A SunCam online continuing education course

External Ballistics Primer for Engineers

Part II: Test Equipment

by

Terry A. Willemin, P.E.

Introduction

In part one of this external ballistics course several factors were introduced which affect the flight of a ballistic projectile. Some basic equations were presented that are useful to a great many engineers outside of ballistics for making approximations with a fair bit of accuracy. Several flow regimes that a ballistic projectile may pass through were also treated, though there were cases that were omitted for either lack of space or complexity. Fast forward to part deux. Projectiles launched from earth generally move through air for most or all of their flight, which means understanding the properties of that air is very important. Besides air there are other flightpath modifiers including Coriolis' effects, so many times it is important to know the direction the projectile is headed in and its speed. The list of measurement equipment used in relation to external ballistics research is extensive, however there are tools which may be more noteworthy or common than others. The balance of this part of the full course delivers descriptions and general information of **some** of the equipment used to measure atmospheric air pressure, humidity, windspeed, wind direction, projectile speed and direction. There are also a few characteristic comparisons in table form appended to a handful of the sections.

The tools that follow are used throughout much of engineering and research rather than being confined to external ballistics, so it is likely you have used at least some of them in relation to your own work.

 Copyright© 2021 Terry A. Willemin, P.E.

MEMS and NEMS

Before jumping into macro-scale measurement devices let's talk about the secret of NEMS and MEMS. MEMS stands for Micro-Electro-Mechanical Systems and NEMS for Nano-…. These systems are micro-scale/nano-scale mechanical components working in conjunction with electronic components to create a micro-scale/nano-scale machine to perform various tasks or measure some phenomenon. Often, they are packaged as integrated circuits for use on a printed circuit board. There are MEMS/NEMS which act as gyroscopes, accelerometers, barometers, magnetometers, hygrometers, mechanical nozzles in ink-jet printers, tilt sensors, microphones, oscillators, and on and on. The need to mention this is driven by the proliferation of MEMS measurement devices. They are found in nearly every new car, cell phones, ink-jet printers, commercial aircraft, IoT devices, and so on. For the purposes of this paper, you can assume there is a MEMS available that will perform the function of the specific atmospheric characteristic measuring device/transducer being discussed. For reference, MEMS/NEMS accuracies can vary widely. The depth of our MEMS and NEMS discussion stops here at the kiddie pool though. All subsequent sections focus on macro-scale measurement equipment. Let's dive in.

Ballistic Chronograph

If you **feel the need, the need for** measuring the **speed** of a projectile, grab a ballistic or gun chronograph. GC's are used to measure the velocity of projectiles. Since at least the 18th century many methods have been devised to measure projectile velocity including, but not limited to: the rotating pair discs, optical chronograph, Doppler radar and the ballistic pendulum. Below we'll consider optical sensor and Doppler radar. A simple ballistic pendulum can be constructed at home and I gave it its own chapter, the final chapter of this course.

Optical Sensor Ballistic Chronometers

Because of simplicity, accuracy, and cost optical sensor chronometers are by far the most commonly used type of projectile chronograph in the world. They are capable tools for small firearms, archery, air guns, paintball guns, and might even work for airsoft guns, slingshot/wrist rocket projectiles, or even potato, rubber band, or marshmallow guns. I

 Copyright© 2021 Terry A. Willemin, P.E.

haven't tried any of the latter group though! The operational concept is very simple, there are two optical "screen" areas set a fixed distance from each other through which the projectile must pass. As the projectile passes through each screen the amount of light to the optical receivers inside the unit's housing, changes slightly. Passing through the first screen triggers a timer to start and passing through the second stops the timer. A velocity is then calculated from the fixed distance between the two sensors and the time the projectile took to traverse it. These chronometers more than adequately measure speeds ranging from as little as 22 ft/s (15 mph) to well over 7000 ft/s (~4800 mph) having accuracies to within 0.5% of the velocity. As with all measurement tools GC's have their shortcomings. A sunny day can be challenging to work in as the optical screens can be adversely affected by ambient light. Also, the projectile must pass through both relatively small screens, which means the distance between it and the gun, bow, etc. must be close enough to do so without risking damage to the chronometer such as by muzzle blast or a projectile colliding with any part of the chronometer. Not so useful if you want to know your velocity just prior to impact on a 100-yard shot unless you happen to be Annie Oakley, or Chris Kyle. The small size of the screens limits the size of projectiles which can pass through them too.

Copyright© 2021 Terry A. Willemin, P.E.

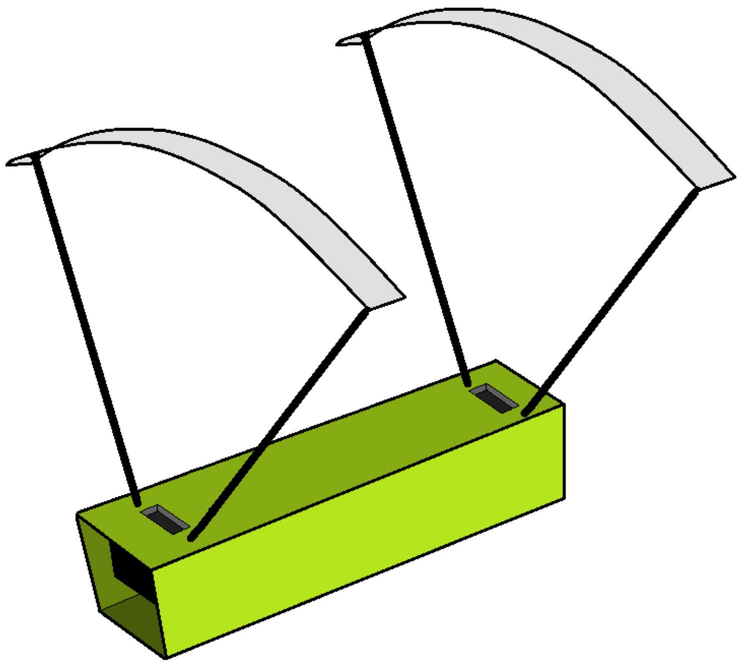


Figure 1, Representation of an electronic ballistic chronometer, or gun chronometer. The optical sensors are mounted just inside the holes in the top of the unit. Each screen is formed by two rods supporting an arc shaped diffuser.

Doppler Radar [(15)]

Until very recently, because of cost, doppler radar systems have not been common in smaller-scale ballistics testing. Units are available now for less than $600US (at the time of writing). See **Figure 2** for an example. The serious engineer or ballistician will rely on doppler radar systems over all others when characterizing a bullet's flight. Doppler radar exploits the Doppler principle which states that a transmitted signal (a waveform with some given frequency) reflected from a projectile will be frequency-shifted by some value depending on its velocity vector relative to the receiver. If the projectile is approaching the receiver, the wave reflected off it will be "compressed", i.e. there is a wavelength decrease (or a frequency increase), whereas a retreating projectile will reflect a lower frequency wave. Acoustically, we've all experienced this phenomenon. As a speeding car approaches, the sound we hear is higher pitched than if the car were sitting next to us

 Copyright© 2021 Terry A. Willemin, P.E.

with the engine running at the same speed. The pitch is lowered when it passes by. The pitch drops after the car passes because of the frequency drop as the waves are being “stretched”. The same happens with waveforms throughout the electromagnetic spectrum. Unlike acoustic/pressure waves in air or water, however, which rely on a medium to propagate, electromagnetic waves can propagate through a vacuum but generally the effects of traveling through a medium are relatively minor and neglected. In the electromagnetic spectrum an approaching source will be shifted toward the higher frequencies, a blue shift, and a retreating source will be red-shifted, i.e. longer wavelength. When Doppler radar is used to measure ballistic velocities radio frequencies are used almost exclusively. **Equation 1** below is a simplified equation (it assumes the radar, target, & projectile form a straight line; and only the projectile is moving) that may be used to determine the velocity of a projectile using Doppler radar.

$$v = \frac{cf_d}{2f_0}$$

Equation 1

Where v is the velocity of the projectile relative to the radar, c is the speed of light ($9.836*10^8$ ft/s), f_0 is the initial frequency transmitted by the radar and f_d is the Doppler shift of the frequency.

As an example, if we have a transmitted frequency of 10MHz with a Doppler shift of about 40.7 Hz, the calculated velocity comes out to be about 2,000 ft/s ($(9.836*10^8*40.7)/(2*10*10^6)$). This equation is in the accompanying spreadsheet to try out.

Typical accuracy variance of a Doppler radar system may be <0.1% over velocities ranging from relatively low subsonic through supersonic. Depending on the system, measurements may be made from tens of yards to hundreds of yards.

There is a table below to finish up this section on BC’s with a comparison between three types. As mentioned we’ll discuss the pendulum in the final chapter.

 Copyright© 2021 Terry A. Willemin, P.E.

Figure 2, A ballistic radar system on a tripod from Labradar. (Picture courtesy of Labradar)

Table 1, General comparisons of ballistic chronometers based on a range of 1-5 with 1 being best/most preferred and 5 worst/least preferred value for the given characteristic.

System	Cost	Accuracy	Ease of Use	Projectile Size	Distance
Optical Sensor[1]	1-3	1-2	2	2-4	5
Radar	3/5	1	2-5	1	1
Ballistic Pendulum	1-3	3-5	1	1	2-4
Notes: [1]Optical systems are susceptible to bright light and muzzle blast.					

Anemometers

Wind can greatly affect the flight path of a projectile. As such we need to know about the air's velocity, direction, density, etc. Anemometers are devices used to measure windspeed and sometimes direction too. There are too many types in existence to cover in any real degree here, so we'll touch lightly on several that may be encountered by the engineer, including: cup, vane, ultrasonic, and pressure difference types. As an added bonus, a couple of windspeed estimation tools and techniques are covered in this section.

 Copyright© 2021 Terry A. Willemin, P.E.

Figure 3, Various Anemometers: Cup (left), Vane (center) and Ultrasonic (right). Picture courtesy of R.M. Young Co.

 Copyright© 2021 Terry A. Willemin, P.E.

Anemo-thermometer (model 8904). Courtesy of Dwyer

Figure 4, Example of a handheld vane type anemometer (picture courtesy of Dwyer Instruments, Inc.)

Cup and Vane Anemometers (3)

Many of us have had the opportunity to construct a cup anemometer in a junior high or high school science class out of Styrofoam cups or halves of ping-pong balls. They are a relatively inexpensive way to measure wind speed. The axis of rotation is normally used in a vertical orientation rather than horizontal. Flows at any angle other than perpendicular to the rotation axis will result in reduced accuracy. Vane type anemometers also mechanically measure windspeed but require the rotational axis to point directly into the flow, thus requiring a vane to keep the propeller aligned. Thirdly, inexpensive handheld units, as seen in **Figure 4**, exist. These are more portable and can be less expensive, but generally suffer the same drawbacks as their predecessors. These anemometers may be encountered at a shooting range or even a test facility though they are less accurate than many other types of anemometer. **Figure 3**'s left side view shows a cup type.

Cup and vane anemometers are subject to several shortcomings including:

1. Accuracy can vary widely.

 Copyright© 2021 Terry A. Willemin, P.E.

2. They generally do not measure flows improperly oriented to the rotational axis very well.
3. Bearings may fail for various reasons rendering the unit useless until repaired.
4. Are subject to freezing up in icy conditions.
5. Slow response time to higher frequency fluctuations in windspeed due to the rotational inertia of the spinning mass.

Ultrasonic Anemometers (3)

Ultrasonic anemometers are relative newcomers in measuring wind speed and direction. They operate by sending pulsed ultrasonic waves from one probe to another and measuring the change in the length of time required to traverse the distance as air velocity changes. There are two-dimensional and three-dimensional versions available. Ultrasonic anemometers can measure high frequency changes in wind speed and direction and are thus suitable for mapping localized turbulence and gusts. An ultrasonic anemometer is capable of measuring a broader range of windspeeds, direction, and fluctuations than most other types. The drawbacks include being relatively expensive and requiring that a computer with data acquisition system be connected to it. When properly set up, these anemometers are very reliable and accurate.

I spent several weeks characterizing flow around a building for reconnaissance UAV research using a 3D ultrasonic anemometer and was very pleased with the results including the ability to construct vector maps for visualization of flows using the abundant data generated by the study.

 Copyright© 2021 Terry A. Willemin, P.E.

Figure 5, A three-dimensional ultrasonic anemometer. Picture courtesy of R.M. Young Co.

Pressure Differential Based Anemometry (1)

As a moving gas's or gas mixture's (liquids will not be treated herein) velocity increases, its static pressure decreases and dynamic pressure increases. In incompressible flows (we'll discuss this further in) the total pressure, aka stagnation pressure, is the sum of the static and dynamic pressure, or: $P_{total(stagnation)} = P_{static} + P_{dynamic}$. Pressure differential measuring devices exploit this phenomenon to indirectly measure air speed usually with the assistance of a differential pressure transducer. The relationship changes a bit as the compressibility effects of a gas come into play, i.e. the density of the gas or gas mixture increases, but there are equations to deal with that too. Practically all aircraft employ a system like this utilizing a special tube for total pressure inputs (aka stagnation pressure) called a pitot tube. Total pressure is also called stagnation pressure or pitot pressure because a pitot tube brings the gas velocity to approximately zero or to stagnate. Bringing it to rest from higher speeds can cause the density change we talked about earlier. For static pressure measurement there are usually taps/ports on the sidewalls of the fuselage that must be oriented perpendicular to the flow. Alternatively, pitot-static probes may be used which allow the two pressures to be taken simultaneously by a single probe. **Figure 6** & **Figure 7** diagram a handheld pitot-static probe, but they may also be permanently mounted to an aircraft (usually having the static port located on the fuselage rather than incorporated into the pitot tube), found in wind tunnels, and have even been used on rockets and missiles such as the Talos SAM (surface-to-air missile). A pitot-static probe has a hole at its upstream tip and holes further down on the cylindrical wall of the tube as shown in **Figure 7**. The two pressures are segregated from each other so that they may be measured individually or differentially. The two pressures can be converted into air-speed if air density is known. These systems are not typically used in very low-speed applications such as a light breeze because the pressure difference between total and static pressures is very small and the increased accuracy/cost of the pressure transducers normally does not warrant it. Systems can be very simple and inexpensive such as in the case of utilizing a simple *u-tube manometer*, as shown in **Figure 8**, or can be very complex and expensive using highly accurate pressure transducers. Accuracy of velocity results can also vary widely. Response times can be very good if distance and fluid volume between pressure transducer(s) and pressure measurement location are minimized, i.e. small diameter, short tubing between pressure pickups and transducer.

Pitot Probe (Model 166T), Courtesy of Dwyer instruments, Inc. (2016)

Figure 6, Handheld pitot-static probe (a pitot probe with static taps/holes on the cylindrical walls). Note the two pressure taps at the end of the tube for total and static pressures. (Picture courtesy of Dwyer Instruments, Inc.)

 Copyright© 2021 Terry A. Willemin, P.E.

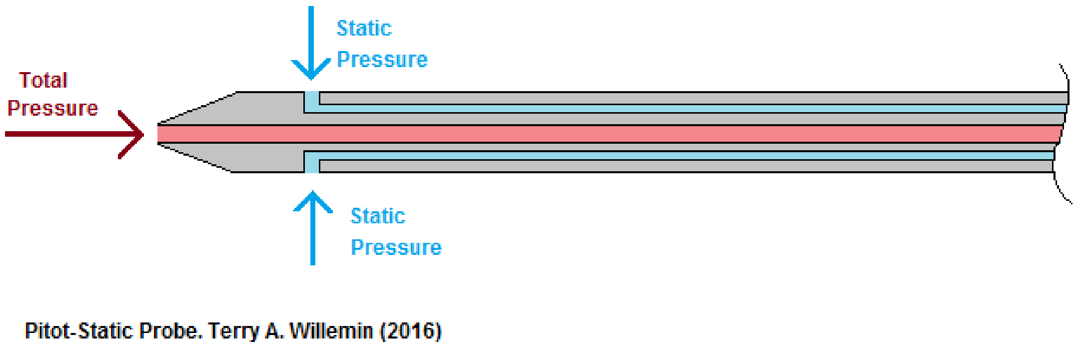


Figure 7, A typical pitot-static probe end showing static and total/stagnation pressure tap orientations. Total pressure is parallel to the flow while static is perpendicular.

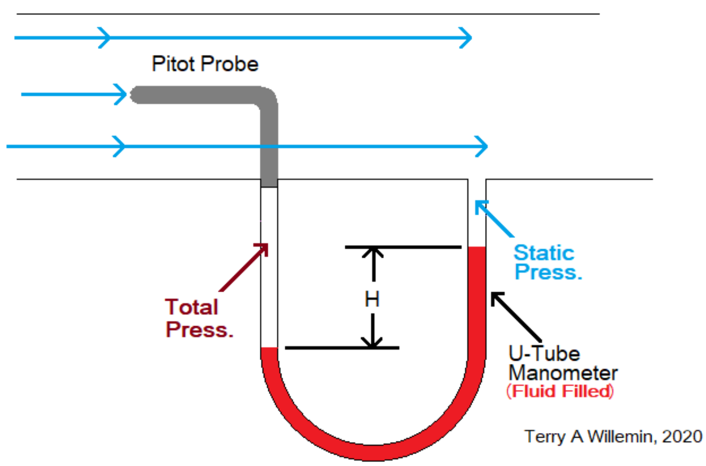


Figure 8, U-tube manometer example when used to measure pressure difference between static and pitot/total/stagnation pressures. The stagnation pressure is obtained from the pitot tube. H is the height difference in the manometer created by the pressure difference.

Pressure to Speed Conversion [(9)]

The next several sections deal with obtaining projectile or air speeds from our pressure measurements. As a side note it doesn't matter if the projectile is moving through the air or the air moving relative to the projectile, just that there is a relative velocity. This is

 Copyright© 2021 Terry A. Willemin, P.E.

what makes wind tunnel testing possible. The sections review some materials presented in part one of this course, but may be partially modified. Before discussing calculating speed from pressure data any further let's do a little review.

Speed of Sound in Air

Unless one is absolutely certain a projectile is moving significantly less than the speed of sound in the fluid it's traveling through then the Mach regime; i.e. subsonic, transonic, or supersonic, etc. must be identified. The reasons for this are many; knowing the correct equations to use to calculate the speed is just one. A propeller driven airplane would certainly be subsonic (traveling below the speed of sound in the air), but what about say… a bullet or rocket? As with many other projectiles, sometimes bullets leave the muzzle (end of the barrel) subsonically and others supersonically (traveling above the speed of sound). Bullets can transition in flight from supersonic to transonic or even subsonic. In the unpowdered portion of a rocket flight it may: be supersonic until it becomes a case for terminal ballistics, fires its engines up for landing, or changes flow regimes in flight like the bullet. Another reason to know the flow regime is if an object's relative speed in a fluid is near or above the speed of sound through it, because shockwaves will be generated. For reference, **Figure 9** below shows a bow shock type shockwave generated by a bullet nose in a supersonic flow [(14)]. Shockwaves cause a new form of drag as discussed in part one of this course, and very abrupt irreversible changes to the fluid, e.g. the upstream (before going through the shock wave) total pressure is always higher than the downstream total pressure through the wave even though no work is done. The total drag on the projectile can be drastic as its goes transonic from subsonic or supersonic making stability an issue. To find the speed of sound in an ideal gas (for our purposes let's assume air is always an ideal gas) we'll use the following simplified equation:

$$a = \sqrt{\gamma RT}$$

Equation 2

Where γ is a unitless value defined as the ratio of specific heats (c_p/c_v) for the gas/gaseous mixture. With air we use a γ of 1.4, R is the gas constant of the specific gas or gaseous mixture, and T is its absolute temperature. Air has a gas "constant" of approximately 53.35 ft-lbf/lbm-˚R in customary U.S. units and 287.03 J/kg-K (287.03

 Copyright© 2021 Terry A. Willemin, P.E.

m^2/s^2-K) in SI units. Note that when working in U.S. customary units, multiply γRT by earth's gravitational "constant" g_c (32.174 lbm-ft/lbf-s2) inside the radical. Once again, and as with all other equations herein, the preceding equation was added to the spreadsheet.

Mach Number

Mach number, named after the Austrian physicist Ernst Mach, is defined as the ratio of the velocity of an object, V, in some medium to the local speed of sound, a which we just discussed. Technically, if a calculated local Mach number is greater than unity it is supersonic and less than would be considered subsonic. However, in the velocity region of roughly $0.8 \leq M \leq 1.2$ there may be regions of flow around a body that are subsonic residing with others which are supersonic. We call this muddled range of speeds transonic. Mathematically, Mach number is defined as:

Copyright© 2021 Terry A. Willemin, P.E.

$$M \equiv \frac{v}{a}$$

Equation 3

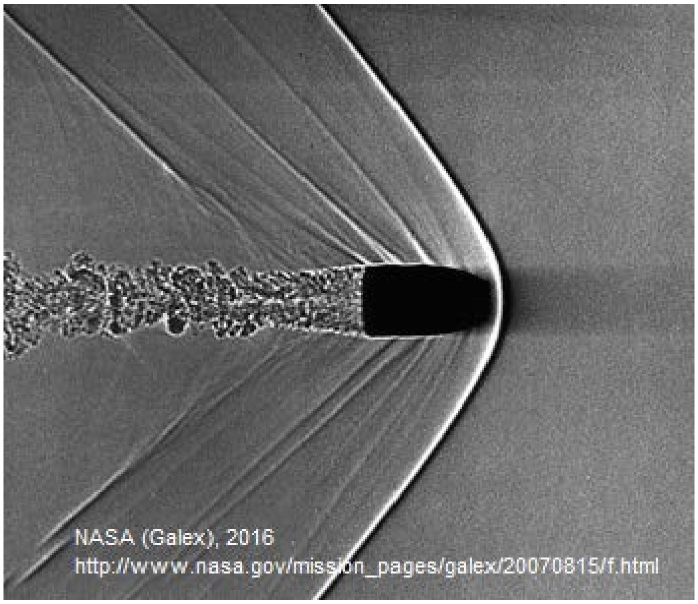


Figure 9, Shock Waves off a Bullet in Supersonic Flight. A detached bow shock resides upstream of the bullet nose. The flow is from right to left. (Courtesy of NASA Galex)

A Few Cases for Calculating Mach Number

The next few sections touch lightly on calculating Mach number or velocity to give a feel of how to use the equations above based with equations specific to the flow regime.

Calculating a Fully Subsonic Mach Number

To use the simple equation that follows the gas or gas mixture must be isentropic, which means:

- Adiabatic - no heat is transferring into or out of the gas
- Frictionless – i.e. reversible. an assumption that may or may not be accurate, but for us it will be appropriate….

 Copyright© 2021 Terry A. Willemin, P.E.

This means the gas must be fully subsonic, i.e. traveling no more than about M=0.3 so compressibility's effects can be ignored. For courser estimation purposes that Mach number may be increased, but be wary of increasing inaccuracy.

Daniel Bernoulli (1700-1782) is a fluid mechanics icon well known for developing the relationship between velocity increase in fluids and an associated fluid static pressure (potential energy) decrease, aka Bernoulli's Principle. What most people don't know though, is that his father, Johann, kicked him out of the house after tying with him for first place in a science contest in Paris. His dad also plagiarized some of his work. Johann held a grudge toward Daniel until his dying day in spite of all that!

Once the criteria are met, this modified version of Bernoulli's equation may be used to determine the projectile velocity. Bernoulli's equation simply states the total energy in a fluid under the conditions listed above, is equal to the sum of the pressure, kinetic energy and potential energy. With a little magic, some hand-waving, and then solving for velocity we can boil Bernoulli's down to give:

$$v = \sqrt{\frac{2(p_t - p_s)}{\rho}}$$

Equation 4

Where p_t is the total, (aka stagnation or pitot) pressure which is obtained from a pitot tube, p_s is the static pressure which comes from one or more static taps/ports oriented normal to the flow and ρ is the mass density of the fluid. Note, **both** pressures must either be absolute or gauge to have any meaning. The difference is that gauge pressure is the pressure above or below local atmospheric pressure and absolute is referenced to absolute-zero pressure or a vacuum. When working with US customary units, e.g. psi, g_c (32.174 lbm-ft/lbf-s^2) should once again be a factor inside the radical.

A Somewhat Lengthy Example:

Civilian Space eXploration Team (CSXT) launched a relatively large rocket equipped with a simple GPS unit, a pitot tube and several static pressure ports. A new engineer on the

 Copyright© 2021 Terry A. Willemin, P.E.

team wants to make a quick hand calculation of its speed from telemetry given at a specific altitude to verify the computer calculations match the GPS generated velocity readings. For simplicity the selected data was pulled as the vehicle was still relatively close to the ground so that it was known to not yet be travelling supersonically. The launch is taking place on a very dry day at an air temperature of 72 °F (288.7K). With the corrected air pressure at the given altitude the engineer obtains an air density of .067 lb_m/ft^3 (1.2254 kg/m^3). The pressure difference between the pitot tube's and static port's pressure transducers give a dynamic pressure; remember dynamic pressure is the difference between total pressure and static pressure (p_t-p_s), of 1 lb_f/in^2. At the coordinated data pull, the GPS is telling her the rocket is travelling at 256 mph. What are the calculated velocity in ft/s and Mach number of the air flow? Is Bernoulli's formula appropriate for this calculation?

1. We'll begin by using **Equation 4**. Note: units must match between square inches and square feet so we add the factor of 144. And because we're using US customary units we also need to multiply by g_c. The equation then becomes: $\sqrt{\frac{2*1*144*32.174}{.067}} \approx 371.9 ft/s\,(253.6\,mph)$. This makes for a pretty good agreement between this velocity and that of the GPS.
2. Determine the speed of sound for the indicated air temperature using **Equation 2** which gives: $\sqrt{1.4*53.35*(72+459.67)*32.174} \approx 1130.3 ft/s$ (~770.7 mph)
3. Finally calculate the rocket's Mach number using **Equation 3:** $M \approx \frac{371.9}{1130.3} \approx .329$
4. Since the Mach number is only a little over 0.3 where air's compressibility effects need to be accounted for, we can say Bernoulli's equation would likely be sufficient since this calculation is only used for verification against GPS readings.

Following is one more potentially useful equation for **subsonic** flows in the realm of 0.3<M<0.8 where gas compressibility effects begin to come into play. This equation, once again from Bernoulli and for an ideal gas, makes use of the case when the free-stream flow is still considered isentropic:

$$M = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{p_t}{p_s}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}-1\right]}$$

 Copyright© 2021 Terry A. Willemin, P.E.

Equation 5

For mathematical simplicity, we'll stick to atmospheric pressure when using this equation.

Transonic Speeds

As mentioned previously the region of $0.8<M<1.2$ is called transonic, and within this band, flow over a projectile may have areas of subsonic and supersonic flow simultaneously. This transition can cause erratic behavior in the flight of a projectile due to varying shifts in the intensity of the types of drag and changes in lift. To make projectile stability easier to deal with velocities should be kept either fully subsonic or fully supersonic for the entire external ballistic flight path. We'll not discuss any equations or formulas relating to transonic flight mostly because of subject's complexity.

Estimating Supersonic Speeds

Care must be taken in selection and location of the probe for transonic to supersonic flows because of vibrations and shockwaves. If the probe is arranged so as to be inside the shock of some object upstream of it this can be completely avoided as it may be placed in a subsonic flow region. However, if the fluid flow of interest is supersonic a pitot-static probe will act as a blunt-nosed body similar to the bullet shown in **Figure 9**, which will cause a similar detached bow shock in front of the tip as shown in **Figure 10**. As a result, the stagnation pressure measured at the tip of the probe is the stagnation pressure of the flow behind the incident normal shock and subsonic because it passed through the shockwave. Fluid velocity immediately downstream of a shockwave is always subsonic, i.e. $M_1>1$ & $M_2<1$ below. **Equation 6** below, can be used to determine the supersonic Mach number upstream of the shockwave.

 Copyright© 2021 Terry A. Willemin, P.E.

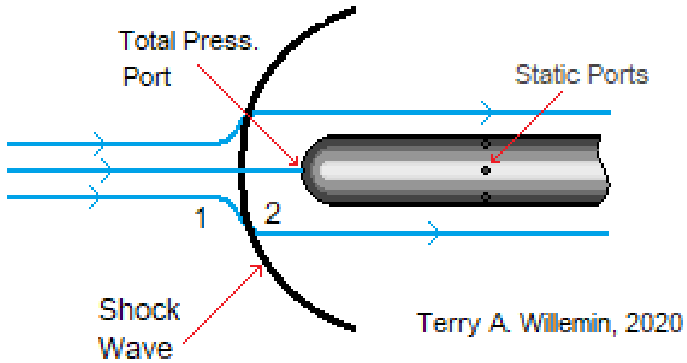


Figure 10, A pitot-static tube in a supersonic flow. A detached bow shock forms upstream of the probe tip. The streamlines are shown to approximate flow paths through the shockwave. The pressures located at points 1 and 2 are referenced in Equation 6 below.

$$\frac{P_t}{P_s} = \frac{\gamma + 1}{2} M^2 \left[\frac{(\gamma + 1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma - 1)}\right]^{\frac{1}{\gamma - 1}}$$

Equation 6

This equation may be used to calculate Mach number, $\mathbf{M_1}$, upstream of a flow given the measured static and total pressures indicated in **Figure 10** and γ, the specific heat ratio (c_p/c_v) for the gas/gaseous mixture. We have now spent enough time on pressure measurements and equations for anemometry purposes, so let's end with estimation tools and techniques.

Wind Socks and Knots

Windsocks (**Figure 11**) are placed in the fields of airports, near helipads on buildings, and a host of other places. They are designed to give not only wind direction, but a windspeed estimate. Yes, windspeed…. Generally, a three-knot breeze is sufficient to orient the sock with the wind and a 15-knot wind will lift it to full horizontal. Socks are typically either entirely orange, or employ alternating orange and white sections to aid in wind-speed estimations. Each section of an alternating orange and white sock which is inflated represents an estimated increase of three knots. The first section, nearest the pole and largest in conic section, is awakened by a wispy waft of only three knots. The next section, the first in white, is buoyed by a billowing breeze of six knots, followed by the second

 Copyright© 2021 Terry A. Willemin, P.E.

orange section swelling in a chinook of nine knots. The fourth section goes in a gentle gust of 12 knots. And, the final section rises in a zealous zephyr of at least 15 knots.

The term knot, not naught or not, is a nautical term hailing from a centuries-ago method for determining sailing vessel speed. Apologies for that sentence and the prior alliteration, but they sure were fun to write! A knot is simply 1.15 miles/hour when rounded to three significant figures; and consequently, the nautical mile is ~1.15 miles. Nautical miles, and the knot, are still used in aircraft and maritime travel because the nautical mile was originally defined as one minute (1/60th of a degree) of latitude along any line of longitude, see **Figure 12**. Since the nautical mile plays so well with global scale travel which is usually either nautical or aeronautical, and because it's based on earth's circumference it has enjoyed continued use and preference over the mile, kilometer, parsec, lightyear, league, AU, furlong, cubit, yard, rope….

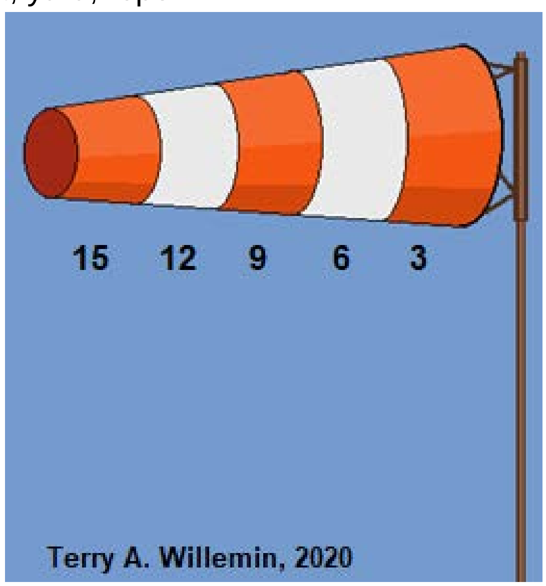


Figure 11, A windsock in a 15 knot or higher wind. The numbers represent windspeed estimates needed to inflate the corresponding section.

 Copyright© 2021 Terry A. Willemin, P.E.

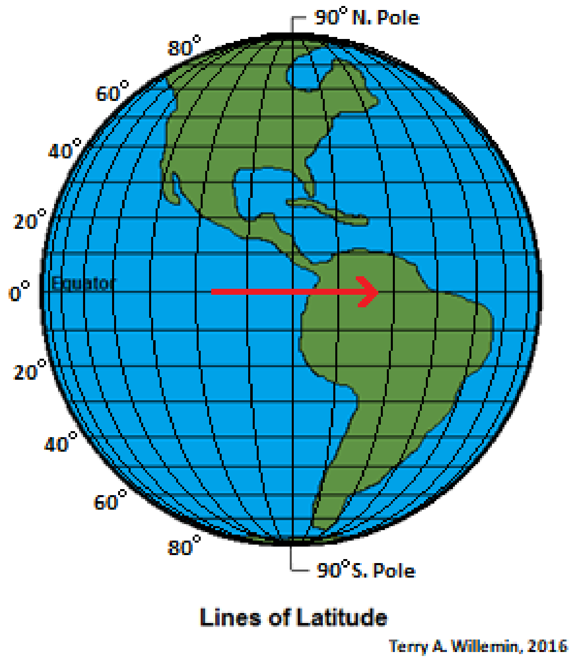


Figure 12, The horizontal lines above represent lines of latitude on earth (shown only at 10-degree intervals and numerically marked at every 20 degrees & the poles). Unlike latitudinal lines which vary in length/circumference, longitudinal lines are for most intents and purposes equidistant. Earth's spin direction is indicated by the red arrow.

Beaufort Wind Force Scale

When neither an anemometer or windsock are available there are useful estimation techniques to gauge windspeed based on the effects the wind has on the surrounding environment. In 1805 Sir Francis Beaufort (1774-1857) of Beaufort Cipher fame and known for locating Hadrian's Gate, exploited the previously mentioned concept to devise a scale for gauging windspeed which became known as the Beaufort Wind Force Scale or simply the Beaufort Scale. This scale is occasionally used by meteorologists because it quickly conveys windspeed effects in practical terms. This particular scale is broken into 12 sections based on the effects on land, but there are also scales used for out at sea. There are many variations of the scale. The lower values in the scale can prove useful in outdoor environments to judge whether or not to perform many activities. As an example,

 Copyright© 2021 Terry A. Willemin, P.E.

if someone is trying to launch a rocket or shoot rifles in gale force winds or higher, they may want to give it a second thought or seek professional help….

Table 2, Excerpt of Beaufort Wind Force Scale [(2)]

Force	Wind (Knots)	WMO Classification	Appearance of Wind Effects on Land
0	<1	Calm	Calm, smoke rises vertically
1	1-3	Light Air	Smoke drift indicates wind direction, still wind vanes
2	4-6	Light Breeze	Wind felt on face, leaves rustle, vanes begin to move
3	7-10	Gentle Breeze	Leaves and small twigs constantly moving, light flags extended
4	11-16	Moderate Breeze	Dust, leaves, and loose paper lifted, small tree branches move
5	17-21	Fresh Breeze	Small trees in leaf begin to sway
6	22-27	Strong Breeze	Larger tree branches moving, whistling in wires
7	28-33	Near Gale	Whole trees moving, resistance felt walking against wind
8	34-40	Gale	Twigs breaking off trees, generally impedes progress
9	41-47	Strong Gale	Slight structural damage occurs, slate blows off roofs
10	48-55	Storm	Trees broken or uprooted, "considerable structural damage"
11	56-63	Violent Storm	
12	64+	Hurricane	

As done previously in the ballistic chronometer section, **Table 3** is provided to generally compare the mentioned methods and tools in this section on anemometry.

Table 3, General comparisons of anemometers based on a range of 1-5 with 1 being best/most preferred and 5 worst/least preferred value for the given characteristic.

System	Low Vel. Accuracy	Hi Vel. Accuracy	Portability	Cost	Response Time	3D Flows*
Cup/Vane	1-3	4	1-2	2	2	4
Ultrasonic	1-2	1-2	2-3	4	1	1
Pitot-Static	3	1	3	4	1	1-3
Wind Sock	3	5	4	3	3	4

 Copyright© 2021 Terry A. Willemin, P.E.

Beaufort	3	3	1	1	3	3
* 3D Flows refers to the ability to measure the flow direction from any orientation						

Barometers & Atmospheric Pressure (4)

There was mention of atmospheric pressure previously, and in treating ballistic flight paths through an atmosphere it is necessary to measure that pressure. Enter barometers. Over hundreds of years many barometry technologies have been invented. Following is an adumbration on just a couple: mercury tube and aneroid. Before that let's clear the atmosphere.

Earth's Atmosphere

Said Rembrandt, "Without atmosphere a painting is nothing." I say, like a painting, without an atmosphere earth would be nothing, or at the very least we would be nothing. Atmospheric pressure decreases as altitude increases (generally), so meteorological stations usually report their barometric pressure "corrected" to sea level when taken at altitudes above or below sea level. What this means is the pressure they report has been modified to match what it would be if it they were measuring it at sea level from their location. In other words, if they were to dig a pit in the station's location down to sea level altitude, that's what the air pressure would measure. The National Weather Service uses three pressure definitions in regards to barometric pressure (16):

- **Station Pressure** – actual, unaltered, pressure as measured by the barometer at a weather station.
- **Altimeter Setting** – a modification to station pressure to change it to a special mean sea level standard pressure used in aviation based on average conditions at 40 degrees latitude in the US. Pilots would normally get this data from a METAR (Meteorological Terminal Aviation Routine Weather Report). Pilots use this value to adjust altimeters. By the way, we're not discussing altimeters in this course.
- **Mean Sea Level Pressure** - a modification to station pressure to change it to what the pressure would be if the station altitude were at sea level. In making the calculation it is based on the average temperature over the previous 12 hours. This is the value you get from a meteorologist.

 Copyright© 2021 Terry A. Willemin, P.E.

Why do that? People in the vicinity of the report can be at various altitudes with respect to the meteorological station and want/need to know the pressure where they are. If you take a barometric pressure reading where you are, and your simple barometer is calibrated, you will get “un-corrected” pressure unless it has the capability of calculating a corrected value. Personally, I’d call the meteorologist’s reported pressure something like “adjusted to sea-level” rather than Mean Sea Level or corrected, and they’d likely call that semantics. To use the reported pressure values at a given altitude the user must then “un-correct” that data to their known altitude. Atmospheric pressure drops *roughly* one inch, or 25 mm, for every 1,000 ft increase in altitude. A slightly more accurate estimate can be simply obtained by multiplying the reported value by the following correction factor, CF: (6)

$$\mathrm{CF} = \frac{760 - (.026 * \mathrm{H})}{760}$$

Equation 7

Where H is the altitude in **feet**. For an altitude of 2,000 ft, for example, that works out to be .932. And if the corrected pressure reading from a nearby weather station is given as 30.0 inHg, then multiplying that by the correction factor gives about 27.95 inHg. This correction factor is independent of the units the pressure is given in. Knowing atmospheric pressure is essential to determining the density of ambient air since air density will have a direct effect on a projectile’s flight.

Alternatively, to un-correct the pressure, one may use a standard temperature lapse rate of -.0065 K/m with the following (3):

$$\mathrm{P} = \mathrm{P_{SL}}\left[1 + \frac{\mathrm{L}}{\mathrm{T_{SL}}}\mathrm{h}\right]^{\frac{-g*M}{R*L}}$$

Equation 8

Where, in SI units, P_{SL} is the pressure corrected to sea level (Pa), L is the standard temperature lapse rate (.0065 K/m), T_{SL} is the sea level corrected temperature (K), h is the height above sea level, T_{SL} is the temperature at sea level, g is the acceleration due

 Copyright© 2021 Terry A. Willemin, P.E.

to gravity (9.80665 m/s^2), M is the molal mass of atmospheric air (0.0289644 kg/mol), and R is the universal gas constant (8.31432 N·m/mol·K).

When no pressure data is available there are also lapse rates in use such as the one shown in **Figure 13** to get a rough idea of what the pressure is based only on altitude and temperature.

The main reason for knowing atmospheric pressure, humidity, and temperature in external ballistics is because the effect it has on air density. Remember from part one, that higher density generally translates to higher drag and increases in the effects wind will have. Since we also need to know the humidity and temperature of the air to get an accurate density reading we will address tools for measuring those parameters.

Another reason for correcting to a standard is for making “apples-to-apples” comparisons between data taken at various locations and atmospheric conditions.

Side Note: many atmospheric condition standards have been generated by a bajillion (I believe that’s the exact number) governing bodies for various uses. In the calculation of ballistic coefficients in the US, data should be corrected to US Army Standard Metro. That is, 59 °F (15 °C), 14.696 psi (101.325 kPa), air density of 0.075126 lb/ft3 (1.203403 kg/m3) and 78% relative humidity.

Copyright© 2021 Terry A. Willemin, P.E.

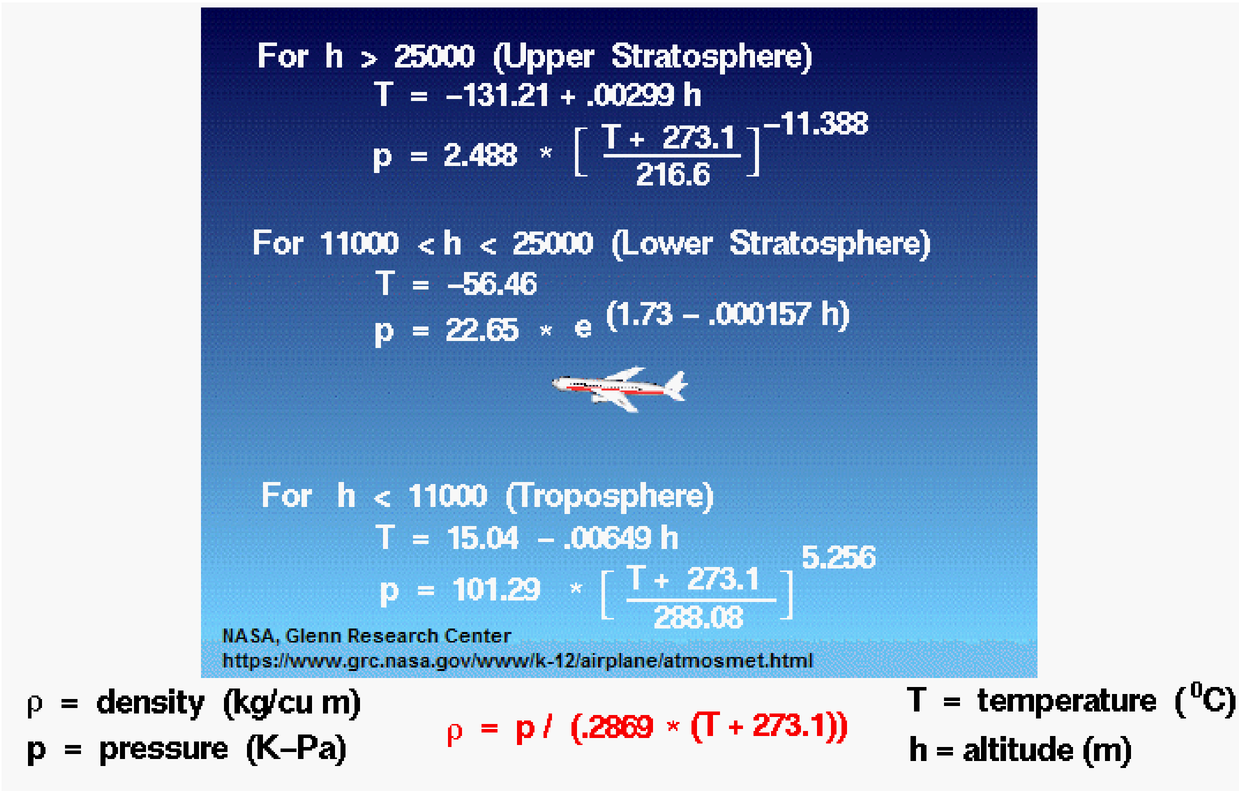


Figure 13, Standard Atmospheric Temperature and Pressure Models at Altitudes in Meters

Mercury Tube Barometer

Mercury barometers are a relatively accurate way to measure barometric pressure when calibrated. They operate by way of atmospheric pressure directly or indirectly forcing mercury up an evacuated cylinder. The simplest, a vertical tube type is shown in **Figure 14**. Mercury and water barometers have been used for hundreds of years and thus often pressure (especially atmospheric) is still reported in units of water or mercury, e.g. in-H20, mmHg etc. Standard atmospheric pressure is set at 760 mmHg exact (29.92 inHg) and is shown in the figure. Since mercury has a low vapor pressure, accuracy of mercury tube barometers is excellent when corrected for temperature and local gravitational acceleration. Some mercury or water barometers allow the scale to be adjusted for correcting the pressure to match readings from weather reports.

 Copyright© 2021 Terry A. Willemin, P.E.

There are water filled barometers of different forms, but finding a water filled fluid tube barometer like that shown in the figure below is highly unlikely because the tube would need to be at least 34 ft tall!

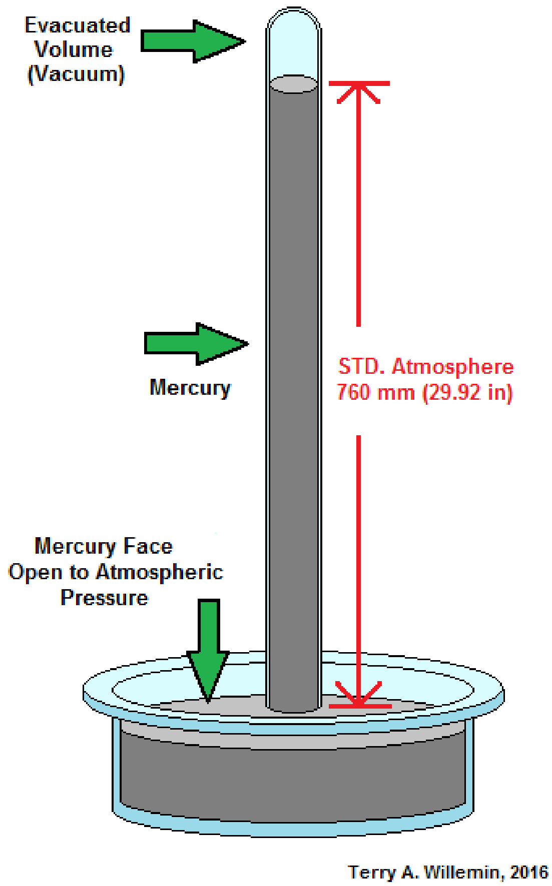


Figure 14, Example of a Simple Mercury Barometer

Aneroid Barometers

Aneroid barometers rely on the expansion or contraction of an evacuated hollow metal container to move levers and gears to register atmospheric pressure. As atmospheric pressure changes, the shape of the tube or diaphragm will correspondingly change, and when connected to an appropriately sized gear-train can rotate an indicator needle around a marked face to indicate pressure. Accuracy of these types of barometers will vary but can be in the vicinity of ±0.02 in-Hg when calibrated. Some aneroid barometers will have bi-metallic coils working in conjunction with the tube or diaphragm to make temperature corrections to the readings. These barometers nearly always have some way to correct the reading for altitude such as with a thumbscrew. You have probably seen some residential or maritime versions of these barometers hanging in people's homes or in seagoing vessels many years ago before weather was generally available on phones and the internet. These will sometimes also include weather predictors on the dial face with indications such as "Stormy", "Rain", "Change", "Fair", and "Very Dry" and an extra needle that can be set anywhere on the gauge to see if pressure is rising or falling over a period of time.

Hygrometers [(11)]

As mentioned before air density is affected by its temperature and humidity. Hygrometers are used to measure atmospheric moisture content. Relative humidity can be very important to us humans as a hot day can become a miserable day when humidity is high. I grew up in what is tied for the most humid city in the contiguous US. It is possibly the most roach and mosquito infested too. So, going anywhere else on the planet for vacation felt great. The temperature, and to a small extent the pressure, of air determines the maximum amount of water vapor it can hold. Relative humidity is then the actual percentage of water vapor actually in the air as a percentage of the maximum it can possibly hold. More technically, it is the ratio of the mole fraction of the vapor in the mixture to the mole fraction of vapor in a saturated mixture at the same temperature and total pressure [(8)]. As with most of the abovementioned measurement equipment, there are many different ways devised to measure humidity in ambient air including: psychrometers, paper-metal, chilled mirror, capacitive, resistive, thermal, optical and other novel types. Gravimetric hygrometers did not make this list because they are usually only used for calibrating other hygrometer types. The following subsections cover a sample of the more commonly used. Please do not feel neglected if one you currently use did not make the list.

Psychrometer

 Copyright© 2021 Terry A. Willemin, P.E.

Psychrometers employ two thermometers to determine relative humidity, RH, with the bulb of one covered by a cloth bag saturated with distilled water and referred to as the wet-bulb. The other thermometer is referred to as the dry-bulb. The sling psychrometer, **Figure 15**, is a common type. It is used by holding the handle (the vertical portion in the picture) and rotating the thermometer pair in the air until the thermometers have stabilized. Unless the RH is 100% or temperature colder than the frost point of the air, the water on wet-bulb thermometer will evaporate resulting in a lower temperature reading than the dry bulb. The rate of evaporation is affected by the RH, so a comparison between the two thermometer readings can be used to derive RH. The accuracy of humidity results is highly dependent on user reading accuracy and getting the thermometers to stabilize. Other psychrometers may use a fan and duct to pass air over the two thermometers (or temperature transducers). Software or a chart is used to then obtain relative humidity. The psychrometric chart, **Figure 21**, is a familiar site to any engineer who has taken a thermodynamics course. It is an easy way to gain relative humidity from the two thermometer readings. To use a psychrometric chart to determine RH simply find the intersection of the wet-bulb and dry-bulb temperatures and follow the relative humidity curves to see what it most closely corresponds to.

An Example: let's say at sea level standard pressure (important information) the dry-bulb temperature is 74°F and the wet-bulb is at 67˚F, then relative humidity is 70% (see **Figure 16**). Note, the psychrometric chart is pressure specific so that is why sea level was mentioned. When more accurate RH readings are needed they should be calculated rather than read from a chart.

 Copyright© 2021 Terry A. Willemin, P.E.

Figure 15, A pocket-sized sling psychrometer.

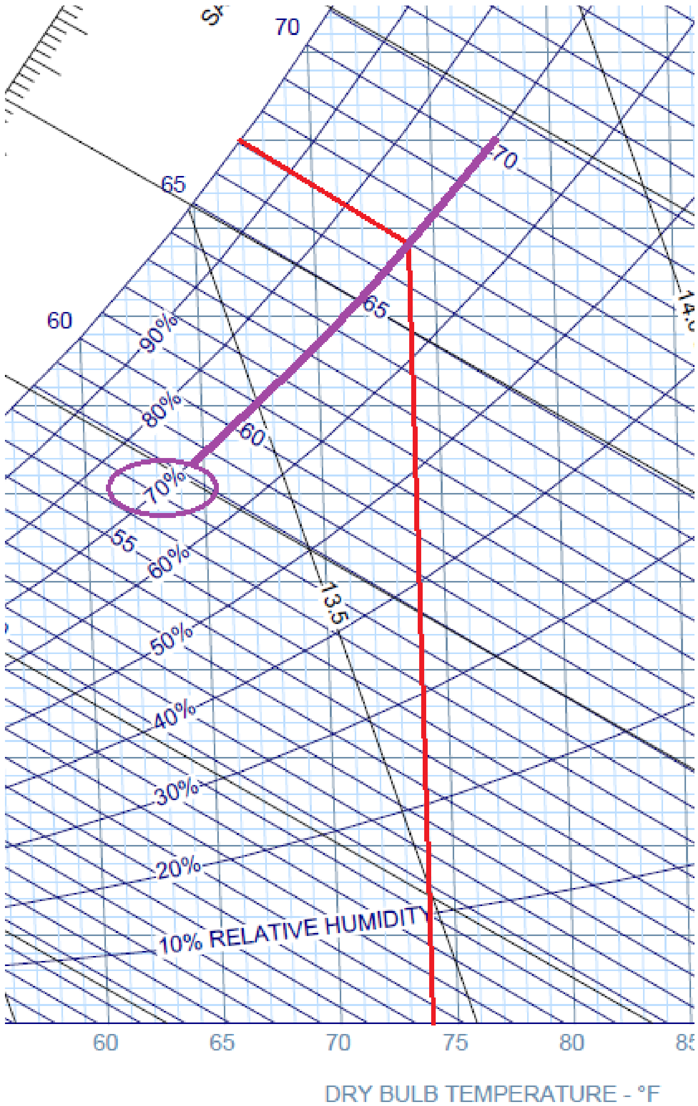


Figure 16, Excerpt of psychrometric chart from the Appendix showing the use of wet-bulb/dry-bulb readings to determine RH.

Chilled Mirror Hygrometers (5)

Chilled mirror hygrometers, CMH's, are the standard for hygrometry; they are used by researchers world-wide. A chilled mirror hygrometer uses a fluid's dewpoint to determine

 Copyright© 2021 Terry A. Willemin, P.E.

humidity. The CMH has a polished metal surface that is cooled to the point that condensation begins to form on it; the dewpoint. Dewpoint, is simply the temperature at which the water vapor in the air will condense in a constant pressure process; the temperature at which the air, if cooled, can no longer hold the amount of water it contains. Typically, optical sensors are used to determine the point at which condensate forms on the mirror and the corresponding temperature is used to calculate the relative humidity. CMH's are more expensive, than other types of hygrometers, but their humidity accuracy is only rivalled by gravimetric humidity measurement. Gravimetric hygrometers are used for calibration of other hygrometer types (17).

Obtaining Relative Humidity from Temperature and Dewpoint

The same psychrometric chart used previously may be used to determine RH from the dewpoint, T_{dew} and air temperature. To determine RH from T_{dew} and the dry-bulb temperature, simply start at the saturation temperature curve and draw a horizontal line that intersects a vertical line from the dry-bulb temperature, that point of intersection is the RH. As an example, once again at sea level, say the dewpoint is 63°F and the air temp (dry-bulb) is 78˚F, then the intersection of the two lines is at approximately 60% RH as shown in **Figure 17** below.

Copyright© 2021 Terry A. Willemin, P.E.

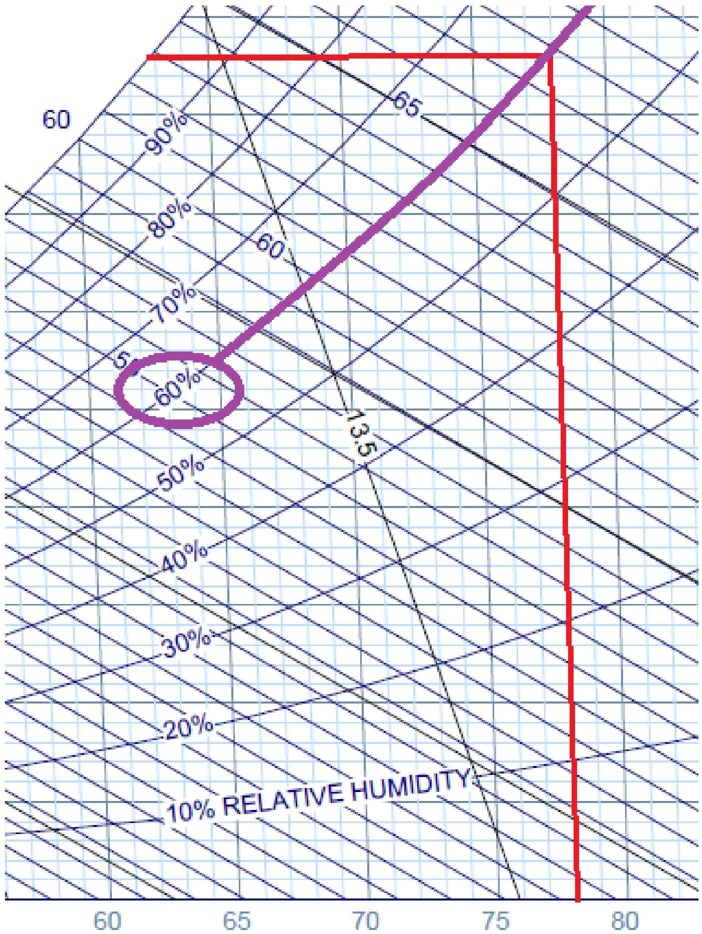


Figure 17, Excerpt of Psychrometric Chart from the Appendix Showing the Use of Dewpoint and Dry-Bulb Readings to Determine RH

Thermometers

With regards to external ballistics there is always a need to know air temperature hence adding thermometers to the equipment list. Additionally, there may be a need to amass surface temperatures of projectiles at specific locations such as at their leading edges, and so on. As thermometric devices are so prolific we are all familiar with at least a couple of types. There are direct and indirect ways to obtain a temperature along with the myriad methods used for gathering that information. Thermometry is profoundly scopic as a

 Copyright© 2021 Terry A. Willemin, P.E.

collective field, but a short-list of thermometer types would likely include: Infrared, mercury/alcohol glass, thermistors, thermocouples, bi-metallic strip, silicone diode and so on. There is no need to expound on thermometry further because of our familiarity with it and that prevalence of measuring devices. At the least, there is a need to make mention because of its importance.

Geo-Orientometers

Since all the preceding sections had really cool -ometer suffixes this one had to follow suit. Unfortunately, there isn't a more pertinent word like geo-orientometer (not a real word) and magnetometer is too exclusive for this category. In ballistics knowing the direction of travel of a projectile is important for obvious reasons in addition to those mentioned in part one of this course. Just as with all the preceding sections there are multiple ways to get a bearing, or heading relative to the earth. Following are ONLY three different tools for geo-orientation: magnetic compass, GPS, and gyrocompass.

GPS

Many years ago, while in the CAP (Civil Air Patrol) I had the privilege of being the in-flight radio operator in a SAREX (Search and Rescue Exercise). In the SAREX we used radio direction finding equipment both onboard a Cessna 182 and, on the ground, to locate an emergency locator transmitter, or ELT transmitting a 121.5 MHZ radio signal. We used radio triangulation to find the ELT. The search took us about 45 minutes as I remember it. Nowadays ELT's emit a 406 MHz signal which more readily facilitates satellite location. With the satellite's aid an ELT's position can be located much more accurately and quickly.

In the US, radio position finding and navigation now relies on a satellite based global positioning system, GPS. Originally this system was available only for military use until 1983 when President Ronald Reagan made it available to civilians, but with reduced locational accuracy. Other countries operate their own similar systems under other names, but all use the same operating principles. With GPS, mobile receivers use signals from at least four satellites to calculate position (including altitude), heading, and course/track. The spatial location of each satellite at any given time is very accurately known. Each also carries extremely accurate clocks which are synchronized with ground-based clocks. Unlike a compass, GPS units generally cannot give direction information

 Copyright© 2021 Terry A. Willemin, P.E.

unless moving. If the receiver is in a continuous-use mode tracking then a path, heading, or direction can be found. There are GPS receivers augmented with magnetic compass or inertial navigation to aid in the accuracy of directional orientation.

Gyrocompass

A gyrocompass is a second non-magnetic way to get geographical direction relative to true north. A gyrocompass relies on torque-induced gyroscopic precession (see part one of the course) and earth's axis of rotation to find true north, i.e. pointing horizontally toward the north pole. Many are susceptible to errors when there are sudden changes in speed or latitude, but there are gyrocompasses which have been designed to overcome those shortcomings. Heading indicators found in smaller or older aircraft work somewhat similarly using a gyroscope but must be manually corrected for drift with relative frequency. An in-depth understanding of the operation and correction of heading indicators and gyrocompasses is beyond the scope of this paper, but they are definitely worth mention since the general population is not familiar with them yet they are still so widely used in navigation and orientation.

Compasses

So, how does a compass really work? For simplicity young children are taught that a compass points north. Soon after we find out the earth has a north and south magnetic pole and that the needle points to that north magnetic pole. But, even that description is not really correct. Yes, a compass needle, like a straight dipole bar magnet, has a north and south pole. Earth's magnetic field is also a dipole magnet, but in the shape of a large sphere. At earth's surface a compass needle aligns with the horizontal component of the magnetic field lines in its location, which usually tend to a **magnetic** north-south orientation. The angular difference between magnetic north and true north is called declination. A magnetic field declination map zoomed in on the contiguous US is shown in **Figure 18**. There is also a vertical component to that field that increases in strength with proximity to earth's magnetic poles; meaning that near the north magnetic pole the compass needle in a compass held horizontally, would be attracted downward more strongly than in any horizontal direction. For that reason, some northern hemisphere compasses will have weight added to the south pole side of the needle to help keep it horizontal; and vice-versa for some compasses made for the southern hemisphere. The angle between horizontal and the total field vector is referred to as inclination. There are

 Copyright© 2021 Terry A. Willemin, P.E.

many drivers which modify local and general field lines, but a compass will "point" to magnetic north sufficiently well enough for navigation. Unfortunately, earth's magnetic field is neither stationary nor aligned with geographical north and south so let's trek a little off course to get our bearing.

A Magnetic Declination Tangent

I had the wonderful opportunity to serve as a scoutmaster in the Boy Scouts of America for about five years. Inside that time my two eldest sons were scouts. Those years meant teaching tons of basic skills including orienteering, i.e. map and compass navigation. Of all the more difficult concepts for youth to grasp taught in all of scouting, the principles of orienteering seemed to me to rank most challenging. That included teaching correcting for magnetic declination until a friend gave me some old grid-reference maps of our area made in 1965. Maps have their compass rose oriented with true north because it doesn't move around like earth's magnetic poles do. Armed with those old maps another key was added for deciphering magnetic declination. For our area the mag-dec had changed a little over 4° in a 45-year span. A 4° difference is enough to make the old maps useless for orienteering unless the current declination is known, but perfect for showing how the magnetic poles are constantly on-the-move. Frequently cartographers will also include a grid north depending on the projection type and grid associated with the map, which can add to the confusion. Magnetic declination is sometimes called grivation, which is pronounced exactly like aggravation without the first "a". That is perfectly fitting since it's the cause of aggravation for newbies when learning to properly correct for it! **Figure 19** gives a 2020 map of magnetic declination for the contiguous US. As of 2020, Barrow, Alaska has a grivation or mag-dec of nearly 13° E and changing by about 0.5° W per year! Also, as of 2020, Van Buren, Maine has a mag-dec of nearly 17° W! The line where true and magnetic north line up is called the *agonic line*. If you're lucky enough to live around Atchafalaya, LA, or anywhere else along that agonic line, and depending on required accuracy, you shouldn't need to correct for mag-dec…at least for a little while!

 Copyright© 2021 Terry A. Willemin, P.E.

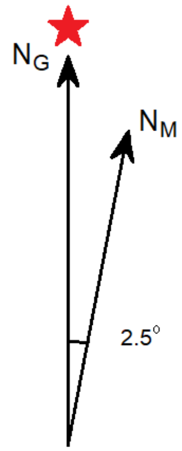


Figure 18, map key showing the magnetic declination, 2.5° in this case, between true/geodetic/geographic north, N_G, and magnetic north, N_M.

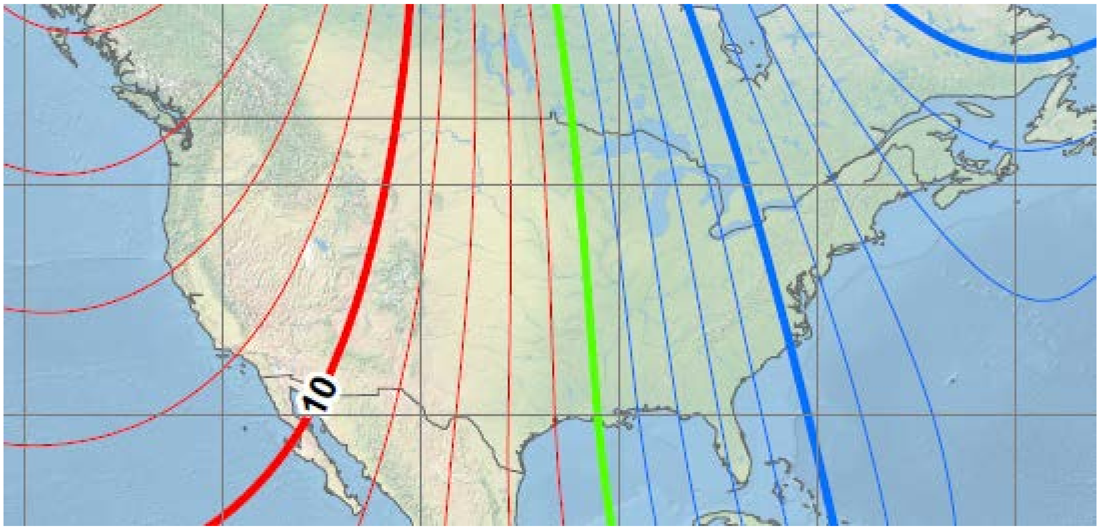


Figure 19, Contiguous US magnetic model for epoch 2020 of main field declination. The agonic line is in green and the thin red and blue lines are spaced at 2° intervals east and west of the agonic. Picture courtesy of NOAA's - National Centers for Environmental Information, NCEI.

Compasses also have one nice advantage over gyrocompasses or GPS systems. One can very easily make a magnetic compass with something as simple as a magnetized

 Copyright© 2021 Terry A. Willemin, P.E.

sewing needle. Just about any ferromagnetic material (many alloys of iron, nickel, and/or cobalt and some rare-earth metals) could potentially become a compass. Accuracy and precision depend on the quality of the housing or bezel's graduations, the magnet's bearing and balance, proximity to external magnetic forces, tilt relative to horizontal and so on.

Distometers and Rangefinders

A distometer is used to measure the distance between the eyelid (actually the apex of the cornea) and back surface of a lens on glasses. What does that have to do with ballistics? Absolutely nothing. By name they continue the coveted criteria of having an 'ometer suffix *and* are used to measure distance. Ergo, mentioning distometers.

There are diverse range-finding and estimating tools available, however, typical external ballistics work usually occurs on a known field over an already known distance to target. For an unknown range, some accurate more modern methods for measuring distance include:

- **Radar** – (born of an acronym for **RA**dio **D**etection **A**nd **R**anging) common in some commercial and military aircraft. Radar is usually an expensive way to go for measuring distances, but it is employed in Tesla vehicles working in conjunction with optical cameras for "autopilot".
- **Lidar** – (an acronym/portmanteau of **Li**ght and ra**dar**) becoming more common in self driving automobiles.
- **Sonar** – (originally an acronym of **SO**und **N**avigation **A**nd **R**anging) makes appearances in watercraft/ocean vessels, oceanographic research vehicles, submarines, etc. Of course, some animals use echolocation as a navigation aid.
- **Laser** – (also began life as an acronym for **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation) very commonly used for hand-held range finding devices.
- **GPS** – (**G**lobal **P**ositioning **S**ystem) GPS is not technically a range finding method, but nonetheless sometimes used for range to target when the target's geographic location is known.
- **Ultrasonic** – (wasn't born from an acronym like AWOL, Scuba, Taser, Maser, Humvee, Zip code, SWAT, BASE jumping, CARE package, BASIC.…)

With the exception of GPS, all of the above-mentioned forms of measurement rely on accurate determinations of the length of time a transmitted electromagnetic or acoustic

 Copyright© 2021 Terry A. Willemin, P.E.

signal's reflection takes to return to a receiver to measure range. Generally speaking, the known or calculated speed of the signal and the time traveled are ordinarily all that are needed to obtain a distance. The use of these signal transceiver range finding tools is limited to line-of-sight. Obstacles or distances where the curvature of the earth removes that line-of-sight generally preclude their use. GPS can overcome some of those nuisances if the geographical position of the target is already known.
Other less accurate and antiquated range finding methods that have been used over time include:

- Split image coincidence telemetry - once found on some range finding cameras
- Stadiametric range finding - still present in some optical rangefinders and rifle scopes
- Stereoscopic range finders

There are other even less accurate methods to judge a range but they are excluded from this paper because some of the signal reflective tools for long distance measurement can be readily obtained at reasonable cost. Some laser range finders can cost less than $50!

Other Instruments

In all of external ballistics research there are other tools used. Some specific to the researchers and others more general to research. As mentioned in the introduction to this course there was a focus on a particular group of them while omitting others such as scales, accelerometers, altimeters, chamber pressure measuring equipment (internal/interior ballistics). If any have been ignored that you feel should be present please let me know why you think it should be included.

The Ballistic Pendulum (7)

In 1742, Benjamin Robins (1707-1751) invented the ballistic pendulum. One of the few things I remember from high school physics was the use of a ballistic pendulum. In class the teacher shot a low velocity .22lr bullet from a attached "gun" into a ballistic pendulum to demonstrate its use. As seen in Robin's patent, **figure 19** below, the ballistic pendulum is a simple device that is used to determine the velocity of a bullet or similar projectile. Prior to his invention the gun designer or engineer had no way of accurately knowing a bullet's speed. His pendulum relied on conservation of momentum to estimate velocity at

Copyright© 2021 Terry A. Willemin, P.E.

a given distance. The gun was fired directly into the swinging mass into which the bullet would lodge and force the mass to swing back some distance. A ribbon, indicated by the letter W in the picture, hanging from the mass was used to record how far the pendulum had swung. Modern pendulums tend to measure the angle the pendulum sweeps rather than using a ribbon.

Figure 20, Benjamin Robbin's Ballistic Pendulum as Invented.

In calculations related to their use we assume there are no losses to: friction, kinetic energy conversion to sound and heat in the collision, drag and so on. Since momentum is conserved, all the mass of the bullet times its velocity (its momentum) just prior to impact, is equal to the combined mass of the bullet and swinging mass after impact. Mathematically this conservation of momentum is shown as $m_b v_b = (m_b + m_p) v_{b+p}$. Where the subscripts b and p denote bullet & pendulum respectively and v_{b+p} refers to the velocity of the two masses combined at the time of impact. Solving for the bullet velocity gives:

$$v_b = \frac{(m_b + m_p)v_{b+p}}{m_b}$$

Equation 9

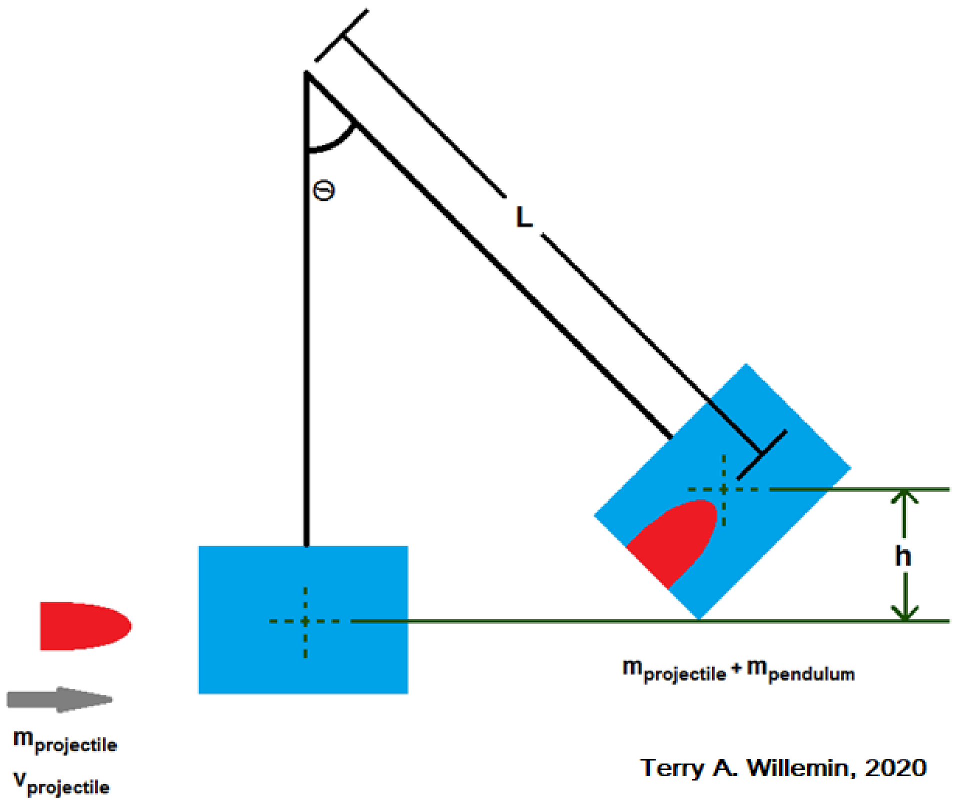


Figure 21, Simplified schematic of a ballistic pendulum

The kinetic energy of the pendulum/projectile combination after impact is completely converted to potential energy at the apex of the swing if we neglect the parasitic frictional losses mentioned above. The maximum height the center of mass attains is denoted by

 Copyright© 2021 Terry A. Willemin, P.E.

h. The velocity at the time of impact can be determined using this energy conversion with the following:

$$\frac{(m_b + m_p)v_{b+p}^2}{2} = (m_b + m_p)gh$$

Equation 10

The velocity of the combined masses at the point of capture then reduces to:

$$v_{b+p} = \sqrt{2gh}$$

Equation 11

If the angle of travel, Θ, is known rather than the height, one may calculate h based on simple trigonometry, using the following:

$$h = L - L\cos\Theta$$

Equation 12

Feel compelled to construct your own ballistic pendulum? A *Ballistic Pendulum* worksheet has been added to the accompanying Excel workbook to calculate velocity, v_b based on height, h or angle of travel of the pendulum. Give it a try. If you build one please let me know how it went and whether or not you have comparison data from a ballistic/gun chronometer.

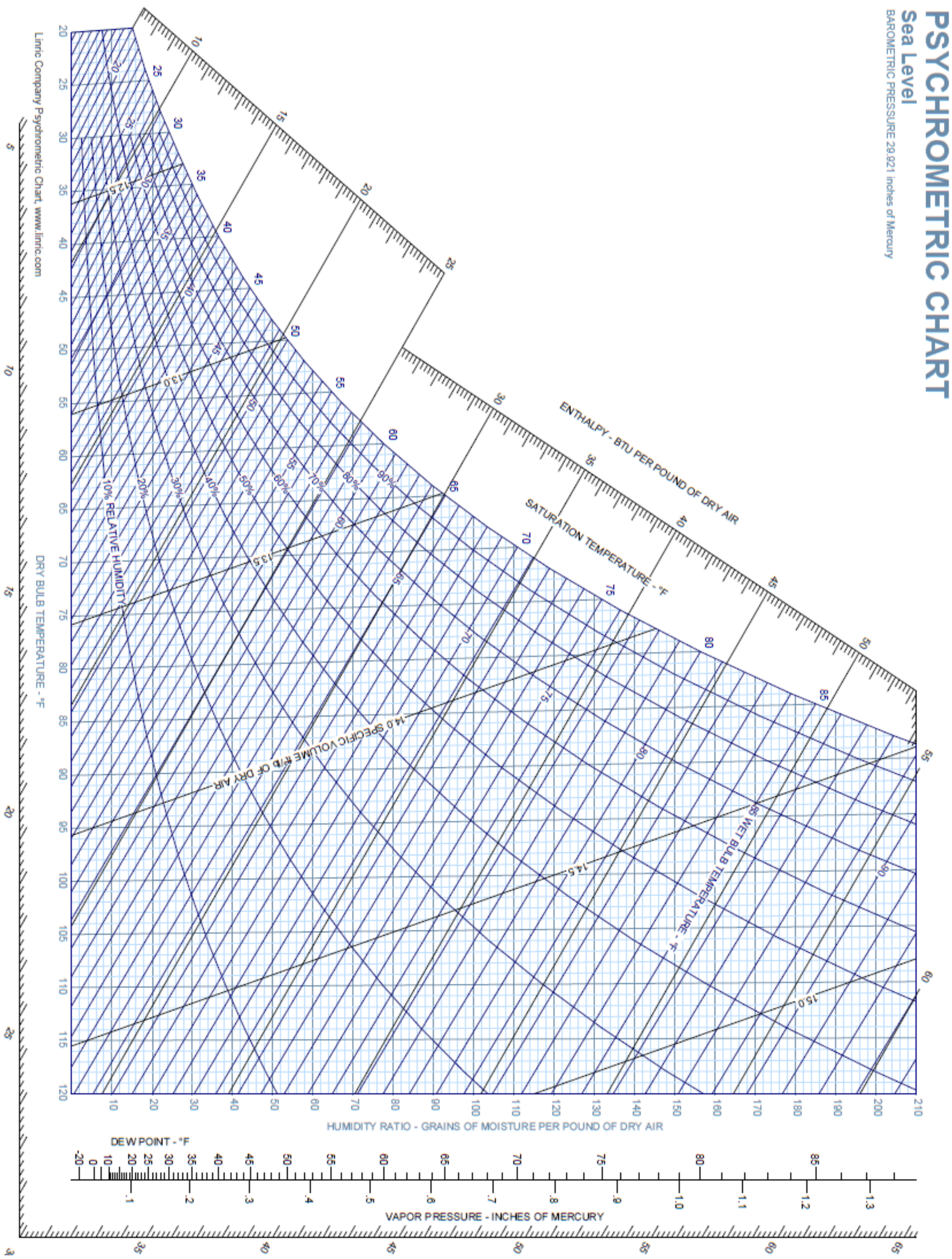
Wrap-up

Phenomenon and physical property/characteristics measurement tools are a wonderful study with depth and breadth. But, hopefully, this more conceptual undertaking did not require performing any mental gymnastics. My desire in this abridged review is that something new was learned in a unique atmosphere despite a lack of deep dives into the tech. Maybe you learned hygrometers are not a dry subject after all, or that "geo-

 Copyright© 2021 Terry A. Willemin, P.E.

orientometers" can give you bearing. Good luck on the test; it should be a good barometer of your understanding of the subject. Finally, thank you for taking this course. And, thank you for bearing with my sense of humor (or the lack thereof)!

 Copyright© 2021 Terry A. Willemin, P.E.



Copyright© 2021 Terry A. Willemin, P.E.

Figure 22, Psychrometric Chart for Sea Level Standard Pressure (Courtesy of Linric, Inc.)

Copyright© 2021 Terry A. Willemin, P.E.

References

1. Flow Measurement Engineering Handbook. R.W. Miller. McGraw-Hill Book Company. 1983. Kingsport Press. New York.
2. Beaufort Wind Scale. National Oceanic and Atmospheric Administration. Retrieved 26 May 2016, from http://www.spc.noaa.gov/faq/tornado/beaufort.html.
3. Windpower Engineering & Development. (2010, August 9). Ultrasonic Wind Sensors or Cup Anemometers? That is the Question. Web. Retrieved 26 May 2016, from http://www.windpowerengineering.com/construction/simulation/ultrasonic-wind-sensors-or-cup-anemometers/.
4. Calibration & Barometric Pressure. Wisconsin Department of Natural Resources. Laboratory Certification. Pressure Adjustments. Web. Retrieved 26 May 2016, from http://dnr.wi.gov/regulations/labcert/BODCalibration2.html.
5. The Chilled Mirror hygrometer: How It Works, Where It Works-and Where It Doesn't. Sensors Online. Questex, LLC. (n.d.). Retrieved 26 May 2016, from http://www.sensorsmag.com/sensors/humidity-moisture/the-chilled-mirror-hygrometer-how-it-works-where-it-works-an-577.
6. Engineering Mechanics: Statics & Dynamics. RC Hibbeler. 9th Edition. Copyright 2001. Prentice Hall.
7. Collins, Dr. A R. "Benjamin Robins on Ballistics." Miscellaneous Technical Articles. Arc.id.au, nd. Retrieved 26 May 2016, from http://www.arc.id.au/RobinsOnBallistics.html.
8. Mechanical Engineering Reference Manual for the PE Exam. Lindeburg, Michael R. PE. 13th edition. Professional Publications, Inc. 2013
9. Modern Compressible Flow. John D. Anderson, Jr. Third edition. McGraw-Hill. 2003. NY, New York.
10. Projectile Supersonic Drag Characteristics. Ilmars Celmins. Army Ballistic Research Lab Aberdeen Proving Ground, MD. Defense Technical Information Center. 1990.
11. Fundamentals of Thermodynamics. Richard E. Sonntag, Claus Borgnakke, Gordon Van Wylen. Fifth edition. John Wiley & Sons, Inc. New York. Copyright 1998.
12. Engineering Design Handbook, Ballistics Series: Interior Ballistics of Guns. US Army Materiel Command. February 1965.
13. Handbook of Firearms and Ballistics: Examining and Interpreting Forensic Evidence. Heard, Brian J. 2nd Edition. 2008. Wiley–Blackwell.

 Copyright© 2021 Terry A. Willemin, P.E.

14. Supersonic Bullet Image. NASA–GALEX. Retrieved 26 May 2016 from https://www.nasa.gov/mission_pages/galex/20070815/f.html.
15. A Unique Personal Radar System. System Image. Labradar. Retrieved 19 November 2020 from https://mylabradar.com/about-us/.
16. Pressure Definitions. National Weather Service. Retrieved 13 November 2020 from https://www.weather.gov/bou/pressure_definitions
17. Handbook of Humidity Measurement: Methods, Materials and Technologies. Ghenadii Korotcenkov. 1st ed. 2019. CRC Press. Boca Raton, FL.

 Copyright© 2021 Terry A. Willemin, P.E.

Errors and Suggestions

When errors are noticed, questions arise or simply to make a suggestion, please email me at: willemta@vt.edu. I'll make every effort to reply in a timely manner. Suggestions for improvements are especially welcome.

 Copyright© 2021 Terry A. Willemin, P.E.

Accompanying Software

By using the accompanying Excel® spreadsheet file (.xlsx or .xlsm) you acknowledge that you have read and agree to the terms and conditions below. If you do not agree, delete the file from all storage media.

Only personal, non-commercial use, of the accompanying file is allowed. By using the file you agree to be bound by the terms of this agreement. You are hereby licensed to make as many copies as you like as long as you do not charge money for, or request donations for such copies. You may not modify, rent or sell the file, or create derivative works based upon it for distribution without the author's express written consent. There are no warrantees or guarantees either expressed or implied, related to the use of this spreadsheet other than that it likely contains errors!

Please email me at: willemta@vt.edu when errors are noticed, questions arise, to use for derivative works, or simply to make a suggestion. I make every effort to reply to emails in a timely manner. Suggestions for improvements are **especially** welcome.

Copyright© 2021 Terry A. Willemin, P.E.