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Engineering Methods in *Microsoft Excel*

Part 2: Applied Optimization

by

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Abstract

This course is part of a series on engineering methods in *Microsoft Excel* tailored to practicing engineers. This course series presents *Microsoft Excel* tools that can be used for a wide range of engineering analyses and data management. This course covers an introduction to applied optimization problems. This course presents a review of the fundamental principles followed by a real-life example encountered by a practicing engineer. The real-life examples are then formulated and implemented in *Microsoft Excel* and worked using the various *Excel* tools, spreadsheet techniques, and built-in functions. Examples from various engineering fields are used to demonstrate the concepts and methods learned throughout this course. Upon completion of this course, practitioners will be able to apply the methods learned to a variety of engineering problems, and also to identify situations in their fields of specialty where the innovative application of these tools and methods will be advantageous to their output and to their work product.



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1. INTRODUCTION

1.1 Overview

Engineers are constantly challenged with solving a wide range of complex analytical and computational problems in their fields of specialty. These problems involve analysis methodologies and the management of data. The application of computers enables repetitive, time-consuming and often tedious calculations to be conducted rapidly, efficiently, and less prone to errors. The application of computer tools also enables the results and outputs of such engineering analyses to be readily transferred and incorporated into reports and other engineering documents. An even greater advantage, in terms of productivity and efficiency, is realized when these calculations and outputs are replicated across numerous projects. Competence in computer skills predisposes engineers to pursue and develop more creative and innovative solutions to problems than otherwise.

Microsoft Excel is an electronic spreadsheet program developed by the *Microsoft* company, and part of the *Microsoft Office* suite of products. A spreadsheet is a grid that organizes data and calculations into columns and rows. The intersection of a column and a row is called a cell. An electronic spreadsheet enables users to store, organize, manipulate, and analyze data in the cells of the spreadsheet. As of this publication, the software is into the 2016 release.

Microsoft Excel is widely and increasingly being used as a tool to assist engineers in conducting and replicating intricate calculations and analysis, designing complex systems, and managing large data sets. This course presents a selection of engineering analysis topics and how to formulate, implement and solve them in *Microsoft Excel*.

This course presents fundamental principles and engineering applications of problems in applied optimization, and demonstrates the *Excel* tools, methods, and strategies that can be used to formulate and solve them.

Upon completion of this course, participants will have gained insight into applying *Excel* tools, methods, and strategies in analyzing, formulating and implementing a variety of engineering related problems, as well as managing large sets of complex data. Participants will also be able to identify professional situations where the application of innovative *Excel* techniques will be of great benefit and advantage, and will enable practitioners to significantly improve their productivity, efficiency, and the quality of their work product.



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2. APPLIED OPTIMIZATION

2.1 Introduction

Engineers and scientists often encounter problems where decisions must be made, such as selecting the “best” or optimal setting of a manufacturing device to obtain the best product, selecting the dimensions of a product or system during the design phase that meets customer expectations, selecting the amount of resources needed to perform a task given the various constraints such as availability of labor and equipment, as well as budgetary constraints etc., etc.

In decision problems of this nature, the overall objective may be expressed as some function (called the **objective function**) of the input variables (or **decision variables**), and the analyst seeks to find values of the decision variables that result in the **optimal** value of the objective function. The optimal value of the objective function is generally based on minimization or maximization of the objective function based on the nature of the problem. For example in optimizing the setting of a manufacturing device the objective of the industrial engineer may be to minimize the amount of defective product, or in the selection the amount of resources to perform a task, the objective may be to maximize productivity and doing so within the constraints imposed by the availability of labor and equipment, as well as the budget.

Optimization (also called **mathematical programming**) involves formulating decision problems and finding decision variable values that optimize (maximize or minimize) the objective function of the decision variables subject to the applicable constraints on the decision variables’ values.

Optimization problems are a core element of the broader field of Operations Research which is concerned with the formulation, analysis, and solution of large and complex mathematical problems found in engineering, the sciences, management, and business.

2.2 Formulation of Optimization Models

Let us use a practical example to illustrate the formulation of an optimization model (or mathematical program) and the general structure used to represent an optimization problem, as well as other fundamental concepts.

Consider a cocoa processing plant in the southern United States that processes raw cocoa beans sourced from three global suppliers to produce four products namely cocoa liquor, cocoa butter,



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low-fat cocoa powder, and fat-free cocoa powder. Cocoa liquor is a solid/ semi-solid substrate used in the manufacture of chocolate. Cocoa butter is an ingredient in chocolate and the main ingredient of white chocolate. Cocoa butter is also used in skincare products and pharmaceutical products. Cocoa powders are used in beverages, baking products, and dyes.

The cocoa processing company obtains the raw material-cocoa beans, from three sources, namely Brazil, Ghana, and Mexico. 1 ton of beans from Brazil yields 0.30 tons of cocoa liquor, 0.30 tons of cocoa butter, 0.35 tons of low-fat cocoa powder, and 0.05 tons of fat-free cocoa powder. 1 ton of beans from Ghana yields 0.20 tons of cocoa liquor, 0.43 tons of cocoa butter, 0.3 tons of low-fat cocoa powder, and 0.07 tons of fat-free cocoa powder. 1 ton of beans from Mexico yields 0.15 tons of cocoa liquor, 0.50 tons of cocoa butter, 0.25 tons of low-fat cocoa powder, and 0.10 tons of fat-free cocoa powder.

The cost per ton of the cocoa beans varies by country due to shipping charges, and other freight related taxes and fees. A ton of raw cocoa beans from Brazil costs \$3,200, whereas a ton of raw cocoa beans from Ghana costs \$3,600. A ton of raw cocoa beans from Mexico costs \$3,000,

Due to factors in the source countries, Brazil can supply up to 20,000 tons per month of raw cocoa beans, Ghana can supply up to 10,000 tons per month, and Mexico can supply up to 15,000 tons per month. In order to maintain a diverse supply base of the raw material it is the company's policy that each source will supply at least 25% of the total monthly supply of the raw material.

The cocoa processing plant has supply contracts with confectionaries, pharmaceutical companies and chemical plants around the world. In order to meet its contractual obligations, the cocoa processing plant must produce and deliver 6,000 tons per month of cocoa liquor, 12,000 tons per month of cocoa butter, 9,000 tons per month of low-fat cocoa powder, and 2,500 tons per month of fat-free cocoa powder.

Like any other company, the cocoa processing facility shall purchase the raw material in such a manner as to minimize its costs. The question is how many tons per month of cocoa beans should the cocoa processing company purchase from each source?

Solution: The overall objective of this operation is to minimize costs resulting from the cost of purchasing the raw material from the three source countries.



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Let x_1 denote the tons per month purchased from Brazil,
Let x_2 denote the tons per month purchased from Ghana, and
Let x_3 denote the tons per month purchased from Mexico.

So x_1, x_2, x_3 are the decision variables.

The objective function can therefore be expressed as

$$\text{minimize } 3200x_1 + 3600x_2 + 3000x_3$$

Achieving this objective is subject to several constraints which we shall group as follows.

Constituents: In order to meet the contractual obligation of supplying at least 6,000 tons of cocoa liquor the processing company shall obtain some amount of cocoa liquor from every ton purchased from each source. This can be expressed as

$$0.3x_1 + 0.2x_2 + 0.15x_3 \geq 6000 \quad \text{[cocoa liquor]}$$

Likewise for the other constituents we obtain

$$0.3x_1 + 0.43x_2 + 0.5x_3 \geq 12000 \quad \text{[cocoa butter]}$$

$$0.35x_1 + 0.3x_2 + 0.25x_3 \geq 9000 \quad \text{[low-fat cocoa powder]}$$

$$0.05x_1 + 0.07x_2 + 0.1x_3 \geq 2500 \quad \text{[fat-free cocoa powder]}$$

Supply: The constraints due to how much each source country can supply per month can be expressed as follow,

$$x_1 \leq 20000 \quad \text{[Brazil supply]}$$



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$$x_2 \leq 10000 \quad \text{[Ghana supply]}$$

$$x_3 \leq 15000 \quad \text{[Mexico supply]}$$

Diversification: The company has decided to maintain a diverse supply of raw material by purchasing a minimum of 25% of its total monthly supply from each source.

Thus, for Brazil,

$$x_1 \geq 0.25 (x_1 + x_2 + x_3)$$

Or

$$0.75x_1 - 0.25x_2 - 0.25x_3 \geq 0 \quad \text{[diversification 1]}$$

Similarly, for the other sources,

$$-0.25x_1 + 0.75x_2 - 0.25x_3 \geq 0 \quad \text{[diversification 2]}$$

$$-0.25x_1 - 0.25x_2 + 0.75x_3 \geq 0 \quad \text{[diversification 3]}$$

Variables: We realize that it may be mathematically possible that we obtain results for the decision variables that are negative values. Though this may be mathematically correct it is practically meaningless in this specific example, as one cannot purchase a negative quantity of raw cocoa beans from say Brazil. So we shall impose constraints that specify the domain over which our decision variables have practical meaning for the specific problem. In this example we shall limit the decision variables to values greater than or equal to zero.

$$x_1 \geq 0 \quad \text{[variable 1]}$$

$$x_2 \geq 0 \quad \text{[variable 2]}$$



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$$x_3 \geq 0 \quad \text{[variable 3]}$$

The constraints that govern the interactions of variables or restrictions on variables, in this case the Constituents, Supply and Diversification, are called the **main constraints**, whereas the constraints that specify the domains of the decision variables, in this example Variables, are called the **variable-type constraints**.

At this juncture we have deciphered all the information in the problem statement. We can now present our optimization model in the **standard form** for optimization models as follows,

Objective function:

$$\text{minimize} \quad 3200x_1 + 3600x_2 + 3000x_3$$

Subject to:

$$\begin{aligned} 0.3x_1 + 0.2x_2 + 0.15x_3 &\geq 6000 && \text{[cocoa liquor]} \\ 0.33x_1 + 0.43x_2 + 0.5x_3 &\geq 12000 && \text{[cocoa butter]} \\ 0.3x_1 + 0.3x_2 + 0.25x_3 &\geq 9000 && \text{[low-fat cocoa powder]} \\ 0.05x_1 + 0.07x_2 + 0.1x_3 &\geq 2500 && \text{[fat-free cocoa powder]} \\ x_1 &\leq 20000 && \text{[Brazil supply]} \\ x_2 &\leq 10000 && \text{[Ghana supply]} \\ x_3 &\leq 15000 && \text{[Mexico supply]} \\ 0.75x_1 - 0.25x_2 - 0.25x_3 &\geq 0 && \text{[diversification 1]} \\ -0.25x_1 + 0.75x_2 - 0.25x_3 &\geq 0 && \text{[diversification 2]} \\ -0.25x_1 - 0.25x_2 + 0.75x_3 &\geq 0 && \text{[diversification 3]} \\ x_1 &\geq 0 && \text{[variable-type 1]} \end{aligned}$$



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$$x_2 \geq 0 \quad \text{[variable-type 2]}$$

$$x_3 \geq 0 \quad \text{[variable-type 3]}$$

2.3 Classification of Optimization Models

In our cocoa processing problem, the objective function and all the constraints are linear functions of the decision variables, and the decision variables are continuous, that is they can take on whole number as well as decimal values. An optimization model that meets all of these properties is called a **linear program** (LP).

A **nonlinear program** (NLP) is an optimization model in which the objective function or any of the constraints are nonlinear functions of the decision variables, and the decision variables are continuous. Examples of nonlinear objective functions or constraints would be those that involve any of the decision variable subject to exponentiation, logarithm, trigonometric functions, step-wise functions etc., etc.

In some problems, the decision variables are limited to taking on only a fixed set of values, typically integer values. In such cases the decision variables are called **discrete variables**, and the optimization model is classified as a **discrete optimization model**. If a discrete variable can take on any non-negative integer value it is called an **integer variable**. In some problems the discrete variables are limited to taking on the value of either zero (0) or one (1) only. These are called **binary variables**. Binary variables are used in problems that involve selection in an all-or-nothing, or either-or manner. For example a bridge engineer selecting bridge projects from a set of candidate projects to consume a given budget, subject to various regulatory and technical constraints. In that scenario, a bridge project is either selected or not selected, there cannot be three-quarters of a project selected, it has to be the entire project or the project is not selected.

If any one of the decision variables in an optimization model is discrete, the optimization model is a discrete optimization model, otherwise it is a **continuous optimization model**. Discrete optimization models are also known as **integer programs**. If all of the decision variables are discrete then the optimization model is referred to as **pure integer model**, otherwise it is called a **mixed integer program**.



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An integer program in which the objective function and all constraints are linear functions of the decision variable is called an integer linear program. If the objective function or any of the constraints of an integer program are non-linear functions of the decision variables, then it is an **integer nonlinear program**.

Optimization problems may be classified as **constrained** or **unconstrained**. In an unconstrained optimization problem, no constraints exist, whereas in a constrained problem, there are one or more constraints.

In some optimization problems a decision variable(s) or a parameter(s) of a constraint may be probabilistic, that is it is random and described by a probability distribution. This leads to stochastic optimization models. Models that do not involve probabilistic variables or parameters are described as deterministic optimization models.

In some optimization problems there may be more than one objective function. For example a large holding company may seek to maximize profits, minimize risk, and minimize cost, all simultaneously. A multiobjective optimization model can be used to address such problems. Other applications of multiobjective optimization include complex engineering designs, and resource allocation problems particularly in the government sector.

The degree to which an optimization model allows for convenient analysis is called its **tractability**, i.e. how much analysis is even practical based on the formulation. Therefore, recognizing and classifying an optimization model is an important step in selecting a particular method that is suited to analyzing and solving it.

2.4 Solutions of Optimization Models

Any combination of values of the decision variables that satisfies all the constraints of the optimization model is called a **feasible solution**. An **optimal solution** is a feasible solution where the objective function reaches the optimal (maximum or minimum) value. It must be noted that an optimization model may have a unique optimal solution or it may have several alternative optimal solutions. An infeasible optimization model is one that has no optimal



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solutions, i.e. there is no combination of decision variables' values that satisfy all of the constraints.

In optimization studies, the optimal solutions are not the only output of interest. It is also important to evaluate how the results may change if the other inputs and data used to formulate the model were also to change. For example, one may ask how the solutions would change if the parameters or coefficients of the objective function were to change. One may also be interested to know how the solutions would be affected by a change to or addition of a constraint. A systematic study of the results to explore how they will change due to changes of the input data and parameters of the optimization model is called a **sensitivity analysis**.

A solution that results from simple formulas or operations on the input data is called a **closed-form solution**. Closed-form solutions yield the most substantial information from sensitivity analyses. However based on the nature of the optimization model, particularly its tractability, a closed-form solution may not be possible.

In some optimization models it may not be possible or practical to find an exact solution. In such cases a **heuristic** approach may be justified. A heuristic is any approach to solving a mathematical program that uses a practical or approximate method that is not guaranteed to yield an exact optimum solution.

2.5 Solving Methodologies

The focus of this chapter is the formulation of optimization problems and how to solve them using the *Excel* tools. In order to apply the *Excel* tools effectively we must familiarize with methodologies used to solve optimization models. In this section we shall conduct a cursory review of some of the methodologies applied to solve optimization models. Participants who are interested in a more in-depth and rigorous presentation of these methods are strongly encouraged to review the operations research literature.

Some optimization models have closed-form solutions. Small optimization models with up to two (2) or three (3) decision variables can be solved by graphical methods. However the overwhelming majority of optimization models are analyzed and solved by **numerical search** methods.



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Numerical search is a process of trying different combinations of values of the decision variables (solutions) repeatedly in some systematically designed way until an optimal solution is found. The most common approach to numerical search is **improving search** in which the process starts at some sub-optimal solution and then the “neighboring“ solutions are tested and if a better or superior one is found the process moves to that solution and repeats.

The improving search process is repeated over and over until no better solutions can be found from the current solution and the search is terminated. In the lingo of the optimization model, an improving search is therefore a numerical algorithm that starts at some feasible solution, and moves along a search path of feasible solutions such that the objective function value continually improves. A search algorithm **converges** if the successive iterations move it gradually towards to the optimal solution.

The simplex search method or **simplex algorithm** is the most widely used algorithm for analyzing linear programs. Another family of algorithms used to analyze linear programs are the **interior point methods**.

Simple discrete optimization models can be analyzed by total **enumeration** in which every possible feasible solution is evaluated and the one yielding the optimal value of the objective function is identified. For larger discrete optimization models, a family of search methods called **branch and bound algorithms** are relevant. Other methods used to analyze discrete optimization models include **tabu search**, **simulated annealing**, and **evolutionary algorithms**. These methods are improving search methods however they allow the search to proceed over non-improving solutions along the search path. Evolutionary algorithms for example, are designed to mimic the biological process of evolution by natural selection where populations of solutions evolve over several generations through reproduction (or crossover), mutation, immigration etc.

For nonlinear optimization models, some of the popular improving search methods include golden section search, the gradient search algorithm, Newton’s method, the Nelder-Mead method, and penalty and barrier methods. Nonlinear optimization models are also amenable to analysis by Calculus.



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2.6 Graphical Solutions

Let us use a graphical approach to a simple optimization problem to illustrate some of the fundamental principles and concepts of optimization models.

Problem: An asphalt plant wants to blend recycled aggregates from two sources with aggregates from a certified quarry to produce two grades of aggregates for asphalt, and sell to local consumers. The premium grade requires a mix of 60% quarry aggregates, 20% Source #1 recycled aggregates, and 20% Source #2 recycled aggregates. The regular grade requires 40% quarry aggregates, 35% source 1 recycled aggregates, and 25% source 2 recycled aggregates. The asphalt plant has 4,000 tons of aggregate from the certified quarry, 3,000 tons of recycled aggregates from Source #1, and 2,000 tons of recycled aggregate from Source #2. The company projects the premium grade will result at a profit of \$35 per ton, and the regular grade will result in a profit of \$25 per ton. How many tons of each mix should the recycling company produce to maximize profits?

Solution:

Let x_1 denote the number of tons of premium grade to produce.

Let x_2 denote the number of tons of regular grade to produce.

Objective function:

$$\text{maximize } 35x_1 + 25x_2$$

Let us create a table to illustrate the relationships between the constituent materials of each grade and the availability of the constituent materials.

Product Grade	Premium (x_1)	Regular (x_2)		Availability (tons)
Quarry	60% * x_1	40% * x_2	\leq	4000
Recycle #1	20% * x_1	35% * x_2	\leq	3000
Recycle #2	20% * x_1	25% * x_2	\leq	2000



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Thus we obtain the constraints

$0.6x_1 + 0.4x_2 \leq 4000$	[Quarry availability]
$0.2x_1 + 0.35x_2 \leq 3000$	[Source #1 availability]
$0.2x_1 + 0.25x_2 \leq 2000$	[Source #2 availability]
$x_1 \geq 0$	[variable-type 1]
$x_2 \geq 0$	[variable-type 2]

We shall now plot each constraint function (x_2 versus x_1) using the equality condition, and determine the region(s) of the graph that are consistent with the inequality condition of the constraint. Let us start with the variable-type constraints.

The first variable-type constraint is,

$$x_1 \geq 0$$

Plotting the equality condition,

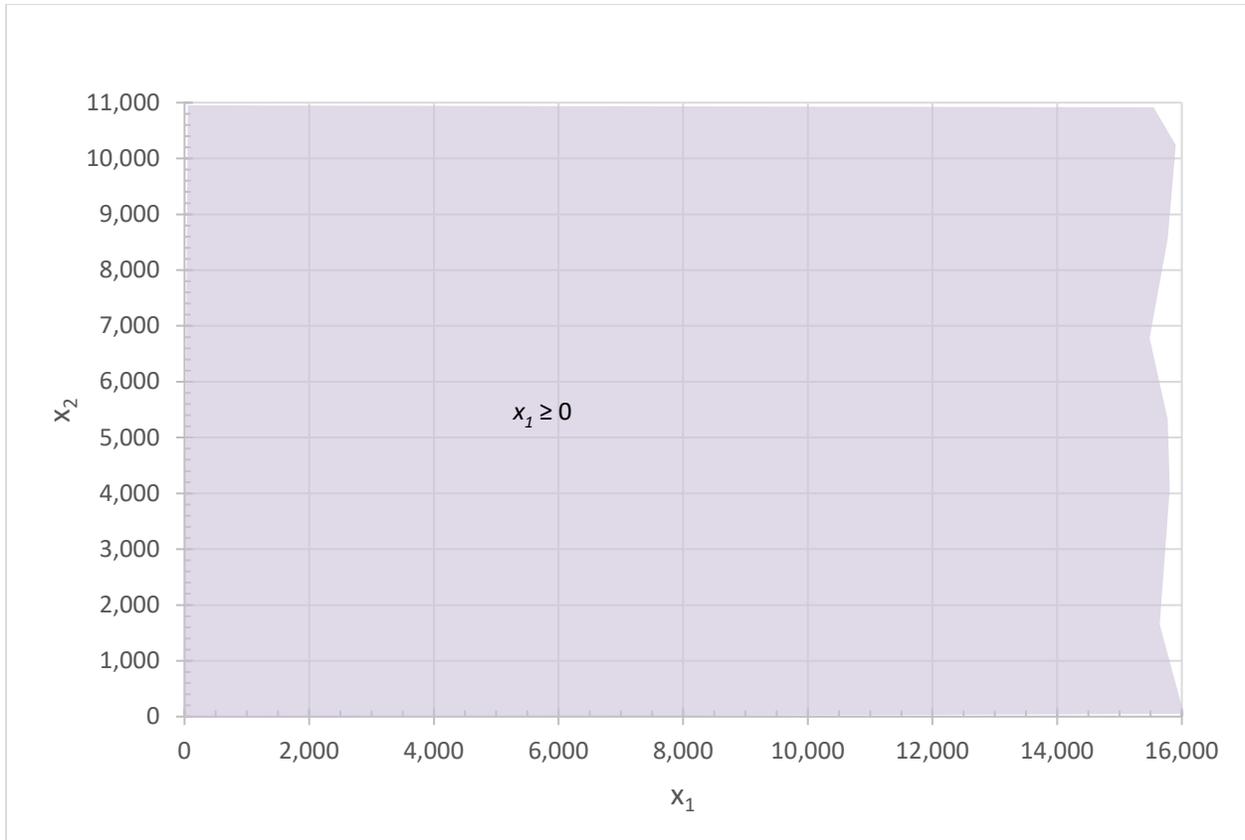
$$x_1 = 0$$

which is the vertical axis, we can see that the vertical axis and the region to the right of the vertical axis satisfy the inequality condition of the constraint.



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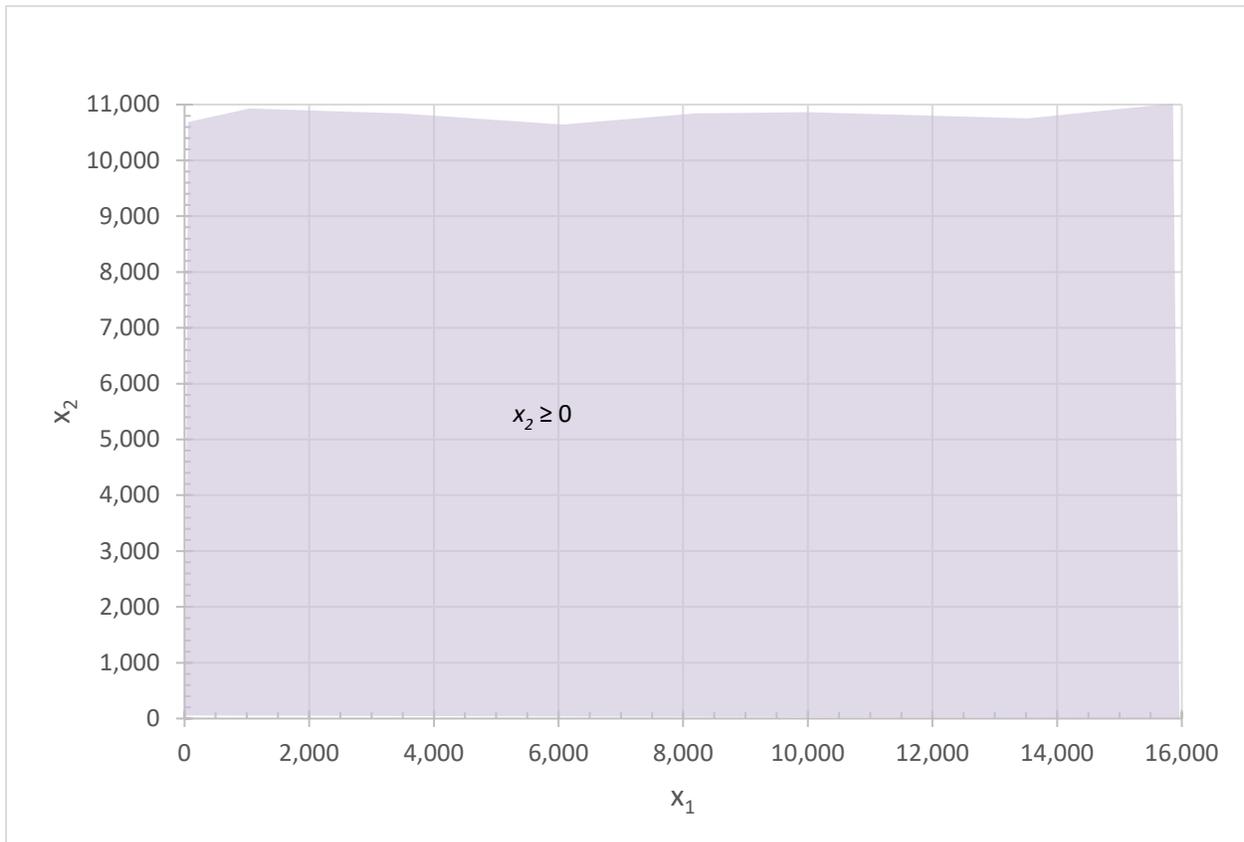
Using non-negative values only, the relevant region for this constraint is therefore the entire quadrant of the x_1, x_2 reference frame.





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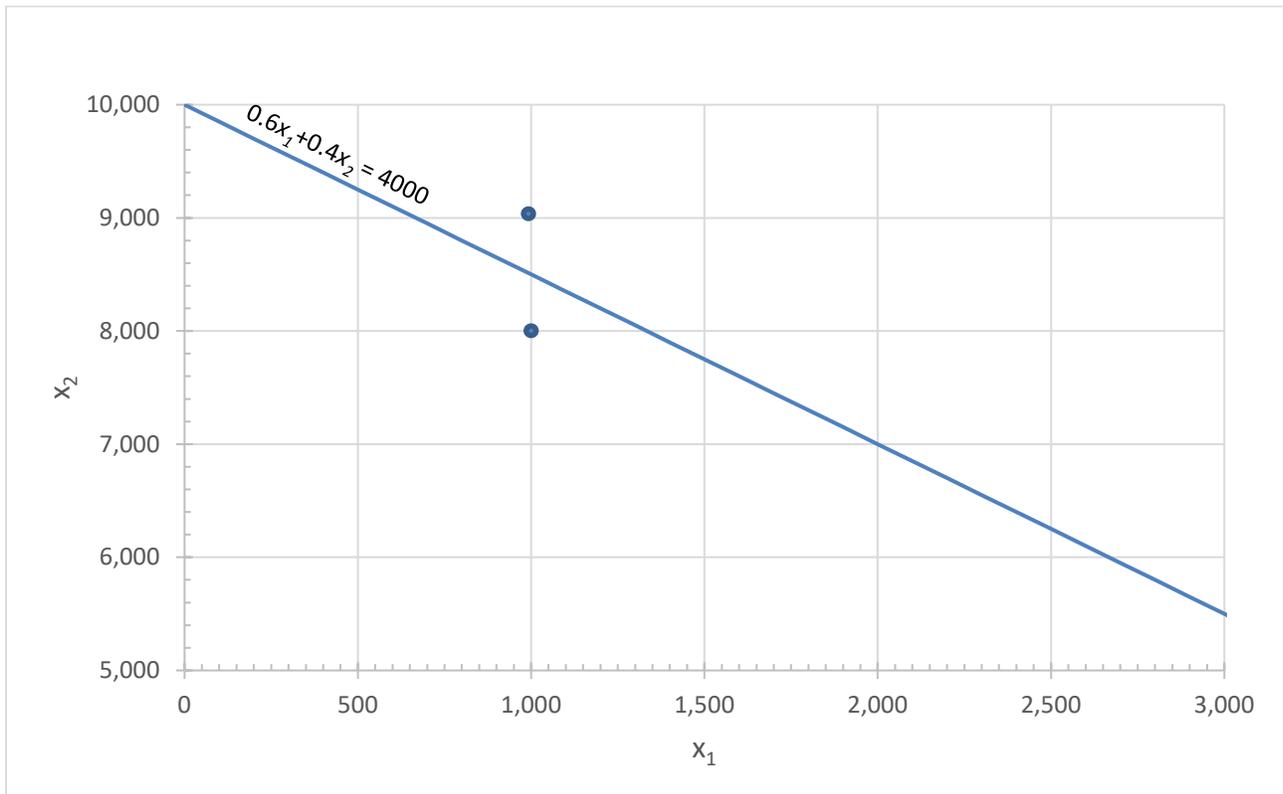
For the second variable-type constraint we obtain,





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We can now move on to the first main constraint – Quarry availability. We must determine which region “above” or “below” the line is the “less than” region, or the “greater than” region. Let us pick two points on the chart (1,000, 9,000) and (1,000, 8,000).



Plugging in the values in the first constraint functions, we obtain,

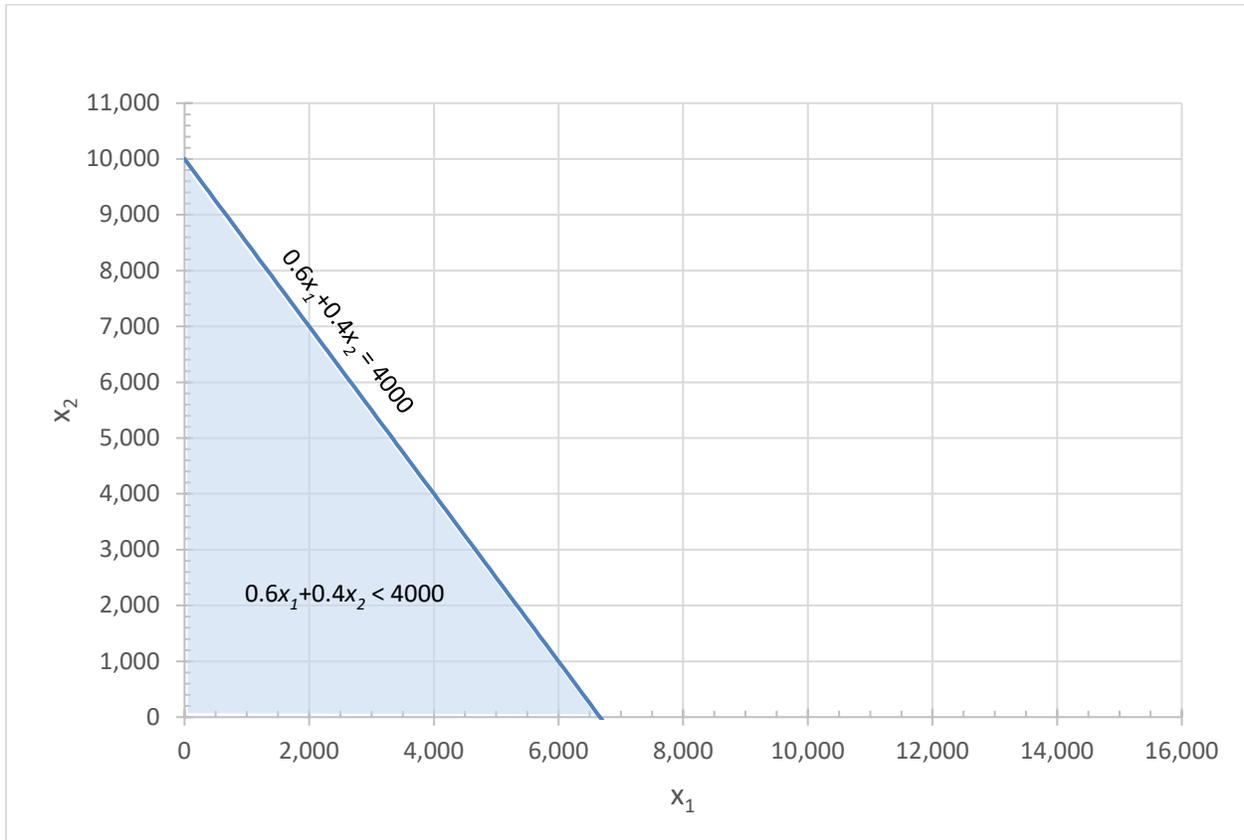
$$0.6 * 1000 + 0.4 * 9000x_2 = 4200 \geq 4000$$

$$0.6 * 1000 + 0.4 * 8000x_2 = 3800 \leq 4000$$



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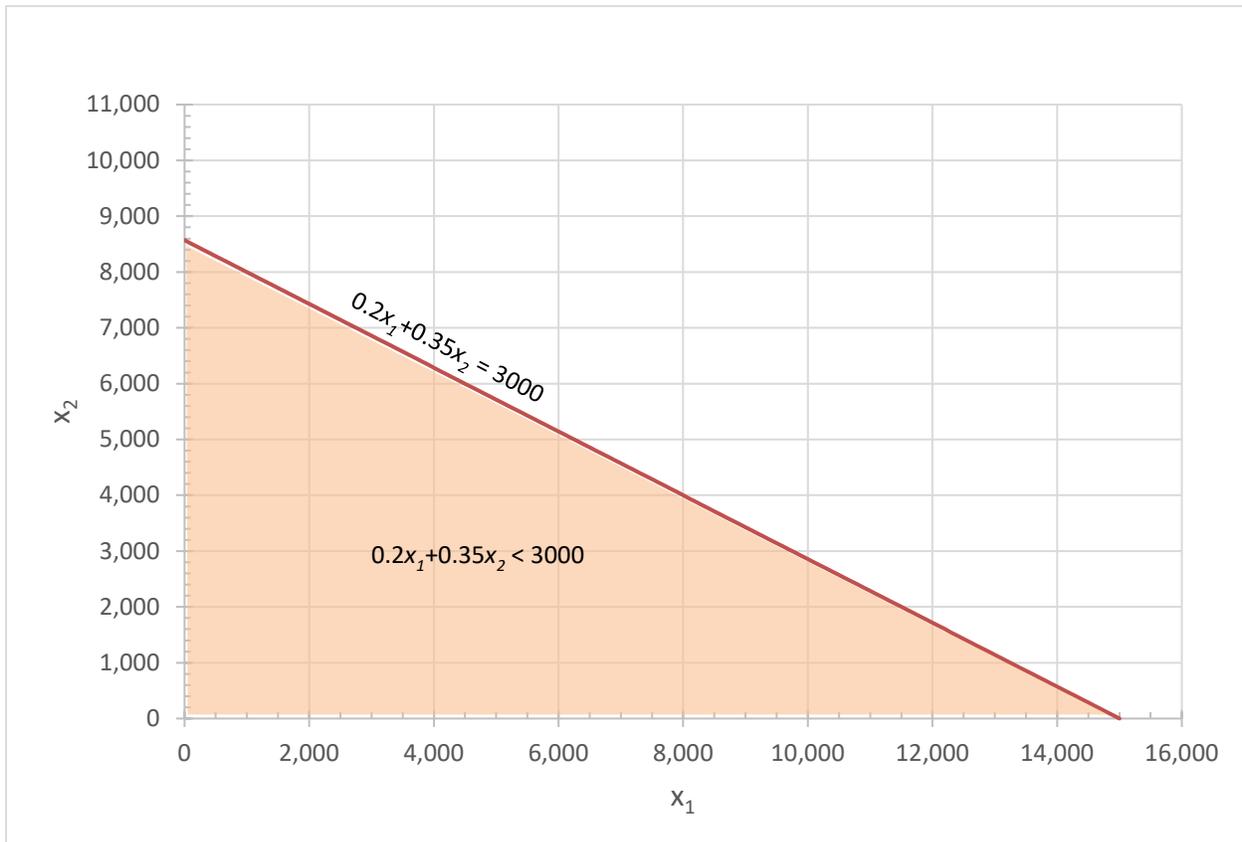
So we see that the region “under” the line is the “less than” region, which is the relevant region for this constraint in this optimization model.





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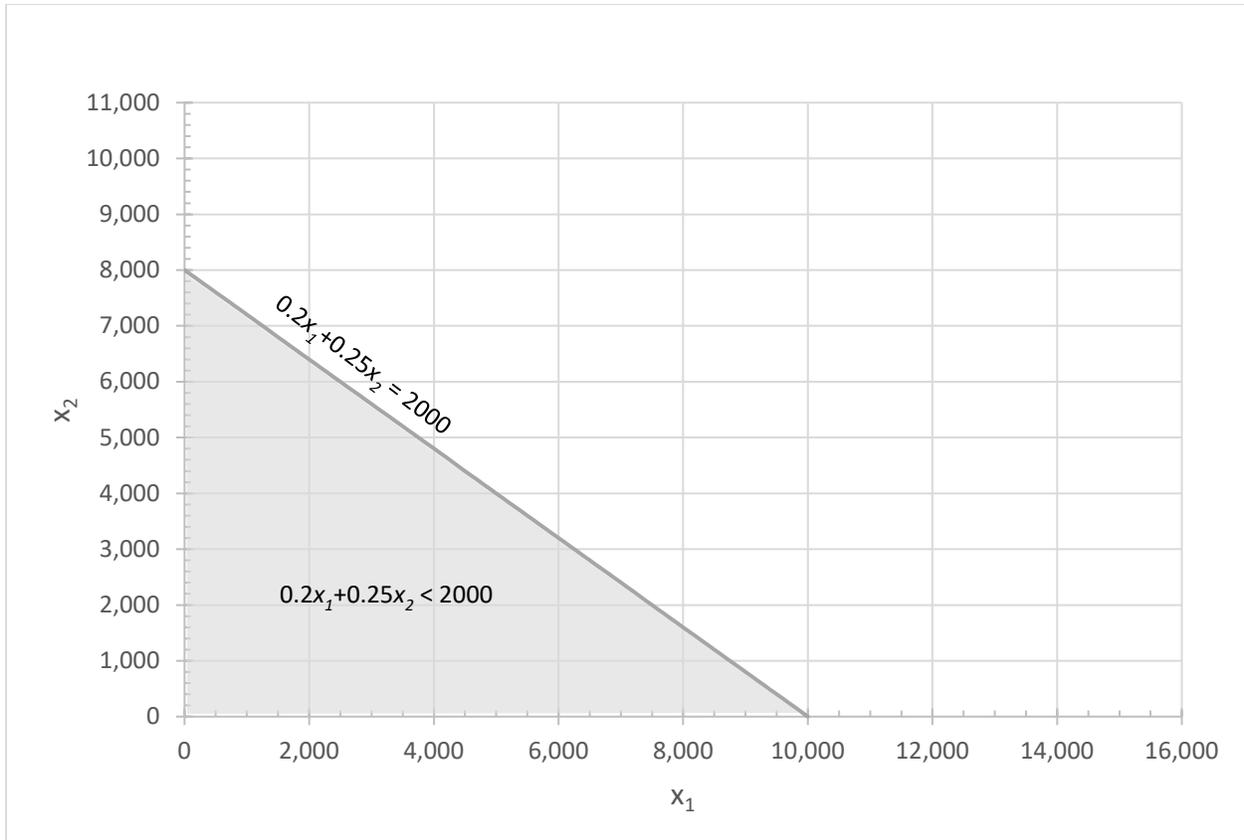
Repeating the process for the main constraint Source #1 availability, we obtain,





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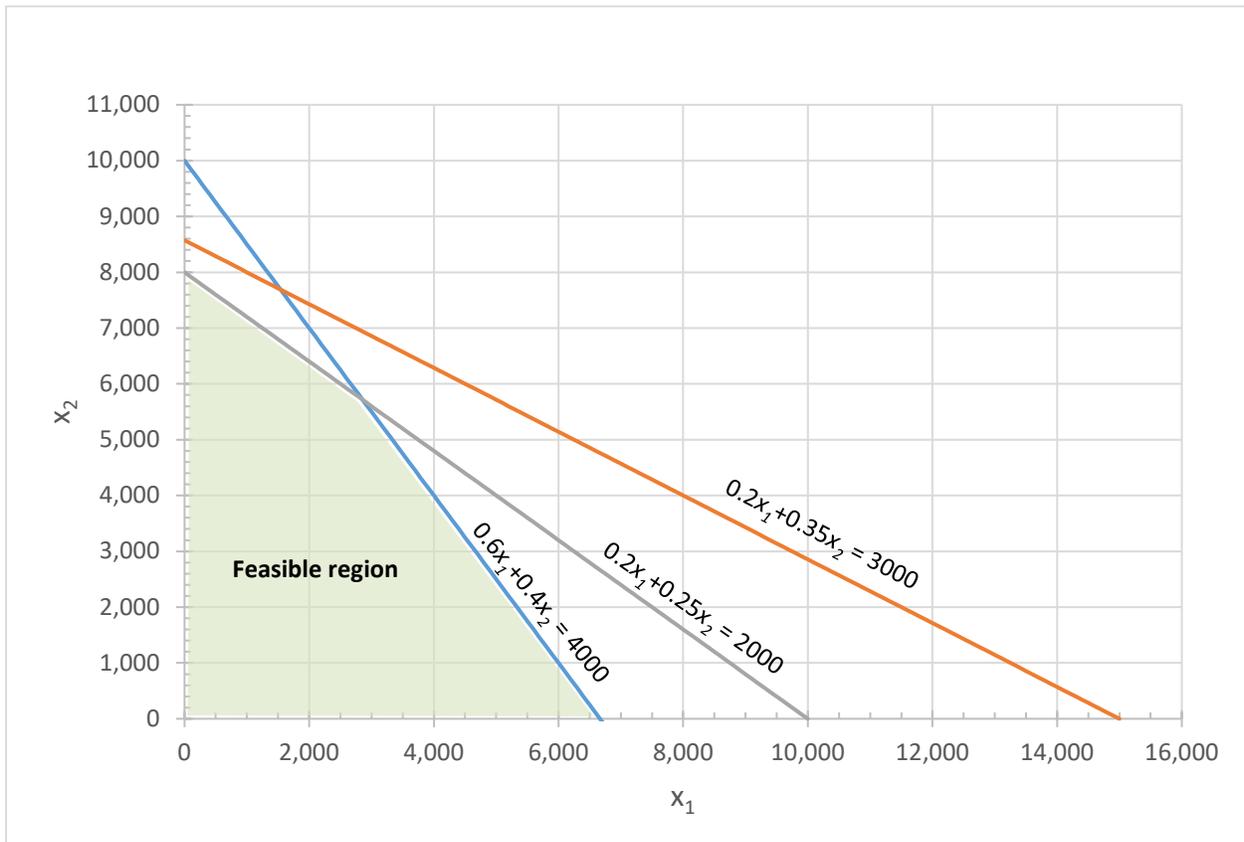
Repeating the process for the main constraint Source #1 availability, we obtain,





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Superimposing the regions associated with each the constraint, we can identify an enclosed region where all of the constraints are satisfied. This is called the **feasible region** of the optimization model. It is only within this region (including the boundaries) that feasible solutions can be found for this optimization model.



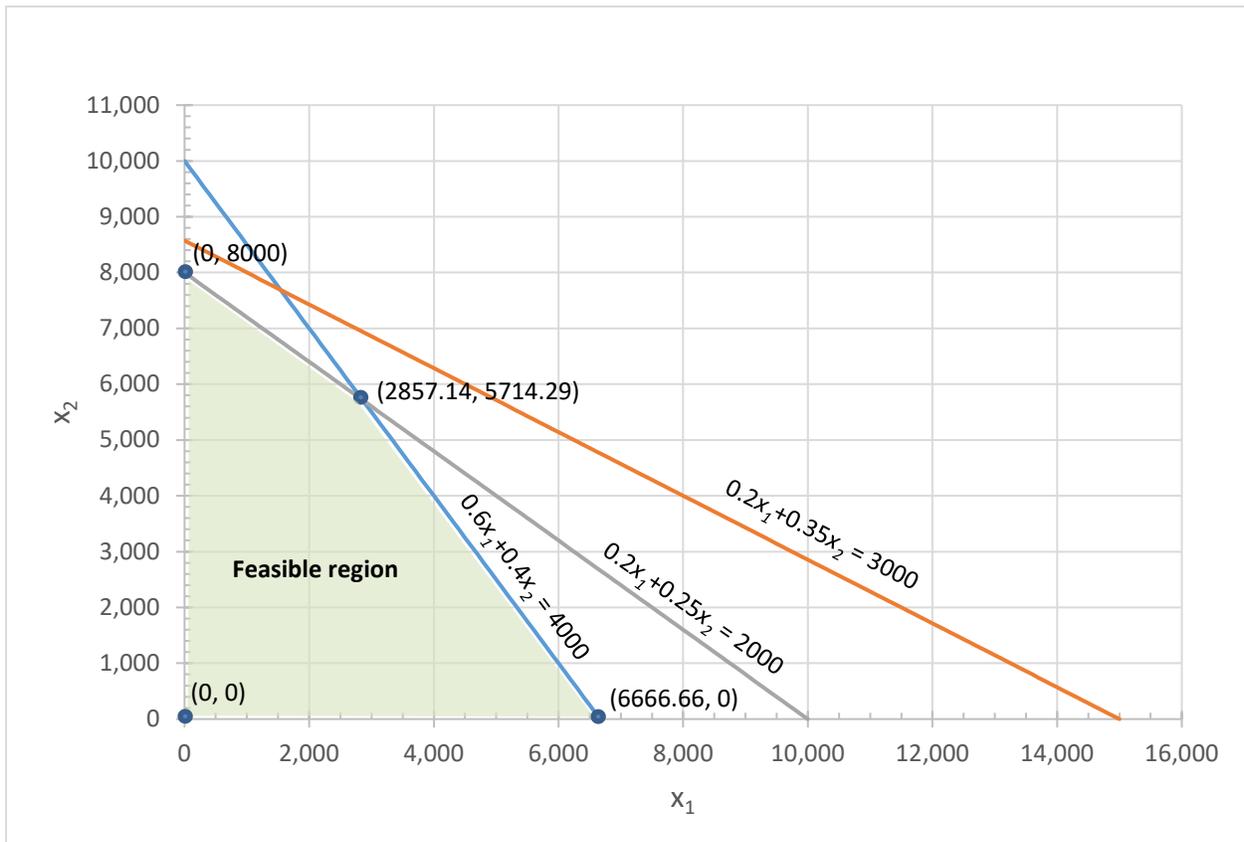
If an enclosed (feasible) region cannot be established from the constraints' functions, then the model is infeasible and will have no feasible solutions. In other words there would be no combination of decision variables that could satisfy all of the constraints.



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The vertices of the feasible region are called corner points or **extreme points**. It can be shown from fundamental principles that if a linear program has any optimal solution, it has one at an extreme point. Also, if a linear program has a unique optimal solution, then that optimal solution must occur at an extreme point.

In our example, the extreme points of this feasible region can be determined by inspecting the graph or by solving the equations simultaneously. The extreme points for this optimization model are as follows.





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We shall now evaluate the objective function using the extreme points to find the extreme point(s) that yield the maximum value of the objective function.

Starting from Extreme point (0, 0) :

$$35 * 0 + 25 * 0 = 0$$

Extreme point (0, 8000) :

$$35 * 0 + 25 * 8000 = 200,000$$

Extreme point (2857.14, 5714.29) :

$$35 * 2857.14 + 25 * 5714.29 = 242,857.15$$

Extreme point (6666.66, 0) :

$$35 * 6666.66 + 25 * 0 = 233,333.10$$

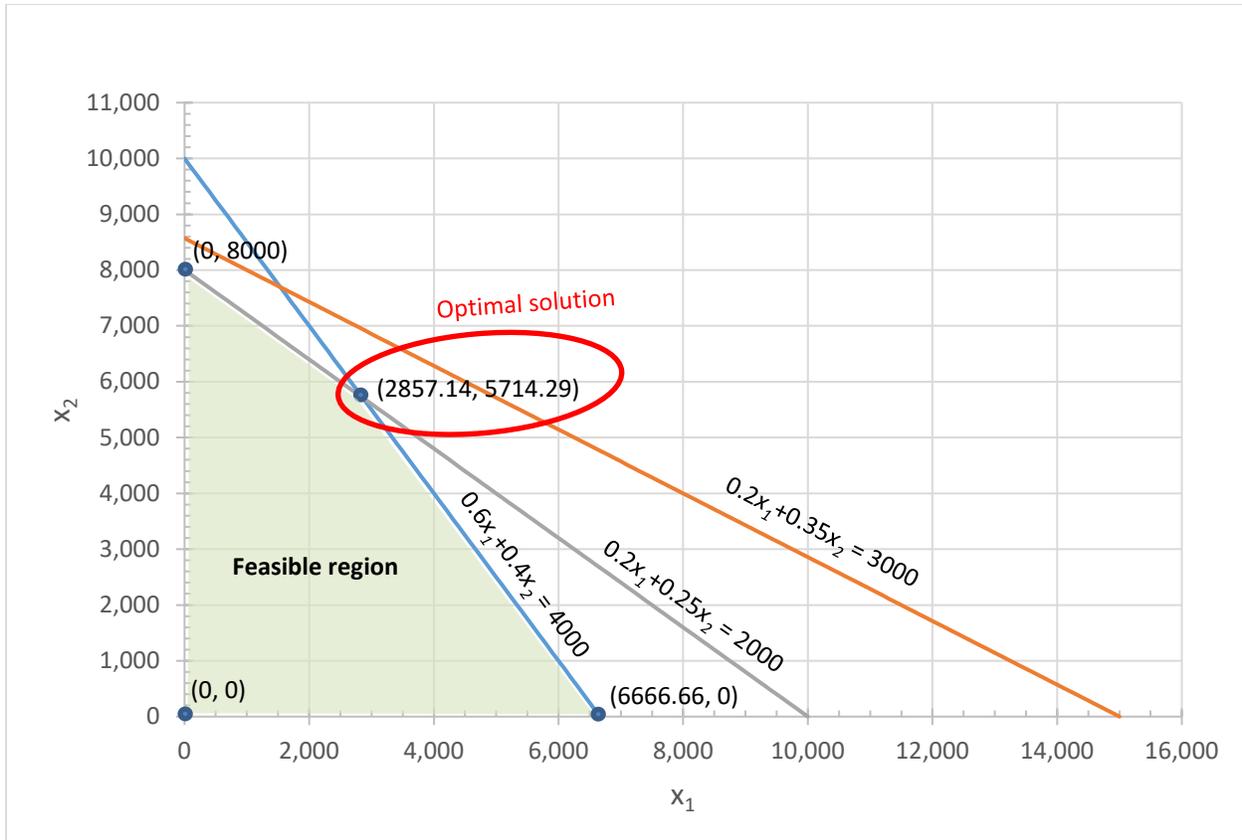
Thus we see that the objective function is maximized at $(x_1, x_2) = (2857.14 \text{ tons}, 5714.29 \text{ tons})$, resulting in maximized revenues of \$242,857.15.

The optimal solution for this optimization model is therefore $(x_1, x_2) = (2857.14 \text{ tons}, 5714.29 \text{ tons})$.



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Our conclusion is that the recycling company should mix 2857.14 tons of premium grade, and 5714.29 tons of regular grade.



In conclusion, the graphical methods are amenable to simpler models with two (or three) decision variables. Beyond that it becomes increasingly difficult, if not, impossible, to visualize graphs and regions in multidimensional space. Therefore, for larger optimization models, analytical and numerical methods are preferred. The simplex algorithm is the most widely used method used to solve many kinds of linear programs. The simplex algorithm is a procedural extension of the graphical approach that sifts through the set of basic feasible solutions, one at a time, until the optimal basic feasible solution is identified, if one exists. Participants in this course are referred to the Operations Research literature for a comprehensive review of the simplex algorithm.



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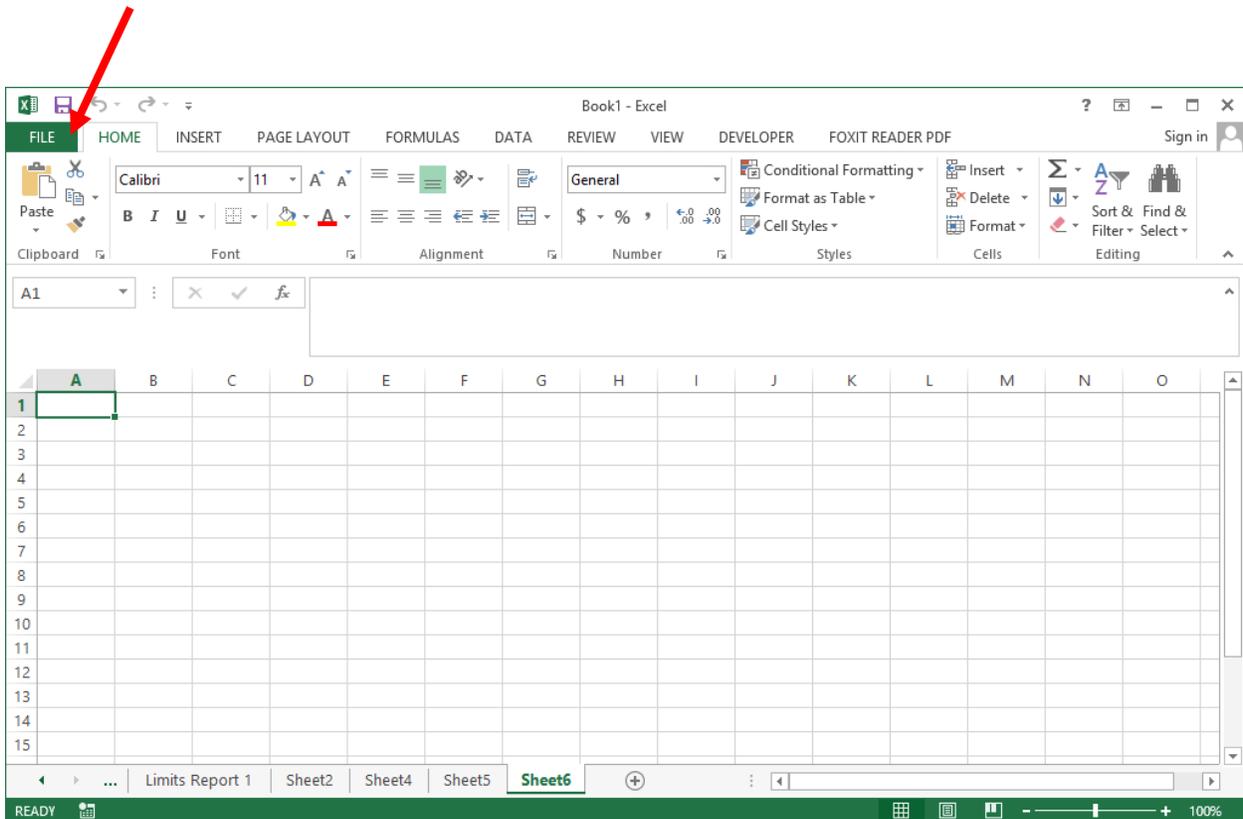
2.7 Excel Solver

The *Excel Solver* Add-In tool can be used to set up and solve optimization models in *Excel*. As an Add-in, *Solver* is not readily available with a regular *Excel* install and must be specifically activated from the Add-in window. Once activated however *Solver* remains available for use unless or until the user deactivates or resets their computer to a setting that predates the *Solver* activation.

Let us perform the following steps to activate *Solver*.

Open *Excel*

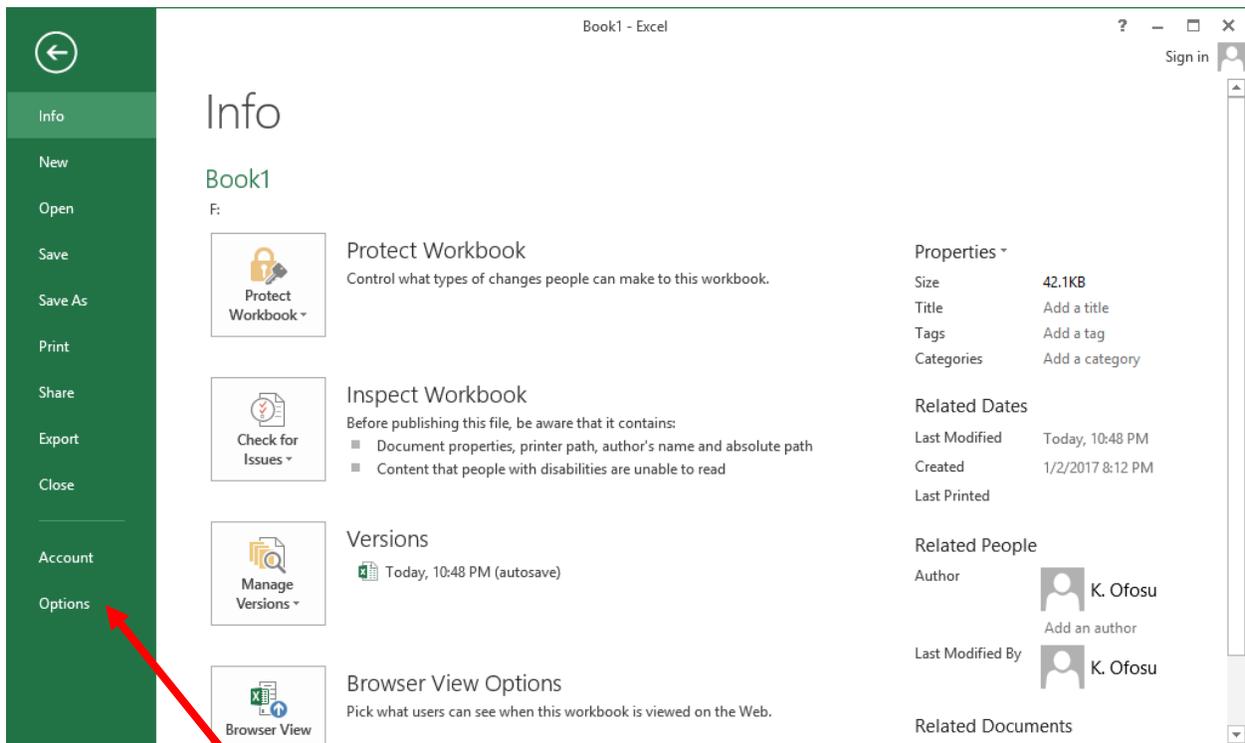
Click on FILE





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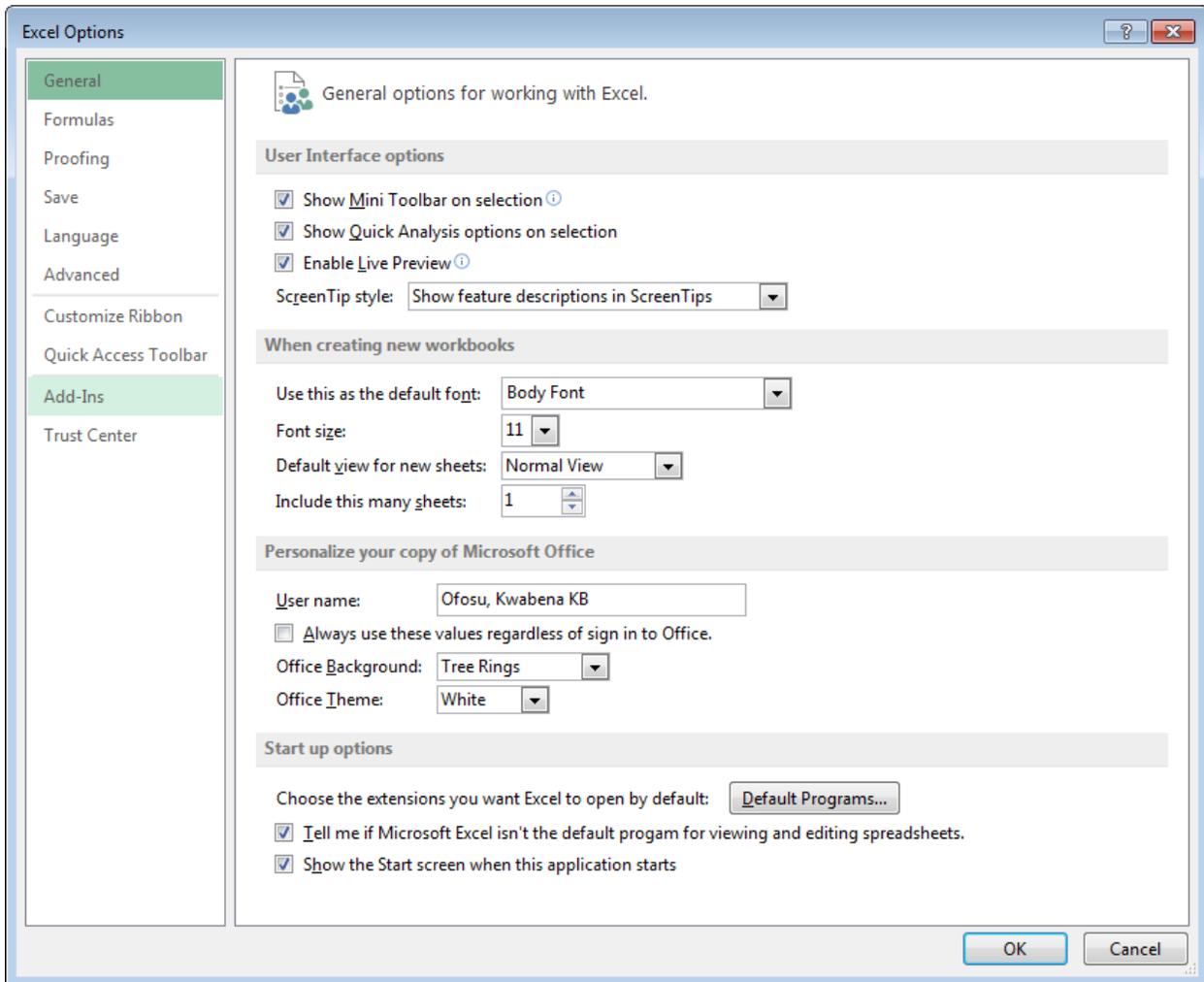
Click on **Options**





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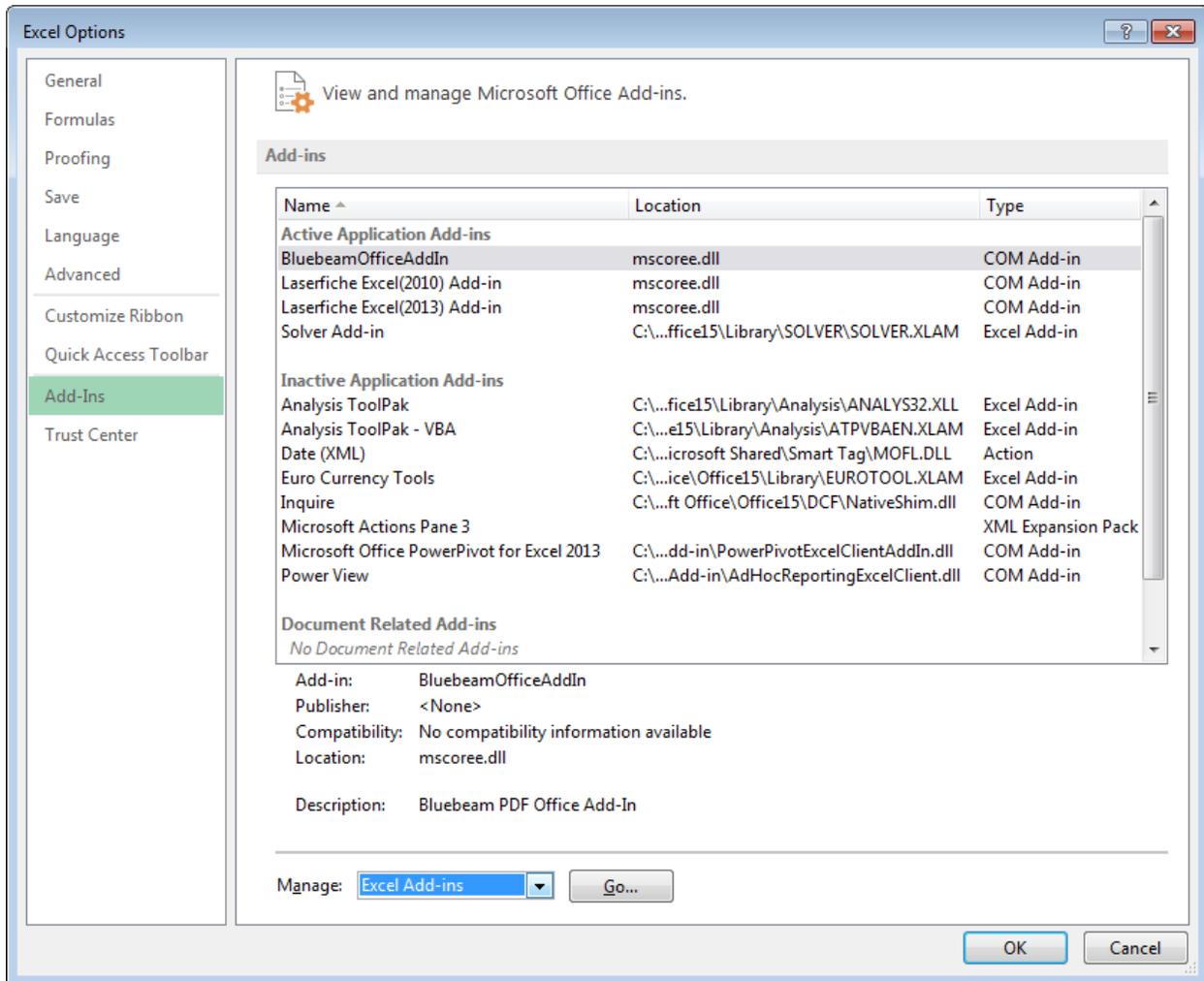
Click on **Add-Ins**





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Under **Manage**, select **Excel-Add-Ins**, and hit **Go**

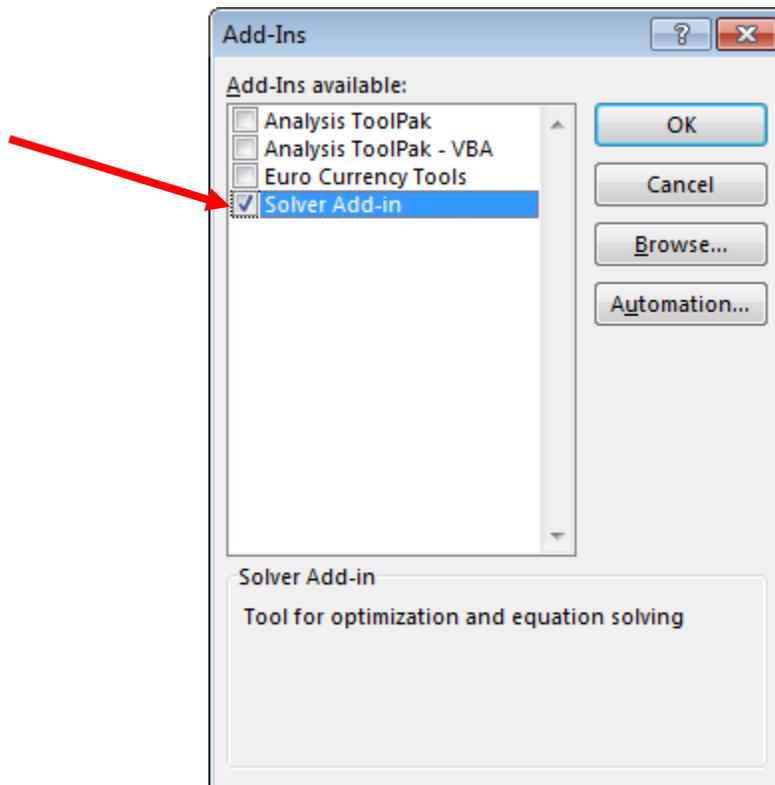


The **Add-Ins** window is opens.



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Under **Add-Ins available**, check the **Solver Add-In** to activate it.



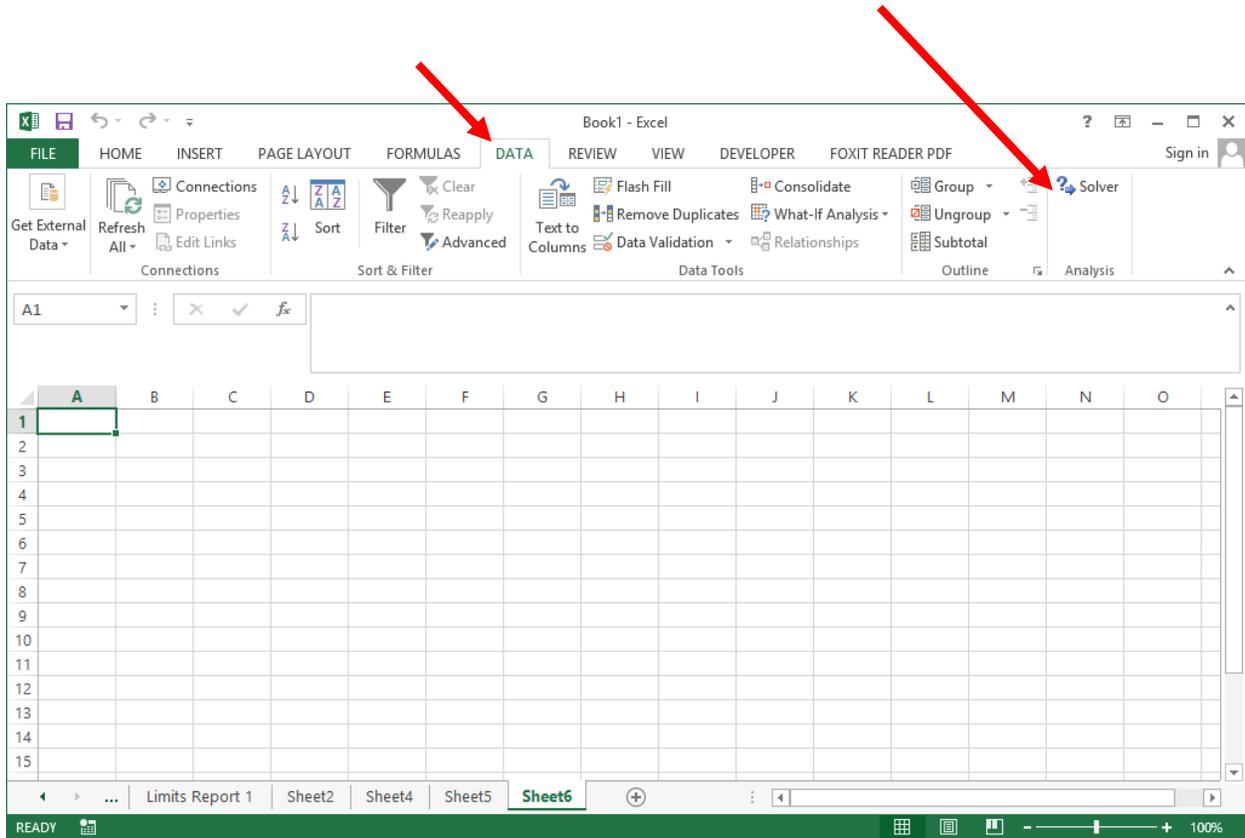
Hit **OK**.

The **Add-Ins** window is dismissed.



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Return to your main *Excel* window and go to the DATA tab.
Notice that the **Analysis** palette now has the **Solver** icon.





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2.8 Optimization Models in Excel

With the *Solver* Add-In activated we may now set up and solve our cocoa processing optimization model. Recall that the optimization model in standard form is as follows.

Objective function:

$$\text{minimize } 3200x_1 + 3600x_2 + 3000x_3$$

Subject to:

$0.3x_1 + 0.2x_2 + 0.15x_3 \geq 6000$	[cocoa liquor]
$0.3x_1 + 0.43x_2 + 0.5x_3 \geq 12000$	[cocoa butter]
$0.35x_1 + 0.3x_2 + 0.25x_3 \geq 9000$	[low-fat cocoa powder]
$0.05x_1 + 0.07x_2 + 0.1x_3 \geq 2500$	[fat-free cocoa powder]
$x_1 \leq 20000$	[Brazil supply]
$x_2 \leq 10000$	[Ghana supply]
$x_3 \leq 15000$	[Mexico supply]
$0.75x_1 - 0.25x_2 - 0.25x_3 \geq 0$	[diversification 1]
$-0.25x_1 + 0.75x_2 - 0.25x_3 \geq 0$	[diversification 2]
$-0.25x_1 - 0.25x_2 + 0.75x_3 \geq 0$	[diversification 3]
$x_1 \geq 0$	[variable-type 1]
$x_2 \geq 0$	[variable-type 2]
$x_3 \geq 0$	[variable-type 3]

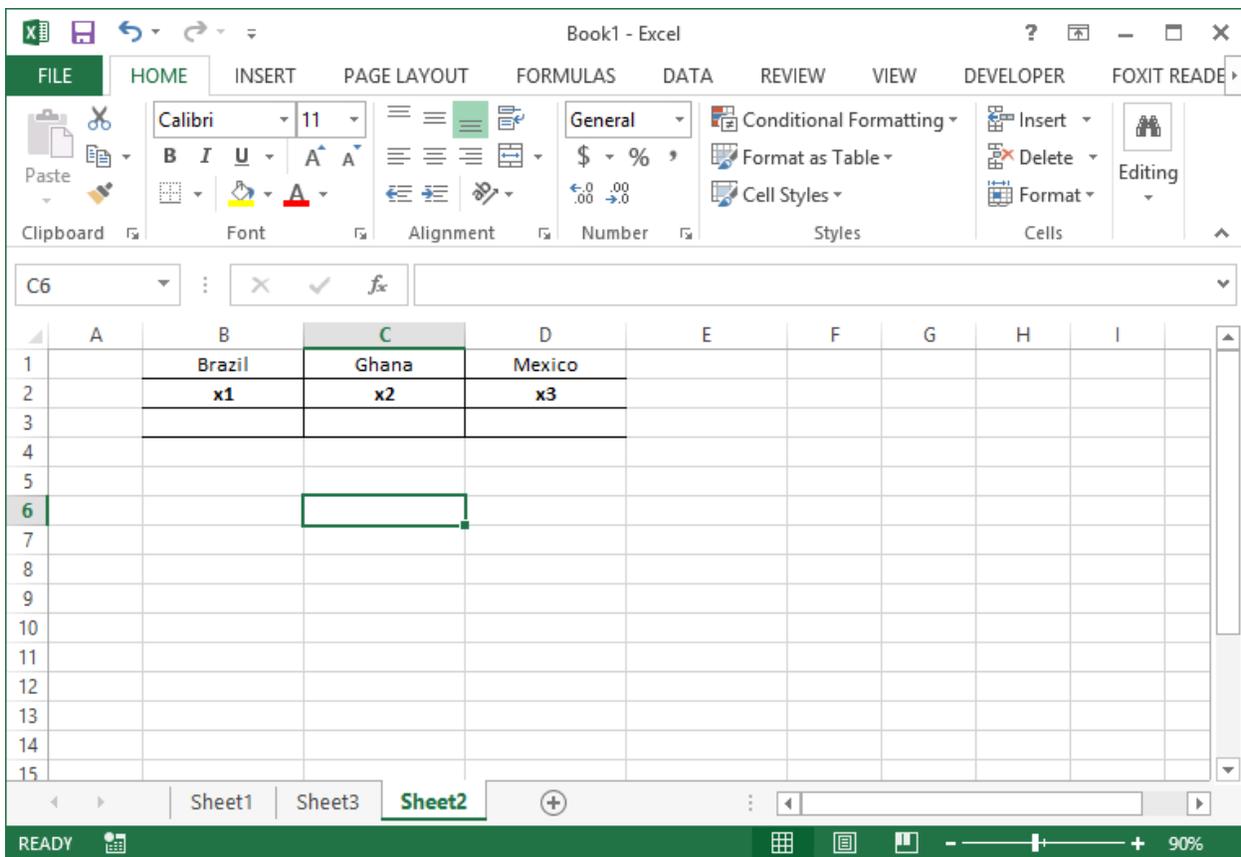


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To set up the problem we shall compile the coefficients of the objective function and all of the constraints into a table as follows.

Open a new Worksheet.

Enter the following table headers for the three source countries for the raw materials and for the respective decision variables.



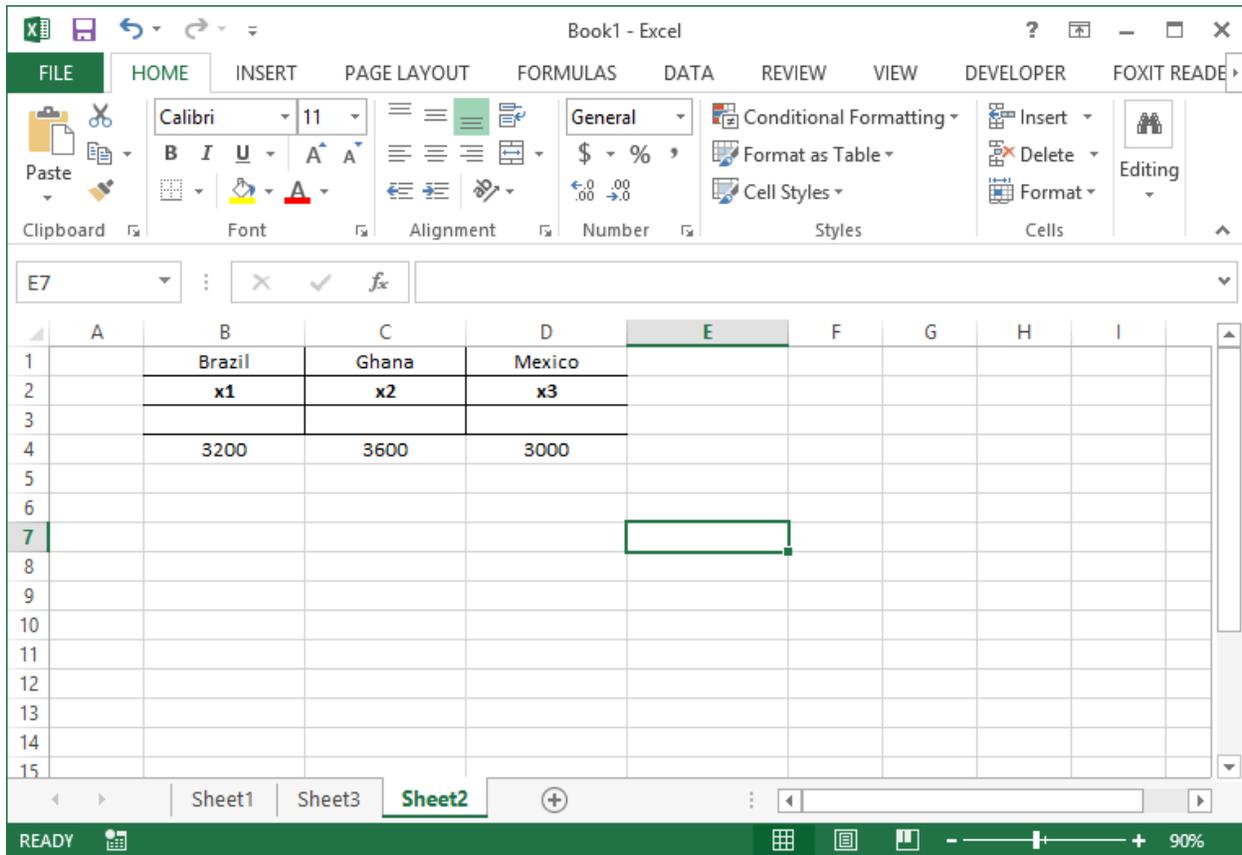
Provide the borders as depicted to enhance distinguishing the various elements.

Note that Row 3 cells are currently blank.



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Enter the coefficients of the objective function



The objective function value is computed by multiplying each decision variable by its respective coefficient and adding up these products.

Use the point-and-click method, with absolute references for the decision variables to set this up as follows.

Select the cell that will hold the objective function value.

Type “=”

Click on decision variable x_1 value cell.

Hit **F4** on your keyboard to make the decision variable value cell an absolute reference.

Type “*”

Click on the cell holding the coefficient of x_1 in the objective function.



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Type “+”

Click on decision variable x_2 value cell.

Hit **F4** on your keyboard.

Type “*”

Click on the cell holding the coefficient of x_2 in the objective function.

Type “+”

Click on decision variable x_3 value cell.

Hit **F4** on your keyboard.

Type “*”

Click on the cell holding the coefficient of x_3 in the objective function.

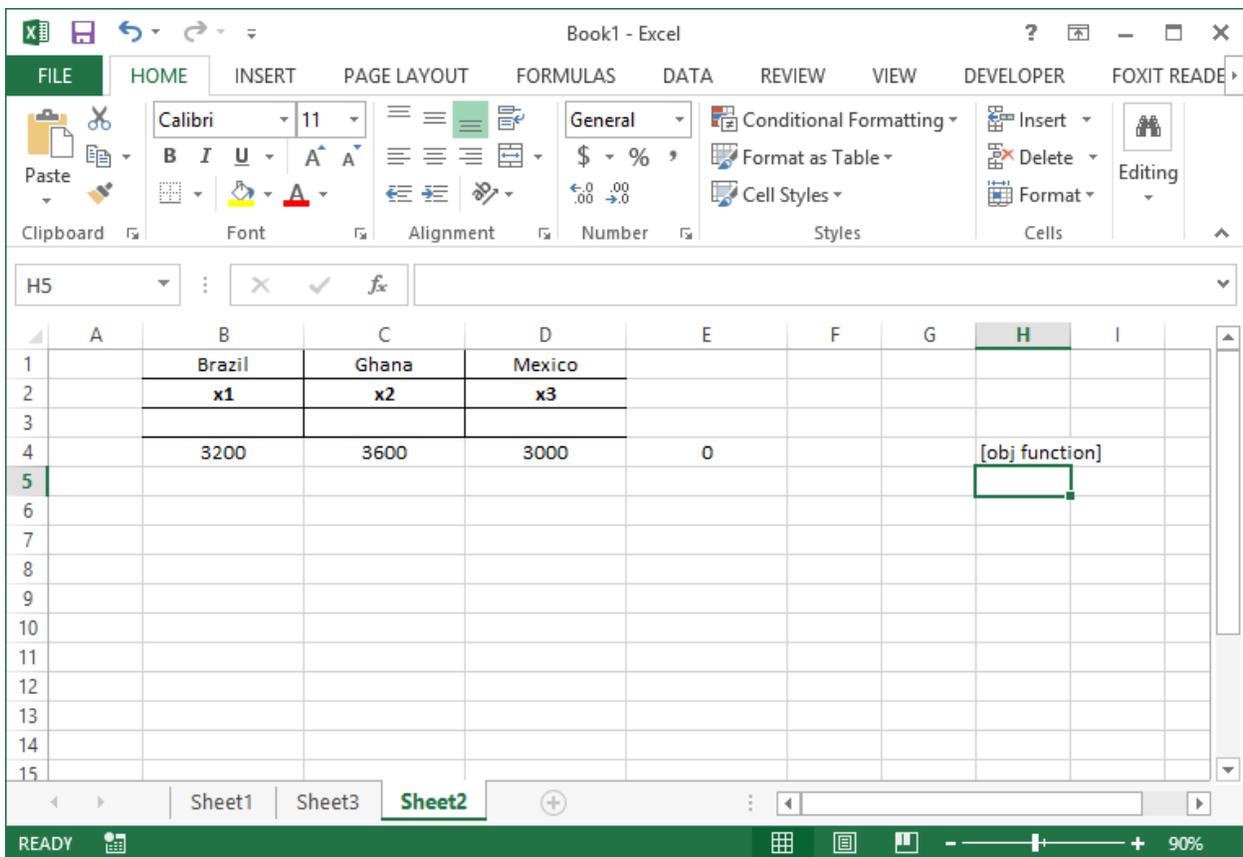
	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		3200	3600	3000					
4					=\\$B\\$3*B4+\\$C\\$3*C4+\\$D\\$3*D4				
5									
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									



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Note that absolute reference (**F4**) was used on the decision variables' values cells so that in calculations we shall perform later on, we may use the fill handle technique while keeping the decision variables' values cell references fixed accordingly.

Hit **Enter** on your keyboard to complete the formula entry.
Add a note for the objective function calculations.





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Let us now enter the coefficients of the first main constraint (cocoa liquor).

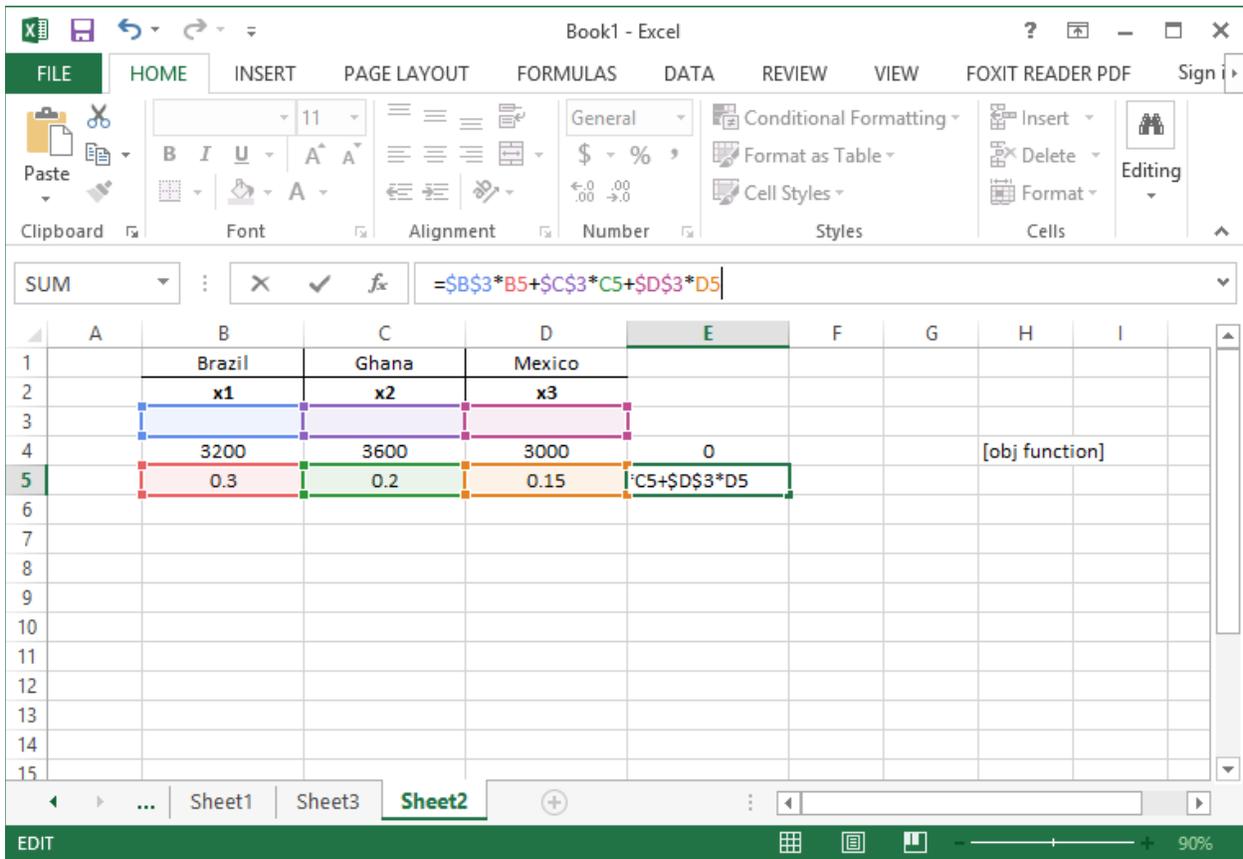
The screenshot shows the Microsoft Excel interface with the following data in the spreadsheet:

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3									
4		3200	3600	3000	0			[obj function]	
5		0.3	0.2	0.17					
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									



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We shall now compute the value of the left hand side (LHS) of this constraint by multiplying each decision variable value by its respective coefficient in the constraint and adding up the products, just as we did in setting up the objective function value.

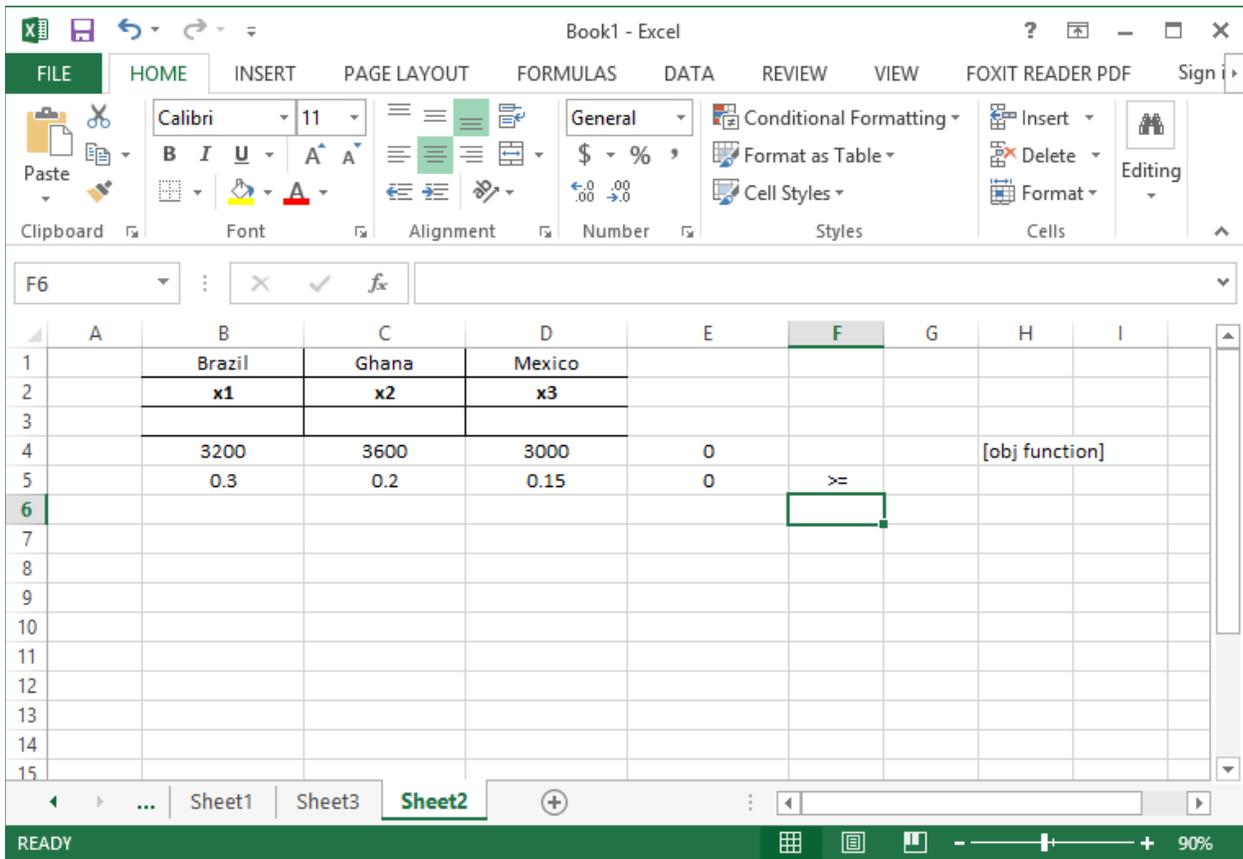


In fact we may have alternately simply performed a fill handle on the objective function value to the relevant cell for the cocoa liquor LHS value.



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The cocoa liquor constraint involves a “greater than or equal to” (“ \geq ”) condition. Type in the “ \geq ” accordingly. For now, this is simply a reminder note but shall guide us once we get into the *Excel Solver* tool itself.





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Enter the value on the right hand side (RHS) of the cocoa liquor constraint.
Add a note on the end of the row.

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3									
4		3200	3600	3000	0			[obj function]	
5		0.3	0.2	0.15	0	>=	6000	[cocoa liquor]	
6									
7									
8									
9									
10									
11									
12									
13									
14									
15									



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The structure for entry of the first main constraint shall be repeated for all constraints in our model. If a decision variable is not present in a particular constraint we simply enter its coefficient value as zero (“0”).

Looking at the optimization model standard form, enter all coefficients for the remaining constraints.

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3									
4		3200	3600	3000	0			[obj function]	
5		0.3	0.2	0.15	0	>=	6000	[cocoa liquor]	
6		0.3	0.43	0.5					
7		0.35	0.3	0.25					
8		0.05	0.07	0.1					
9		1	0	0					
10		0	1	0					
11		0	0	1					
12		0.75	-0.25	-0.25					
13		-0.25	0.75	-0.25					
14		-0.25	-0.25	0.75					
15		1	0	0					
16		0	1	0					
17		0	0	1					



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Enter the computations for the LHSs by point-and-click with absolute reference for the decision variables values cells, or simply fill handle the first main constraint down the entire table.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3									
4		3200	3600	3000	0			[obj function]	
5		0.3	0.2	0.17	0	>=	6000	[cocoa liquor]	
6		0.33	0.43	0.5	0				
7		0.35	0.3	0.4	0				
8		0.05	0.07	0.15	0				
9		1	0	0	0				
10		0	1	0	0				
11		0	0	1	0				
12		0.75	-0.25	-0.25	0				
13		-0.25	0.75	-0.25	0				
14		-0.25	-0.25	0.75	0				
15		1	0	0	0				
16		0	1	0	0				
17		0	0	1	0				

The formula bar shows the formula for cell E5: $=\$B\$3*B5+\$C\$3*C5+\$D\$3*D5$



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For each constraint in the standard form of our optimization model (on page 32), enter the relevant equality or inequality condition – less than, greater than or equal to, etc. etc.

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3									
4		3200	3600	3000	0			[obj function]	
5		0.3	0.2	0.15	0	>=	6000	[cocoa liquor]	
6		0.3	0.43	0.5	0	>=			
7		0.35	0.3	0.25	0	>=			
8		0.05	0.07	0.1	0	>=			
9		1	0	0	0	<=			
10		0	1	0	0	<=			
11		0	0	1	0	<=			
12		0.75	-0.25	-0.25	0	>=			
13		-0.25	0.75	-0.25	0	>=			
14		-0.25	-0.25	0.75	0	>=			
15		1	0	0	0	>=			
16		0	1	0	0	>=			
17		0	0	1	0	>=			



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For each constraint, enter the value on the RHS of the constraint.
Add a note on the end of each row.

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3									
4		3200	3600	3000	0				[obj function]
5		0.3	0.2	0.15	0	>=	6000		[cocoa liquor]
6		0.3	0.43	0.5	0	>=	12000		[cocoa butter]
7		0.35	0.3	0.25	0	>=	9000		[low-fat cocoa powder]
8		0.05	0.07	0.1	0	>=	2500		[fat-free cocoa powder]
9		1	0	0	0	<=	20000		[Brazil supply]
10		0	1	0	0	<=	10000		[Ghana supply]
11		0	0	1	0	<=	15000		[Mexico supply]
12		0.75	-0.25	-0.25	0	>=	0		[diversification 1]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 2]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 3]
15		1	0	0	0	>=	0		[variable type 1]
16		0	1	0	0	>=	0		[variable type 2]
17		0	0	1	0	>=	0		[variable type 3]



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Since the objective function in this case is money, let us apply an appropriate number format to the cell holding the computed objective function value.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3									
4		3200	3600	3000	0				[obj function]
5		0.3	0.2	0.15	0	>=	6000		[cocoa liquor]
6		0.3	0.43	0.5	0	>=	12000		[cocoa butter]
7		0.35	0.3	0.25	0	>=	9000		[low-fat cocoa powder]
8		0.05	0.07	0.1	0	>=	2500		[fat-free cocoa powder]
9		1	0	0	0	<=	20000		[Brazil supply]
10		0	1	0	0	<=	10000		[Ghana supply]
11		0	0	1	0	<=	15000		[Mexico supply]
12		0.75	-0.25	-0.25	0	>=	0		[diversification 1]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 2]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 3]
15		1	0	0	0	>=	0		[variable type 1]
16		0	1	0	0	>=	0		[variable type 2]
17		0	0	1	0	>=	0		[variable type 3]

The formula bar shows the formula for cell E4: $=\$B\$3*B4+\$C\$3*C4+\$D\$3*D4$. The ribbon is set to 'DATA', and the 'Number' section is selected. The status bar at the bottom indicates 'READY' and '90%' zoom.



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Select an appropriate number or currency format.

	A	B	C	D
1		Brazil	Ghana	Mexico
2		x1	x2	x3
3				
4		3200	3600	3000
5		0.3	0.2	0.15
6		0.3	0.43	0.5
7		0.35	0.3	0.25
8		0.05	0.07	0.1
9		1	0	0
10		0	1	0
11		0	0	1
12		0.75	-0.25	-0.25
13		-0.25	0.75	-0.25
14		-0.25	-0.25	0.75
15		1	0	0
16		0	1	0
17		0	0	1



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The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3									
4		3200	3600	3000	0.00				[obj function]
5		0.3	0.2	0.15	0	>=	6000		[cocoa liquor]
6		0.3	0.43	0.5	0	>=	12000		[cocoa butter]
7		0.35	0.3	0.25	0	>=	9000		[low-fat cocoa powder]
8		0.05	0.07	0.1	0	>=	2500		[fat-free cocoa powder]
9		1	0	0	0	<=	20000		[Brazil supply]
10		0	1	0	0	<=	10000		[Ghana supply]
11		0	0	1	0	<=	15000		[Mexico supply]
12		0.75	-0.25	-0.25	0	>=	0		[diversification 1]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 2]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 3]
15		1	0	0	0	>=	0		[variable type 1]
16		0	1	0	0	>=	0		[variable type 2]
17		0	0	1	0	>=	0		[variable type 3]



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The values of the optimal decision variables (the solution) are currently unknown. Ultimately these cells will contain the actual values of the decision variables (the solution) that we are solving for. For now, to get *Solver* started enter any random values such as all zeros or ones, which will serve as a starting point for the improve search algorithm.

Apply a red font to the decision variables values cells for distinction purposes.

Format the decision variables to 2 decimal places for convenience.

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		0.00	0.00	0.00					
4		3200	3600	3000	0.00				[obj function]
5		0.3	0.2	0.15	0	>=	6000		[cocoa liquor]
6		0.3	0.43	0.5	0	>=	12000		[cocoa butter]
7		0.35	0.3	0.25	0	>=	9000		[low-fat cocoa powder]
8		0.05	0.07	0.1	0	>=	2500		[fat-free cocoa powder]
9		1	0	0	0	<=	20000		[Brazil supply]
10		0	1	0	0	<=	10000		[Ghana supply]
11		0	0	1	0	<=	15000		[Mexico supply]
12		0.75	-0.25	-0.25	0	>=	0		[diversification 1]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 2]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 3]
15		1	0	0	0	>=	0		[variable type 1]
16		0	1	0	0	>=	0		[variable type 2]
17		0	0	1	0	>=	0		[variable type 3]



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Though not required to solve this problem, let us add a calculation of the sum total of the decision variables, in other words total number of tons of raw cocoa beans purchased.

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		0.00	0.00	0.00	=B3+C3+D3				
4		3200	3600	3000	0.00				[obj function]
5		0.3	0.2	0.15	0	>=	6000		[cocoa liquor]
6		0.3	0.43	0.5	0	>=	12000		[cocoa butter]
7		0.35	0.3	0.25	0	>=	9000		[low-fat cocoa powder]
8		0.05	0.07	0.1	0	>=	2500		[fat-free cocoa powder]
9		1	0	0	0	<=	20000		[Brazil supply]
10		0	1	0	0	<=	10000		[Ghana supply]
11		0	0	1	0	<=	15000		[Mexico supply]
12		0.75	-0.25	-0.25	0	>=	0		[diversification 1]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 2]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 3]
15		1	0	0	0	>=	0		[variable type 1]
16		0	1	0	0	>=	0		[variable type 2]
17		0	0	1	0	>=	0		[variable type 3]



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Format the sum total of the decision variables to two decimal places for consistency.

To enhance the distinction of elements of the table, let us color code the objective function and the calculated LHSs of the constraints as follows.

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		0.00	0.00	0.00	0.00				
4		3200	3600	3000	0.00			[obj function]	
5		0.3	0.2	0.15	0	>=	6000	[cocoa liquor]	
6		0.3	0.43	0.5	0	>=	12000	[cocoa butter]	
7		0.35	0.3	0.25	0	>=	9000	[low-fat cocoa powder]	
8		0.05	0.07	0.1	0	>=	2500	[fat-free cocoa powder]	
9		1	0	0	0	<=	20000	[Brazil supply]	
10		0	1	0	0	<=	10000	[Ghana supply]	
11		0	0	1	0	<=	15000	[Mexico supply]	
12		0.75	-0.25	-0.25	0	>=	0	[diversification 1]	
13		-0.25	0.75	-0.25	0	>=	0	[diversification 2]	
14		-0.25	-0.25	0.75	0	>=	0	[diversification 3]	
15		1	0	0	0	>=	0	[variable type 1]	
16		0	1	0	0	>=	0	[variable type 2]	
17		0	0	1	0	>=	0	[variable type 3]	

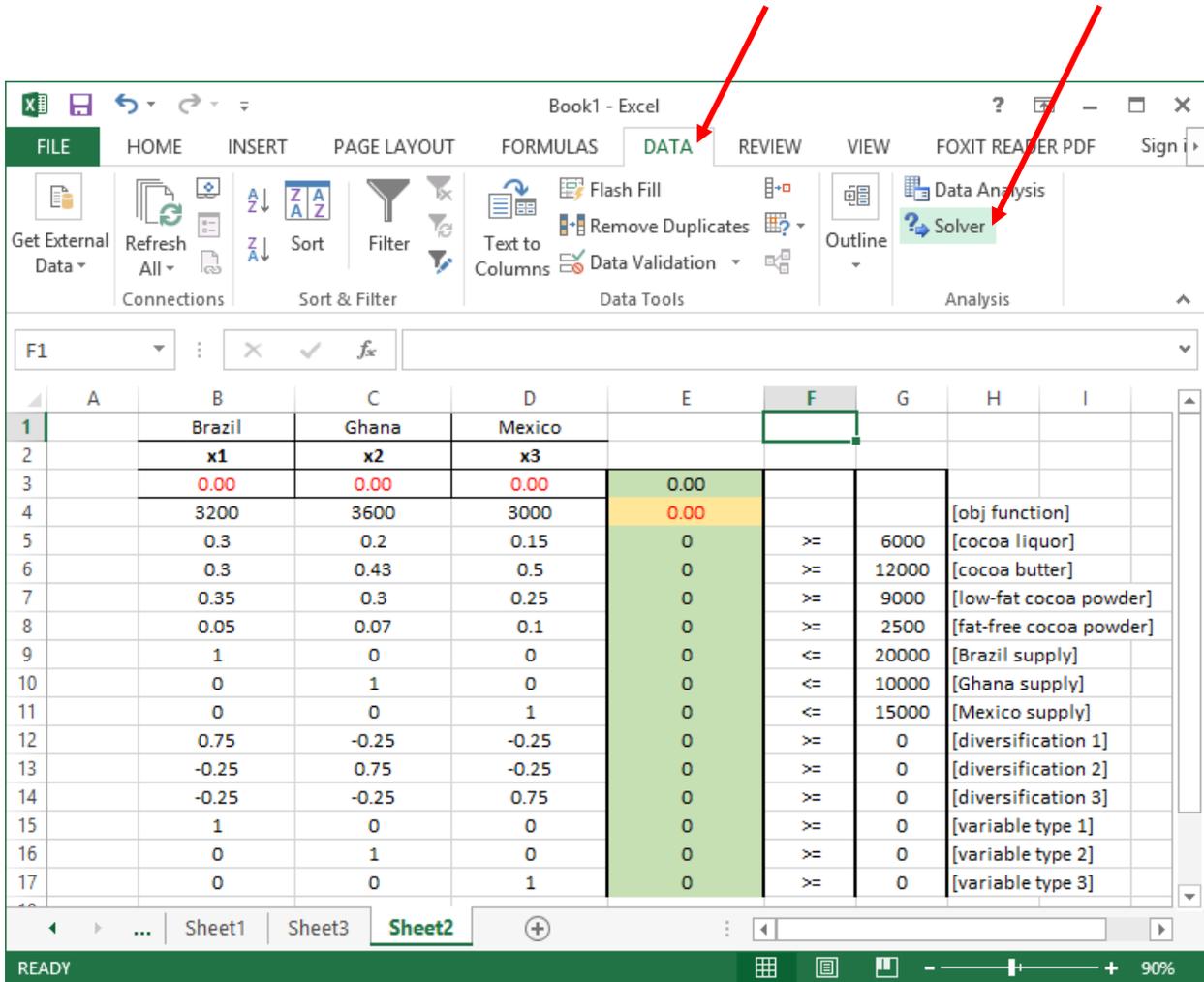
At this juncture we have all the information in place needed by *Excel Solver* to solve the optimization model. We shall now move ahead with setting up *Solver* so it can search for the optimal solution.



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Click on DATA tab.

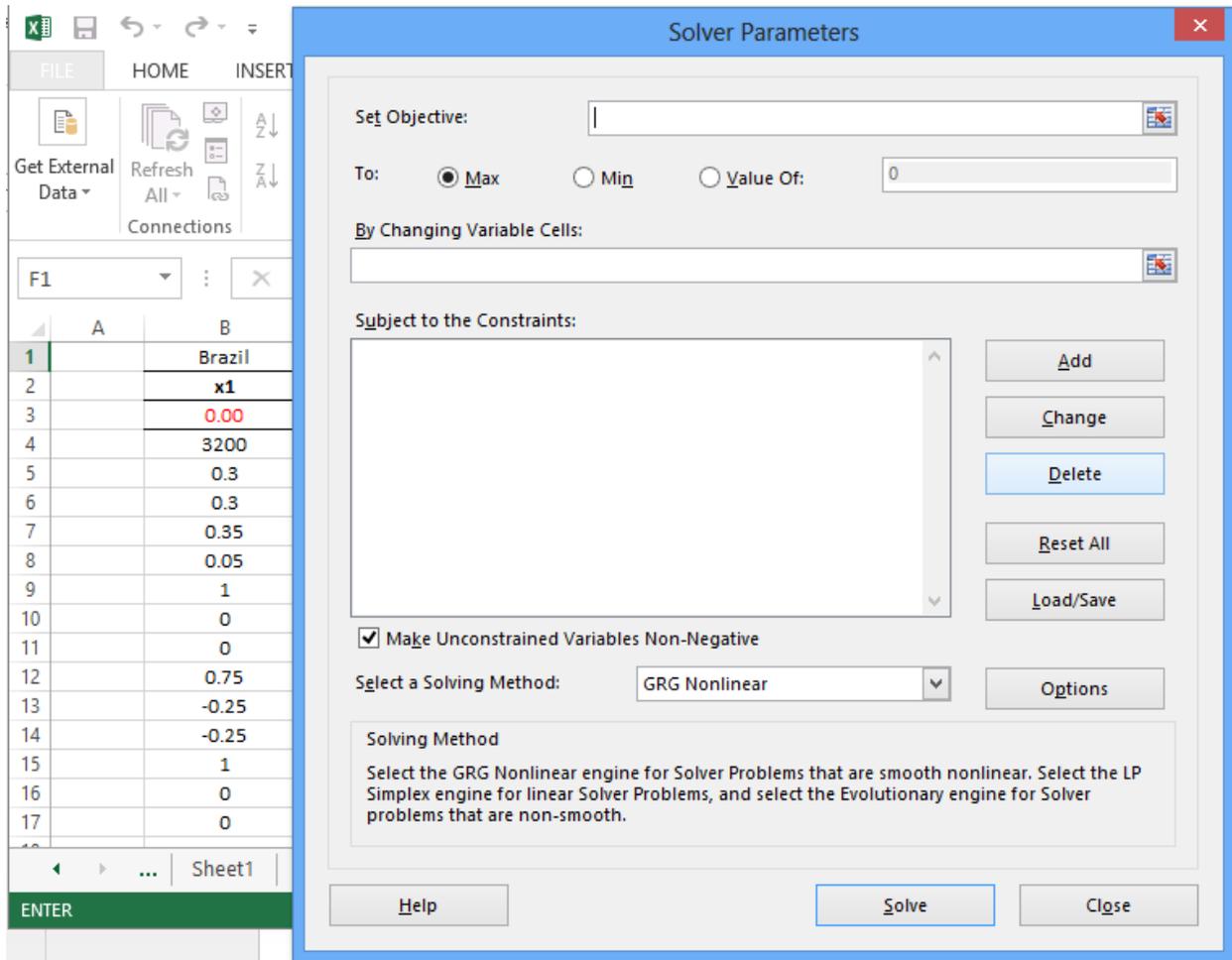
In the **Analysis** palette click on **Solver** to open the *Excel Solver*.





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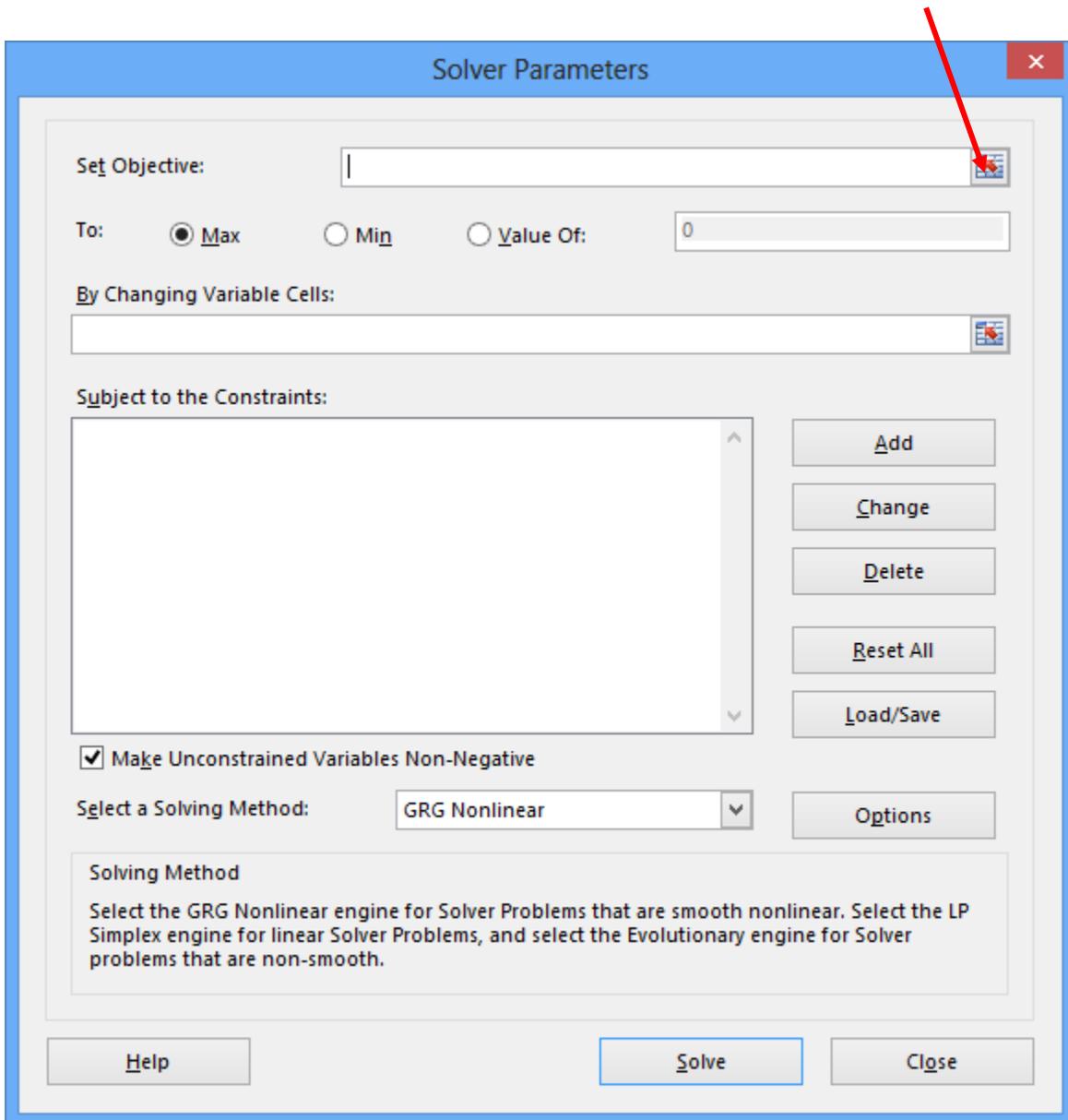
The *Excel Solver* (**Solver Parameters** window) opens.





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To set the objective function, click on the **Set Objective** textbox tab.





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This shrinks the **Solver Parameters** (*Excel Solver*) window to focus on selecting the location of the objective function on the spreadsheet.

The screenshot shows the Excel Solver Parameters window overlaid on a spreadsheet. The window is titled "Solver Parameters" and is currently focused on the "Set Objective" field, which is empty. The spreadsheet data is as follows:

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		0.00	0.00	0.00	0.00				
4		3200	3600	3000	0.00				[obj function]
5		0.3	0.2	0.15	0	>=	6000		[cocoa liquor]
6		0.3	0.43	0.5	0	>=	12000		[cocoa butter]
7		0.35	0.3	0.25	0	>=	9000		[low-fat cocoa powder]
8		0.05	0.07	0.1	0	>=	2500		[fat-free cocoa powder]
9		1	0	0	0	<=	20000		[Brazil supply]
10		0	1	0	0	<=	10000		[Ghana supply]
11		0	0	1	0	<=	15000		[Mexico supply]
12		0.75	-0.25	-0.25	0	>=	0		[diversification 1]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 2]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 3]
15		1	0	0	0	>=	0		[variable type 1]
16		0	1	0	0	>=	0		[variable type 2]
17		0	0	1	0	>=	0		[variable type 3]



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On the spreadsheet click on the cell that holds the computed objective function value.
Confirm in the *Solver* window that the correct cell reference is in place.

The screenshot shows the Excel Solver Parameters dialog box open over a spreadsheet. The dialog box title is "Solver Parameters" and the objective function cell reference is set to "\$E\$4". The spreadsheet contains the following data:

	Brazil	Ghana	Mexico			
1						
2	x1	x2	x3			
3	0.00	0.00	0.00	0.00		
4	3200	3600	3000	0.00		[obj function]
5	0.3	0.2	0.15	0	>=	6000 [cocoa liquor]
6	0.3	0.43	0.5	0	>=	12000 [cocoa butter]
7	0.35	0.3	0.25	0	>=	9000 [low-fat cocoa powder]
8	0.05	0.07	0.1	0	>=	2500 [fat-free cocoa powder]
9	1	0	0	0	<=	20000 [Brazil supply]
10	0	1	0	0	<=	10000 [Ghana supply]
11	0	0	1	0	<=	15000 [Mexico supply]
12	0.75	-0.25	-0.25	0	>=	0 [diversification 1]
13	-0.25	0.75	-0.25	0	>=	0 [diversification 2]
14	-0.25	-0.25	0.75	0	>=	0 [diversification 3]
15	1	0	0	0	>=	0 [variable type 1]
16	0	1	0	0	>=	0 [variable type 2]
17	0	0	1	0	>=	0 [variable type 3]



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Click on the tab on the objective function textbox to return to the main Solver window.

The screenshot shows the Excel Solver Parameters dialog box open over a spreadsheet. The dialog box title is "Solver Parameters" and it has a tab labeled "SES4". A red arrow points to the "Objective Function Range" tab in the dialog box. The spreadsheet data is as follows:

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		0.00	0.00	0.00	0.00				
4		3200	3600	3000	0.00				[obj function]
5		0.3	0.2	0.15	0	>=	6000		[cocoa liquor]
6		0.3	0.43	0.5	0	>=	12000		[cocoa butter]
7		0.35	0.3	0.25	0	>=	9000		[low-fat cocoa powder]
8		0.05	0.07	0.1	0	>=	2500		[fat-free cocoa powder]
9		1	0	0	0	<=	20000		[Brazil supply]
10		0	1	0	0	<=	10000		[Ghana supply]
11		0	0	1	0	<=	15000		[Mexico supply]
12		0.75	-0.25	-0.25	0	>=	0		[diversification 1]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 2]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 3]
15		1	0	0	0	>=	0		[variable type 1]
16		0	1	0	0	>=	0		[variable type 2]
17		0	0	1	0	>=	0		[variable type 3]



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The full **Solver Parameters** window reopens.

The screenshot shows the Excel Solver Parameters dialog box overlaid on a spreadsheet. The spreadsheet has columns A and B, and rows 1 through 17. The Solver Parameters dialog box is open, showing the following settings:

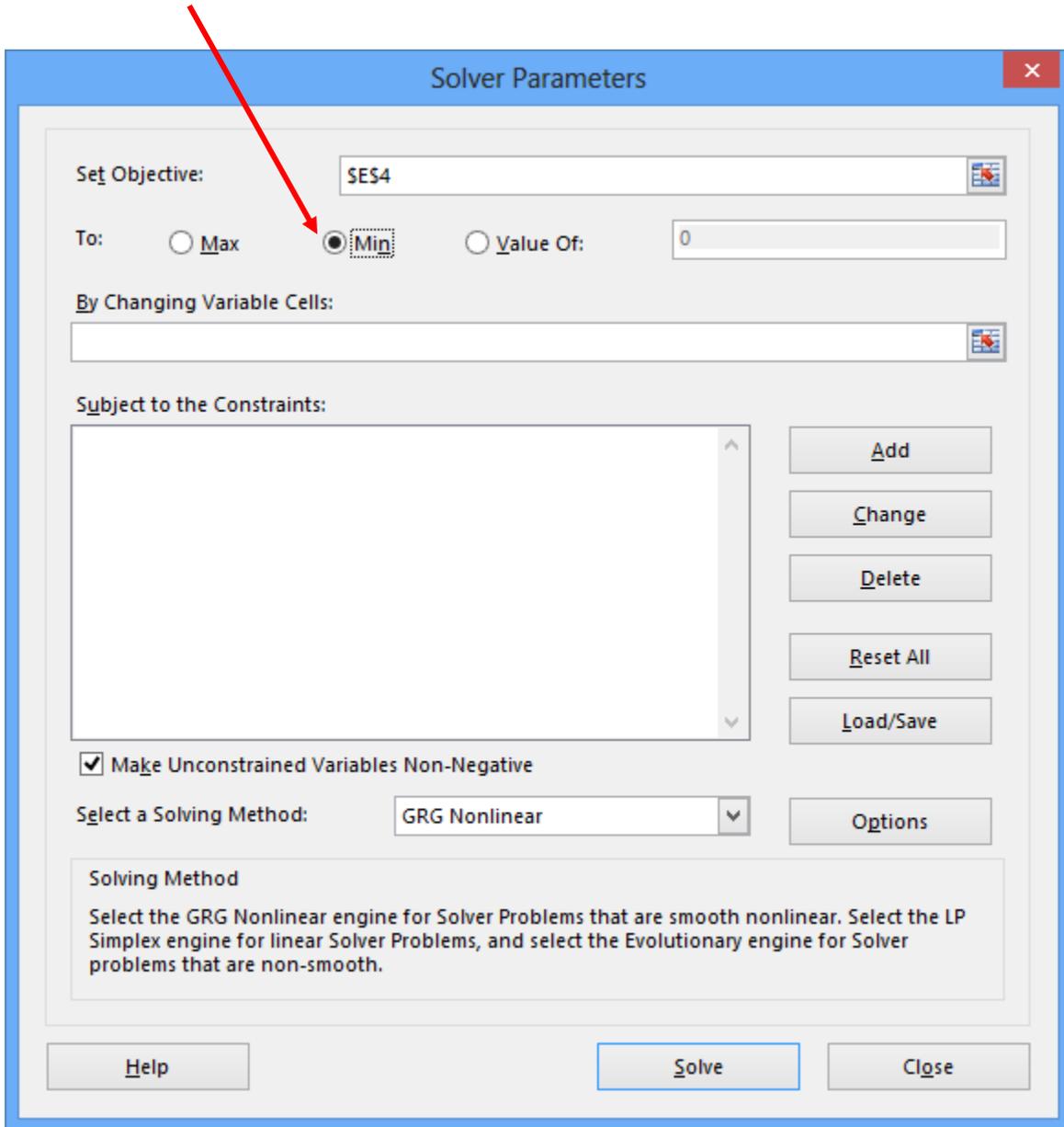
- Set Objective:** SES4
- To:** Max Min Value Of: 0
- By Changing Variable Cells:** [empty]
- Subject to the Constraints:** [empty]
- Make Unconstrained Variables Non-Negative
- Select a Solving Method:** GRG Nonlinear
- Solving Method:** Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
- Buttons: Add, Change, Delete, Reset All, Load/Save, Options, Help, Solve, Close

	A	B
1		Brazil
2		x1
3		0.00
4		3200
5		0.3
6		0.3
7		0.35
8		0.05
9		1
10		0
11		0
12		0.75
13		-0.25
14		-0.25
15		1
16		0
17		0



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The objective of this optimization model is to minimize costs.
Therefore under **To:** select the **Min** option.

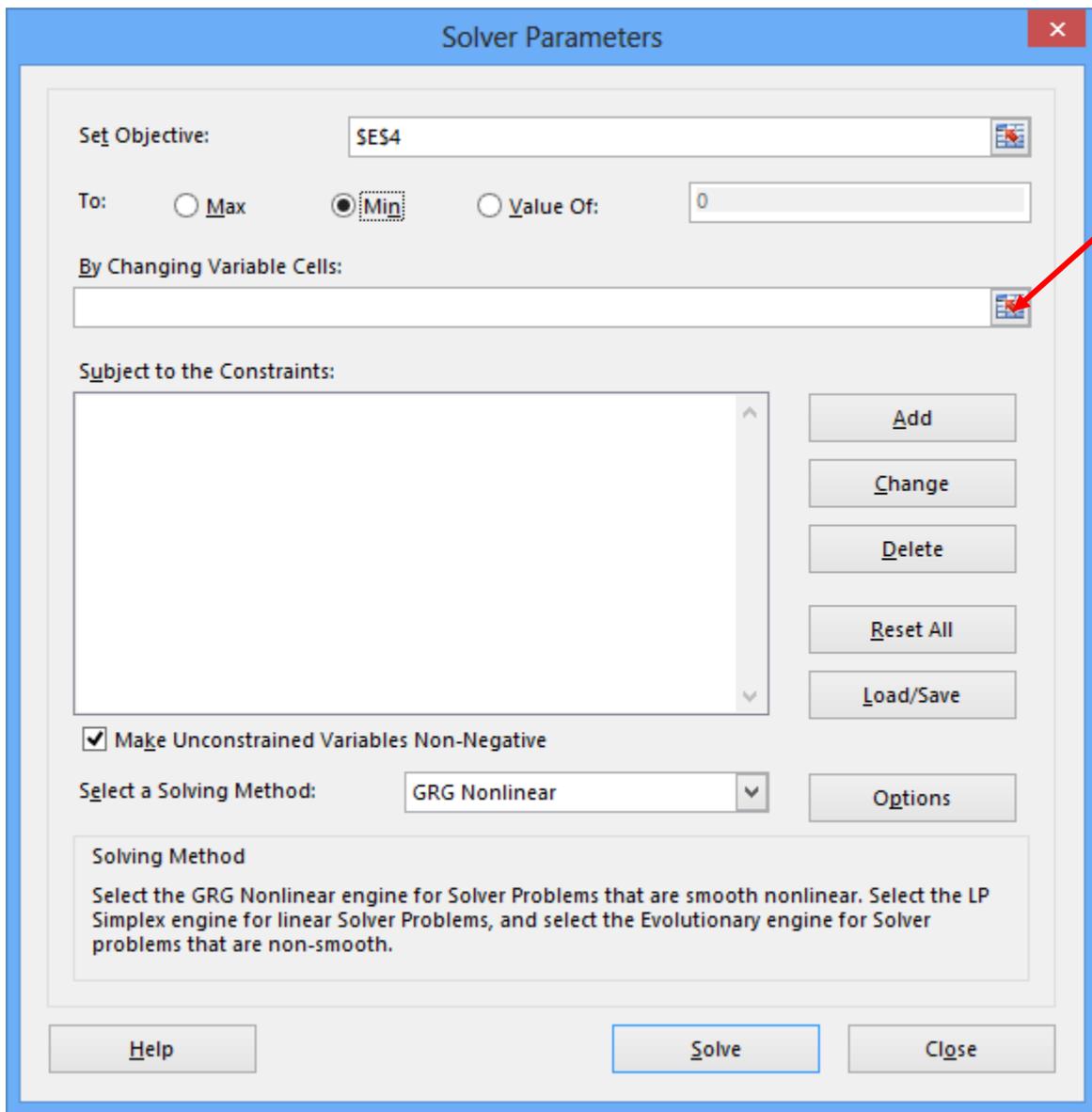




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In order to achieve the objective function (minimization of costs) the improving search algorithm shall change the decision variables values repetitively until the optimal combination of decision variable (or optimal solution) is found.

Under **By Changing Variable Cells:** click on the tab to the right of the textbox.





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On the spreadsheet, select the range of cells (“block of cells”) that contain all of the decision variables.

The screenshot shows the Excel Solver Parameters dialog box open over a spreadsheet. The dialog box title is "Solver Parameters" and the target cell is "\$B\$3:\$D\$3". The spreadsheet data is as follows:

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		0.00	0.00	0.00	0.00				
4		3200	3600	3000	0.00				
5		0.3	0.2	0.15	0	>=	6000		[obj function]
6		0.3	0.43	0.5	0	>=	12000		[cocoa liquor]
7		0.35	0.3	0.25	0	>=	9000		[cocoa butter]
8		0.05	0.07	0.1	0	>=	2500		[low-fat cocoa powder]
9		1	0	0	0	<=	20000		[fat-free cocoa powder]
10		0	1	0	0	<=	10000		[Brazil supply]
11		0	0	1	0	<=	15000		[Ghana supply]
12		0.75	-0.25	-0.25	0	>=	0		[Mexico supply]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 1]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 2]
15		1	0	0	0	>=	0		[diversification 3]
16		0	1	0	0	>=	0		[variable type 1]
17		0	0	1	0	>=	0		[variable type 2]
18									[variable type 3]



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Click on the tab to the right of the **By Changing Variable Cells:** textbox to return to the main Solver window.

The screenshot shows the Excel Solver Parameters dialog box overlaid on a spreadsheet. The dialog box title is "Solver Parameters" and the "By Changing Variable Cells:" field contains "\$B\$3:\$D\$3". A red arrow points to a small icon in the bottom right corner of the dialog box. The spreadsheet below shows a linear programming problem with the following data:

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		0.00	0.00	0.00	0.00				
4		3200	3600	3000	0.00				[obj function]
5		0.3	0.2	0.15	0	>=	6000		[cocoa liquor]
6		0.3	0.43	0.5	0	>=	12000		[cocoa butter]
7		0.35	0.3	0.25	0	>=	9000		[low-fat cocoa powder]
8		0.05	0.07	0.1	0	>=	2500		[fat-free cocoa powder]
9		1	0	0	0	<=	20000		[Brazil supply]
10		0	1	0	0	<=	10000		[Ghana supply]
11		0	0	1	0	<=	15000		[Mexico supply]
12		0.75	-0.25	-0.25	0	>=	0		[diversification 1]
13		-0.25	0.75	-0.25	0	>=	0		[diversification 2]
14		-0.25	-0.25	0.75	0	>=	0		[diversification 3]
15		1	0	0	0	>=	0		[variable type 1]
16		0	1	0	0	>=	0		[variable type 2]
17		0	0	1	0	>=	0		[variable type 3]



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Solver Parameters [X]

Set Objective: [icon]

To: Max Min Value Of:

By Changing Variable Cells:
 [icon]

Subject to the Constraints:

Make Unconstrained Variables Non-Negative

Select a Solving Method: [v]

Solving Method

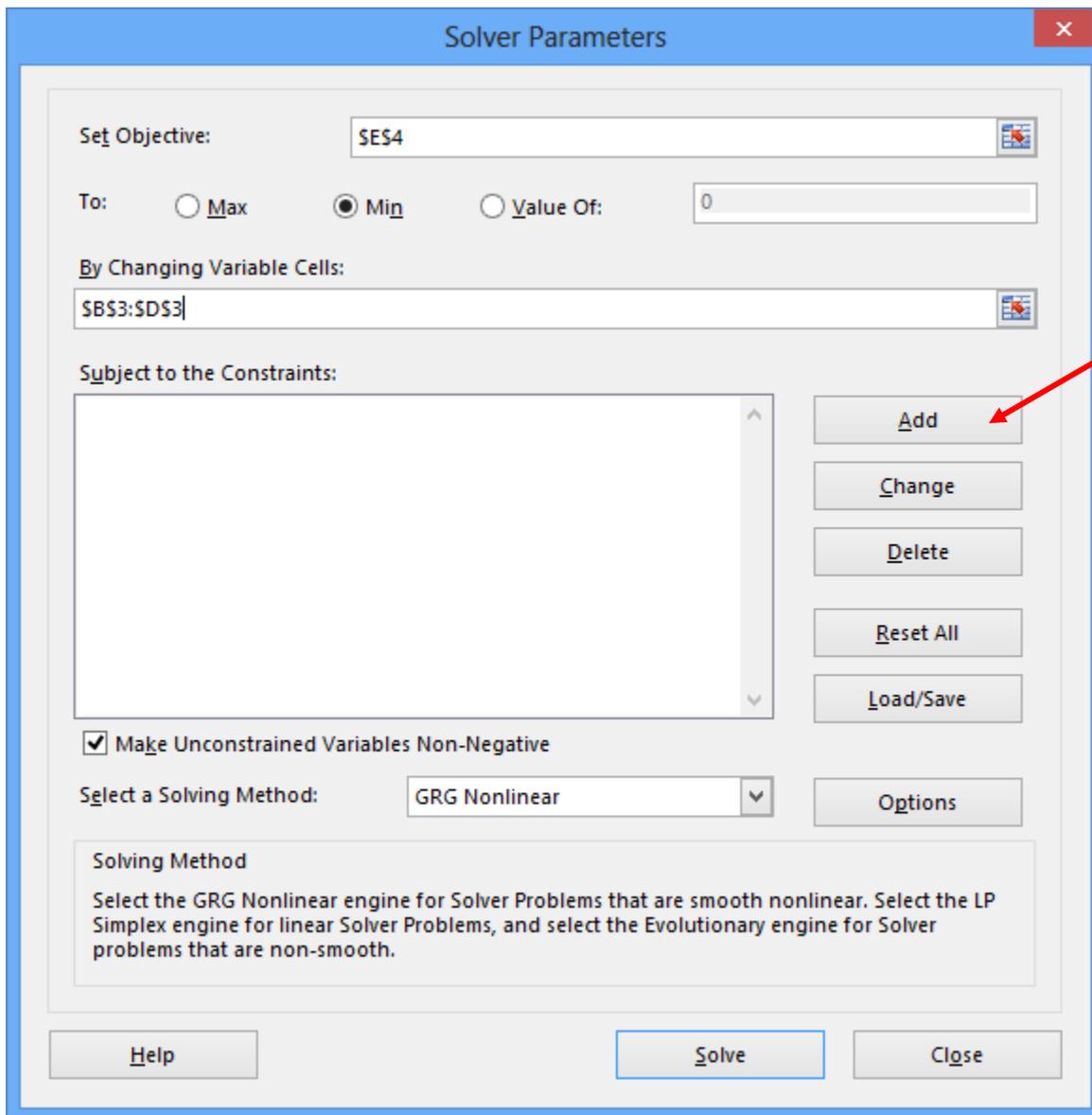
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.



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We shall now add the constraints.

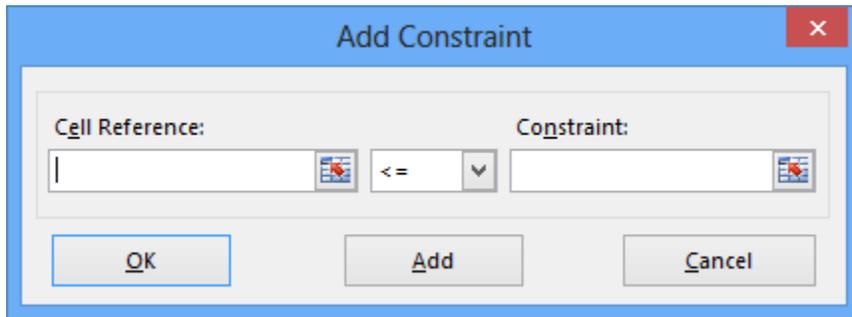
Click on the **Add** button.



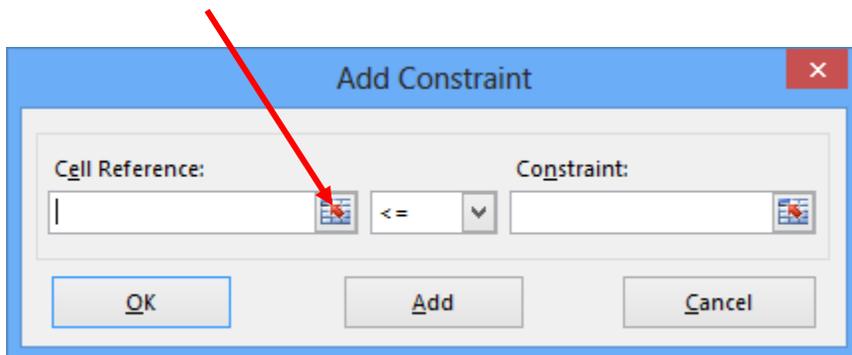


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This opens the **Add Constraint** dialogue window.



Under **Cell Reference:** click on the tab to the right of the textbox.





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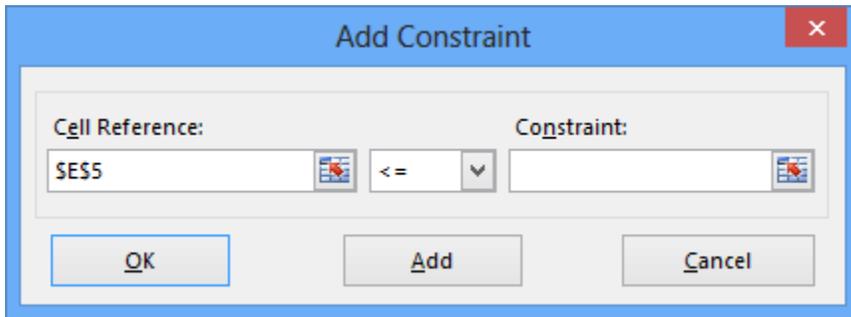
On the spreadsheet click on the cell holding the computed LHS of the first constraint.

	Brazil	Ghana	Mexico			
1						
2	x1	x2	x3			
3	0.00	0.00	0.00	0.00		
4	3200	3600	3000	0.00		[obj function]
5	0.3	0.2	0.15	0	>=	6000 [cocoa liquor]
6	0.3	0.43	0.5	0	>=	12000 [cocoa butter]
7	0.35	0.3	0.25	0	>=	9000 [low-fat cocoa powder]
8	0.05	0.07	0.1	0	>=	2500 [fat-free cocoa powder]
9	1	0	0	0	<=	20000 [Brazil supply]
10	0	1	0	0	<=	10000 [Ghana supply]
11	0	0	1	0	<=	15000 [Mexico supply]
12	0.75	-0.25	-0.25	0	>=	0 [diversification 1]
13	-0.25	0.75	-0.25	0	>=	0 [diversification 2]
14	-0.25	-0.25	0.75	0	>=	0 [diversification 3]
15	1	0	0	0	>=	0 [variable type 1]
16	0	1	0	0	>=	0 [variable type 2]
17	0	0	1	0	>=	0 [variable type 3]



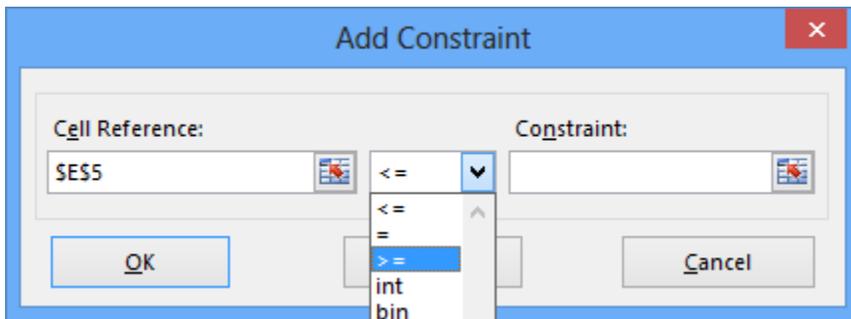
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Click on the tab to the right of the textbox to return to the main **Add Constraint** window.



Select the applicable equality or inequality condition for this constraint.

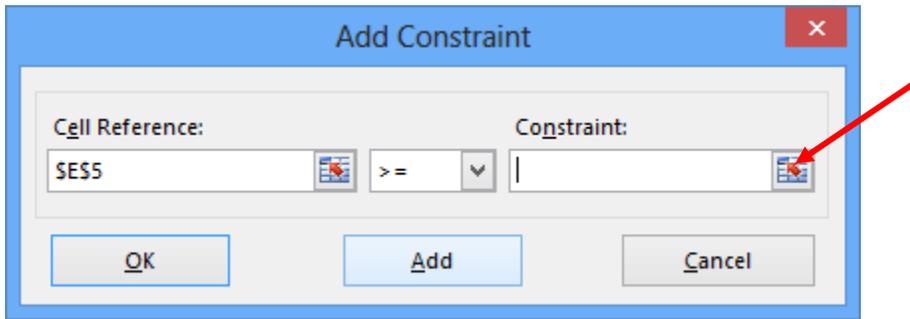
For the cocoa liquor constraint it is “greater than or equal to”.



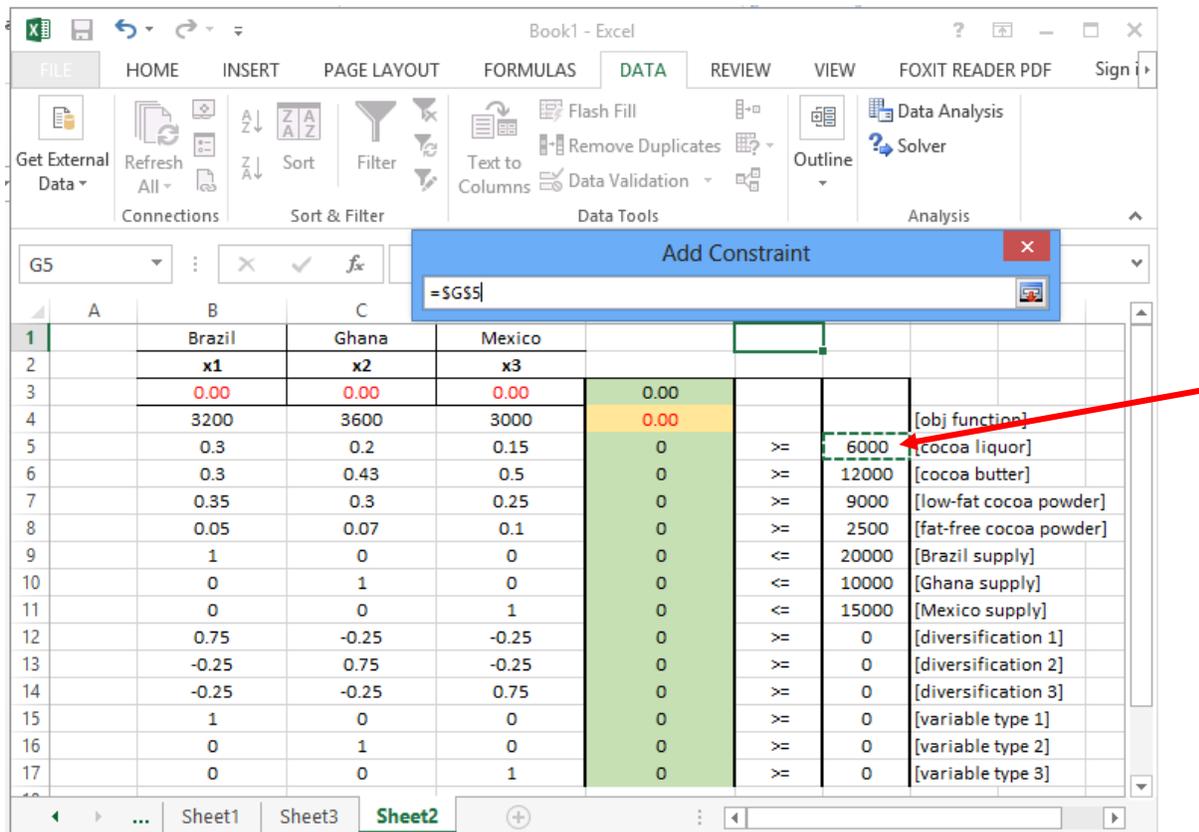


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Under **Constraint**: click on the tab to the right of the textbox.



On the spreadsheet click on the cell holding the RHS value of the first constraint.





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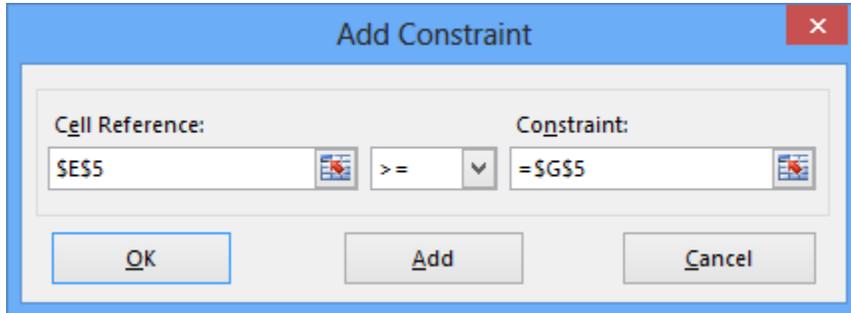
Click on the tab to the right of the textbox to return to the main **Add Constraint** window.

The screenshot shows the Excel 'Add Constraint' dialog box. The spreadsheet data is as follows:

	Brazil	Ghana	Mexico			
1						
2	x1	x2	x3			
3	0.00	0.00	0.00	0.00		
4	3200	3600	3000	0.00		[obj function]
5	0.3	0.2	0.15	0	>=	6000 [cocoa liquor]
6	0.3	0.43	0.5	0	>=	12000 [cocoa butter]
7	0.35	0.3	0.25	0	>=	9000 [low-fat cocoa powder]
8	0.05	0.07	0.1	0	>=	2500 [fat-free cocoa powder]
9	1	0	0	0	<=	20000 [Brazil supply]
10	0	1	0	0	<=	10000 [Ghana supply]
11	0	0	1	0	<=	15000 [Mexico supply]
12	0.75	-0.25	-0.25	0	>=	0 [diversification 1]
13	-0.25	0.75	-0.25	0	>=	0 [diversification 2]
14	-0.25	-0.25	0.75	0	>=	0 [diversification 3]
15	1	0	0	0	>=	0 [variable type 1]
16	0	1	0	0	>=	0 [variable type 2]
17	0	0	1	0	>=	0 [variable type 3]



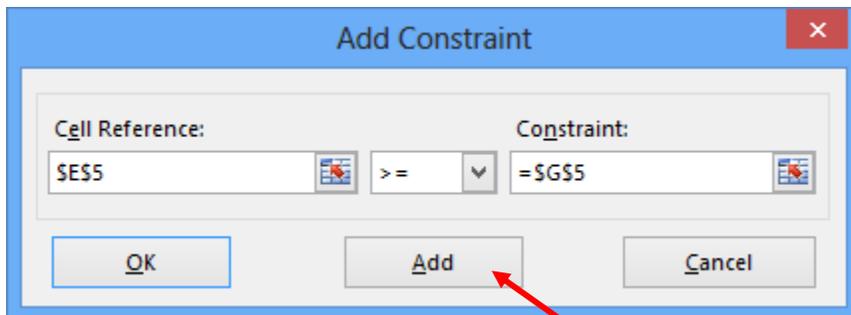
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From here we may choose one of two options.

We may click on **OK** to return to the main *Solver* window, or we may click on **Add** to enter data for the next constraint.

Click on **Add** to enter the next constraint.





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Repeat the process to add all remaining constraints.

After entering the last constraint, click on **OK** to return to the main *Solver* window.

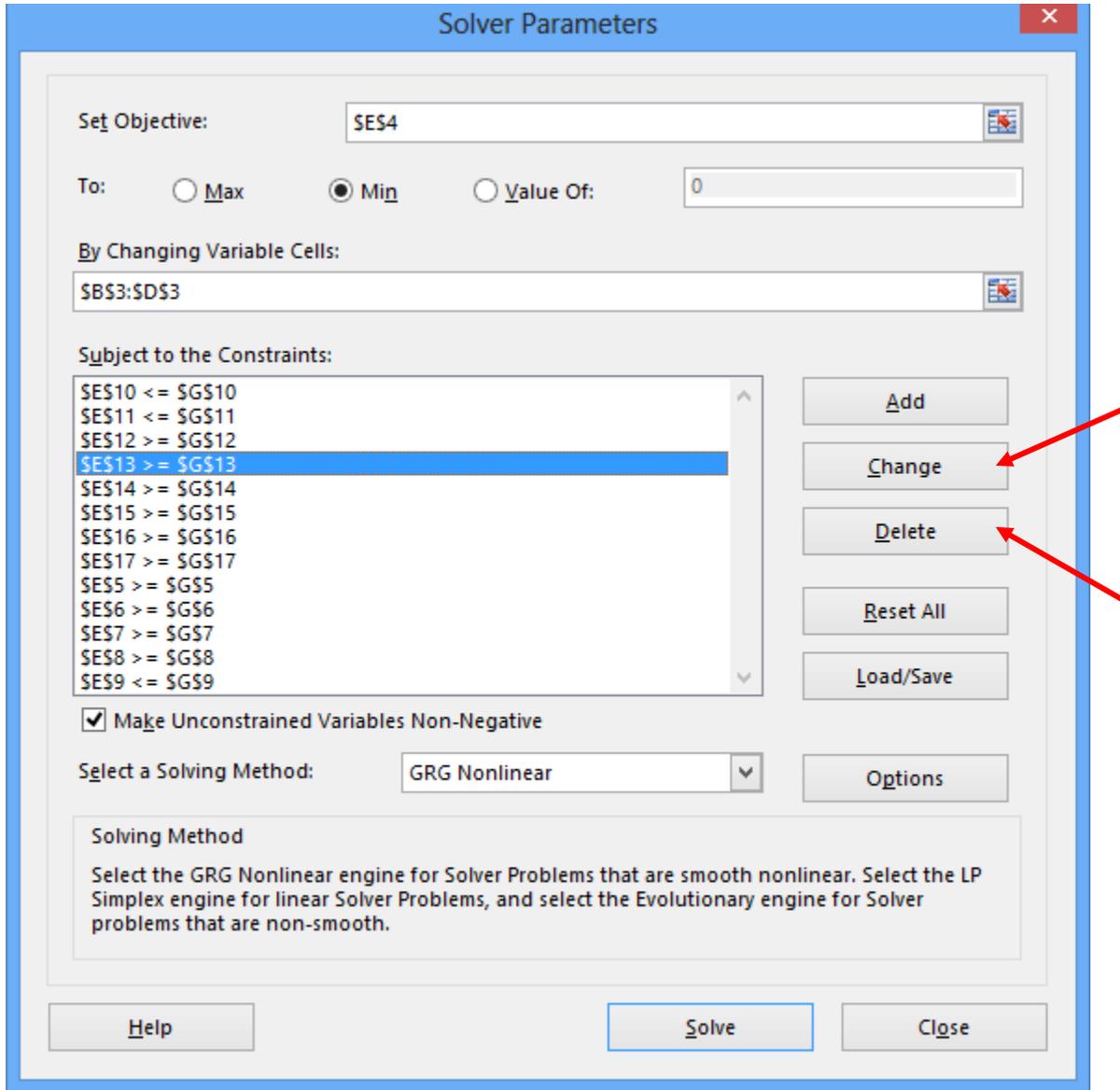
The screenshot shows the 'Add Constraint' dialog box in Microsoft Excel. The 'Cell Reference' is set to '\$E\$17' and the 'Constraint' is set to '>= =\$G\$17'. The 'OK' button is highlighted with a red arrow. The background spreadsheet shows a linear programming problem with the following data:

	Brazil	Ghana	Mexico			
1						
2	x1	x2	x3			
3	0.00	0.00	0.00	0.00		
4	3200	3600	3000	0.00		[obj function]
5	0.3	0.2	0.15	0	>=	6000 [cocoa liquor]
6	0.3	0.43	0.5	0	>=	12000 [cocoa butter]
7	0.35	0.3	0.25	0	>=	9000 [low-fat cocoa powder]
8	0.05	0.07	0.1	0	>=	2500 [fat-free cocoa powder]
9	1	0	0	0	<=	20000 [Brazil supply]
10	0	1	0	0	<=	10000 [Ghana supply]
11	0	0	1	0	<=	15000 [Mexico supply]
12	0.75	-0.25	-0.25	0	>=	0 [diversification 1]
13	-0.25	0.75	-0.25	0	>=	0 [diversification 2]
14	-0.25	-0.25	0.75	0	>=	0 [diversification 3]
15	1	0	0	0	>=	0 [variable type 1]
16	0	1	0	0	>=	0 [variable type 2]
17	0	0	1	0	>=	0 [variable type 3]



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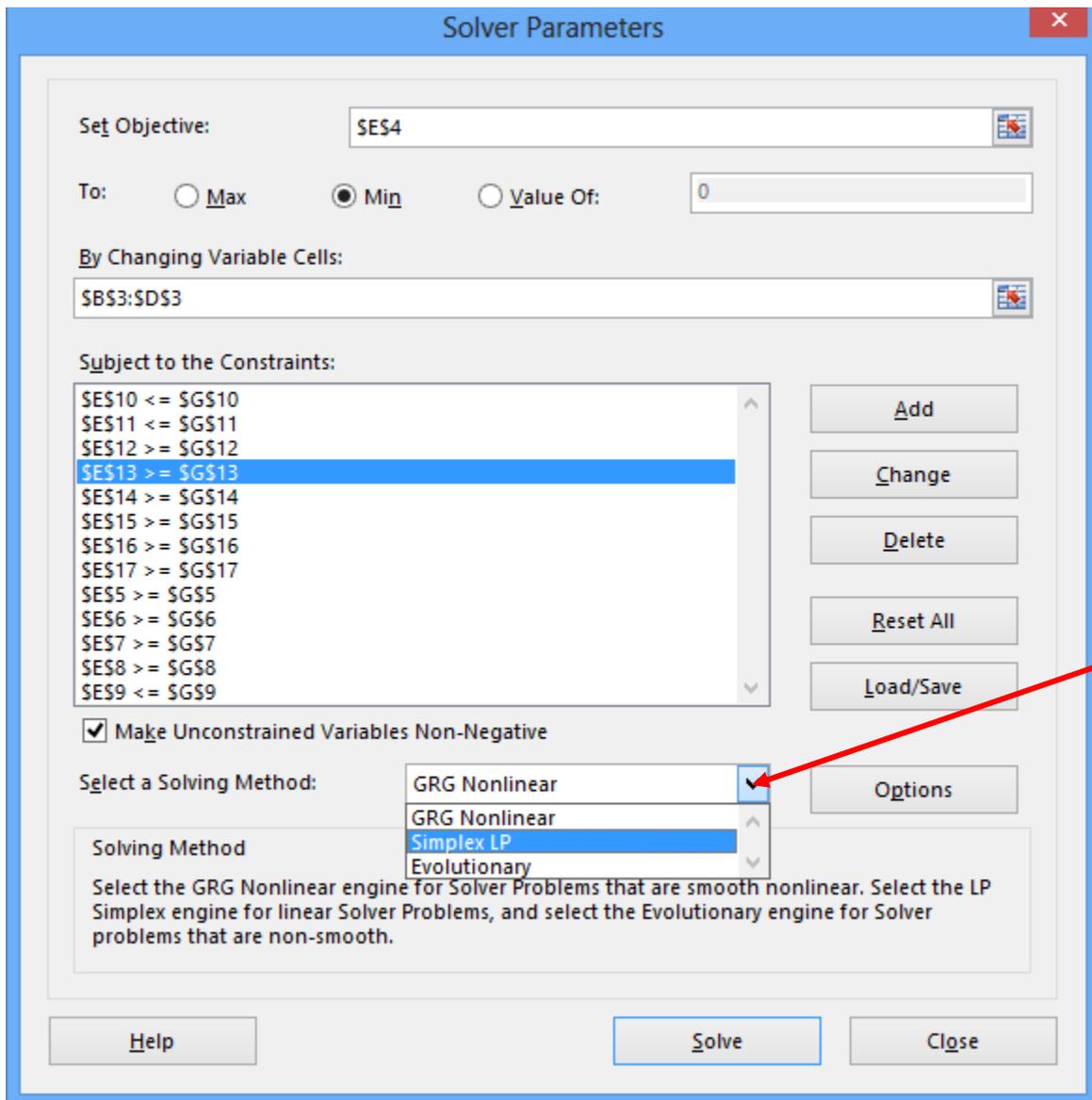
A constraint may be selected and then modified or deleted by clicking on the **Change** or **Delete** buttons respectively.





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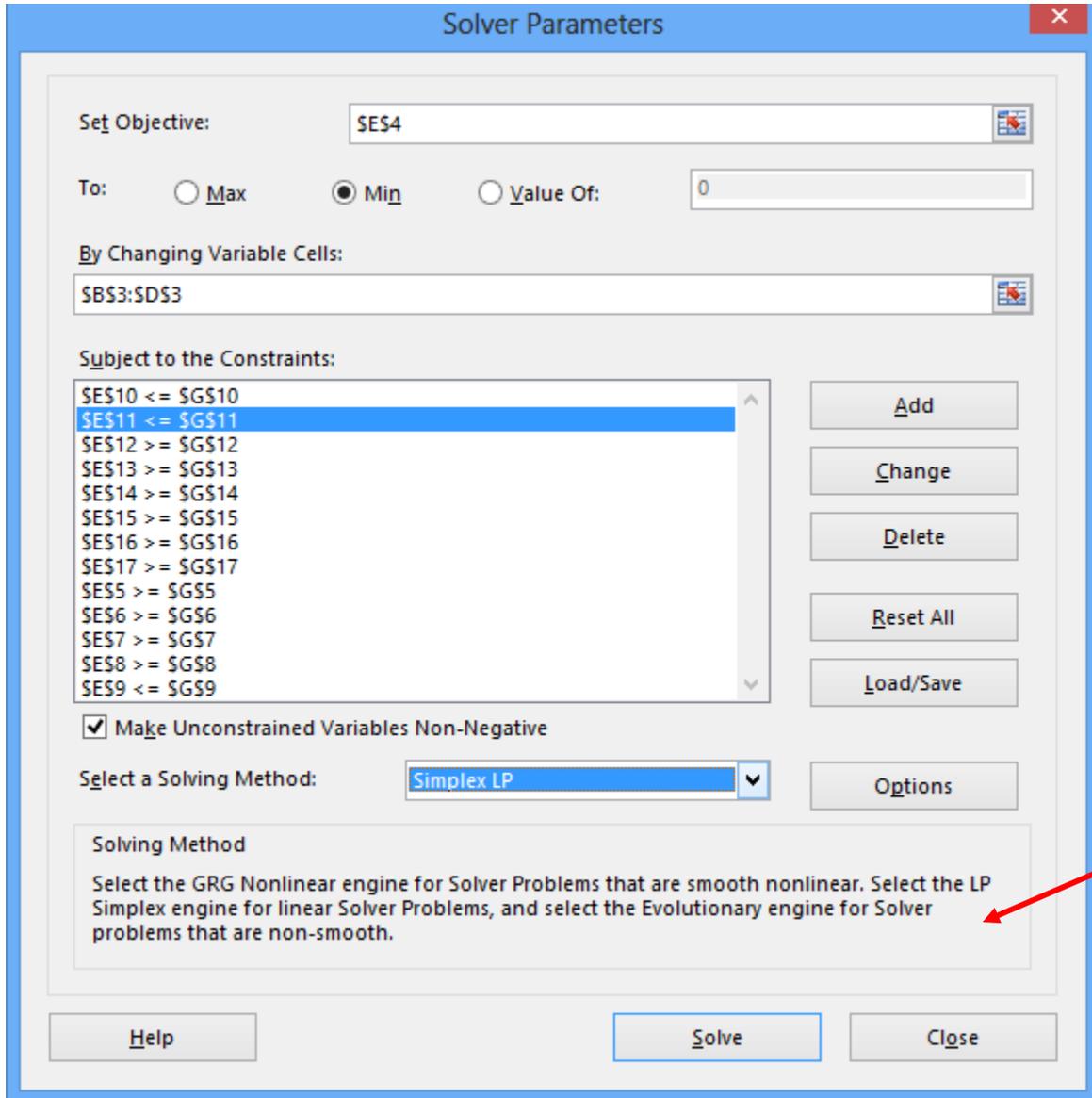
Under **Select a Solving Method**: select a method from the drop down. As mentioned in the description of the cocoa processing problem, this is a linear program, therefore the simplex algorithm is the relevant solving method.





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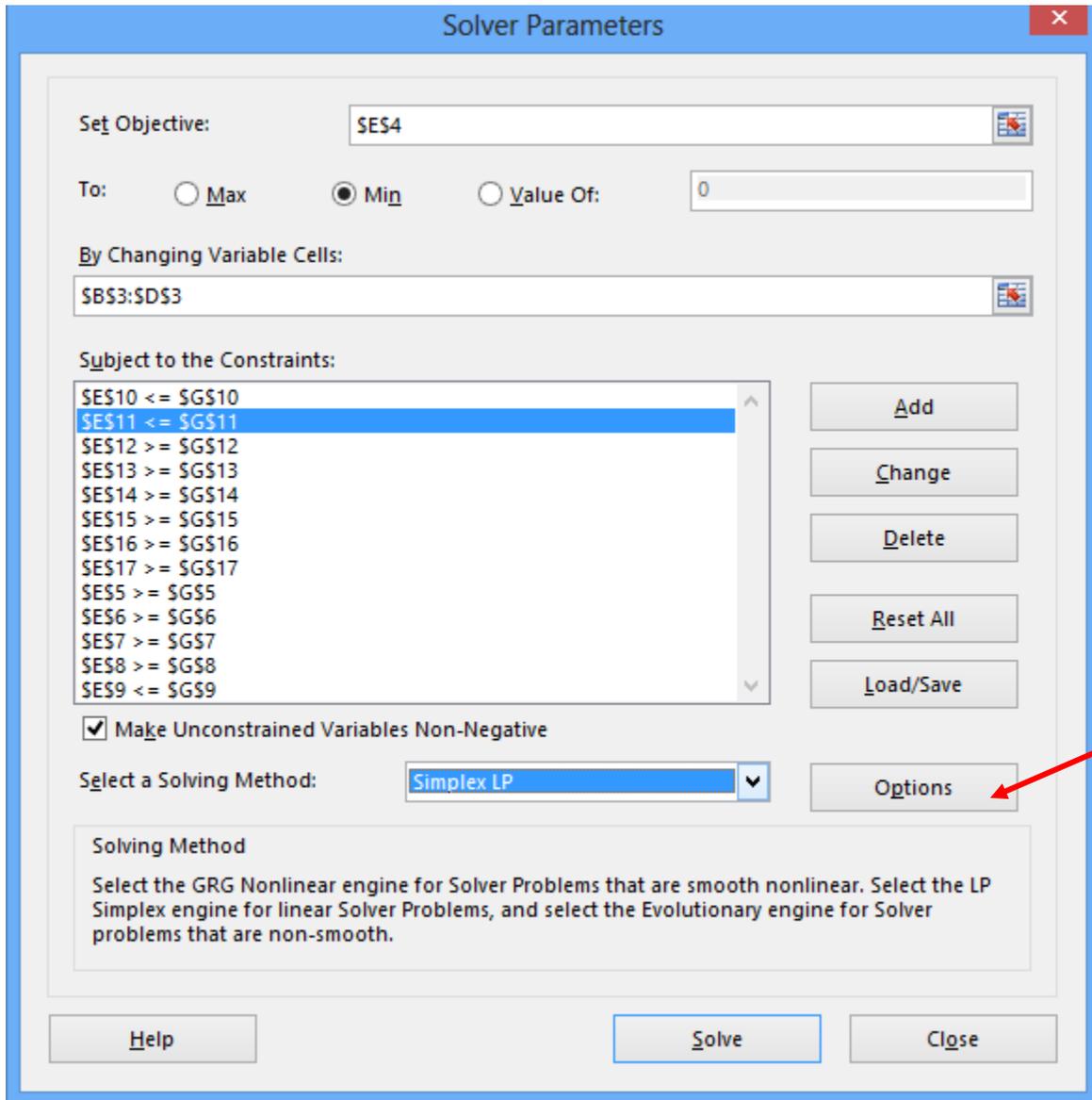
Note that the label below the **Select Solving Method:** pick list provides a brief description of each option and its applicability.





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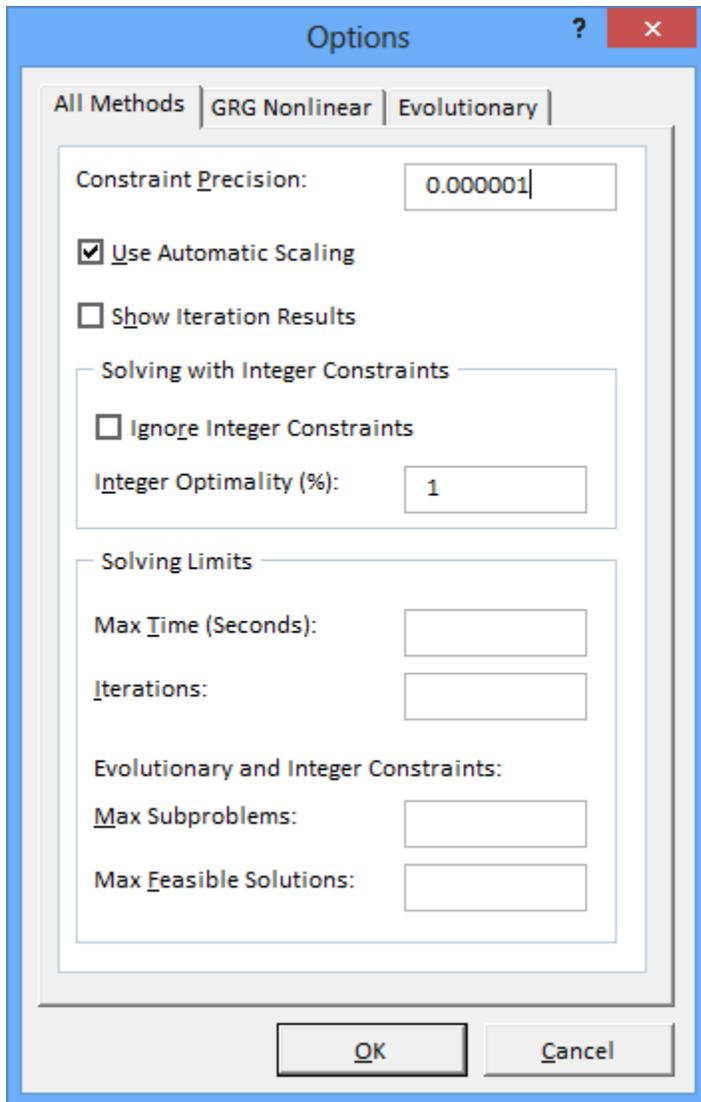
With **Simplex LP** selected, click on the **Options** button.





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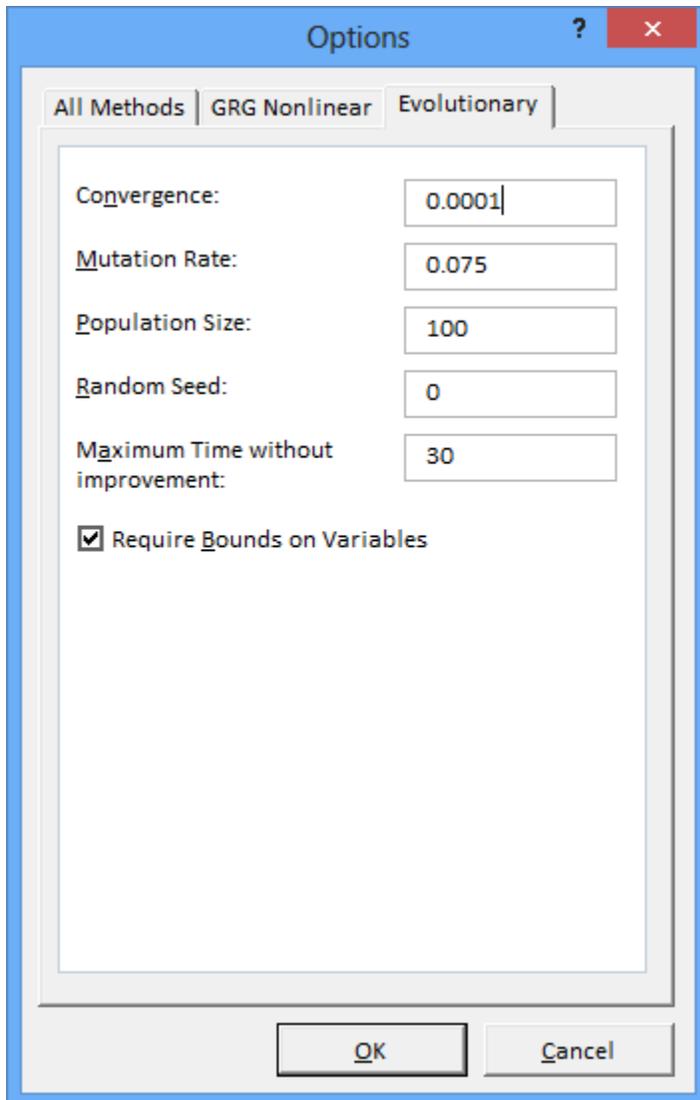
The **Options** window enables the user to change settings or parameters of the solving algorithm itself.





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For example a user seasoned in Evolutionary algorithms may decide to change the Mutation Rate or the Population Size.

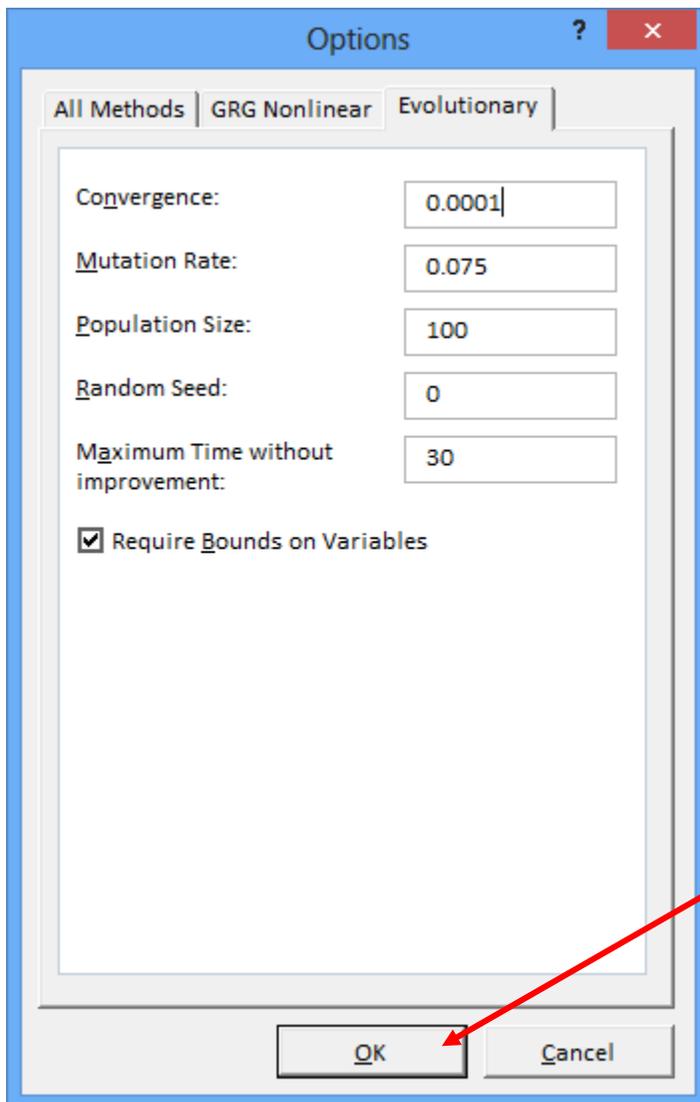




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In this example we shall leave all default **Options** settings in place.

Click **OK** or **Cancel** to dismiss the **Options** window.

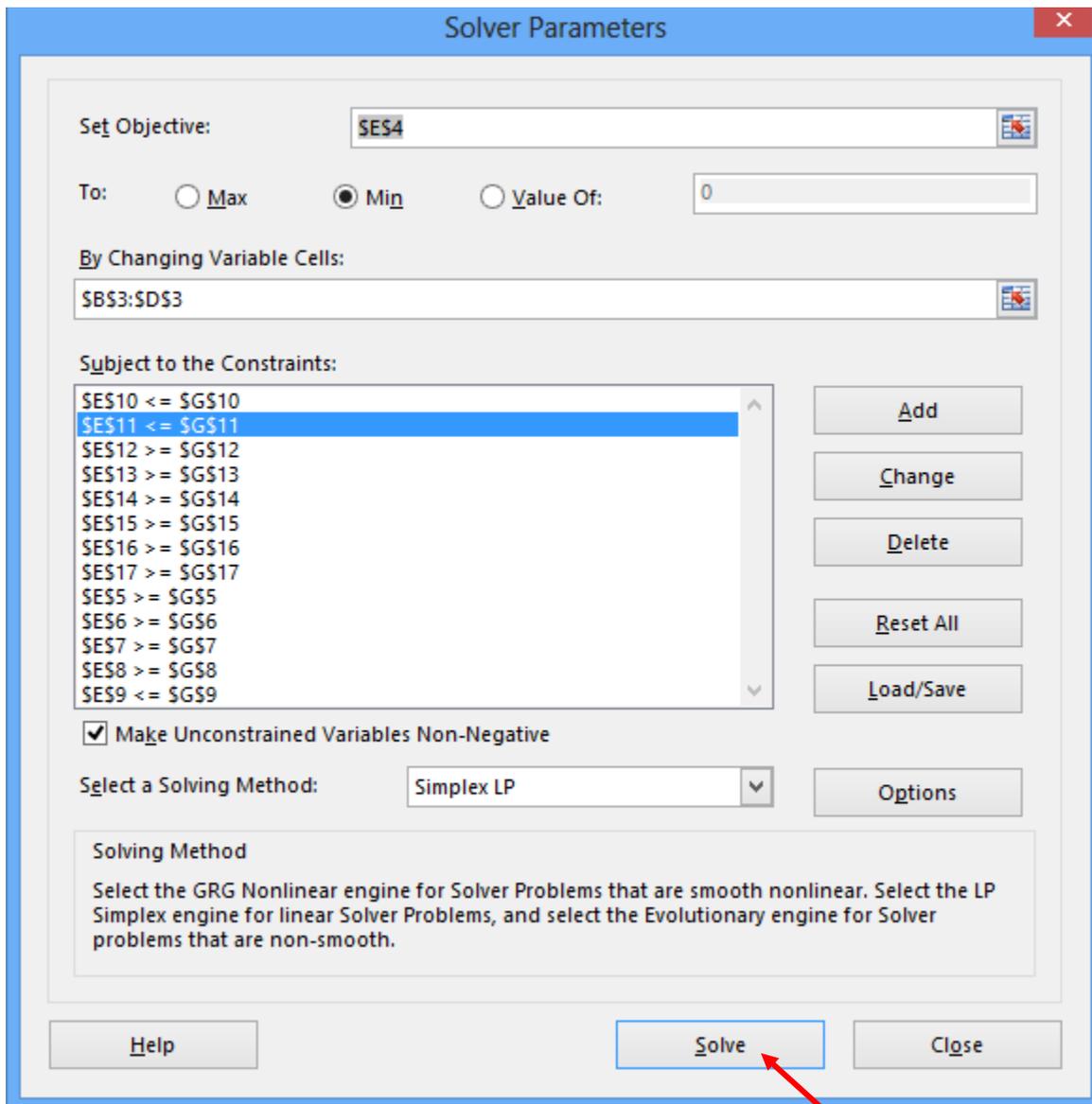




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We are now ready to solve the cocoa processing problem.

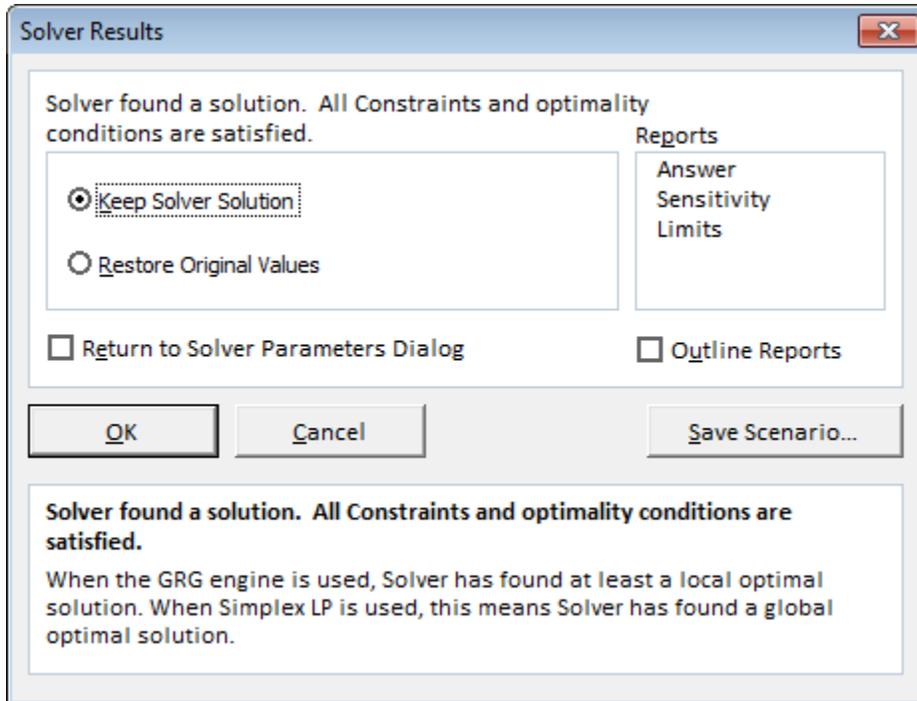
Press the **Solve** button to start the algorithm.





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Once an optimal solution has been found the **Solver Results** message notifies the user that a solution meeting all constraints and optimality conditions has been found.



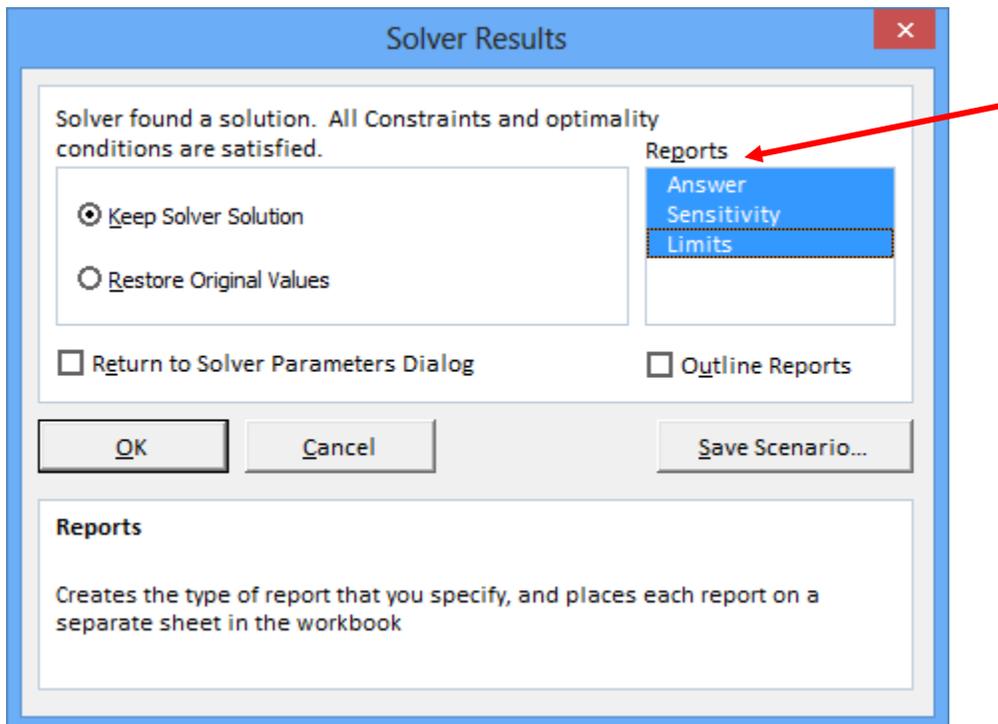
If a feasible solution could not be found the message notifies the user to that effect and provides general suggestions for the user to review and address, for example, checking or adding variable-type constraints etc.

In the **Reports** panel, the user may select any of the reports concerning the performance of the algorithm.



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For demonstrative purposes, select all three reports by clicking on each one.



Hit **OK** to dismiss the **Solver Results** message.



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Notice the reports created in separate worksheets.

	Brazil	Ghana	Mexico				
1							
2	x1	x2	x3				
3	7884.62	8653.85	15000.00	31538.46			
4	3200	3600	3000	101384615.38			[obj function]
5	0.3	0.2	0.15	6346	>=	6000	[cocoa liquor]
6	0.3	0.43	0.5	13587	>=	12000	[cocoa butter]
7	0.35	0.3	0.25	9106	>=	9000	[low-fat cocoa powder]
8	0.05	0.07	0.1	2500	>=	2500	[fat-free cocoa powder]
9	1	0	0	7885	<=	20000	[Brazil supply]
10	0	1	0	8654	<=	10000	[Ghana supply]
11	0	0	1	15000	<=	15000	[Mexico supply]
12	0.75	-0.25	-0.25	0	>=	0	[diversification 1]
13	-0.25	0.75	-0.25	769	>=	0	[diversification 2]
14	-0.25	-0.25	0.75	7115	>=	0	[diversification 3]
15	1	0	0	7885	>=	0	[variable type 1]
16	0	1	0	8654	>=	0	[variable type 2]
17	0	0	1	15000	>=	0	[variable type 3]

(Your reports number suffixes may differ if you have previously used *Excel Solver* on your computer).



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Click on each report to view its contents.

The **Answer Report**.

The screenshot shows the Microsoft Excel interface with the Solver Options and Answer Report 2 displayed. The Solver Options are as follows:

Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
\$E\$4		0.00	101384615.38

Variable Cells

Cell	Name	Original Value	Final Value	Integer
\$B\$3	x1	0.00	7884.62	Contin
\$C\$3	x2	0.00	8653.85	Contin
\$D\$3	x3	0.00	15000.00	Contin

The taskbar at the bottom shows the following reports: Answer Report 2 (selected), Sensitivity Report 2, and a plus sign to expand more reports. The status bar at the bottom indicates 'READY' and '100%' zoom.



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The **Sensitivity Report**.

The screenshot shows the Microsoft Excel interface with the 'DATA' ribbon selected. The active window is 'Book1 - Excel'. The 'Sensitivity Report' is displayed, showing the following information:

Worksheet: [Book1.xlsx]Sheet2
Report Created: 12/31/2017 9:22:22 AM

Variable Cells

Cell Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$3 x1	7884.615385	0	3200	1E+30	628.5714286
\$C\$3 x2	8653.846154	0	3600	880	1645.714286
\$D\$3 x3	15000	0	3000	2215.384615	1E+30

Constraints

Cell Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
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The Limits Report.

The screenshot shows an Excel spreadsheet titled 'Microsoft Excel 15.0 Limits Report'. The report is structured as follows:

1 Microsoft Excel 15.0 Limits Report
2 Worksheet: [Book1.xlsx]Sheet2
3 Report Created: 12/31/2017 9:22:22 AM

Objective		
Cell	Name	Value
\$E\$4		101384615.38

Variable			Lower	Objective	Upper	Objective
Cell	Name	Value	Limit	Result	Limit	Result
\$B\$3	x1	7884.62	7884.62	101384615.38	10961.54	111230769.23
\$C\$3	x2	8653.85	8653.85	101384615.38	8653.85	101384615.38
\$D\$3	x3	15000.00	15000.00	101384615.38	15000.00	101384615.38



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Return to your optimization model worksheet.

	A	B	C	D	E	F	G	H	I
1		Brazil	Ghana	Mexico					
2		x1	x2	x3					
3		7884.62	8653.85	15000.00	31538.46				
4		3200	3600	3000	101384615.38			[obj function]	
5		0.3	0.2	0.15	6346	>=	6000	[cocoa liquor]	
6		0.3	0.43	0.5	13587	>=	12000	[cocoa butter]	
7		0.35	0.3	0.25	9106	>=	9000	[low-fat cocoa powder]	
8		0.05	0.07	0.1	2500	>=	2500	[fat-free cocoa powder]	
9		1	0	0	7885	<=	20000	[Brazil supply]	
10		0	1	0	8654	<=	10000	[Ghana supply]	
11		0	0	1	15000	<=	15000	[Mexico supply]	
12		0.75	-0.25	-0.25	0	>=	0	[diversification 1]	
13		-0.25	0.75	-0.25	769	>=	0	[diversification 2]	
14		-0.25	-0.25	0.75	7115	>=	0	[diversification 3]	
15		1	0	0	7885	>=	0	[variable type 1]	
16		0	1	0	8654	>=	0	[variable type 2]	
17		0	0	1	15000	>=	0	[variable type 3]	

We see that the optimal cost is \$101,384,615.38, which is due to purchasing 7,884.62 tons of cocoa beans from Brazil, 8,653.85 tons from Ghana, and 15,000.00 tons from Mexico. We also see that this amounts to a total of 31,538.46 tons per week from our sources altogether. By inspecting the LHSs versus RHSs, we see that all constraints are indeed satisfied.

On your own, consider applying relevant number formats to your output table. For example, the objective function value is money. Also, consider adding the comma style for every three digits to the cocoa tonnage values to enhance readability.



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Successful completion!

Assuming operational circumstances change in the future, the user may simply update the table and re-run the algorithm for the new scenario. If changes involve new constraints, then the user will have to reopen the *Solver* window and make the necessary updates.

It is pertinent to note that *Excel Solver* can solve optimization problems with up to 200 decision variables and up to 200 constraints. Beyond these limits the user will have to use *Excel Macros*. Using *Macros* requires the user has a thorough understanding of the details of the algorithm, and then write code in the *Excel* programming language called *Visual Basic for Applications*, to implement the algorithm. This author can confirm *Excel's* capabilities in this regard having successfully programmed an evolutionary algorithm involving 15,000 plus variables, and 258 constraints. The program ran for 9 days to successful completion.

For a given optimization problem, *Excel Macros* may be the appropriate approach to solving the problem if it involves multiobjective optimization or stochastic optimization.

For further information on coding, participants in this course are recommended to review the four-part series on *Programming in Excel Visual Basic for Applications* by this author, available on SunCam.



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3. CONCLUSION

This course is part of a series on engineering methods in *Excel*. This course presented a broad overview and introduction of fundamental principles and engineering applications in the field of applied optimization, and how to use *Excel* tools to analyze and solve these types of problems.

We introduced ourselves to optimization problems and how they are used to make decisions in planning and production of engineering systems. We learned how to formulate such problems, and how to implement the formulation on an *Excel* spreadsheet. We learned how to use the *Excel Solver* to analyze the formulations and obtain the optimal solution.

This course has enabled participants to identify professional situations where the innovative application of techniques learned in this course are relevant and will be of benefit to the productivity, efficiency, and quality of the work output of a practitioner.

Practitioners are strongly encouraged to look out for situations in their domains of expertise where engineering methods in *Excel* are applicable and will be of benefit to their work and their organization.

A successful application of engineering methods in *Excel* requires a careful and meticulous approach, and can only be mastered and retained by practice and repetition.

It has been my utmost pleasure presenting this topic to you. Thank you.



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Images were all drawn/ prepared by Kwabena. Ofosu

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