?
    - Who will be responsible for the maintenance?
6. Fish passage is a consideration on some culverts. If this is the case, the engineer should check with the appropriate authorities as to what is required. (Generally, a three-sided culvert with no bottom is better for fish passage than are other configurations. For instance, the large, bottomless culvert shown in the photograph below would not present an obstruction to fish flow whereas a culvert with a concrete bottom in this location might present an obstacle to fish, especially during low-flow conditions).

7. When culvert velocities are high, erosion or scour can be a factor at the downstream end of the structure. This is often addressed by adding riprap or other conduit outlet protection such as gabions or a revetment blanket. The photograph below shows riprap at the outlet of an elliptical culvert.

In short, there are several factors that must be considered in culvert analysis and design. However, the hydraulic capabilities of the culvert certainly rank at the very top of the list.