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WHAT EVERY ENGINEER SHOULD KNOW ABOUT ENGINEERING ECONOMIC ANALYSIS II

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Introduction

This course is sequel to course, "What Every Engineer should Know about Engineering Economic Analysis I". In that course we examined some of the basic principles and underlying assumptions of Engineering Economic Analysis. However, in this course, our focus is on some of the fundamental issues that make it easy to transition from the basic concepts developed in the earlier course to more robust and complex issues that undergird the framework of Engineering Economic Analysis. These fundamental issues build upon the basic principles and assumptions to form the building blocks that are central to understanding the subject matter of Engineering Economic Analysis.

Engineering Economy is the study of the feasibility and evaluation of the cost of possible solutions to engineering problems. When benefits outweigh costs, the alternative becomes an acceptable option. The lowest cost among alternates can be determined by using different methods that we would discuss in this course. This follow-up course will focus primarily on the important areas of decision making among alternatives and decisions in the certainty-uncertainty spectrum.

Engineering economics is concerned with the use and application of economic principles in the analysis of engineering decisions. Part of its focus is on what informs the behavior of individuals and firms as they allocate limited resources to alternative portfolios. In this sense, it focuses on the decision-making process, its context and environment, as well as the decision aid that help guide the decision. It is pragmatic in nature and integrates economic theory and the logical framework of economics and the analytical strengths of mathematics and statistics in formulating solutions.
Fundamentally, engineering economics revolves around formulating, estimating, and evaluating the economic viability of outcomes in the presence of alternatives. Central to economic engineering analysis is the notion of the time value of money (TVM) which makes it possible for cash flows to be discounted using an interest rate.

Decision making, and more specifically, financial decision-making is at the core of engineering economic analysis. Any decision is a conceptualization or eventual realization of a choice situation, whether through a mental image of what the decision ought to be or through an explicit formal model. Because decisions affect a company or an individual's reality, it is often a simplification of reality due to the difficulty of conceptualizing and contextualizing the choice situation inherent in the complexity and high dimensionality of the reality space. Most decisions are abstract, complicated and involve multiple changing variables on different fronts. A rational decision is made when the marginal benefit of an action is greater than or equal to the marginal cost.

Marginal benefit and marginal cost are two important metrics which give some insight into how the cost or value of a product changes. Marginal benefit is the maximum amount a consumer will pay for one additional good or service. Marginal benefit generally decreases as consumption increases. Marginal benefit impacts the customer, whereas marginal cost impacts the producer. Both metrics must be taken into consideration as far production, pricing, and marketing are concerned.

Marginal costs on the other hand refer to the change in total costs per unit change in output. In marginal cost, the focus is on the increased total cost that will arise from the production of one more unit of output. For example, the pecan pie making company, Natures Pie, currently makes 5,000 12-inch pies at a cost of $12,500, so that the average cost per pie is $2.50. However, if the production line makes 5,001 units, the total cost is $12,502, so that the marginal cost of the one additional unit is only $2. This is to be expected because there is hardly any additional overhead cost associated with a single unit of output, thus resulting in a lower marginal cost.

The marginal costs specifically refer to one more unit of output over the planned output. In general, marginal benefits of repeated activities decrease over time which then changes the decision-making option. In benefit-cost terms, a rational decision is made when the marginal benefit of an action is greater than or equal to the marginal cost.

Opportunity costs refer to the best available alternative that is relinquished to make room for a preferred alternative since, as a result of scarcity, not all alternatives can be supported. With each choice comes a cost or a missed opportunity. The opportunity cost of a choice is the next best alternative not taken.

Decisions are a necessity as a result of scarcity. Scarcity is a basic life condition that exits when unlimited wants exceed limited resources such as limited funds to invest in new equipment or software. At the very basic level, engineering economic analysis is about scarcity of resources. Part of the human condition is that we have unlimited wants but the resources to fulfill those wants are limited. As a result, we must decide on how to best use the limited resources. Individuals, businesses, governments, and countries all face scarcity. Money, goods, and services can become scarce due to the limited amount and quantity available. Anything that is desirable that has multiple uses and limited availability is considered scarce. As a result, we are bound to forego certain things (due to cost and due to lack of availability), thus creating opportunity costs and eventual tradeoffs. There are models, decision aids and processes that help to explicate the decision process and to provide some clarity so that decisions are made thoughtfully and rationally. Most of those focus on rational decision making.

1.1 The Rational Decision-Making

The Rational Decision-Making Model applies explicitly logical steps to making decisions and achieving a solution. It is data centric and involves comparing a range of options and alternatives that use verifiable (non-biased) research and facts to assist the decision maker to choose among the alternatives. Rational decisions are knowing, logical, explainable decisions rather than emotional or random decisions. To choose rationally is to choose in an explainable way, that is, explainable to the decision maker <u>and to others</u>. Rational decision making is different from intuitive decision making. It is a procedure that utilizes objective knowledge and logic to arrive at a decision. Because it uses logic, rational decision-making most often follows an identifiable path or process to arrive at a decision.

For a given problem, there are usually many possible alternative solutions. One option that must be considered in each analysis is the "do nothing" alternative. And for each alternative, the opportunity cost of making one choice over another as well as the marginal costs must also be considered.

1.2 Purpose of Rational Decision-Making

The purpose of a rational decision-making tool is to design step-by-step decision aids which would ultimately result in good outcomes from the decision taken and reduce or eliminate bias to the greatest extent possible. Rational decision making is a precise and thorough process and takes longer to achieve than intuitive decision-making. The rational decision-making process is a step-by-step process that utilizes the scientific approach and is evidence based.

1.2.1 Steps in Rational Decision-Making

About six, and sometimes up to a nine-step process that have typically been proposed as part rational decision-making model in order to achieve optimal results. These include problem identification, data gathering, identification of options and alternatives, analysis, examination of all the relationships among alternatives, and selecting the best alternative, among others. More, specifically, we have:

1. *Define and Identify the Problem*

Problem identification is the first step in the decision-making model. This step requires data collection from the specific area of interest need. This is accompanied by a detailed analysis to determine if there are any discernable trends or patterns that would help to accurately define the problem. Once the problem is defined then the desired outcome can be better understood. Understanding and defining the problem and the attendant ramifications is key to the rational decision-making process.

2. *Identify the Decision Criteria*

In this step, the decision criteria that is necessary to make a rational decision is identified. The criteria must derive from the problem and the expected outcome. The criteria that are chosen should also reflect the organization's vision and values as well as the decision maker's values

3. *Criteria Weighting*

Weightings are assigned to the criteria and the criteria should be ranked to reflect their degree of importance not only to the problem but also to the decision.

4. *Develop Alternatives*

Alternatives or options are important especially if the decision or the problem is very difficult. The options should be listed in order of based on their impact on the problem. The choices selected must be reasonable and should be ranked ordered based on their ability to meet the criteria listed.

5. *Evaluating the Alternatives*

Evaluating the alternatives requires a lot of time, research, and reflection to confirm that the alternatives identified would lead to the best rational decision.

6. *Select the Best Alternative*

There are several ways to select the best alternative from a set of viable alternatives. The three that most commonly utilized are Experience, Experimentation, and Research & Analysis.

i. Experience: The existence in the organization of a good and relevant experience is preferable. However, past experience can also be a limitation because it tends to cloud open-minded, objectivity and flexibility.
ii. Experimentation: Experimentation is a small-scale implementation to see if the thing works as expected before full lunch. This is the case with test marketing before product lunch. During this phase, changes and adjustments can be made to enhance features on a limited scale without disrupting the entire enterprise.
iii. Research and Analysis: This step represents the most common method of selecting alternatives, especially for critical decisions. Research and analysis critically explore the relationship between the problem variables and constraints and by so doing it provides a clear picture of the relationship among the system variables.

7. *Choose the best solution and test it*.

Based on the evaluation of the potential solutions, choose the best one in the set and test it. If the test solves the problem, implement the choice. If not, cycle back to the beginning to refine the problem definition as necessary and test again.

It is important to realize that although numerous approaches exist for choosing an alternative(s), the decision maker's values and aspirations tend to determine the choice that is made. As indicated earlier, there are usually many possible alternatives for any given problem. One option that must be considered in each analysis is the do nothing alternative. Additionally, the opportunity cost of making one choice over another as well as the marginal costs must be considered.

8. *Track and analyze the results of your test*
9. *If the test solves your problem, implement the solution. If not, test a new one*

1.3 The Nature of Management Decisions

Most managers and analysts would be happy to know how best to codify their decision-making process because most of their decision can best be described as intuitive. Part of what a decision aid does for decision making is to provide clarity, verbalization, and better communication from those tasked to make decisions. Efforts expended at understanding decisions and the development of decision aids are critical because they lead to rational decisions. Hence a rational or good decision must be consistent or aligned with the data available to the decision maker and must be consistent with the decision makers' values and world view as well as his/her goals and objectives. It is important to note that while a good decision does not guarantee a good outcome, a good outcome does not necessarily imply the outcome was borne out of good decision.

The objective of any decision analysis is to construct decision aids to ultimately increase the probability of a good outcome from the decision taken. Decision aids are methods, techniques, tools, and models that have the potential and promise to enhance the effectiveness and/or efficacy of the decision. Some of those include problem simplification, ambiguity reduction, providing structure to the problem, clarifying the stated goals, and coping with uncertainty.

Research on management decision-making shows that decision-makers integrate the notions of instincts, judgment, and perspectives in their decision-making process. Instinct in this case relates to the intuitive or gut feeling about a situation or stimulus. Judgment on the other hand is applying both data and experience in the process of analyzing a problem or scenario. Accommodating different perspectives seeks to bring other viewpoints and expertise into the equation with the goal of ultimately expanding, influencing, or even changing the decision makers point of view about the situation under reference either partially or completely.

There is some correlation between the complexity of a decision and how instinct, judgment, and perspectives are applied to the decision-making process. If the problem is simple and straight forward, then only an instinctive response would be adequate. If the decision is more complex, such as staff hire or equipment purchase, then a combination of both instinct and judgment would be sufficient. However, in the case of very higher order or complex decision, then the integration of all three, namely instinct, judgement, and perspective would definitely be needed.

The reason for integration in this instance is to allow for the nuanced interplay of opposing ideas, opinions, experiences, and new directions. It requires adept negotiations by employing intuition, data, opinions (personal and those of others), and widely held views. This requires discipline, self-awareness, vulnerability, and an openness to new ideas to be able arrive at a rational decision.

Financial Decision Making Among Alternatives

Projects are undertaken for some benefit, in the form of money or otherwise. In that sense they are considered as investment opportunities or investment proposals. Every investment proposal can be looked upon as an investment alternative more so because to "do nothing" is also an alternative. An investment alternative on the other hand may consist of a set or a group of investment proposals. Capital budgeting is the vehicle by which potential projects or investments are evaluated. Any major investment or venture would require capital budget assessment before they are either approved or rejected. As part of the capital budgeting process, a company might assess or appraise a prospective project's lifetime cash inflows and outflows to determine whether the potential returns that would be generated from the investment would sufficiently meet the proposed target or in the alternative, the minimum attractive rate of return, also known as the MARR.

Different types of proposals are typically proffered for capital budgeting, each proposal depending on the need and circumstance. Projects can be mutually exclusive, non-mutually exclusive, independent, or dependent.

Borrowing from the notion of events in probability theory, two events are said to be mutually exclusive when their occurrence is not simultaneous. In other words*, each excludes or precludes the other and they can't both exist, be true, or happen at the same time. Based on this, t*wo projects or proposals are said to be mutually exclusive, when they cannot both exist or when both cannot simultaneously be realized.

Mutually exclusive projects are projects where **only one project** can be selected out of all the possible investments or projects. Similarly, non-mutually events or projects are those where an investor has different alternatives proposals or projects available, but more than one project can be selected consistent with capital or budget constraints.

An independent project is one whose acceptance or rejection is independent of the acceptance or rejection of another project. In other words, it is a project that is not part of or dependent on any other project. With respect to cash flows, an independent project is one whose cash flows have no impact on the acceptance or rejection of other projects **(not mutually exclusive)**. Thus, the funding of an independent project does not depend on another project receiving funding first. A dependent project is one whose acceptance or selection affects the occurrence or the selection of the other projects. Dependent projects depend on what projects have already been selected before. They are affected by the outcomes that had already occurred previously.

From the point of view of probability, mutually exclusive events are disjoint events. But by definition two events cannot be independent if they are disjoint events. Hence, we know that independent events are not mutually exclusive. However, if events are mutually exclusive, they may also be dependent (but not independent).

2.1 Mutually Exclusive (ME)Projects

In mutually exclusive projects, the different projects are meant to accomplish the same task. Therefore, such projects cannot be undertaken simultaneously. Hence, while choosing among mutually exclusive projects**,** while **more than one** project may satisfy the Capital Budgeting criterion only one project can be accepted to accomplish the task under consideration. Which project is eventually accepted depends on different factors like initial investment, time period required for completion, strategic importance of the project, etc. Which project is accepted depends on different factors like initial investment, the time period required for completion, strategic importance of the project, etc. Usually, the project which adds more value to the business in the long run will be selected. Capital budgeting techniques would typically result in the same acceptance or rejection decisions regarding independent projects, but conflict may arise in case of mutually exclusive projects. If conflicts arise while making decisions regarding mutually exclusive projects, the Net Present Value method is the preferred analytical tool and is given preference due to its more conservative or realistic reinvestment rate assumption. While the Net Present Value approach and the Internal Rate of Return method are both superior to the payback period method, the Net Present Value is superior to the Internal Rate of Return method when analyzing mutually exclusive projects.

The idea of projects being mutually exclusive is quite often used or applied in capital budgeting. Companies may have to choose between multiple projects that would add value to the company's profile. Based on its background or origins in probability, two events are said to be mutually exclusive when their occurrence is not simultaneous. In other words*, each excludes or precludes the other and they can't both exist, be true, or happen at the same time*. For example, a company that is considering investing in a CNC machining center with a budget of $500,000. CNC type A costs $450K and CNC type B costs $450k. A dryer for finished products cost $50K. In this case Projects A and B are mutually exclusive because they cannot both exist at the same time given the budget constraint. Project C in this context can be looked at as an independent project because the decision to purchase either CNC A or B does not affect Machine C. So, the purchase of CNC A or B does not have any impact on Machine C and the decision to purchase machine C does not have any bearing on the viability of machine A or B. Also in some ways, based on our description of the relationship of mutually exclusive projects and dependent projects, we can also say in a very limited sense that B is dependent on A because once we purchase CNC A, we would have no need to purchase CNC B.

2.2 Probability Aspect of Projects

We know from probability that if two nonzero events A and B are independent, then $P(A \cap B) = P(A)P(B)$. Also, we know that the same events A and B, are disjoint (mutually exclusive) only if $P(A \cap B) = 0$. Thus, taking the two assumptions together we can say that if P(A) and P(B) are different from zero then independent events are not disjoint events, hence they are not mutually exclusive events since it is not possible for $P(A \cap B) = P(A)P(B)$ and at the same time $P(A \cap B) = 0$

From the point of view of probability, mutually exclusive events are by definition disjoint events. Also, from the viewpoint of probability two events cannot be independent if they are disjoint events. Hence, we know that independent events are not mutually exclusive. However, if events are mutually exclusive, they may also be dependent (but not independent). Independent projects are those not affected by the cash flows of other projects. Mutually exclusive projects, however, are different. If two projects are mutually exclusive, it means that there are two ways of accomplishing the same result. A Project whose cash flows have no impact on the acceptance or rejection of other projects is termed as Independent Project (not mutually exclusive). Thus, all such projects which meet this criterion should be accepted.

Dependent events in probability are events whose occurrence of one affects the probability of occurrence of the other. **Dependent events** are those which depend upon what happened before. These events are affected by the outcomes that had occurred previously.

2.2.1 Mutually Exclusive Versus Independent Projects.

In mutually exclusive projects (ME), **all projects are to accomplish the same task**. Thus, a set of projects from which at most one will be accepted is termed as Mutually Exclusive Projects. That is, projects among which only one project can be selected out of all the possible investments or projects. In mutually exclusive projects, cash flows of one project can be adversely affected by the acceptance of the other projects. Therefore, such projects cannot be undertaken simultaneously. When we have mutually exclusive projects, we have a budget constraint, hence we can only choose one project. So, we need to evaluate all projects and select the best project that is economically satisfactory.

Hence, while choosing among Mutually Exclusive (ME)Projects, more than one project may satisfy the Capital Budgeting criterion, but **only one project** can be accepted. Which project should be accepted depends on different factors like initial investment, the time period required for completion, strategic importance of the project, etc. Usually, the project which adds more value to the business in the long run is selected.

The capital budgeting technique which is preferred for evaluating mutually exclusive projects is the Net Present Value (NPV) method which yields a value indicative of net change in the wealth of investors if a given project is undertaken.

Independent projects on the other hand result in the same acceptance or rejection decisions regardless of the Capital budgeting evaluation technique employed. However, in the case of mutually exclusive projects there may be conflicts since the results from the different evaluation techniques may not be consistent. When such conflicts arise, the NPV method is usually preferred because it is quite conservative with a more realistic reinvestment rate assumption. The NPV and Internal Rate of Return (IRR) are superior to the payback period because of the time value of money (TVM) and their results are consistent with the discounted payback period method. Due to the approximations involved in estimating the IRR, the NPV is superior and perhaps even more practical than the IRR.

2.2.2 Mutually Exclusive Versus Dependent Projects

Mutually exclusive projects are projects that compete with each other and require choosing only one, while dependent projects are projects where tasks are interrelated and must be completed in a specific order. To evaluate mutually exclusive projects, we compare the NPV or IRR of each option and choose the one with the highest value.

Dependent projects on the other hand are projects or tasks that are interrelated and must be completed in a specific order. Dependent tasks are interrelated project activities. Task dependency is when a project task or milestone can't begin until the completion of a separate task. Some tasks may have multiple dependencies, while others only have one. Common types of dependencies include finish one set of tasks-then- start the next set, finish one set -then-finish the next set, start the set of tasks-then- start the next set.

Discount Rate and Time Value of Money

Time value of money is based on the idea that people would rather have money now, today, rather than sometime in the future. Money is more valuable in the present than in the future because it can earn interest which is compounded with time going forward. More specifically, Time Value of Money (TVM) is an important concept in devising an investment strategy because **money on hand today is worth more than the same amount promised in the future due to the compounding factor (based on the discount rate, namely the Minimum Attractive Rate of Return: MARR) and because of inflation**. Again, so long as money can earn interest, TVM provides the assurance that any amount of money is worth more the sooner it is received. At the most basic level, the time value of money (TVM) demonstrates that, all things being equal, it is better to have money now rather than later. Thus, money on hand today can be invested **and can earn interest resulting in capital gain (CG). Thus, money at hand today is worth more than the same amount of money tomorrow, next week, next month, or next year.** TVM has three major components, namely:

i). **Present Value (PW)**
ii)**. Annual Worth (AW) or Equivalent Annual worth**
ii). F**uture Value (F)**.

3.1 Interest and Interest Formulas

The word interest in the context of engineering economics means the extra amount earned by the investor along with the investment (or) the amount owed by the borrower along with the amount borrowed. It is the monetary charge for the privilege of borrowing money, typically expressed as an annual percentage rate (APR). Interest is the amount of money a lender or financial institution receives for lending out money. In the broader area of economics, interest can also refer to the amount of ownership a stockholder has in a company, usually expressed as a percentage. For our purposes, Interest is the cost of borrowing money, where the borrower pays a fee to the lender which is the cost of the loan. On the other hand, it can be viewed as the benefit of an investment in the form of extra amount earned from the investment. In general, there are two types of interests, namely, simple interest and compound interest.

Simple interest is based on the principal amount of a loan or the deposit in a savings account. Simple interest doesn't compound, which means a creditor will only pay interest on the principal amount and a borrower would never have to pay more interest on the previously accumulated interest. Simple interest is calculated only on the principal amount of a loan or deposit, so it is easier to determine than compound interest.

Compound interest (also known as compounding interest) is the interest on a loan or deposit calculated based on both the initial principal and the accumulated interest from previous periods. It is based on the principal amount and the interest that accumulates on it in every period. The idea of compound interest is believed to have originated in Europe in the 17th century. Compound interest can be thought of as "interest on interest and makes an amount grow at a faster rate than in the case of simple interest. The rate at which compound interest accrues depends on the frequency of compounding. The higher the number of compounding periods, the greater the compound interest. For example, the amount of compound interest accrued on $1000 compounded at 8% annually will be lower than that on $1000 compounded at 8% semi-annually over the same time period. Compound interest is calculated on the accumulated principal and interest and hence it is different for every span of time period as it is calculated on the amount not the principal.

Define the following:

Let $P = principal$

$i = nominal\ annual\ interest\ rate\ in\ percentage\ terms$, $n = number\ of\ compounding\ periods$

F= future value after interest has been earned

The compound interest is the total amount of principal and interest in future (or future value) minus principal amount at present (or present value). Let t=1 to n

Then, $For\ t = 1, F_1 = P + Pi = P(1+i)$

$For\ t = 2,$

$F_2 = P(1+i) + i[P(1+i)] = P(1+i)(1+i) = P(1+i)^2$

$For\ t = 3$

$F_3 = P(1+i)^2 + iP(1+i)^2 = P(1+i)^2[1+i] = P(1+i)^3$

$For\ t = n$

$F_n = P(1+i)^{n-1} + iP(1+i)^{n-1} = P(1+i)^{n-1}[1+i] = P(1+i)^{(n-1)+1} = P(1+i)^n$

Compound Interest earned, CI= $F - P \Rightarrow P(1+i)^n - P = P[(1+i)^n - 1]$

The compounding Factor C = $[(1+i)^n - 1]$

Example 1: An investment requires a deposit of $10,000 for 5 years at an interest rate of 12% compounded annually. What is the compound interest and what is the compound interest factor.

Compound Interest CI= $P[(1+i)^n - 1] = \$10{,}000[(1+0.12)^5 - 1] = \$10{,}000[1.7623 - 1]$

So, the compound interest earned in 5 years is $= \$10{,}000[0.7623] = \$7623.42$

The compound factor C=$[(1+i)^n - 1] = [1.7623 - 1] = 0.7623$

3.2 Nominal and Effective Interest Rates

The **nominal interest rate** (NIR) is the stated interest rate of a financial instrument such as a loan or investment and signifies the actual monetary price borrowers pay lenders to use their money or the money that accrues t the investor. If the nominal rate (also called the coupon rate) on a loan is 10%, then the debtor can expect to pay $100 of interest if the amount of the loan is loaned $1000.

The nominal interest rate (NIR or simply r) is the annual interest rate that does not consider the effect of compounding of the interest or the discount rate. It is also known as the simple, announced or stated interest rate. Please note that in practice, the announced or stated interest rate is the nominal annual rate. If it is not annual, then a different period would have to be specified. So, a stated interest rate of 15% compounded monthly, says that the annual nominal rate is 15% but compounded monthly. Please note that if the period of interest for the stated interest is not specified, then it is assumed to be the **annual rate**

The **effective interest rate**, also known as the effective annual rate (EAR) is the interest rate which takes compounding into account during the year. The effective interest rate is the actual percent interest that a borrower pays on their loan or earns on their investment. It is the actual annual interest considering compounding

If the interest is compounded annually, the normal interest rate and the effective interest rate will be the same. In other words, if there is compounding, the effective interest rate is higher than the nominal rate. If the rate is compounded once a year, then the rates would be the same since the compounding is once. If the interest is compounded more than once a year, then the effective interest rate will be higher than the nominal rate. The effect of the more frequent compounding is that the effective interest rate per year is higher than the nominal interest rate. So, the effective interest rate for daily>monthly>quarterly>semi-annual>annual.

As an example, suppose the savings account policy of a bank says as follows: "15 % interest rate compounded monthly." In this case what is the interest i). in the first month, and ii). how much would the total amount be at the end of one year if the amount deposited is $1,000.

A few things to note about this simple example. The 15 % interest is for one year since there is no period qualifier. Of course, if the interest is not for one year, then it would be necessary to state the period of interest.

i. Compounded monthly indicates that there are 12 interest periods per year (m=12), with an interest period being 1 month.
ii. Since we have 12 interest periods (due to 12 months in a year), then the interest rate per interest period (1 month in this case) is 1.25% (15÷12). For the one-year duration, we have 12 interest periods.

Example 2 Data: P=principal=$1,000, i=1.25%, n =1*12=12, F=future value

$F = P(1+i)^m$

For 1 month, m=1, i=1.25%, $F = P(1+i)^1 = \$1{,}000(1.0125)^1 = \$1000(1.0125) = \$1{,}012.5$ thus 1 month interest =$12.5=(1,012.5-1,000)

For 1 year, m=12

$F = P(1+i)^m = \$1{,}000(1.0125)^{12} = \$1{,}000(1.1607) = \$1{,}160.75.$ So, the accrued interest on the $1,000 is $160.75 in one year.

Thus, the interest paid is $[(1.1607) - 1] = 0.1607\ or\ 16.7\%$. which is the effective interest rate.

Alternatively, the interest can be computed as follows $[\$(160.75) \div \$1{,}000] = 0.1607 = 16.07\%$

3.2.1 Computation of Effective Interest Rate and Nominal Interest Rate

Let *r = the nominal interest rate (in percentage) for the interest period (typically one year horizon)*
m = the number of interest compounding periods/subperiods per year
i_a *= the effective interest rate for the period of interest*
n = number of years or number of annual periods

Let the nominal interest rate compounded quarterly $=r$, then the equivalent quarterly interest rate is $\frac{r}{m}$. The future amount at the end of the year is: $F = P\left(1+\frac{r}{m}\right)^m$. We showed that this amount is as a result of compounding which yields the effective interest rate.

Let i_a be the effective annual interest rate per year in where (n=1) periods,

Then: $F = P(1+i_a)^n = P(1+i_a)^1$

Since the two quantities are identical, $P(1+i_a)^1 = P\left(1+\frac{r}{m}\right)^m \Rightarrow\Rightarrow P(1+i_a) = P\left(1+\frac{r}{m}\right)^m$

Cancelling P on both sides, we have ; $(1+i_a) = \left(1+\frac{r}{m}\right)^m \Rightarrow \left[i_a = \left(1+\frac{r}{m}\right)^m - 1\right]$

Hence (EAR), the effective interest rate: $i_a = \left(1+\frac{r}{m}\right)^m$, where *r* is the nominal rate.

$$(1+i_a) = \left(1+\frac{r}{m}\right)^m \Rightarrow \left[\left(1+\frac{r}{m}\right)^m = (1+i_a)\right] \Rightarrow \left[\frac{r}{m} = (1+i_a)^{\frac{1}{m}} - 1\right]$$

Therefore, $\left[\boldsymbol{\frac{r}{m} = (1+i_a)^{\frac{1}{m}} - 1}\right] \Rightarrow \left\{\boldsymbol{r = m\left[(1+i_a)^{\frac{1}{m}} - 1\right]}\right\}$

Hence the nominal interest rate r, given the effective interest rate $\boldsymbol{i_a}$, is $\boldsymbol{r = m\left[(1+i_a)^{\frac{1}{m}} - 1\right]}$

<u>Problem</u>: What is nominal interest rate compounded monthly if you are charged **10%** compounded quarterly?

i) First convert quarterly to effective rate (m=4 for 4 times a year)

ii) The convert the effective rate to nominal annual rate

iii). Then compute the nominal monthly or periodic rate

i). The effective annual interest rate $i_a = \left[\left(1+\frac{r}{m}\right)^m - 1\right] = \left[\left(1+\frac{0.10}{4}\right)^4 - 1\right] = 0.1038 = 10.38\%$

ii). Nominal <u>annual</u> interest rate: $r = m\left[(1+i)^{\frac{1}{m}} - 1\right] = 12\left[(1+0.1038)^{\frac{1}{12}} - 1\right] = 9.9\%$

iii). Nominal monthly interest rate: $\frac{r}{12} = \frac{1}{12}\left[(1+0.099)^{\frac{1}{12}} - 1\right] = \frac{0.099}{12} = 0.00825 = 0.825\%$

3.3 Interest Formulas-Compound Interest (CI)

Principal: *P = The present principal sum or the amount that was initially borrowed from the bank or invested.*

Rate: ***r*** *= nominal annual interest rate, the rate of interest at which the principal amount is loaned or invested.*

Time: *n = the number of annual periods for which the principal amount is loaned or invested.*

Amount: *A = single payment in a series of n equal payments made at the end of annual period*

Amount: *F = the future sum after compounding interest*

Compounding periods: m = number of interest compounding periods per year

The compound interest is calculated, after calculating the total amount over a period of time, based on the rate of interest, and the initial principal. For an initial principal P, rate of interest(in percent) per annum of *r*, time period *n* in years, the frequency, or the number of times the interest is compounded annually is *m,* then the formula for calculating the future amount F is as follows:

1. *Compounding Annually* : $F = P(1+r)^n$
2. *CompoundingSemiannually*: $F = P\left(1+\frac{r}{2}\right)^{2n}$
3. *Monthly Compounding*: $F = P\left(1+\frac{r}{12}\right)^{12n}$

Relationship Between PW, AW, and FW

We can in the same way establish the relationship between the Annual Equivalent (AW) and the Present Worth (PW), and the Annual Equivalent (AW)and the Future Worth (FW)

$P = A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$, **<u>hence:</u>** $A = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$

Also: $F = A\left[\frac{(1+i)^n-1}{i}\right]$, **hence**: $\boldsymbol{A} = \boldsymbol{F}\left[\frac{i}{(1+i)^n-1}\right]$
As previously shown : $\boldsymbol{F} = P(1+r)^n,$ ***hence*** $\boldsymbol{P} = F(1+r)^{-n}$

3.4 Summary of cash Flows and Interest Formulas

Future Cash Flow (F), given Present Cash Flow (PW or P): $\mathrm{F} = \mathrm{P}\left(^{\mathrm{F}}/_{\mathrm{P}}, \mathrm{i\%}, \mathrm{n}\right) = \mathrm{P}[(1+i)^n]$

Present Worth Cash Flow (or P) given Future Cash Flow (F): $\mathrm{P} = \mathrm{F}\left(^{\mathrm{P}}/_{\mathrm{F}}, \mathrm{i\%}, \mathrm{n}\right) = \mathrm{F}[(1+\mathrm{i})^{-\mathrm{n}}]$

Future Cash Flow (F), given Annual Cash Flow (PW or P): $F = \mathrm{A}\left(^{F}/_{A}, i\%, n\right) = \mathrm{A}\left[\frac{(1+i)^n-1}{i}\right]$

Annual Cash Flow(A), given Future Cash Flow (F): $\mathrm{A} = \mathrm{F}\left(^{\mathrm{A}}/_{\mathrm{F}}, \mathrm{i\%}, \mathrm{n}\right) = F\left[\frac{i}{(1+i)^n-1}\right]$

Present Worth (PW or P), given Annual Cash Flow(A): $P = \mathrm{A}\left(^{\mathrm{P}}/_{\mathrm{A}}, \mathrm{i\%}, \mathrm{n}\right) = \mathrm{A}\left[\frac{(1+\mathrm{i})^{\mathrm{n}}-1}{\mathrm{i}(1+\mathrm{i})^{\mathrm{n}}}\right]$

$$P = \mathrm{A}\left(^{\mathrm{P}}/_{\mathrm{A}}, \mathrm{i\%}, \mathrm{n}\right) = \mathrm{A}\left[\frac{(1+\mathrm{i})^{\mathrm{n}}-1}{\mathrm{i}(1+\mathrm{i})^{\mathrm{n}}}\right] \Rightarrow P = A\left[\left(1-\frac{1}{(1+\mathrm{i})^{\mathrm{n}}}\right)/(i)\right]$$

Annual Cash Flow(A), given Present Cash Flow (P): $\mathrm{A} = \mathrm{P}\left(^{\mathrm{A}}/_{\mathrm{P}}, \mathrm{i\%}, \mathrm{n}\right) = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$

$$\mathrm{A} = \mathrm{P}\left(^{\mathrm{A}}/_{\mathrm{P}}, \mathrm{i\%}, \mathrm{n}\right) = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] \Rightarrow A = P\left[(i)/\left(1-\frac{1}{(1+i)^n}\right)\right]$$

3.4.1 Important Notes About Interest Rates

Note: If the interest is compounded annually, the nominal interest rate and the effective or periodic interest rate will be the same. If the interest is compounded more than once per year, say semi-annually, quarterly, or monthly, then the effective or periodic interest rate would be higher than the original nominal annual interest rate. Please note the following distinctions as they are important in understanding interest rates. Examples: "12% interest" means that the interest rate is 12% per year, compounded annually. "12% interest compounded monthly" means that the interest rate is 12% per year (not 12% per month), compounded monthly. Thus, the interest rate is 1% (12% / 12) per month.

Ranking Methods for Decision Among Alternatives

Typically, projects that undergo assessment of economic viability are of two types, namely Non-Mutually Exclusive and Mutually Exclusive projects. Among these two categories projects may be classified as independent or dependent. So, the first step in the process is to classify the alternatives as either mutually exclusive or independent, and second as either revenue alternative or (distribution, service-only) alternatives. Such classification determines how the alternatives would be evaluated and compared. There are only four (really three because DN is not compared or evaluated against itself) types of alternatives based on these classifications as shown in Table 1.

Types of Proposals or Alternatives	Evaluation Basis/criteria
Do Nothing (DN)	**N/A (Never implemented)**
Independent	**Evaluate only against Do-Nothing (DN)**
Revenue Generating and Implied Benefit (such as in public works, e.g, i). dollar worth of the number of lives saved because of new instrumentation ii). travel time saved in dollars because of a new bridge) Mutually Exclusive Projects	**Against DN first, then against each other in ascending First Cost order**
Service or Cost only Mutually Exclusive Project	**Only Against each other**

Table 1: Types of Alternatives and Evaluation Basis

For non-mutually exclusive case, there are several alternatives from which more than one alternative may be chosen. In that scenario, the portfolios are ranked based on some acceptable criteria such as NPV, AW or IRR and based on that ranking (where the higher the ranking the more economically desirable the portfolio) the acceptable investments are chosen.

For mutually exclusive projects, there is usually a budget constraint and so of the several investment alternatives, one and ONLY one alternative is chosen. When more than one alternative can be selected from those available, the alternatives are considered NOT mutually exclusive (non-mutually exclusive)

4.1 Please Note the Following about ROR (IRR) and Multiple Alternatives

i. All alternatives will be considered as mutually exclusive unless otherwise indicated. **Independent alternatives are compared only against the do-nothing (DN) alternative**. In such a case, all the alternatives which have a rate of return that exceeds the MARR are selected.

ii. For mutually exclusive alternatives, the do-nothing (DN) is a viable option **when revenue alternatives are involved.** When more than one alternative is involved, the alternatives must be compared against each other on an incremental basis, which is simply the economic analysis of the *difference* in cash flow between *two alternatives*. The DN comparison with the qualifying alternative after ranking the initial cost from lowest to highest is the first starting point

iii. When the alternatives under consideration have **only disbursements (service alternatives), the do-nothing (DN) alternative cannot be considered (see table).** In that case the alternatives are compared just against each other.

iv. For alternatives having **only negative cash flows**, the only way to compare them using ROR is on an incremental cash flow basis.

v. When the cash flow of the alternative with the lower initial investment is subtracted from that with the higher initial investment, a rate of return on the incremental cash flow that equals or exceeds the MARR **means the lower-initial investment alternative is the more attractive.**

vi. When alternatives **are ranked** on the basis of a present worth (PW) analysis, the same ranking will always be obtained with an annual worth (AW) **but NOT always** with rate of return (ROR) analysis. Under certain circumstances, ROR analysis can yield different **rankings** than a PW and AW analysis**.**

vii. When an incremental rate of return analysis is conducted, the alternative **identified as best** by the present worth (PW) method will also be identified as best by the incremental ROR analysis.

viii. When **more than one alternative** can be selected from those available, then alternatives are said to be **NOT** mutually exclusive (They are not Mutually Exclusive).

ix. The rate of return (ROR) on the extra investment is interpreted as the percentage return on the amount invested in year 0 for the larger initial investment alternative.

There are several methods used to evaluate mutually exclusive projects, and they serve as the criterion on which the acceptance or rejection decision shall be made.

- Incremental Method
- Present Worth (PW)
- Annual Worth (AW)
- Future Worth (FW)
- Net Present Worth (NPV)
- Rate of Return (ROR)
- Internal Rate of Return (IRR)
- External Rate of Return (ERR)
- Profitability Index Method
- Payback Period Method
- Discounted Payback Period Method
- Benefit/Cost Analyses Method

4.2 Incremental Method

When there are mutually exclusive alternatives to consider, such as investment, equipment acquisition, production, hiring, or asset management, a decision must be made regarding what action to take or what option to choose. Incremental analysis is a useful tool for determining which decision will make the most economic sense or yield the best return on investment (ROR) for the company.

Incremental analysis is a comparative decision-making process. It is used to compare multiple options when the goal is to determine the most cost-effective action between two or more mutually exclusive alternatives. When making decisions between mutually exclusive alternatives, it is the differences between the alternatives that are relevant.

For mutually exclusive alternatives, the do-nothing (DN) is a viable option when revenue alternatives are involved. When there is more than one alternative involved, the alternatives must be compared against each other on an incremental basis.

An incremental analysis refers to an economic analysis of the difference in cash flow between two alternatives. Such an analysis is based on the fact that if the extra investment required in the alternative which has the higher initial investment does not earn at least the minimum attractive rate of return (MARR), then that investment should not be made because that increment of money could be better invested elsewhere (where it would earn at least the MARR).

As a side note, selecting the option with the higher ROR may not necessarily be the best decision because it may not yield the highest return on the capital available for the project. As such it is important to consider the weighted average of the total available capital since the understanding is that any unused funds are invested and assumed to earn at MARR

4.2.1 Example of Incremental Analysis

The sum of $500,000 is available to a small business owner for investment at MARR of 15%. Two projects are under consideration as follows:

Project P1: Investment of $500K with earnings of 28% per year.

Project P2: Investment of $500K with earnings of 32% per year

Based on the return or yield profile of the projects and (assuming no other information is available), then the project with the highest yield, in this case P2, must be selected. If the total amount available is more than $500K, or if the project costs different from each other and less than $500K, then an incremental analysis must be done to determine the value that the unused capital adds to the overall rate of return.

Consider a different scenario where the total amount available for investment is $500K:

Project P1: Investment of $400K required with earnings of 28%

Project P2: Investment of $250K required with earnings of 32%

You will recognize in this case that that the decision regarding which project to choose is not as straightforward as the earlier case.

For project 1, with $400K invested with a return of 28% per year, there is left $100K to be invested at a discount rate of 15%. In the case of project 2, $250K is invested with a fantastic return of 32%. However, the remining amount of $150K must be invested at 15%. So, to determine the project to choose, we must look at the average or weighted return on both projects. And we compute that as follows:

$ROR_{P1} = [\,\$400{,}000(0.28) + \$100{,}000(0.15)]/(\$500{,}000) = 25.4\%$

$ROR_{P2} = [\,\$250{,}000(0.32) + \$250{,}000(0.15)]/(\$500{,}000) = 23.5\%$

Although project 2 shows a good return on the investment, the incremental investment as a result of the total available funds shows an overall weighted yield of 23.5%. On the other hand, while the yield on the initial investment for project 1 is less than that of project, the incremental investment due to the total available funds for project 1 has improved the weighted yield project 1 equal to 25.4% which is undoubtedly higher than project 2. So, it is important to critically examine the conditions associated with each alternative portfolio using incremental analysis before making any decision about portfolio selection.

Please note that if the Present worth, (or FW or AW) on the additional investment required by one alternative over another is positive, then that incremental investment is economically viable.

It is also important to note that the relationship between the decision rules of the two present worth criteria, namely the Present worth on total investment and the Present worth on incremental investment. For example, if the objective is to maximize the Present worth on total investment and say PW(A2)>PW(A1), then the criteria for the Present worth on total investment would suggest selecting Alternative A2. So, if PW(A2)>PW(A1), then PW_{A2-A1} must be positive and the decision rule on incremental investment is also to accept A2, rather accept A1 since PW_{A2-A1}>0

4.2.2 The Incremental ROR Procedure

In order to choose between multiple mutually exclusive alternatives, we will employ the challenger-defender model which compares alternatives two at a time or pairwise comparisons using incremental analysis. The first step in the incremental ROR procedure is to order all the alternatives from the smallest to the largest initial investment. The winner of the comparison becomes the defender for the next comparison and takes on a new challenger. The procedure is as follows:

1. First order the alternatives based on their initial investment or first cost (P_0 at t=0); from lowest first cost to highest first cost. This ensures that for each pairwise comparison, the increment is an investment. If the alternatives are revenue alternatives (or benefits, un-benefits, and not just costs alone), then do-nothing (DN) is added and is the first alternative. Identify the first two alternatives as A and B.
2. The lowest cost option is always the first defender (A or DN), while the next highest is the challenger (B). Then determine if the IRR(i*) of the (B-A) is greater than the MARR (i.e., IRR>MARR)
   a. If yes, then first challenger is better than the DN and it is now the defender
   b. If no, then reject the first challenger, keep the defender, and continue through the sequence until a new defender emerges, whose IRR is greater than MARR. If there is no alternative whose IRR is greater than the MARR, then the DN is the alternative
3. If A is the defender and B is the challenger, the incremental analysis is (B-A).
   If IRR of (B-A) is greater than the MARR, then B becomes the new defender and then continue through other challenges

   In general, in the incremental ROR analysis for multiple mutually exclusive alternatives, if the incremental ROR (or IRR) is equal to or greater than the MARR, the "Challenger" is selected and becomes the "Defender" for the next round of analysis.

4.3 Net Present Worth (NPV)

Net present value (NPV) is the difference between the present value of cash inflows because of an investment or project and the present value of the initial cost or cash outflows from the investments over a period. As such, it is used to analyze the profitability of a proposed investment or project and represents the current total value of a future stream of payments or income. If the NPV of a project or investment is positive, then it means that the discounted present value of all future cash flows related to such a project or investment would be positive, and hence the NPV for such a project or investment is considered as acceptable or worthwhile. In order words, any potential project or investment with a negative NPV should not be considered.

The whole notion of NPV is anchored around the idea of the importance of time value of money (TVM). This makes it a particularly valuable tool in comparing similar investment alternatives. NPV assesses the profitability of an investment based on TVM, namely, the idea that an amount of money in the future is not worth the same amount today because money loses its value over time due to inflation. However, money invested today can earn a return, making its future value higher than the same amount received at some point in time in the future. NPV seeks to determine the present value of an investment's future cash flows over and above the investment's initial cost. The discount rate element of the NPV formula is derived from the cost of the capital required to make the investment and is used to discounts the future cash flows to the present-day value.

Subtracting the initial cost of the investment from the sum of the cash flows in the present day would result in a remainder referred to as NPV. If the quantity NPV is positive, then the investment is considered worthwhile. The NPV dictum is that only projects with positive NPV are to be given further consideration. Define:

- PV= Present Value of Invested Cash=Initial Cash Investment
- PVECF= Present Value of Expected Cash Flow
- R_t=Net cash inflow-outflows during a single period t
- i=Discount rate or return that could otherwise be earned in alternative investments
- t=Number of time periods

Let: $PVECF = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t}$

The NPV formula is given as: $\boldsymbol{NPV} = -\boldsymbol{PV} + PVECF \ = -\boldsymbol{PV} + \sum_{t=1}^{n} \frac{R_t}{(1+i)^t}$

$$\begin{Bmatrix} If\ NPV \ > 0, & Then\ Investment\ is\ acceptable/worthwhile \\ If\ NPV \ < 0, & Then\ Investment\ is\ NOT\ acceptable/Not\ worthwhile \end{Bmatrix}$$

NPV is obtained calculating the present value of all cash flows over the life of a project. The present value of those cash flows is subtracted from the investment's initial investment. If the difference is positive (NPV> 0), then the project is considered profitable, otherwise (i.e., if NPV<0) it is not considered as profitable.

<u>Example:</u> The company, Organic Poultry Inc. is setting up an organic poultry farm requiring an initial investment (P_0) of $500,000 (year 0). This investment represents an initial cash outlay and is considered a net negative value cash outflow. After the first year, the poultry farm generated $100,000 during year one (second year), $200,000 in year two (third year), $300,000 in year three, $400,000 in year four, $500,000 the year five (6th year), and $650,000 in year six (7th year) which represented the final year of the pilot. Following pilot phase, a new and more comprehensive phase was implemented. Table 2 shows the cash flows. The discount rate applied is 10%

Year	Cash flow
0	($500,000)
1	$100,000
2	$200,000
3	$300,000
4	$400,000
5	$500,000
6	$650,000

Table 2: Cash Flows for the Poultry Farm

$$NPV = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t} - P_0$$

$$NPV = \left[\frac{R_1}{(1+i)^1} + \frac{R_2}{(1+i)^2} + \frac{R_3}{(1+i)^3} + \frac{R_4}{(1+i)^4} + \frac{R_5}{(1+i)^5} + \frac{R_6}{(1+i)^6}\right] - P_0$$

$$NPV = [\$90{,}909.09 + \$165{,}289.26 + \$225{,}394.44 + \$273{,}205.36 + \ldots + \$336{,}166.89] - P_0$$

$$NPV = \$1{,}432{,}166.89 - \$500{,}000 = \$932{,}166.89$$

Year	Cash flow	PV
0	($500,000)	($500,000.00)
1	$100,000	$90,909.09
2	$200,000	$165,289.26
3	$300,000	$225,394.44
4	$400,000	$273,205.38
5	$500,000	$310,460.66
6	$650,000	$366,908.05
	NPV(MARR=10%)	$932,166.89

Table 2A: EXCEL compuation of NPV

Table 2A shows the result of the EXCEL computation of NPV using the SUM (PV), from t=0,6). Note the following for NPV:

- NPV = All Cash Inflows – Cash Outflows
- Always choose the project(s) with the highest NPV.
- If NPV is the same for all the projects, then choose the project with highest IRR.
- If NPV and IRR are the same for all projects, then choose the project(s) with the smallest early payback period.

4.3.1 NPV and Mutually Exclusive Alternatives

Assume that an investor has two portfolios under consideration for investment and the total cash available for the investment is $500,000. Other alternatives exist to invest at MARR =10% Project life for each is 5 years. The cashflow for investments are:

A: Initial cash outflow P_0=-$80K, Annual cash inflow =$50K/year, Salvage =$50K

B: Initial cash outflow P_0=-$500K, Annual cash inflow =$150K/year, Salvage =$500K

$$NPV_A = -\$80{,}000 + \$50{,}000(P/A, 15\%, 5) + \$50{,}000(P/F, 15\%, 5)$$

$NPV_A = -\$80{,}000 + \$50{,}000(3.79078) + \$50{,}000(0.62092)$=$140,585.00

$NPV_B = -\$500{,}000 + \$150{,}000(P/A, 15\%, 5) + \$500{,}000(P/F, 15\%, 5)$

$NPV_B = -\$500{,}000 + \$150{,}000(3.79078) + \$500{,}000(0.62092)$=$379,077

Since the NPV for project A is positive and that for project B is positive, then we can conclude that both projects are economically viable. Also, since the NPV of project B is higher than the NPV for project A, then we can further conclude that Project B is the more economically viable alternative. See table 2B.

As a reminder about the figures in EXCEL table 2B, the values were computed using the NPV and IRR utilities in EXCEL: $NPV_A = G12 + NPV(0.1, G13{:}G17),\ \ IRR_A = IRR(G12{:}G17)$

$$NPV_B = H12 + NPV(0.1, H13{:}H17),\ \ IRR_B = IRR(H12{:}H17)$$

Note: The rows and columns in EXCEL were reproduced here for emphasis as shown in table 2B.

F	G	H
Year	**Cash Flows-Project A**	**Cash Flows-Project B**
0	**-$80,000.00**	**-$500,000.00**
1	**$50,000.00**	**$150,000.00**
2	**$50,000.00**	**$150,000.00**
3	**$50,000.00**	**$150,000.00**
4	**$50,000.00**	**$150,000.00**
5	**$100,000.00**	**$650,000.00**
NPV	**$140,585.40**	**$379,078.68**
IRR	**60%**	**30%**

Table 2 B. NPV and IRR estimates of the Cashflow for Projects A and B

4.3.2 Incremental Analysis for NPV and IRR

Since the two projects are economically viable, the investor is faced with the decision about which to pick. Although we agreed that the larger the NPV, the better the investment, it is possible for the project with a Lower NPV to have a higher internal rate of return IRR (or ROR). In our previous example with MARR of 10%. As shown in table 2B (generated from EXCEL), the NPV of Project A is $140,585.40, with an IRR of 60% as compared to project B whose NPV is $379,078.68 and IRR of 30%. So, in this case, project A uses up 16% of the money ($80K out of $500K) with a yield of 60%, while project B uses up 100% of the capital but yields only 30%. Hence the investor has a binary choice as follows, either

1. Choose project A spend $80,0000 on the project and then invest the remaining $(500K-80K=420,000) in another project at 10% MARR and no more, **OR**
2. Choose project B which is equivalent to investing in project A + invest in the incremental project (B-A)

So, we need one more step in the form of incremental analysis to be able to compare the projects in order to determine which of the two projects to fund. In incremental analysis the incremental fund is invested and if it yields an IRR, i* >MARR, then the project is deemed economically viable.

To conduct the incremental analysis, we use the challenger-defender concept which compares alternatives in pairs (pairwise comparisons). When performing the "Incremental ROR analysis" of multiple mutually exclusive alternatives, the first step is to order all the alternatives from the smallest to the largest initial investment or first cost. After that we then create the incremental project by subtracting the cashflows of the project with the smaller initial first cost from the cashflows of the project with the larger first cost. We create this incremental project for each pair of projects after arranging them in increasing order of first cost. Do the same subtraction for all the cash flow categories.

Perform incremental IRR analysis by pairwise comparison in a defender/challenger approach. At each comparison, choose the higher cost alternative (challenger) if the incremental IRR exceeds the MARR. Otherwise choose the lower cost alternative (defender). Continue until all alternatives have been considered.

The projects considered in the example are three, namely, DN (Do Nothing), project A, and project B. In this example, project B has the largest initial first cost, followed by A, and then DN. The first set of increment analysis is (A v DN). The incremental cashflow set up is as shown in table 2 C.

1. (A-DN)

Defender is DN, Challenger is A

$$NPV_{A-DN} = -\$80{,}000 + \$50{,}000(P/A, 0.1{,}5) + \$50{,}000(P/F, 0.1{,}5)$$

$$NPV_{A-DN} = -\$80{,}000 + \$50{,}000(3.790786769) + \$50{,}000(0.62092) = \$140{,}585.41$$

Since NVP >0, IRR (60%) >MARR (10%), drop DN, A= New Defender

	Incremental Cash Flow=Cash FlowA-Cash Flow Where larger initial investment is Project A			
	year	**DN**	**A**	**A-DN**
First cost/Initial Investment	**0**	**\$0.00**	**-\$80,000.00**	**-\$80,000.00**
Annual Cash Inflow-1	**1**	**\$0.00**	**\$50,000.00**	**\$50,000.00**
Annual Cash Inflow-2	**2**	**\$0.00**	**\$50,000.00**	**\$50,000.00**
Annual Cash Inflow-3	**3**	**\$0.00**	**\$50,000.00**	**\$50,000.00**
Annual Cash Inflow-4	**4**	**\$0.00**	**\$50,000.00**	**\$50,000.00**
Salvage + Annual Cash Inflow-5	**5**	**\$0.00**	**\$100,000.00**	**\$100,000.00**
NPV				**\$140,585.40**
IRR				**60%**

Table 2 C: Incremental Analysis (A-DN)

2. (B-A)

Defender is A, Challenger is B

$$NPV_{B-A} = -\$420{,}000 + \$100{,}000(P/A, 0.1{,}5) + \$450{,}000(P/F, 0.1{,}5)$$

$$NPV_{B-A} = -\$420{,}000 + \$100{,}000(3.790786769) + \$450{,}000(0.62092) = \$238{,}493.28$$

Since NVP >0, IRR (25%)>MARR (10%), drop A, and B= New Defender. However, since we have exhausted the list of projects, we will choose B since it is economically viable. See table 2D

	Incremental Cash Flow= Cash Flow B-Cash Flow A Where larger initial investment is Project B			
	year	A	B	B-A
First cost/Initial Investment	0	-$80,000.00	-$500,000.00	-$420,000.00
Annual Cash Inflow-1	1	$50,000.00	$150,000.00	$100,000.00
Annual Cash Inflow-2	2	$50,000.00	$150,000.00	$100,000.00
Annual Cash Inflow-3	3	$50,000.00	$150,000.00	$100,000.00
Annual Cash Inflow-4	4	$50,000.00	$150,000.00	$100,000.00
Salvage + Annual Cash Inflow-5	5	$100,000.00	$650,000.00	$550,000.00
NPV				$238,493.27
IRR				25%

Table 2 D: Incremental Analysis (B-A)

Note that incremental NPV is equal to the difference between the NPV of the project pair
$(NPV_B - NPV_A) = NPV_{B-A} \Longrightarrow (\$379{,}078.68 - \$140{,}585.40) = \$238{,}493.28 = NPV_{B-A}$

4.4 Present Worth (PW) Analysis for Mutually Exclusive Projects

The data in Table 3 shows the cash flow of four mutually exclusive projects. We will use the data to compute the PW(PV), FW, AW and IRR, the B-C analysis approach, Payback period and discounted payback period, the Profitability Index (PI) using first principles and where necessary EXCEL. We will later employ incremental analysis to demonstrate its application for mutually exclusive projects with a discount rate (MARR) of 10%

$(A/P\,, 10\%, 5) = \left[\frac{i}{1-(1+i)^{-n}}\right] = 0.26379748 \Longrightarrow (P/A\,, 10\%, 5) = 3.790786769$

$(A/F\,, 10\%, 5) = \left[\frac{i}{(1+i)^n - 1}\right] = 0.163797 \Longrightarrow (F/A\,, 10\%, 5) = 6.1051179$

$(F/P\,, 10\%, 5) = (1+i)^n = 1.61051 \Rightarrow (P/F\,, 10\%, 5) = 0.620921$

YEAR	A1	A2	A3	A4
0	-$12,000.00	-$10,000.00	-$12,000.00	-$15,000.00
1	$20,000.00	$2,500.00	$1,750.00	$1,900.00
2	$20,000.00	$2,500.00	$1,750.00	$1,900.00
3	$20,000.00	$2,500.00	$1,750.00	$1,900.00
4	$20,000.00	$2,500.00	$1,750.00	$1,900.00
5	$20,000.00	$2,500.00	$1,750.00	$1,900.00
End of Life Salvage	$2,000.00	$1,000.00	$1,800.00	$3,000.00

Table 3: Cash Flow of Mutually Exclusive Projects

$PW_{A1} = -\$12000 + \$2000(P/A\,, 10\%, 5) + \$2000(P/F\,, 10\%, 5) =?$

$PW_{A1} = -\$12000 + \$2000(3.790786769) + \$2000(0.62092) = -\$3{,}176.58$
$\boldsymbol{PW_{A2} = -\$10000 + \$2500(P/A, 10\%, 5) + \$1000(P/F, 10\%, 5) =?}$
$\boldsymbol{PW_{A2} = -\$10000 + \$2500(3.790786769) + \$1000(0.62092) = \$97.89}$ √
$PW_{A3} = -\$12000 + \$1750(P/A, 10\%, 5) + \$1800(P/F, 10\%, 5) =?$
$PW_{A3} = -\$12000 + \$1750(3.790786769) + \$1800(0.62092) = -\$4{,}248{,}46$
$PW_{A4} = -\$15000 + \$1900(P/A, 10\%, 5) + \$3000(P/F, 10\%, 5) =?$
$PW_{A4} = -\$15000 + \$1900(3.790786769) + \$3000(0.62092) = -\$5{,}934.74$

4.5 Future Worth Analysis for Mutually Exclusive Projects

$FW_{A1} = -\$12000(F/P, 10\%, 5) + \$2000(F/A, 10\%, 5) + \$2000 =?$
$FW_{A1} = -\$12000(1.61051) + \$2000(6.1051179) + \$2000 = -\$5{,}115.92$
$FW_{A2} = -\$10000(F/P, 10\%, 5) + \$2500(F/A, 10\%, 5) + \$1000 =?$
$\boldsymbol{FW_{A2} = -\$10000(1.61051) + \$2500(6.1051179) + \$1000 = \$157.65}$ √
$FW_{A3} = -\$12000(F/P, 10\%, 5) + \$1750(F/A, 10\%, 5) + \$1800 =?$
$FW_{A3} = -\$12000(1.61051) + \$1750(6.1051179) + \$1800 = -\$6{,}842{,}20$
$FW_{A4} = -\$15000(F/P, 10\%, 5) + \$1900(F/A, 10\%, 5) + \$3000 =?$
$FW_{A4} = -\$15000(1.61051) + \$1900(6.1051179) + \$3000 = -\$9{,}557.96$

4.6 Annual Worth Analysis for Mutually Exclusive Projects

$AW_{A1} = -\$12000(A/P, 10\%, 5) + \$2000 + \$2000(A/F, 10\%, 5) =?$
$AW_{A1} = -\$12000(0.263797) + \$2000 + \$2000(0.163797) = -\$837.97$
$AW_{A2} = -\$10000(A/P, 10\%, 5) + \$2500 + \$1000(A/F, 10\%, 5) =?$
$\boldsymbol{AW_{A2} = -\$10000(0.263797) + \$2500 + \$1000(0.163797) = \$25.82}$ √
$AW_{A3} = -\$12000(A/P, 10\%, 5) + \$1750 + \$1800(A/F, 10\%, 5) =?$
$AW_{A3} = -\$12000(0.263797) + \$1750 + \$1800(0.163797) = -\$1{,}120.73$
$AW_{A4} = -\$15000(A/P, 10\%, 5) + \$1900 + \$3000(A/F, 10\%, 5) =?$
$AW_{A4} = -\$15000(0.263797) + \$1900 + \$3000(0.163797) = -\$1{,}565.57$

Based on the results, it is clear that project **A2** is the one worthy of funding because its Present Worth Value (PW) is positive. The same is true for Annual Worth (AW), and Future Worth (FW). **So, the present worth (PW) on total investment will give a result consistent with the annual worth (AW) or the future worth (FW).**

Since the projects in our example are mutually exclusive alternatives then we must carry out an incremental analysis to confirm that the results are indeed optimal or actionable.

Tables 3 and 4 were obtained using EXCEL and then the results were verified from first principles. EXCEL has functions for PV (PW), FV (FW). However, there is none specifically implemented for AW. The closest one is PMT which we found not to be very responsive or convenient and so we used the relationship between AW and PW and AW and FW to confirm the values of AW computed from first principles. To do this, we computed all three parameters (i.e., PW, FW, AW) from first principles. We then used the compounding factor for PW and AW as well as the factor for FW and AW and the values of PW and AW from EXCEL to compute the numerical values for AW as a way of confirmation. This approach is often quite useful because the published tables sometimes give incorrect values. Recall that $F = P(1+i)^n \Rightarrow P = F(1+i)^{-n}$

$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \Rightarrow A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$

$but \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = \left[\frac{i}{1-(1+i)^{-n}}\right], Thus, A = P\left[\frac{i}{1-(1+i)^{-n}}\right]$

Also, $F = A\left[\frac{(1+i)^n - 1}{i}\right]$, **<u>hence</u>:** $\boldsymbol{A} = \boldsymbol{F}\left[\frac{i}{(1+i)^n - 1}\right]$

Using these relationships, we can confirm that the values from EXCEL are consistent with those from first principles. This is especially important for AW since we do not have a straightforward way of computing AW using EXCEL. As can be seen from table 4, the values of AW are in alignment with the ones estimated for PW and FW using EXCEL in table 5..

	A1	A2	A3	A4
AW(From Defining Equations/First Principles	-$837.97	$25.82	-$1,120.73	-$1,566.57
PW	-$3,176.58	$97.89	-$4,248.46	-$5,934.74
A/P FACTOR=(A/P,10%,5)=0.263797	0.263797	0.263797	0.263797	0.263797
AW(P)=PW*(A/P Factor)				
FW	-$5,115.92	$157.65	-$6,842.20	-$9,557.96
A/F FACTOR=(A/F,10%,5)=0.163797	0.163797	0.163797	0.163797	0.163797
AW(F)=FW*(A/F Factor)	-$837.97	$25.82	-$1,120.73	-$1,565.57

Table 4: Computation of AW using the Compound Factors for PW and FW

	A/P=0.2638	A/F=0.16379	F/P=1.6101							
	P/A=3.791	F/A=6.105	P/F=0.6209							
	MARR=10%	10%	A/P	0.263797						
	P/F	0.620921	A/F	0.163797						
		YEAR	A1	A2	A3	A4	FV-A1	FV-A2	FV-A3	FV-A4
	5	0	-$12,000.00	-$10,000.00	-$12,000.00	-$15,000.00	($19,326.12)	($16,105.10)	($19,326.12)	($24,157.65)
	4	1	$2,000.00	$2,500.00	$1,750.00	$1,900.00	$2,928.20	$3,660.25	$2,562.18	$2,781.79
	3	2	$2,000.00	$2,500.00	$1,750.00	$1,900.00	$2,662.00	$3,327.50	$2,329.25	$2,528.90
	2	3	$2,000.00	$2,500.00	$1,750.00	$1,900.00	$2,420.00	$3,025.00	$2,117.50	$2,299.00
	1	4	$2,000.00	$2,500.00	$1,750.00	$1,900.00	$2,200.00	$2,750.00	$1,925.00	$2,090.00
	0	5	$2,000.00	$2,500.00	$1,750.00	$1,900.00	$2,000.00	$2,500.00	$1,750.00	$1,900.00
Salvage	0	0	$2,000.00	$1,000.00	$1,800.00	$3,000.00	$2,000.00	$1,000.00	$1,800.00	$3,000.00
	Salvage Factor (PW)		$1,241.84	$620.92	$1,117.66	$1,862.76				
		PW	-$3,176.58	$97.89	-$4,248.46	-$5,934.74				
		FV	-$5,115.92	$157.65	-$6,842.20	-$9,557.96	($5,115.92)	$157.65	($6,842.20)	($9,557.96)
		AW (P)	-$837.97	$25.82	-$1,120.73	-$1,565.57				
		AW(F)	-$837.97	$25.82	-$1,120.73	-$1,565.57				
		AW	-$837.97	$25.82	-$1,120.73	-$1,565.57				

Table 5. Analysis of Alternatives using PV, FV, EAC (Equivalent Annual Cost)

4.7 Internal Rate of Return (IRR)

All alternatives will be considered mutually exclusive unless indicated otherwise. Independent alternatives are compared only against the do-nothing alternative wherein all alternatives which have a rate of return that exceeds the MARR are selected. For mutually exclusive alternatives, the do-nothing (DN) is a viable option when revenue alternatives are involved. When there is more than one alternative involved, the alternatives must be compared against each other on an incremental basis as described below. As indicated earlier an incremental analysis refers to an economic analysis of the difference in cash flow between two alternatives. Such an analysis is based on the fact that if the extra investment required in the alternative which has the higher initial investment does not earn at least the minimum attractive rate of return, then that increment of investment should not be made because that increment of money could be better invested elsewhere (where it would earn at least the MARR). The procedure for comparing mutually exclusive alternatives can be summarized as follows

i. Rank the alternatives in terms of increasing First Cost-FC (initial investment cost). If the alternatives are revenue alternatives, then do-nothing (DN) is added and considered the first alternative.

ii. Identify the first two alternatives as A and B

iii. Tabulate the difference in cash flow between the first two alternatives (i.e. the incremental cash flow) by subtracting the cash flow for alternative A from the cash flow for alternative B (i.e. B-A) over their least common multiple of lives.

iv. Find the rate of return on the incremental cash flow. If the (IRR= i*) ≥ MARR, eliminate A, or vice versa.

v. Compare the survivor with the next-in-line alternative per steps (iii) and (iv) above.

vi. Continue steps (iii) thru (v) until only one alternative remains

The internal rate of return IRR is the discount rate that would make all of the present values of cash flows equal to the initial outlay. IRR is the discount rate at which the NPV of the project equals zero. Companies often have MARR (or a hurdle rate or a required rate of return) that serves as the benchmark. $\sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} - C_0 = 0$, Where:

CF_t =Cash flow at time t

C_0 = Initial outlay at time zero

n = number of periods

The decision criteria therefore are:

- Accept if (IRR= i*) >MARR.
- Reject if (IRR= i*) < MARR)

4.7.1 Example of IRR Analysis of Mutually Exclusive Alternatives

The following table (Table 6) shows the cash flow of 4 mutually exclusive revenue alternatives with MARR=15%

	A1	A2	A3	A4
Intial Investment/First Cost ($)	-$70,000.00	-$170,000.00	-$105,000.00	-$125,000.00
Annual Maintenance Cost/year ($)	-$40,000.00	-$30,000.00	-$45,000.00	-$45,000.00
Annual Inflow/Income ($)	$85,000.00	$115,000.00	$100,000.00	$115,000.00
End of Life Salvage	$25,000.00	$35,000.00	$25,000.00	$25,000.00
Life/Years	3	3	6	3

Table 6: IRR Analysis of Mutually Exclusive Alternatives

We will reorganize the cash flows, specifically the two annual cash flows into a single resultant annual cash flow. Also, we will add the salvage to the resultant cash flow for year three (So for A1, we have $45K plus $25K=$70K, For A2 we have $85K plus $35K=$120K, for A3 we have $55K plus $25K =$80K). The reorganization of the cash flows is shown in table 6A.

Year	A1	A2	A3	A4
0	-$70,000.00	-$170,000.00	-$105,000.00	-$120,000.00
1	$45,000.00	$85,000.00	$55,000.00	$70,000.00
2	$45,000.00	$85,000.00	$55,000.00	$70,000.00
3	$70,000.00	$120,000.00	$80,000.00	$95,000.00

Table 6A: IRR analysis of Mutually Exclusive Alternatives (Adjusted)

To start, we rank the alternatives in terms of increasing initial investment as follows: DN, A1, A3, A4, A2. We include the DN (do nothing) as an alternative because the alternatives are revenue alternatives)

1. For the first incremental analysis, the alternatives are (A1-DN), as shown in table 6B.

Year	DN	A1	A1-DN
0	$0.00	-$70,000.00	-$70,000.00
1	$0.00	$45,000.00	$45,000.00
2	$0.00	$45,000.00	$45,000.00
3	$0.00	$70,000.00	$70,000.00

Table 6B: Incremental Analysis (A1-DN)

$0 = -\$70{,}000 + \$45{,}000(P/A,\ i^*, 2) + \$70{,}000(P/A,\ i^*, 3)$
$i^* = IRR = 50.847\% \approx 51\%.$
Since IRR (51%) >MARR (15%), drop DN, Now A1 = new defender

2. For the next pairwise comparison, the alternatives are (A3-A1), see table 6C.

Year	A1	A3	A3-A1
0	-$70,000.00	-$105,000.00	-$35,000.00
1	$45,000.00	$55,000.00	$10,000.00
2	$45,000.00	$55,000.00	$10,000.00
3	$70,000.00	$55,000.00	$10,000.00
4	-$25,000.00	$55,000.00	$80,000.00
5	$45,000.00	$55,000.00	$10,000.00
6	$70,000.00	$80,000.00	$10,000.00

Table 6C: Incremental Analysis (A3-A1)

For this comparison, we note that the lives of the two projects are different (A1=3 years while A3=6 years). In that case we use the LCM of the project years to represent the years. In this case the LCM 6 years. In that case project A1 will continue for another 3 years after it expires. It will then incur the initial investment cost (-$70,000) in year 4 as well as the resultant annual cash inflow of $45,000. This gives a total cash flow of -$25,000 in year 4 for A1. In year 5 for A1, the cash flow is

\$85K-\$40K=\$45K, and in year 6 it will be \$45K plus \$25K=\$70K. Similarly, the cash flows for A3, A4 are obtained in the same manner as shown in tables 6C and 6D.

$0 = -\$35{,}000 + \$10{,}000(P/A,\ i^*, 6) + \$70{,}000(P/F,\ i^*, 4)$

$i^* = IRR = 45.868\% \approx 46\%$

Since IRR (46%) >MARR (15%), drop A1, A3 = new defender

3. For the next pairwise comparison, the alternatives are (A4-A3), see table 6D.

Year	A3	A4	A4-A3
0	-\$105,000.00	-\$125,000.00	-\$20,000.00
1	\$55,000.00	\$70,000.00	\$15,000.00
2	\$55,000.00	\$70,000.00	\$15,000.00
3	\$55,000.00	\$95,000.00	\$40,000.00
4	\$55,000.00	-\$10,000.00	-\$65,000.00
5	\$55,000.00	\$70,000.00	\$15,000.00
6	\$80,000.00	\$95,000.00	\$15,000.00

Table 6D: Incremental Analysis (A4-A3)

$0 = -\$20{,}000 + \$15{,}000(P/A,\ i^*, 6) + \$25{,}000(P/F,\ i^*, 3) - \$80{,}000(P/F,\ i^*, 4)$

$i^* = IRR = 44.185\% \approx 44\%$

Since IRR (44%) >MARR (15%), drop A3, A4 = new defender

4. For the next pairwise comparison, the alternatives are (A2-A4), see table 6E.

Year	A4	A2	A2-A4
0	-\$125,000.00	-\$170,000.00	-\$45,000.00
1	\$70,000.00	\$85,000.00	\$15,000.00
2	\$70,000.00	\$85,000.00	\$15,000.00
3	\$95,000.00	\$120,000.00	\$25,000.00

Table 6E: Incremental Analysis (A2-A4)

$0 = -\$45{,}000 + \$15{,}000(P/A,\ i^*, 3) + \$10{,}000(P/F,\ i^*, 3)$

$i^* = IRR = 9.79\% \approx 9.8\%$

Since IRR (9.8%) < MARR (15%), drop A2 (The current challenger), A4 = final defender. However, since we have exhausted the list of pairwise comparisons, A4 emerges the winner. Furthermore, an NPV analysis also shows that A4 has the highest NPV of all four projects, that is **(NPV (A4)= \$56,263.6640**) which confirms that it is the most economically viable (See table 6F)

Year	A1	A2	A3	A4
0	-$70,000.00	-$170,000.00	-$105,000.00	-$120,000.00
1	$45,000.00	$85,000.00	$55,000.00	$70,000.00
2	$45,000.00	$85,000.00	$55,000.00	$70,000.00
3	$70,000.00	$120,000.00	$80,000.00	$95,000.00
NPV	$49,183.0360	$47,087.2031	$37,015.2872	$56,263.6640

Table 6F: NPV of Mutually Exclusive Alternatives

Please note: When an incremental rate of return ROR(IRR) analysis is conducted correctly, the alternative identified as best by the present worth method (NPV)will also be identified as best by the incremental ROR (IRR) analysis.

4.8 External Rate of Return (ERR)

The External Rate of Return (ERR) is the equivalent rate at which the present worth of the investment expenditures is considered to be invested to yield a future worth equal to the future worth of the revenues invested at the (MARR) rate. ERR directly considers the interest rate external to a project at which the net cash flows generated or required by the project over its life can be reinvested or borrowed. The ERR is the ROR on a project where any excess cash from a project is assumed to earn interest at a pre-determined explicit rate —usually the Minimum Acceptable ROR (MARR).

The ERR is best used when it is apparent that there is the possibility of multiple IRRs. Unfortunately, it is sometimes difficult to know in advance if there are multiple IRRs. However, most investments will have a cash flow structure which excludes multiple IRRs. If a project is a simple investment, it would have at most one positive IRR. With non-conventional cashflows however, there is a great possibility of multiple IRR. Non-conventional cash flows are characterized by sort of cyclic, sinusoidal, or up and down movement of cash outflow followed by cash inflow, alternating over the life of the project. This alternating sequence tends to produce several zeros for any polynomial function. The zeros of a polynomial p(x) are all the x-values that make the polynomial equal to zero We know this from the Descartes rule of signs. Descartes' rule of sign suggests that the number of positive real zeros in a polynomial function corresponds approximately to changes in the sign of the coefficients. And this is usually the case with what is commonly referred to as non-conventional cash flows.

We have a case of multiple rates of return MIRR when more than one rate of return from the same project will result in the net present value (NPV)being equal to zero. This situation arises when the IRR method is used for a project in which negative cash flows is followed by positive cash flows, and vice versa, repeatedly. This means that when there are cash flow periodically changing up (cash inflow) and down (cash outflow), then there is a likelihood of multiple IRRs.

The IRR does not assume, nor does it implicitly require the reinvestment of the intermediate cash flows of an investment, while the ERR does assume and explicitly require the reinvestment of the cash flows. ERR method directly accounts for the interest rate external to a project at which the net cash flows (inflows and outflows) generated or required by the project over its life lifetime can be reinvested or borrowed.

The IRR is used to estimate the profitability of potential investments. It is the discount rate that equates the net present value (NPV) of all cash flows from a particular project to zero. IRR is used essentially to plan for future growth and expansion and in computing the IRR, taking into consideration only the internal factors, rather than external factors (such as inflation and cost of capital). The higher the IRR for a project, the more desirable or economically viable that project or investment is considered to be. If the costs of investment are equal among the various projects, the project with the highest IRR would probably be considered the best among equals. A way to think about IRR is that it provides a projection of the potential growth of the enterprise.
The formula and calculation used to determine this figure follows.

If ERR=IRR, then the ERR will produce a result identical to the IRR

If ERR≥IRR, then the project is economically justified

The ERR is the ROR on a project where any excess cash from a project is assumed to earn interest at a pre-determined explicit rate, usually the Minimum Acceptable ROR (MARR).

4.8.1 Computation of ERR

If all net project receipts are taken forward at the MARR to the time of the last cash inflow and all net project disbursements are taken forward at an unknown interest rate to provide a $0 future worth, the unknown interest rate that enables that equality to take place is the approximate. ERR.

Stated differently, if ALL the project cash inflows are taken forward to the last inflow and summed to yield FV (at MARR), and ALL the project outflows are taken back to the origin and summed to yield the present value PV (at MARR), then the unknown interest rate that would make the sum of the Future values of the inflow equal to the sum of the present values of the out flow is ERR. The reverse of this is also true. In other words, the unknown interest rate that would make the sum of the present value of the cash outflows equal to the future value of the sum of the cash inflows is the ERR.

The formula for computing EER is given as:

$$\sum_{k=0}^{n} E_k(P/F, MARR\%, k)(F/P, ERR\%, n) = \sum_{k=0}^{n} R_k(P/F, MARR\%, n-k), \text{ OR}$$

$$\sum_{k=0}^{n} E_k(P/F, MARR\%, k) = \sum_{k=0}^{n} R_k(P/F, MARR\%, n-k)(P/F, ERR\%, n), \text{ where}$$

$R_k = Excess\ receipts\ over\ and\ above\ the\ expenses\ in\ period\ k$

$E_k = Excess\ expenditures\ over\ and\ above\ receipts\ in\ period\ k$

n= number of periods, MARR=Maximum Attractive Rate of Return, ERR=External Rate of Return

The essence of the formula is demonstrated in figure 2 and 3. The idea of the excess receipts or excess expenditures is especially important when the cash inflows (plus sign) and cash outflows (negative sign) are coincident, namely they occur at the same time period. When that is the case, the cash flows are added. If the result from the addition is positive, then the resultant cash flow at that time period is a cash inflow and it will have a positive sign. If the result of the addition is negative, then the resultant cash flow at that particular period is a cash outflow and will have a negative sign.

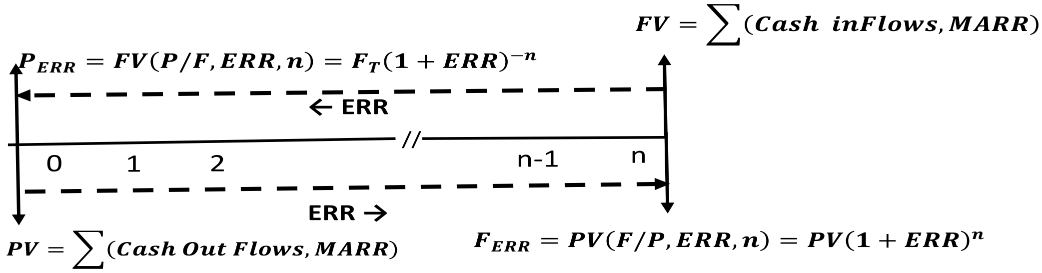

Figure 1: Estimating ERR Based on the Resultant Excess Cash Inflows & Excess Cash Outflows

Figure 2 represents the cash outflow and cash inflow for a certain project. The nature of the cash flows would lead us to believe that this is an un-conventional cash flow with a possibility of multiple IRR. As a result, we will deploy the ERR method

Table 7 was derived from the EXCEL solution to the problem. First, we determine the sum of out flows at MARR. This is the PV. Second, we determine the sum of inflows at MARR. This is the FV. Next, we determine the value of ERR that would make the cumulative sum of the FVs equal to the cumulative value of the PV. We made an initial guess of 0.02 or 2%. Use the *Gold Seek utility* in the data Section of EXCEL to search for the value of ERR that would give us the equality. As shown on Table 7, that value is 9.9%. Incidentally, we have also solved the problems using the definition and first principles (see figure 3)

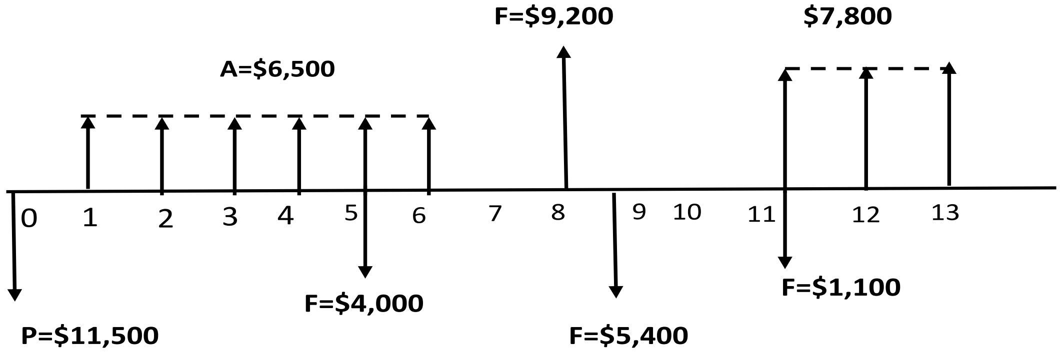

Figure 2: Cash Flow Diagram for External Rate of Return (ERR)

		Rate	0.035	3.50%		
Year	Years until Year 13	AMOUNT	NET OUTFLOW	NET INFLOW	PV of outflow at (i=MARR)	FV of inflow at (i=MARR)
0	13	-$11,500.00	$11,000.00	$0.00	$11,000.00	$0.00
1	12	$6,500.00	$0.00	$6,500.00	$0.00	$6,500.00
2	11	$6,500.00	$0.00	$6,500.00	$0.00	$6,500.00
3	10	$6,500.00	$0.00	$6,500.00	$0.00	$6,500.00
4	9	$6,500.00	$0.00	$6,500.00	$0.00	$6,500.00
5	8	$2,500.00	$0.00	$2,500.00	$0.00	$2,500.00
6	7	$6,500.00	$0.00	$6,500.00	$0.00	$6,500.00
7	6	$9,200.00	$0.00	$9,200.00	$0.00	$9,200.00
8	5	$0.00	$0.00	$0.00	$0.00	$0.00
9	4	-$5,400.00	$5,400.00	$0.00	$3,962.15	$0.00
10	3	$0.00	$0.00	$0.00	$0.00	$0.00
11	2	$6,700.00	$0.00	$6,700.00	$0.00	$6,700.00
12	1	$7,800.00	$0.00	$7,800.00	$0.00	$7,800.00
13	0	$7,800.00	$0.00	$7,800.00	$0.00	$7,800.00
				SUM	$14,962.15	$50,900.00
					ERR =	0.099
ERR	0.099					$14,962.15

Table 7: Computation of ERR Using the Data from the Cash Flow Diagram on Figure 1

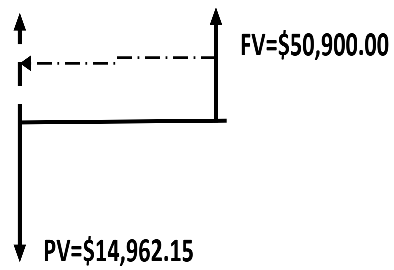

Figure 3: Equivalency Diagram to Determine ERR

$$PV = FV(P/F, ERR, 13) \Leftrightarrow FV = PV(F/P, ERR, 13)$$
$$PV = FV(P/F, ERR, 13) = FV\left(\frac{1}{(1 + ERR)^{13}}\right)$$
$$(1 + ERR)^{13} = \frac{FV}{PV} = \frac{\$50{,}900.00}{\$14{,}962.12} = 3.401924$$
$$(1 + ERR) = (3.401924)^{\frac{1}{13}} \Longrightarrow (1 + ERR) = 1.099$$
$$(1 + ERR) = 1.099 \Longrightarrow ERR = (1.099 - 1) = 0.099 = 9.9\%$$

4.9 Profitability Index (PI)

The Profitability Index (PI) is the ratio of payoffs to investment of a proposed project and represents the relationship between the costs and benefits of a proposed project. Also known as profit investment ratio or value investment ratio, it is an appraisal technique used in capital budgeting to evaluate the economic viability of potential capital outlays. It is the ratio of the present value of future expected cash flows to the initial amount invested in the project.

PI measures the monetary benefits (from cash inflows) received for each dollar invested (from cash outflow), with the resultant cash flows discounted back to the present. It compares the present value (PV) of future cash flows received from a project to the initial cash outflow (investment) required to fund the investment. PI is computed as follows

$$PI = \begin{bmatrix} Present\ value\ (PV) \\ of\ Future\ Cash\ Flows \end{bmatrix} \div [Initial\ Investment\ or\ Cash\ Outflows]$$

PI is a useful tool for ranking potential investments because it quantifies the value of such investment. Ideally, a PI index should be greater than or equal to unity. An index of 1.0 is the lowest acceptable index because any lower value is an indication that the present value (PV) of the investment is less than the initial investment. Values of PI higher than 1.0 is an indication that the future anticipated discounted cash inflows of the project are greater than the anticipated discounted cash outflows. So, as the value of PI increases, so does the economic viability or financial acceptability of the proposed investment.

Because PIs cannot be negative, the sign of the initial investment must be changed to positive before they are computed. Calculations resulting in PI values greater than 1.0 indicate the future anticipated discounted cash inflows of the project are greater than the anticipated discounted cash outflows. Values less than 1.0 indicate the deficit of the outflows is greater than the discounted inflows, and the project should not be accepted. Calculations that equal 1.0 bring about situations of indifference where any gains or losses from a project are minimal. A higher PI means that a project will be considered more attractive. In other words.

If PI >1, then the project is economically viable

If PI <1, project is not considered viable

If PI =1, project will breakeven

When capital is limited, and projects are mutually exclusive, the project with the highest profitability index is to be accepted which is an indication that the project makes the most productive use of limited capital. The profitability index is also called the benefit-cost ratio for this reason. Although some projects result in higher net present values, those projects may be passed over because they do not have the highest profitability index and do not represent the most beneficial use of company assets. Again, under capital constraints and mutually exclusive projects, only those with the highest PIs should be undertaken

4.9.1 Example

An investment portfolio has the following profile, with MARR=15%:

Initial investment =$35,000 (or -$35,000)

Inflow year 1=$10,000

Inflow year 2=$12,000

Inflow year 3=$15,000

Inflow year 4=$15,000

Inflow year 5=$8,000

Arranging the cash flows with a base of $10,000 per year for year 1-4, we have the following PV estimates:

Inflow years 1-4 = A= $10,000/year = $\$10{,}000(P/A, 15\%, 4) = \$28{,}549.78363$

Inflow for year 2 = $2,000 = (P/F, i, 2) = $\$2{,}000(P/F\,, 15\%, 2) = \$15{,}12.28733$

Inflow for year 3 = $5,000 = (P/F, i, 3) = $\$5{,}000(P/F\,, 15\%, 3) = \$3{,}287.58116$

Inflow for year 4 = $5,000 = (P/F, i, 4) = $\$5{,}000(P/F\,, 15\%, 4) = \$2{,}858.76612$

Inflow for year 5 = $8,000 = (P/F, i, 5) = $\$8{,}000(P/F\,, 15\%, 5) = \$3{,}977.41388$

Total PV of inflow = $\$40{,}185.83212$

$$PI = \$(40{,}185.83212 \div \$35{,}000) = 1.148 \Longrightarrow Project\ is\ economically\ viable$$

Table 7 shows a set of mutually exclusive portfolios with initial investment and several yearly inflows. We want to use the PI metric to assess which of the portfolios can be considered the most viable economically at MARR=10%

YEAR	A1	A2	A3	A4
0	-$12,000.00	-$10,000.00	-$12,000.00	-$15,000.00
1	$2,000.00	$2,500.00	$1,750.00	$1,900.00
2	$2,000.00	$2,500.00	$1,750.00	$1,900.00
3	$2,000.00	$2,500.00	$1,750.00	$1,900.00
4	$2,000.00	$2,500.00	$1,750.00	$1,900.00
5	$2,000.00	$2,500.00	$1,750.00	$1,900.00
salvage	$2,000.00	$1,000.00	$1,800.00	$3,000.00

Table 7: Mutually Exclusive Alternatives with Zero Salvage Value

The salvage value which occurs at end of life (in this case year 5) was added to the Year 5 cash inflow as shown in Table 7A, after which further analysis was performed using EXCEL to obtain the PV for each alternative

	YEAR	A1	A2	A3	A4
	0	-$12,000.00	-$10,000.00	-$12,000.00	-$15,000.00
	1	$2,000.00	$2,500.00	$1,750.00	$1,900.00
	2	$2,000.00	$2,500.00	$1,750.00	$1,900.00
	3	$2,000.00	$2,500.00	$1,750.00	$1,900.00
	4	$2,000.00	$2,500.00	$1,750.00	$1,900.00
Year 5 + Salvage	5	$4,000.00	$3,500.00	$3,550.00	$4,900.00

Table 7A. Mutually Exclusive Alternatives with the Salvage added to year 5

Table 7B shows the PV analysis and the values of the PI for each alternative. Project A2 is the choice due to it having the highest value of PI among all the alternatives with **PI=1.0098**

	PV(A1)	PV(A2)	PV(A3)	PV(A4)
	$1,818.18	$2,272.73	$1,086.61	$1,179.75
	$1,652.89	$2,066.12	$1,086.61	$1,179.75
	$1,502.63	$1,878.29	$1,086.61	$1,179.75
	$1,366.03	$1,707.53	$1,086.61	$1,179.75
	$2,483.69	$2,173.22	$2,204.27	$3,042.51
SUM(PV)=SPV	$8,823.42	$10,097.89	$6,550.72	$7,761.52
INVESTMENT (IINVEST)	-$12,000.00	-$10,000.00	-$12,000.00	-$15,000.00
PI=(SPV)/(-IINVEST)	0.7353	1.0098	0.5459	0.5174

Table 7B. Computation of PI for the Mutually Exclusive Alternatives

4.10 Payback Period (PB)

In non-discounted method of capital budgeting the cash flows do not incorporate the time value of money (TVM) and solely consider the current value of cash flows when it comes to making investment decisions. In other words, all dollars earned in the future are assumed to have the same value as today's dollars. Since, non-discounted cash flows do not consider the reduction in the value of money over time, it does not support realistic or accurate investment decisions because it tends to overstate the Net Present Value (NPV). An example of a non-discount method is the payback method that is often utilized because of the simplicity in its application and implementation. The payback method simply computes the number of years it would take for an investment to payback an amount equal to the amount invested. The resulting number of years is referred to as the payback period.

As an example, suppose an investor invests $200,000 today in a project with the expectation that it generates cash inflow of $40,000 for four years followed by $30,000 per year for two additional years, and $20,000 in years seven through eight.
($40,000 + $40,000+ $40,000+ $40,000 +$30,000+$10,000=$200,000).

Note: If we sequentially add the cashflows from year one on until we get to the initial investment of $200,000, we will get to year 5 with a total of $190K as follows:

$40K(y1)+$40k(y2)+$40K(y3)+$40K(y4)+$30K(y5)=$190K. If we add $30K(y6), we would have exceeded $200K so we must stop at year 5 and only add $10K to get to $200K. Since the cash inflow in year 6 is equal to $2500/month, then the number of months to accumulate $10,000 in year 6 is 4 months ($2500/month multiplied by 4=$10,000). Hence, the payback period is 5 years and 4 months

Assume that another investment of $100,000 generates cash inflow of $20,000 per year for two years and then generates cash inflows of $40,000 per year for six additional years. The payback period is approximately 3.5 years computed as follows: ($20,000 + $20,000 + $40,000 + half of $40,000).

The payback method answers only one question: How long before the cash invested is returned? The payback method does not address which investment is more profitable. Note from our examples that the payback method not only ignores the time value of money, but it also ignores all of the cash received after the payback period.

4.10.1 Discounted Payback Period (DPB)

The payback period for a discounted cash flow is given by the following.

$$Payback\ Period = \left[Years\ Before\ Breakeven + \frac{Unrecovered\ Amount}{Cash\ Flow\ in\ Recovering\ Year}\right]$$

Assume that we want an investment with an initial cost of $80,000 and annual cash inflow of $20,000/year for 8 years at MARR =10% and 15%.

For the simple (undiscounted) Payback Period (PB), $Payback\ Period = \frac{\$100{,}000}{\$20{,}000} = 5\ years$

Table 8, shows the EXCEL solution of the problem using the PV function

MARR	10%			15%	
Year	**Cash flow**	**PV of Cash Flow**	**Cumulative PV of Cash Flows**	**PV of Cash Flow**	**Cumulative PV of Cash Flows**
0	-$80,000.00	-$80,000.00	-$80,000.00	-$80,000.00	-$80,000.00
1	$20,000.00	$18,181.82	$18,181.82	$17,391.30	$17,391.30
2	$20,000.00	$16,528.93	$34,710.74	$15,122.87	$32,514.18
3	$20,000.00	$15,026.30	$49,737.04	$13,150.32	$45,664.50
4	$20,000.00	$13,660.27	$63,397.31	$11,435.06	$57,099.57
5	$20,000.00	$12,418.43	$75,815.74	$9,943.53	$67,043.10
6	$20,000.00	$11,289.48	$87,105.21	$8,646.55	$75,689.65
7	$20,000.00	$10,263.16	$97,368.38	$7,518.74	$83,208.39
8	$20,000.00	$9,330.15	$106,698.52	$6,538.04	$89,746.43

Table 8: Computation of Discounted Payback Period

$Payback\ Period = 5 + [(80{,}000 - 75.815.74) \div 11{,}289.48] = 5.37 yrs$, MARR =10%
$Payback\ Period = 6 + [(80{,}000 - 75{,}689.65) \div 7{,}518.74] \quad = 6.57\ yrs$, MARR =15%

For this example, the MARR (or discount rate) is 10% with a discounted payback period is of 5.4 years. With discount rate of 15%, the discounted payback period is 6.6 years. In either case the project should be funded because the payback is less than the life of the project. But the simple payback period is 5 years in both cases. This means that as the discount rate increases, the difference in payback periods of a discounted pay period and simple payback period increases.

The payback period approach is of interest in equipment replace strategies because of the significance of the payback period for equipment with specific cash flow profiles as part of the capital budgeting decision. Whether for alternatives or independent equipment, the payback period analysis is an important component of any equipment replacement decisions.

Example:

Due to increasing demand, a manufacturing company is considering replacing its existing machine with new fully automated CNC equipment. Three popular models (DMG-machine A, FANUC -machine B, and MAZAK-machine C) are available at the cost of $ 60k, $62k, and $65k for machines A, B, and C respectively. The salvage value of the existing machine is estimated at $90k. The auxiliary electrical/electronic infrastructure of the existing equipment can be upgraded to support Machine A at an additional cost of $110k. For machine B, the existing electrical, cabling, and other support structure can be used but would require an additional expense of $120k for renovations. In the case of machine C, a new support platform would be required at a cost of $140k due to its unique, cabling, electrical and electronic requirements. If the cabling, electrical and other support platform for the old machine is entirely scrapped, the salvage value is $30,000. The expected cash inflows for the three machines are as shown in table 9A. Assume a discount rate of MARR of 10%. Determine the Payback period for each machine type. The expected salvage for the new machines is 60k, 62k, 65k respectively. The salvage is added to end of life (year 5) cash flows in table 9 and 9B. The summary of the costs is as shown on table 9A.

Year	A	B	C
1	$120,000	$220,000	$300,000
2	$170,000	$240,000	$320,000
3	$195,000	$210,000	$330,000
4	$220,000	$200,000	$290,000
5	$200,000	$100,000	$200,000
Salvage	$60,000	$62,000	$65,000

Table 9. Cash Inflow for Machines A, B, C

.

Particulars	A	B	C
Cost of machine	$600,000	$620,000	$650,000
Cost of utilities	$110,000	$120,000	$140,000
Salvage of old machine	($90,000)	($90,000)	($90,000)
Salvage of old machine			($30,000)
Total Cost	$620,000	$650,000	$670,000

Table 9A: Resultant Total Cost of Machines after accounting for Salvage

	MARR	10%							
Year	Cash inflows (Mach A)	PV (Mach A)	Cum PV (Mach A)	Cash inflows (Mach B)	PV (Mach B)	Cum. PV (Mach B)	Cash inflows (Mach C)	PV (Mach C)	Cum. PV (Mach C)
0	$620,000	$620,000	$620,000	$650,000	$650,000	$650,000	$670,000	$670,000	$670,000
1	$120,000	$109,091	$109,091	$220,000	$200,000	$200,000	$300,000	$272,727	$272,727
2	$170,000	$140,496	$249,587	$240,000	$198,347	$398,347	$320,000	$264,463	$537,190
3	$195,000	$146,506	$396,093	$210,000	$157,776	$556,123	$330,000	$247,934	$785,124
4	$220,000	$150,263	$546,356	$200,000	$136,603	$692,726	$290,000	$198,074	$983,198
5	$260,000	$161,440	$707,796	$162,000	$100,589	$793,315	$265,000	$164,544	$1,147,742

Table 9B: Discounted Payback Period Computation for Machine Replacement/Selection

$$Payback\ Period\ (Mach\ A) = 4 + [(620{,}000 - 546{,}356) \div 161{,}440] = 4.46 yrs$$
$$Payback\ Period (Mach\ B) = 3 + [(650{,}000 - 556{,}123) \div 136{,}603] = 3.69\ yrs$$
$$Payback\ Period\ (Mach\ C) = 2 + [(670{,}000 - 537{,}190) \div 247{,}934] = 2.54 yrs$$

From Table 9B, we observe after some minor calculations that Machine C has the fastest payback period of 2.54 years and is thus recommended, everything else being equal.

4.11 Benefit-Cost Analysis (BCA) for greater than two alternatives

The nature of the cash flows determines the type of the analysis as shown in table 10.

In terms of benefits, there two types of benefits that require different incremental B/C analysis. The first type of benefits is the usage cost estimates. This is the benefit derived from usage cost. It is defined as the implied benefit accrued as a result of the differences in the cost estimates of the alternatives. In this case the comparisons will be only between the alternatives, no DN. The second type of benefits is the direct benefit estimate where the benefits are estimated for each alternative. In this case the alternatives compared with DN first.

Types of Proposals or Alternatives	Evaluation Basis/criteria
Independent	**Evaluate only against Do-Nothing (DN)**
Revenue Generating and Implied Benefit (such as in public works, e.g, i). dollar worth of the number of lives saved because of new instrumentation ii). travel time saved in dollars because of a new bridge) Mutually Exclusive Projects	**Against DN first, then against each other in ascending First Cost order**
Usage cost Estimate. This is the implied benefits based on differences in cost estimates of alternatives	**Only Against each other**
Direct Benefit Estimate This is where benefits are estimated or stated for each alternative	**Comparison of alternatives but Comparison with DN first**

Table 10: Types of Alternatives and Evaluation Basis for B/C Analysis

4.11.1 The Procedure for Incremental B/C Analysis

1. Find equivalent values for costs, benefits (and disbenefits D if estimated), that is, PW, AW, FW.
2. Order the alternatives by increasing total equivalent cost (for direct benefit alternatives, add DN first)
3. For each pair of alternatives, say A2, A1, determine incremental B (benefit) and incremental C(cost), that is
   $\Delta B = (Benefit_{A2} - Benefit_{A1},)\ also, (\Delta C = Cost_{A2} - Cost_{A2})$
4. Determine: $\Delta B/\Delta C \quad OR\ \Delta(B - D)/\Delta C$
5. If $\Delta B/\Delta C\ > 1.0, eliminate\ A1,\ and\ A2\ is\ the\ survivor$, Else, A1 is survivor
6. Compare survivor with next alternative, continue steps (3)-(5) until only one alternative is let. That alternative becomes the choice or the desirable investment alternative out of all the alternatives.

4.11.2 Example of incremental B/C (Benefit Cost Ratio) Analysis

Table 10A represents a bid for the construction of a roadway with expected life of **35 years** Indicated are cost of construction and annual maintenance. Also indicated are expected annual benefits that are to be derived from the roadway expressed in dollars, hence this is a direct benefit investment. As a result, we will use incremental B/C Analysis (Benefit to Cost Ratio, where B=Benefit, and C=Cost) with DN as the first alternative. Assume MARR is 5%

			Proposed Annual Benefits		
Contractor #	Construction Cost	Annual Routine Maintenance	Annual Travel Time Reduction	Annual Operating Costs Reduction	Annual Savings Due to Reduced Accidents
A	$1,650,000	$42,000	$60,000	$30,000	$80,000
B	$1,700,000	$46,000	$70,000	$30,000	$80,000
C	$1,950,000	$40,000	$81,000	$36,000	$80,000
D	$1,855,000	$51,000	$80,000	$35,000	$85,000

Table 10A: Benefit-Cost Analysis for Road Improvement

1. Find equivalent values for Costs, Benefits (and Disbenefits D if estimated) in terms of PW (Table 10B)

		(P/A, 0.05,35)=	16.3742	
Contractor	Benefit/ COST	PV	AW	PW
A	B		$170,000.00	$2,783,614.00
	C	$1,650,000.00	$42,000.00	$2,337,716.40
B	B		$180,000.00	$2,947,356.00
	C	$1,700,000.00	$46,000.00	$2,453,213.20
C	B		$197,000.00	$3,225,717.40
	C	$1,955,000.00	$40,000.00	$2,609,968.00
D	B		$200,000.00	$3,274,840.00
	C	$1,855,000.00	$51,000.00	$2,690,084.20

Table 10B: Benefit Cost Ratio (BCR) Computation for the Road Improvement Data

2. Order the alternatives by increasing total equivalent cost (Table 10C)

	A	B	C	D
Equivalent Benefits	$2,783,614.00	$2,947,356.00	$3,225,717.40	$3,274,840.00
Equivalent costs	$2,337,716.40	$2,453,213.00	$2,636,839.00	$2,690,084.20

Table 10C: Equivalent Values for Benefits and Cost in with Costs in Ascending Order

3. For each pair find
$$\Delta B_{2-1} = (Benefit_2 - Benefit_1)\ also, (\Delta C_{2-1} = Cost_2 - Cost_1)$$
4. Compute $\Delta B/\Delta C$

For: A v B: $\Delta B/\Delta C \quad = (2947356 - 2783614) \div (2453213 - 2337716) = 1.42$

Since: $\Delta B/\Delta C > 1, B\ is\ the\ survivor$
For B v C: $\Delta B/\Delta C \quad = (3225717.4 - 2947356)\ \div (2636839 - 2453213) = 1.52$
Since: $\Delta B/\Delta C > 1, C\ is\ the\ survivor$
For C v D: $\Delta B/\Delta C \quad = (3274840 - 3225717.4)\ \div (2690084.2 - 2636839) = 0.922$
Since $\Delta B/\Delta C < 1, reject\ D$
Hence C is the survivor and becomes the choice

Replacement Analysis

Replacement analysis is one of the crucial elements of capital budgeting. An asset's life may be reduced or may become obsolete as a result of physical impairment, changes in economic requirements and rapid changes in technology. The replacement of assets provides an opportunity for an organization to take corrective economic to prevent interruptions in productive activities. In replacement analysis there is two alternatives:

i. The assets that are currently in use: The defender
ii. The assets that are to be acquired to replace current assets: The challenger

Some important considerations that are pertinent to replacement strategies include:

i. Sunk costs do not play any role
ii. Values of existing assets are not part of the equation.
iii. The optimal replacement cycle is one with the lowest equivalent annual cost (EAC)
iv. Changing technologies can render an equipment obsolete, thus shortening the replacement cycles. This means that one asset is not being replaced by one exactly similar
v. It is important to understand how often to replace productive assets. Replacing in long intervals delays incurring the cost a new equipment. However, of course means keeping an asset whose value is declining but which costs more to maintain.

5.1 The Equivalent Annual Cost and Equipment Replacement

The Formula for the Equivalent Annual Cost is given by: $EAC = Asset\ Price * Annuity\ Factor$

Where Annuity Factor $= \left[\frac{1+\frac{1}{(1+i)^n}}{i}\right]$

$$also: [A/P, i\%, n] = \left[\frac{i}{1 + \frac{1}{(1+i)^n}}\right]$$

Consider two alternative investments on two competing equipment:
Equipment E1:
The initial capital outlay $I = \$400{,}000$

Expected life of 5 years
The annual operating and maintenance cost (OM): $O\&M = \$15{,}000$

Equipment E2:
An initial capital outlay $I = \$525{,}000$
Expected life of 8 years
The annual operating and maintenance cost (OM): $O\&M = \$20{,}000$

The cost of capital for the company or the MARR = 10%.

$EAC = I(A/P, i, n) + OM$

For E1

$$EAC_{E1} = I_{E1}(A/P, 10\%, 5) + OM_{E1}$$

$$(A/P, 10\%, 5) = \left[\frac{i}{1 + \frac{1}{(1+i)^n}}\right] = \left[\frac{0.1}{1 + \frac{1}{(1.1)^5}}\right] = 0.263797$$

$$EAC_{E1} = \$400{,}000(0.263797) + \$15{,}000 = \$120{,}518.993$$

For E2

$$EAC_{E2} = I_{E2}(A/P, 10\%, 8) + OM_{E2}$$

$$(A/P, 10\%, 5) = \left[\frac{0.1}{1 + \frac{1}{(1.1)^8}}\right] = 0.187444$$

$$EAC_{E2} = \$525{,}000(0.187444) + \$20{,}000 = \$118{,}408.109$$

The use of equivalent annual cost as a common metric for the capital budgeting decision makes easy to compare the performance of the equipment. In this case, if cost is the only consideration, then E2 would be selected has an EAC that is about \$2172 lower than Machine E1.

5.2 Economic Service Life

Economic service life of a system or project/equipment is the remaining useful life of an asset that results in the minimum annual equivalent cost. The annual equivalent cost is defined as:

Equivalent Annual Cost (EAC) = Capital Cost + Operating Cost

The mathematical relationship among the different components of the EAC is as follows, Please note that the use of EAC obviates the need to worry about equipment life since the costs are expressed in annual terms.

<u>Capital Recovery:</u> Capital recovery (CR) is the cost difference between the initial investment (I) and equivalent cash inflow stream including the salvage (S)

$CR(i) = I(A/P, i, n) - S(A/F, i, n)$

$$But\ (A/F) = \left[\frac{i}{(1+i)^n - 1}\right]$$
$$and\ (A/P) = \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$
$$(A/F) + i = \left[\frac{i}{(1+i)^n - 1} + i\right] = \left[\frac{i + i\,[(i+i)^n - 1]}{(1+i)^n - 1}\right] = \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = (A/P)$$
$$= S[(A/F\,,i,n) + i] = [S(A/F\,,i,n) + iS] = S(A/P\,,i,n)$$
$$\Rightarrow S(A/F\,,i,n) = S(A/P\,,i,n) - iS$$
$$CR = I(A/P,i,n) - [S(A/P\,,i,n) - iS] = [I(A/P,i,n) - S(A/P\,,i,n) + iS]$$
$$CR = (I - S)\,(A/P,i,n) + iS$$

Operating Cost (OC): The operating Cost is the combined operating and maintenance cost (O&M) for running the system expressed in annual equivalent cost (OC)

$$OC(i) = \left(\sum_{n=1}^{T} O\&M(P/F,i,n)\right)(A/F,i,T)$$

Hence, $EAC(i) = CR(i) + OC(i)$

The value (n) that minimizes the EAC(i) is the economic or useful life.

5.3 Example: Service Life for a Heavy-Duty Road Grader

Hillsborough county Florida is in negotiations for a heavy-duty road grader for use in southern part of the county. The county ordinance states that the service or economic life of any heavy-duty equipment be specified during negotiations and agreed upon before actual purchase. The equipment in question has the following parameters

Initial investment cost I=P_0 = \$50,000, cost of capital = 10%, Salvage value decreases by -25% (25% deterioration) over previous year, O&M = \$5,000 during the first year, and increases by 20% over the previous year and going forward

For n=1

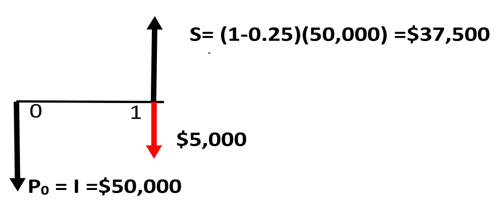

Figure 4: Cash flow for the Initial cost, Salvage & Operating Cost (n=1)

$$CR = (I - S)\,(A/P,i,n) + iS = \$(50000 - 37500)(A/P,10\%,1) + \$0.1(37{,}50) = \$17{,}500$$
$$OC = \big(\$5{,}000(P/F)\big)(A/P,10\%,1) = \$5000(0.90909)(1.1) = \$5{,}000$$

$EAC = \$17{,}500 + \$5{,}000 = \$22{,}500$

For n=2

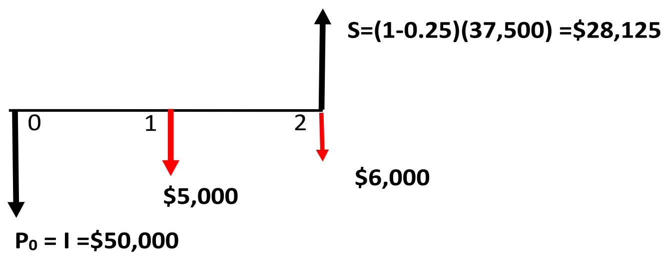

Figure 5: Cash flow for the Initial cost, Salvage & Operating Cost (n=2)

$CR = (I - S)\ (A/P, 10\%, 2) + iS = \$(21{,}875)(0.5761904) + \$0.1(28{,}125) = \$15{,}416.67$
$OC = [\$5{,}000(P/F, 10{,}1) + \$6{,}000(P/F, 10{,}2)](A/P, 10\%, 2) =$ \$5,476.19
$EAC = \$15{,}416.67 + \$5{,}476.19 = \$20{,}892.86$

For n=3
S(n=3) = \$(1-0.25)28,215=\$21,093.75, O&M= \$(1.20)*5000=\$7,200
$CR = (I - S)\ (A/P, 10\%, 3) + iS = \$(28{,}906.25)(0.402114) + \$0.1(21{,}093.75)$
$CR = \$13{,}733.01$
$OC = [\$5{,}000(P/F, 10{,}1) + \$6{,}000(P/F, 10{,}2) + \$7{,}200(P/F, 10{,}3)](A/P, 10\%, 3)$
$OC = \$5{,}996.98$
$EAC = \$13{,}733.01 + \$5{,}996.98 = \$19{,}729.99$

We continue this approach for n=1, to 14 as shown in Table 11. From the table, we observe that the minimum value of EAC (n*) occurs between (n=8 and n=9). The optimum service life is 8+ years. Figure 6 is a plot of EAC(i) versus n.

n	Market Value (MV)	O&M COST	CR(10%)	OC(10%)	EAC(10%)	
0	$50,000.00					
1	$37,500.00	$5,000.00	$17,500.00	$5,000.00	$22,500.00	
2	$28,125.00	$6,000.00	$15,416.67	$5,476.19	$20,892.86	
3	$21,093.75	$7,200.00	$13,733.01	$5,996.98	$19,729.98	
4	$15,820.31	$8,640.00	$12,364.72	$6,566.47	$18,931.20	
5	$11,865.23	$10,368.00	$11,246.38	$7,189.15	$18,435.53	
6	$8,898.93	$12,441.60	$10,327.00	$7,869.91	$18,196.91	
7	$6,674.19	$14,929.92	$9,566.78	$8,614.07	$18,180.85	
8	$5,005.65	$17,915.90	$8,934.49	$9,427.46	**$18,361.95**	n=8
9	$3,754.23	$21,499.08	$8,405.56	$10,316.42	$18,721.99	
10	$2,815.68	$25,798.90	$7,960.60	$11,287.88	$19,248.48	
11	$2,111.76	$30,958.68	$7,584.20	$12,349.38	$19,933.58	
12	$1,583.82	$37,150.42	$7,264.10	$13,509.15	$20,773.26	
13	$1,187.86	$44,580.50	$6,990.49	$14,776.20	$21,766.69	
14	$890.90	$53,496.60	$6,755.46	$16,160.31	$22,915.77	

Table 11: Computation of EAC(10%)

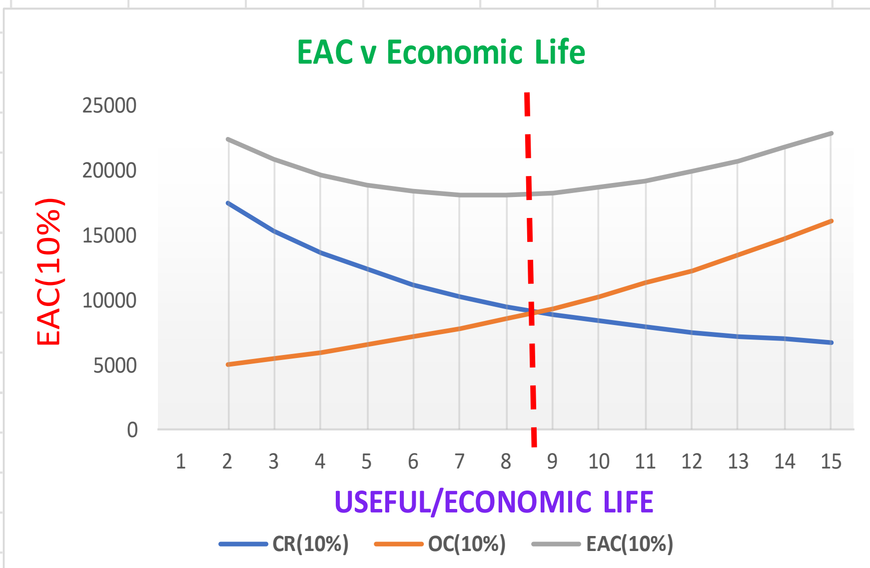

Figure 6. Plot of EAC(10%) v Economic Life

The Nature of Decision Making

Decision making is the idea of exploring the choice space of a problem situation for the purpose of determining which among the choice options is(are) the most appropriate. This requires a systematic or step-by-step framework that is built around the three pillars of problem identification, information gathering for the purpose of explicating the data nuances, and the assessment of the possible alternatives and solutions in the choice spectrum. The use of a systematic or step-by-step decision-making framework often results in consistent and deliberate decisions. There is some empirical evidence to suggest that a systematic approach to decision making, even when highly simplified, is still much better than no method whatsoever is brought to bear.

Depending on the available information, generally, there are three conditions that are associated with any decisions, namely, uncertainty, risk and certainty.

6.1 Uncertainty

This is the situation where the decision maker or analyst has absolutely no knowledge or idea about the likelihood of the occurrence of the event in question. In this scenario, the behavior of the decision maker/analyst is strictly based on their predisposition towards the unknown.

When in a decision-making situation the decision maker or the analyst is not sure whether to take a certain decision or its alternative then there exists a case of uncertainty. As an example, the decision to fly or drive represents uncertainty. The existence of inflation creates uncertainty with respect to the price of goods and services.

In general, the causes of uncertainty would include the following

i. Too much information or knowledge
ii. The subjectivity of information or opinions based on subjective rather objective interpretation
iii. The non-existence of information or knowledge
iv. Conflicting information

Recognizing that uncertainty cannot be wished away, there are two practical methods to deal with it from the point of view of decision making. Depending on the circumstances, one may decide to cope with it through some well-known coping mechanisms or one may decide to reduce its overall effect.

6.1.1 Coping with Uncertainty

Coping with uncertainty exposes the decision maker to a wide spectrum of uncertainties. Any effort to cope with uncertainties would require certain actions designed to limit exposure to uncertainty

There are several coping mechanisms associated with uncertainty

- adaptation
- diversification

- imitation
- flexibility
- collaboration
- partnership and possible integration with other organizations
- avoiding uncertainty altogether

6.1.2 Reducing Uncertainty

Reduction in uncertainty ultimately results in minimizing exposure to uncertainty. This requires a change in both actions and strategies. Some of the ways to reduce rather than cope with uncertainties include the following

a). Gathering relevant information to ensure consistency and completeness
b). Proactive and sustained engagement and collaboration with stakeholders
c). Ensure information alignment and robustness by strengthening professional connections through networking

6.2 Risk

If the decision maker is making decisions under the presumption of and awareness of the implication of possible risks, then this implies that the decision maker has knowledge, even if subjectively, of the probability space and can reasonably assign those probabilities to each event in the decision space.

6.3 Certainty

In certain situations, it is reasonable and even practical to assume that there is complete and unvarnished information and as such there is no form of uncertainty associated with the decision or the analysis through which the decision was made. Conditions of certainty are at the beginning of the certainty-uncertainty spectrum while uncertainty are at the other end of the spectrum. It is a general expectation that the bottle of water from the refrigerator be cold. In this scenario, for example, the decision to drink a bottle of water from the refrigerator is done with certainty because of the expectation that the refrigerator houses cold bottles of water. For most people the decision to order a pack of McDonald French fries is one in which the expectation is that indeed the quantity and quality of the fries is the same or as expected. For these foregoing types of decisions, it is expected that one can make such decisions with certainty. Thus, in certainty-based decisions, the outcome usually has 100% probability of occurrence. It is important not to lose sight of the fact that the certainty in this case is as a result of the simplification of certain realities by suppressing certain uncertainties. For example, whether the pack of French fries would be full or three-quarters full is not a consideration because the assumption is that the package would have the desired number of fries.

6.4 Decision making under Certainty

A condition of certainty is said to exist when there is reasonable certainty about the alternatives, the conditions associated with the alternative have been well articulated, and the outcome of the alternatives are well known. Decision under certainty is typically one in which accurate, objective/measurable, and reliable types of information are available to the decision maker. In this situation the future outcome is highly and reliably predictable. The more credible and accurate the information, the more certain the decision. An example is the routine, seat of the pants everyday type of personal or business operation decisions.

6.5 Decision making under Uncertainty

Uncertainty is when the future and/or the outcomes are not predictable. Everything is unpredictable. The alternatives are unknown and are not transparent. In the case of uncertainty, the alternatives, the risks associated with each alternative, the attendant consequences of each alternative and the likelihood of occurrence are unclear or in a state of flux. Information about the outcomes of decisions under uncertainty is usually lacking and does not exist because there are just too many unknowns. A way to improve decision making under uncertainty is to utilize some techniques that would further enhance the ability to explicate the uncertainty. Two particular methods are used, namely risk analyses and decision trees.

6.6 Decision Making Under Risk

When decisions are made with incomplete or insufficient information, where only the certainty is uncertainty, then the decision under such a condition is considered a decision under risk. In the current world of enormous amounts of data being processed at increasingly rapid rates, identification, management, and risk mitigation are a major challenge to any organization. Risk is the effect of uncertainty on an objective and can include anything that generates uncertainty related to an organization's objectives or causes a deviation from the expected norm. In undertaking a decision under risk, there is usually an understanding that there exists some knowledge of the likelihood of the outcome in the form of probabilities. The use of probabilities to represent the likelihood of occurrences is a substitute or surrogate for the measure of certainty or complete knowledge that is not present.

The ISO (The International Standards Organization) and the IEC (International Electrotechnical Commission) provide guidance on the selection and application of techniques for assessing risk in a wide range of situations. The techniques are used to assist in making decisions where there is uncertainty, to provide information about particular risks and as part of a process for managing risk. In those situations where the only certainty is uncertainty, the IEC and ISO 'risk management toolbox' have been designed to assist organizations to help forestall threats that could undermine their success.

The ISO 31000 standard provides directions on how a company can implement risk-based decision making into its operations. Using ISO 31000 can help organizations increase the possibility of achieving objectives with respect to its decision making in order to enhance corporate performance, improve the identification of potential threats, and hence proactively and effectively allocate resources for the purposes of risk mitigation.

6.7 Uncertainty and Risk`

Decision making under risk involves an attempt to predict the possibility of a future occurrence. For uncertainty however, such a prediction is not possible or realistic. Risks can be managed by assigning probabilities to the different possible outcomes based on their likelihood of occurrence but uncertainty by its very nature implies uncontrollability which means that such probability assignment is not feasible. As a result, risk is seen to be present when future events occur with a given likelihood or numerical probability whereas for uncertainty the probability of a future outcome is both indefinite and incalculable. As we may recall from the ISO 3100 definition of risk, it is the effect of uncertainty on an objective. Risk on its own is not uncertainty. It is, however, the potential impact of uncertainty on certain outcomes. This impact can be represented in the form of probabilities. Thus, the information gap between what is known (Certainty) and what is not known (uncertainty) can be accomplished by employing probabilities in order to ameliorate the negative effect of uncertainty.

Framing a problem as a risk or uncertainty can result in a significant difference to the conclusion derived from the analysis of the problem and ultimately affects the decision making going forward. Based on the ISO definition of risk, the objectives are key to proper problem identification as well as the evaluation of alternative solutions.

6.7.1 Decision Making Under Risk

There are two possible extremes in decision-making along the certainty-uncertainty spectrum. At one extreme is the certainty domain while the other extreme is the uncertainty domain. The amount and quality of information is key to a good decision. Thus, a decision can be adjudged as good based on the outcome alone when there is certainty. The opposite end of the spectrum is uncertainty and with less knowledge and information. Between these opposite extremes is risk. Obviously, the closer decision is to the uncertainty extreme part of the spectrum, the riskier the decision because of reduced or lack of knowledge.

In certain instances, risk is looked upon as a negative concept or idea that should be avoided or transferred to others. However, risk is indeed a fact of life that cannot simply be avoided or wished away but can be managed by the use of proper tools such as knowledge acquisition and management as well as probability encoding.

Managing risk requires taking the necessary steps to reduce threats or uncertainties by utilizing proper tools. It is the process of identifying risks and taking actions necessary to manage the risks, assessing the severity of the risk, prioritizing the risk based on the importance or the effect on the ensuing decisions. Ultimately, the goal is to enhance the value of the decision through the management of the uncertainties that detract from stated objectives.

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