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*A SunCam online continuing education course*

# The Hardy Cross Method and its Successors in Water Distribution Modeling

by

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# Course Description

In the 1930s the Hardy Cross method provided a breakthrough in pipe network analysis. Later, the advent of the modern computer allowed for analysis of even larger distribution systems using the method. Despite the development of more efficient computer algorithms, the Hardy Cross method remains as the pipe network analysis method taught to most undergraduate civil engineering students. This course covers the history, basic principles, assumptions, step-by-step procedures, advantages, and disadvantages for solving pipe network problems using the Hardy Cross method. Multiple sample problems are solved using the Hardy Cross method. The three predominate analysis methods used by water distribution modeling software over the past 40 years are introduced and explained (the Newton-Raphson, Linear Theory, and Gradient methods). Examples for two of these three methods are provided. Finally, five leading software developers and their water distribution modeling software programs are discussed. A total of ten example problems and solutions are presented throughout the course as topics are introduced to reinforce learning.

# Learning Objectives

This course teaches the following specific knowledge and skills:

1. Hydraulic concepts necessary for understanding various pipe network analysis methods
2. The history behind the development of the Hardy Cross method
3. The basic principles the Hardy Cross method is based on and the step-by-step procedures used for pipe network analysis
4. The differences between the Hardy Cross and Modified/Improved Hardy Cross methods
5. Some of the advantages and disadvantages of using the Hardy Cross method
6. The differences between loop methods, node methods, flow methods, and gradient/node-loop methods for analyzing pipe networks
7. The basic concept of the Newton-Raphson method
8. The basic concept and application of the Linear Theory method
9. The basic concept and application of the Gradient method
10. A timeline of water distribution modeling advances from 1960 to 2020
11. The head loss methods and hydraulic balancing methods used by common hydraulic modeling software programs
12. Introduces leading modeling software developers, programs, and their basic capabilities

# Introduction

Water distribution system models have become very important and practical tools for civil engineers. They are often used to optimize the design of new distribution systems or analyze major extensions or modifications to existing distribution systems. Computer models can help engineers answer many common questions. For example, what is the maximum fire flow at a given point in the existing system? How long can that fire flow be provided for? What size pipe installation would be necessary between two points in a system to increase the pressure at one of the points to the minimum pressure required? If a subdivision or commercial development is built, will adequate pressures and flows exist? If not, what length and size of water mains must be upgraded by the developer to allow for the proposed construction?

Most undergraduate water resources classes introduce the basic principles used in current hydraulic modeling software for distribution systems, but do not cover the contemporary methods used in the software. Generally, undergraduate engineers are taught the Hardy Cross method since the method's steps can be calculated manually with relative ease and subsequently the concepts behind the method can be more easily understood. In this course the Hardy Cross method will be reviewed, followed by an overview of the newer methods which have in recent decades supplanted the Hardy Cross method as the method of choice for use in modeling software. At the end of the material a brief summary of the methods used by some popular public-domain and commercial hydraulic models is presented.

Figure 1 shows a sample schematic of a water distribution system consisting of pipes, a reservoir, a pump, and a storage tank. A simplified schematic like this is often said to be *skeletonized*. This skeletonized distribution system is a fairly small example of a system that might be analyzed using one of the newer methods.

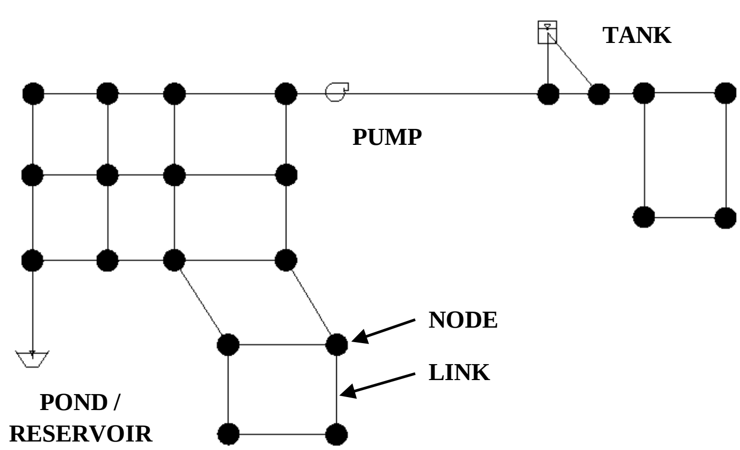



**Figure 1: Sample Schematic of a Water Distribution System**

# Basic Hydraulic Review

Prior to beginning our discussion on pipe networks a brief overview of pipe hydraulics is probably warranted. At the very least this will provide clear definitions of terms used later in the course. At most it may briefly summarize terms and equations you have not used or thought about in years.

**Energy Head in Pipe Flow**

The majority of the energy in water flowing through pressurized pipes exists in three fundamental forms. For convenience these forms are often all expressed in terms of head, with the units of head being a measure of length. These forms are shown below with each respective form's expression for head displayed in parenthesis.

1. Kinetic Energy $\left(\frac{V^2}{2g}\right)$
2. Potential Energy $(h)$
3. Pressure Energy $\left(\frac{P}{\gamma}\right)$

Where: $V$ = Mean velocity
$g$ = Gravity
$h$ = Elevation
$P$ = Pressure
$\gamma$ = Specific weight of water

Graphically the energy head at two different locations in a pressurized pipe is often depicted as shown in Figure 2. The energy loss experienced as the water moves from location 1 to location 2 is captured by the head loss term, $h_L$.

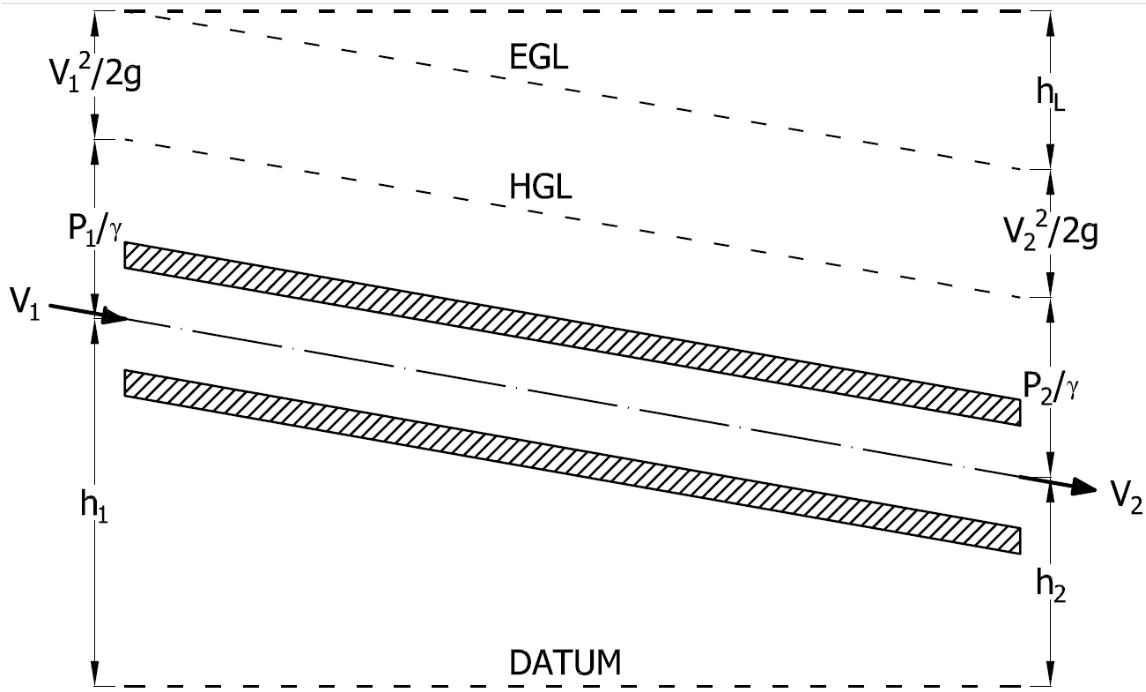



**Figure 2: Energy Head and Head Loss in Pressurized Pipe Flow**

The energy head at location 1 is the sum of the kinetic, pressure, and potential heads at that location. The energy head at location 2 is the sum of the kinetic, pressure, and potential heads plus the head loss between the two locations. This relationship if often described by the *energy equation* shown below (also called Bernoulli's equation by some, though more traditionally Bernoulli's equation does not include a head loss term).

$$\frac{V_1^2}{2g} + \frac{P_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2 + h_L \quad (Equation\ 1)$$

**Grade Lines**

In pressurized pipe flow the hydraulic grade line (HGL) is the sum of the potential, or elevation, head and pressure head. The energy grade line (EGL) for pressurized pipe flow is the sum of the potential head, pressure head, and kinetic, or velocity, head. These grade lines are show in Figure 2.

**Converting Between Pressure and Pressure Head**

Conversion between pressure and pressure head for any fluid can be accomplished using the following equation.

$$P = \gamma h \quad (Equation\ 2)$$

Where: $P$ = Pressure
$\gamma$ = Specific weight of the fluid
$h$ = Pressure head

EXAMPLE:
Convert a pressure of 60 pounds per square inch (psi) to an equivalent pressure head for water. Assume the specific weight of water is 62.4 lb/ft$^3$.

SOLUTION:
Use Equation 2 and rearrange it so that *h* is by itself on one side of the equation. Then use the appropriate conversation factor so that units are consistent.

$$h = \frac{P}{\gamma} = \frac{\left(60\ \frac{lb}{in^2}\right)\left(\frac{144\ in^2}{1\ ft^2}\right)}{\left(62.4\ \frac{lb}{ft^3}\right)} = 138.5\ ft$$

**Hydraulic Radius**

The hydraulic radius is defined as the water cross-sectional area, *A*, divided by the wetted perimeter, *P*. The water cross-sectional area is simply the area of a given cross-section comprised of water and the wetted perimeter is the distance over which the water cross-section is in contact with a surface other than air. In the case of pressurized pipe flow the water cross-sectional area is the same as the internal pipe area in cross-section and the wetted perimeter is the internal perimeter of the pipe cross-section.

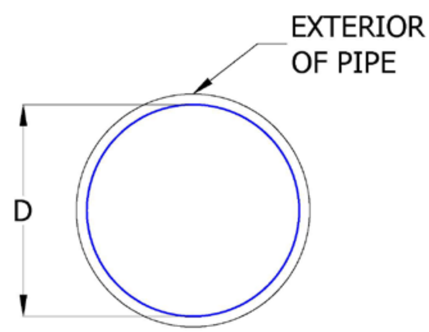


**Figure 3: The wetted perimeter, *P*, is shown in blue for a circular pipe flowing full. In this situation the internal perimeter or circumference is the same as the wetted perimeter. *D*, is the internal pipe diameter.**

The equation for calculating the hydraulic radius of a pressurized, full-flowing circular pipe is given below.

$$R = \frac{A}{P} = \frac{\left(\frac{\pi D^2}{4}\right)}{\pi D} = \frac{D}{4} \quad (Equation\ 3)$$

Where: $R$ = Hydraulic radius
$A$ = Water cross-sectional area

$P$ = Wetted perimeter
D = Internal pipe diameter

**Mean Velocity**
For many hydraulic computations the mean velocity for a given cross-section of water is assumed to be the discharge divided by the water cross-sectional area.

$$V = \frac{Q}{A} \quad (Equation\ 4)$$

Where: $V$ = Mean velocity
$Q$ = Discharge (sometimes referred to as the flow rate or simply flow)
$A$ = Water cross-sectional area

**Formulas for Friction Head Loss**
Many different formulas for estimating friction loss in pipe flow have been developed. The most common formulas used in the United States (U.S.) are probably the Hazen-Williams formula, the Darcy-Weisbach formula, the Kutter formula, and the Manning formula. Sometimes the Manning formula is called the Chezy-Manning formula since it is in part based on Antoine Chezy's previously developed formula. Only two of these formulas are covered in this course.

## Hazen-Williams Formula

The Hazen-Williams formula is an empirical formula developed for pipes with diameters of approximately two inches or larger. It is valid for mean velocities less than about 10 feet per second. It is only appropriate for water flow, not any other fluid.

In English units the Hazen-Williams formula is usually written:

$$V = 1.318 C_{HW} R^{0.63} S^{0.54} \quad (Equation\ 5)$$

Where: $V$ = Mean velocity, in ft/s
$C_{HW}$ = Hazen-Williams coefficient, a dimensionless coefficient generally obtained from a table
$R$ = Hydraulic radius, in ft
$S$ = Slope of the energy grade line (EGL), in ft/ft

Based on the definition of $S$ above another way to write the slope of the EGL is:

$$S = \frac{h_L}{L} \quad (Equation\ 6)$$

Where: $S$ = Slope of the energy grade line (EGL)
$h_L$ = Head loss
$L$ = Length of pipe

Typical Hazen-Williams coefficients for various pipe materials are shown in Table 1. As illustrated by the differing cast-iron coefficient values in the table some coefficient values can changes with time due to corrosion, scaling, etc. For the materials in Table 1 that do not have multiple values the values shown should be considered average values unless otherwise stated. The Hazen-Williams coefficient is not dependent on the flow condition (i.e. laminar or turbulent flow). This is different than the Darcy-Weisbach formula which does take into account whether flow is laminar or turbulent when estimating the friction factor for a given pipe material.

The form of the Hazen-Williams formula in S.I. units is usually presented as:

$$V = 0.849 C_{HW} R^{0.63} S^{0.54} \quad (Equation\ 7)$$

Where: $V$ = Mean velocity, in m/s
$C_{HW}$ = Hazen-Williams coefficient, a dimensionless coefficient generally obtained from a table
$R$ = Hydraulic radius, in m
$S$ = Slope of the energy grade line (EGL), in m/m

| Pipe Material | $C_{HW}$ |
|---|---|
| Asbestos cement | 140 |
| Brass | 130 - 140 |
| Brick sewer | 100 |
| Cast-iron | |
| New, unlined | 130 |
| 10 yr. old | 107 - 113 |
| 20 yr. old | 89 - 100 |
| 30 yr. old | 75 - 90 |
| 40 yr. old | 64 - 83 |
| Concrete or concrete lined | |
| Steel forms | 140 |
| Wooden forms | 120 |
| Centrifugally spun | 135 |

| Pipe Material | $C_{HW}$ |
|---|---|
| Copper | 130 - 140 |
| Ductile iron | 130 |
| Galvanized iron | 120 |
| Glass | 140 |
| Lead | 130 - 140 |
| Plastic | 140 - 150 |
| Steel | |
| Coal-tar enamel lined | 145 - 150 |
| New unlined | 140 - 150 |
| Riveted | 110 |
| Tin | 130 |
| Vitrified clay (good condition) | 110 - 140 |
| Wood stave (average condition) | 120 |

**Table 1: Values of Hazen-Williams Coefficients (from Hwang Table 3.2 and Bentley's WaterGEMS software)**

EXAMPLE:

Use the Hazen-Williams formula to calculate the mean velocity, in ft/s, for a 12 inch internal diameter circular pipe with a centrifugally spun concrete lining that is 400 feet long and has a pressure loss of 5 psi over its 400 foot length. The elevations at the beginning and end of the pipe are the same. Assume the specific weight of water is 62.4 lb/ft$^3$.

SOLUTION:

First calculate the hydraulic radius using Equation 3. Make sure to convert the diameter from inches to feet since the empirical formula, as shown, requires the diameter to be in feet.

$$R = \frac{D}{4} = \frac{1}{4} = 0.25\ ft$$

Next use Equation 2 to convert the pressure loss to pressure head loss.

$$h = \frac{P}{\gamma} = \frac{\left(5\ \frac{lb}{in^2}\right)\left(\frac{144\ in^2}{1\ ft^2}\right)}{\left(62.4\ \frac{lb}{ft^3}\right)} = 11.54\ ft$$

Then use Equation 6 to calculate the slope of the EGL.

$$S = \frac{h_L}{L} = \frac{11.54}{400} = 0.02885\ ft/ft$$

Finally use Equation 5 to calculate the mean velocity. The value for $C_{HW}$ is found in Table 1 based on the description of the pipe material provided in the example problem statement.

$$V = 1.318 C_{HW} R^{0.63} S^{0.54} = (1.318)(135)(0.25)^{0.63}(0.02885)^{0.54} = 10.95\ ft/s$$

The calculated mean velocity is greater than 10 ft/s. The valid range for use of the Hazen-Williams formula is about 10 ft/s or less, therefore use of the formula in this specific situation may be questionable.

EXAMPLE:

For the previous example calculate the mean velocity, in ft/s, if the beginning of the pipe is 5 feet above an arbitrarily selected datum and the end of the pipe is 8 feet above the datum.

SOLUTION:

Since the pipe is now sloped (instead of flat) the slope of the EGL changes. First the head loss must be calculated. This can be accomplished by rearranging Equation 1 to solve for head loss, $h_L$. The two velocity head terms cancel each other out because the velocity is the same at both ends of the pipe (since the diameter of the pipe is the same throughout its length). The 144 shown in the equation below is the conversion factor between lb/in$^2$ and lb/ft$^2$.

$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + \frac{P_1}{\gamma} - \frac{P_2}{\gamma} + h_1 - h_2 = \frac{P_1 - P_2}{\gamma} + h_1 - h_2 = \frac{(5)(144)}{62.4} + 5 - 8 = 8.54\ ft$$

Divide the head loss by the pipe length to find the slope of the EGL.

$$S = \frac{h_L}{L} = \frac{8.54}{400} = 0.02135\ ft/ft$$

Use Equation 5 to calculate the mean velocity.

$$V = 1.318 C_{HW} R^{0.63} S^{0.54} = (1.318)(135)(0.25)^{0.63}(0.02135)^{0.54} = 9.31\ ft/s$$

Compared to the first example the mean velocity is slower. This makes sense because as velocity decreases so does discharge (recall Equation 4 where $Q = VA$) and given the same pressure drop we would expect less water to be forced up to a higher elevation.

## Darcy-Weisbach Formula

Only a basic review of the Darcy-Weisbach formula is provided here as a comprehensive review is beyond the scope of the course.

In English and S.I. units the Darcy-Weisbach form is typically written:

$$h_f = \frac{P_1 - P_2}{\gamma} = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) \quad (Equation\ 8)$$

Where:
- $h_f$ = Head loss due to friction along the length of pipe, in ft [m]
- $P_1$ = Pressure at point 1 of the pipe, in lb/ft$^2$ [N/m$^2$]
- $P_2$ = Pressure at point 2 of the pipe, in lb/ft$^2$ [N/m$^2$]
- $\gamma$ = Specific weight of the fluid, in lb/ft$^3$ [N/m$^3$]

$f$ = Darcy-Weisbach friction factor, a dimensionless factor
$L$ = Length of pipe, in ft [m]
$D$ = Internal pipe diameter, in ft [m]
$V$ = Mean velocity, in ft/s [m/s]
$g$ = Gravity, in ft/s$^2$ [m/s$^2$]

The friction factor term is a function of pipe diameter, mean velocity, kinematic viscosity, and relative roughness. The friction factor can be obtained for a given flow situation in a number of ways. Often the Moody Diagram is used. Alternatively, a number of different implicit equations have been developed. These implicit equations cannot isolate the friction factor term on a single side of the equations which makes them more difficult to solve (hence the Moody Diagram is commonly used). In practice the Darcy-Weisbach formula is probably the best choice for pipe network analysis, but for simplicity the Hazen-Williams formula is used primarily in this course.

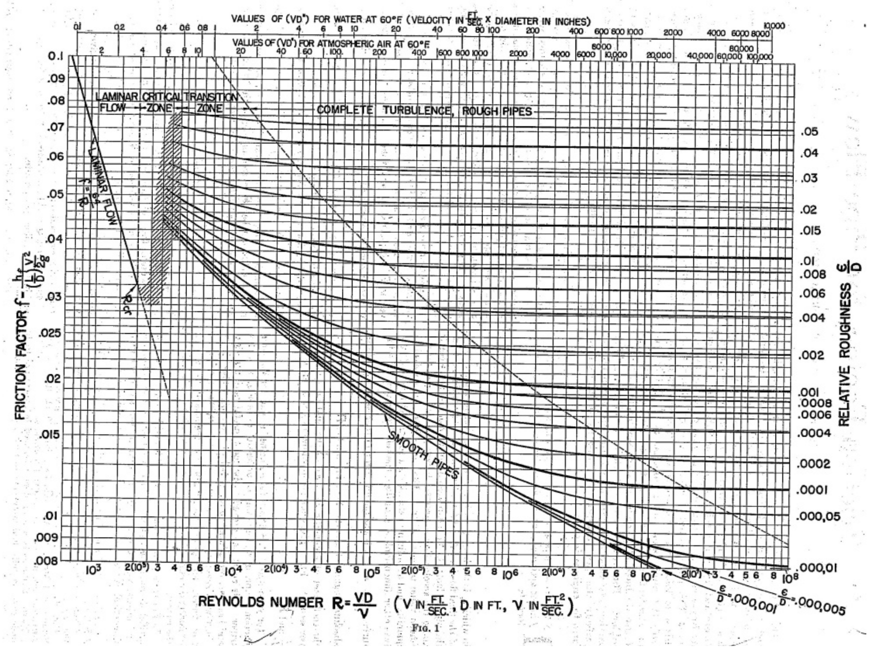



**Figure 4: Moody Diagram from Lewis Moody's 1944 *Transactions of the A.S.M.E.* paper entitled "Friction Factors for Pipe Flow"**

## EXAMPLE:

Solve the first example problem presented in the section on the Hazen-Williams formula. Use the Darcy-Weisbach formula to calculate the mean velocity, in ft/s, for a 12 inch internal diameter circular pipe with a centrifugally spun concrete lining that is 400 feet long and has a pressure loss of 5 psi over its 400 foot length. Assume the specific weight of the water is 62.4 lb/ft$^3$. Since the Moody Diagram in the course text is difficult to read, use a friction factor of 0.0202.

SOLUTION:

First use Equation 8 to convert the pressure loss to head loss.

$$h_f = \frac{P_1 - P_2}{\gamma} = \frac{\left(5\ \frac{lb}{in^2}\right)\left(\frac{144\ in^2}{1\ ft^2}\right)}{\left(62.4\ \frac{lb}{ft^3}\right)} = 11.54\ ft$$

Then rearrange Equation 8 to solve for mean velocity.

$$V = \sqrt{\frac{2gh_fD}{fL}} = \sqrt{\frac{(2)(32.2)(11.54)(1)}{(0.0202)(400)}} = 9.59\ ft/s$$

The mean velocity calculated using the Darcy-Weisbach formula is approximately 10 percent slower than the velocity calculated using the Hazen-Williams formula.

**Minor Losses**

In many situations the majority of head loss occurs in straight lengths of pipe due to friction. However additional head losses can occur as the result of many different conditions such as:

1. Contraction – pipe diameter decreasing abruptly or gradually
2. Pipe Entrance – water entering a pipe from a reservoir, tank, etc.
3. Enlargement – pipe diameter increasing abruptly or gradually
4. Pipe Fittings – bends, tees, crosses, etc.
5. Valves

Head losses from these conditions are usually called minor losses since they are often small compared to the losses in long, straight sections of pipe. Typically the head loss is estimated by multiplying the velocity head (kinetic head), $V^2/2g$, by a coefficient, *K*.

$$h_{minor} = K\frac{V^2}{2g} \quad (Equation\ 9)$$

The *K* coefficients are normally obtained from a table of values or from the manufacturer of the specific valve, fitting, etc. The recommended method is to acquire the value from the part manufacturer. A few generic values for *K* are shown in Table 2 and Table 3. Please note these tables should only be used for preliminary calculations. In many cases *K* values for valves are dependent on valve size and other factors.

| Ratio of Centerline Bend Radius to Pipe Diameter, R/D | K |
|---|---|
| 1 | 0.35 |
| 2 | 0.19 |
| 4 | 0.17 |
| 6 | 0.22 |
| 10 | 0.32 |
| 16 | 0.38 |
| 20 | 0.42 |

| Valve Type (all fully open) | K |
|---|---|
| Gate | 0.15 |
| Globe | 10.0 |
| Rotary | 10.0 |
| Check | |
| Swing Type | 2.5 |
| Ball Type | 70.0 |
| Lift Type | 12.0 |

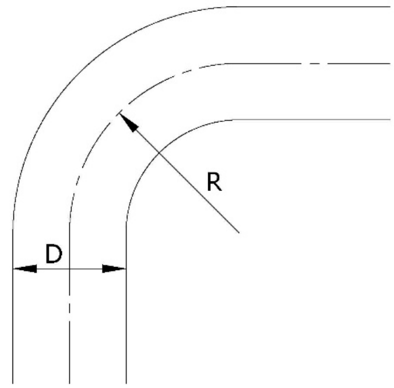



**Table 2: Generic Values of *K* Coefficients (from Hwang Chapter 3)**

| Fitting | K |
|---|---|
| Pipe Entrance - Bellmouth | 0.03 - 0.05 |
| Pipe Entrance - Rounded | 0.12 - 0.25 |
| Pipe Entrance - Sharp Edged | 0.50 |
| Pipe Entrance - Projecting | 0.80 |
| Contraction - Sudden, $D_2/D_1 = 0.80$ | 0.18 |
| Contraction - Sudden, $D_2/D_1 = 0.50$ | 0.37 |
| Contraction - Sudden, $D_2/D_1 = 0.20$ | 0.49 |
| Contraction - Conical, $D_2/D_1 = 0.80$ | 0.05 |
| Contraction - Conical, $D_2/D_1 = 0.50$ | 0.07 |
| Contraction - Conical, $D_2/D_1 = 0.20$ | 0.08 |
| Expansion - Sudden, $D_2/D_1 = 0.80$ | 0.16 |
| Expansion - Sudden, $D_2/D_1 = 0.50$ | 0.57 |
| Expansion - Sudden, $D_2/D_1 = 0.20$ | 0.92 |
| Expansion - Conical, $D_2/D_1 = 0.80$ | 0.03 |
| Expansion - Conical, $D_2/D_1 = 0.50$ | 0.08 |
| Expansion - Conical, $D_2/D_1 = 0.20$ | 0.13 |

| Fitting | K |
|---|---|
| Mitered Bend, $\theta = 15°$ | 0.05 |
| Mitered Bend, $\theta = 30°$ | 0.10 |
| Mitered Bend, $\theta = 45°$ | 0.20 |
| Mitered Bend, $\theta = 60°$ | 0.35 |
| Mitered Bend, $\theta = 90°$ | 0.80 |
| Tee, Line Flow | 0.30 - 0.40 |
| Tee, Branch Flow | 0.75 - 1.80 |
| Cross, Line Flow | 0.50 |
| Cross, Branch Flow | 0.75 |
| 45° Wye, Line Flow | 0.30 |
| 45° Wye, Branch Flow | 0.50 |

**Table 3: Generic Values of K Coefficients (from Walski Chapter 6)**

EXAMPLE:

The velocity head in an 18 inch diameter pipe is 0.60 feet. A gate valve, a 90 degree bend (with 18 inch radius), and a second gate valve are present in the layout of the 18 inch pipe. Estimate the total minor losses.

SOLUTION:

First calculate the ratio of the bend radius to pipe diameter which in this case is 1.0 (R/D = 18 inch/18 inch = 1). From Table 2 the *K* value for a R/D of 1.0 is 0.35. Also from Table 2 the *K* value for a gate valve is 0.15. Plug the *K* value and the given velocity head into Equation 9 to calculate the minor loss for each valve or fitting. The minor losses for each individual valve or fitting can be added together to find the total minor loss.

$$h_{minor} = K_1 \frac{V^2}{2g} + K_2 \frac{V^2}{2g} + K_3 \frac{V^2}{2g}$$
$$h_{minor} = \frac{V^2}{2g}(K_1 + K_2 + K_3) = (0.60)(0.15 + 0.35 + 0.15) = 0.39\ ft$$

# Taylor Series

Multiple methods discussed later in this course utilize a technique taught in most calculus classes called Taylor series, or Taylor polynomials. This is a technique that approximates the value of functions at a given point using an infinite sum of polynomial terms. It is useful because non-polynomial functions can be approximated using polynomial terms that are much easier to calculate and/or solve, especially when trying to solve multiple equations simultaneously. For example, the Taylor series for the function $e^x$ is:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots$$

Table 4 compares the approximated value of $e^x$ using six terms of its Taylor series to the true value of $e^x$ for $x$ values of 0, 1, and 2.

| | Taylor Series Terms | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| x Values | 1 | x | $x^2/2!$ | $x^3/3!$ | $x^4/4!$ | $x^5/5!$ | Approximated Value | True Value (Rounded) |
| 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0.5 | 0.167 | 0.042 | 0.008 | 2.717 | 2.718 |
| 2 | 1 | 2 | 2 | 1.333 | 0.667 | 0.267 | 7.267 | 7.389 |

**Table 4: Approximated Values of Taylor Series for $e^x$**

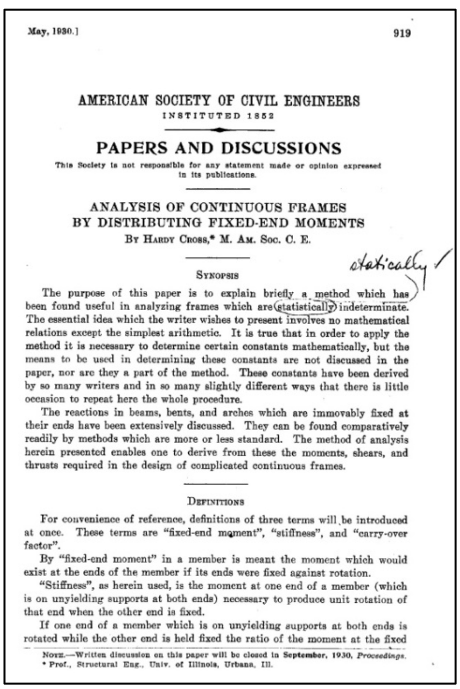
  

As you can see from the above table, the approximated values using six terms are quite good for this function at the values of $x$ shown. Methods used to solve water distribution piping problems typically only use the first few terms of a Taylor series.

# Hardy Cross Method

### History of the Hardy Cross Method

Hardy Cross was an engineering consultant and professor of structural engineering. His most notable work occurred while teaching at the University of Illinois at Urbana-Champaign. Much of his efforts focused on analyzing statically indeterminate structures (structures which cannot be solved using statics principles only, but rather also require material or deformation information to solve). In 1930 he published a paper on an iterative method he developed for calculating the magnitude of moments in indeterminate structures entitled, "Analysis of Continuous Frames by Distributing Fixed-End Moments". His moment distribution method was used as the primary method for designing indeterminate structures for decades. Cross realized a similar iterative approach could be used to solve problems in other engineering fields as well. In 1936 he published a paper entitled, "Analysis of Flow in Networks of Conduits or Conductors". In the paper he described how to analyze incompressible flow in a network regardless if it was a fluid network or an electrical network. In the case of electricity, amperage is analogous to fluid flow (flow rate or discharge), voltage to friction loss, and resistance to a roughness coefficient or friction factor.



May, 1930.] 919

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PAPERS AND DISCUSSIONS

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ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

By Hardy Cross,* M. Am. Soc. C. E.

statically

Synopsis

The purpose of this paper is to explain briefly a method which has been found useful in analyzing frames which are statistically indeterminate. The essential idea which the writer wishes to present involves no mathematical relations except the simplest arithmetic. It is true that in order to apply the method it is necessary to determine certain constants mathematically, but the means to be used in determining these constants are not discussed in the paper, nor are they a part of the method. These constants have been derived by so many writers and in so many slightly different ways that there is little occasion to repeat here the whole procedure.

The reactions in beams, bents, and arches which are immovably fixed at their ends have been extensively discussed. They can be found comparatively readily by methods which are more or less standard. The method of analysis herein presented enables one to derive from these the moments, shears, and thrusts required in the design of complicated continuous frames.

Definitions

For convenience of reference, definitions of three terms will be introduced at once. These terms are "fixed-end moment", "stiffness", and "carry-over factor".

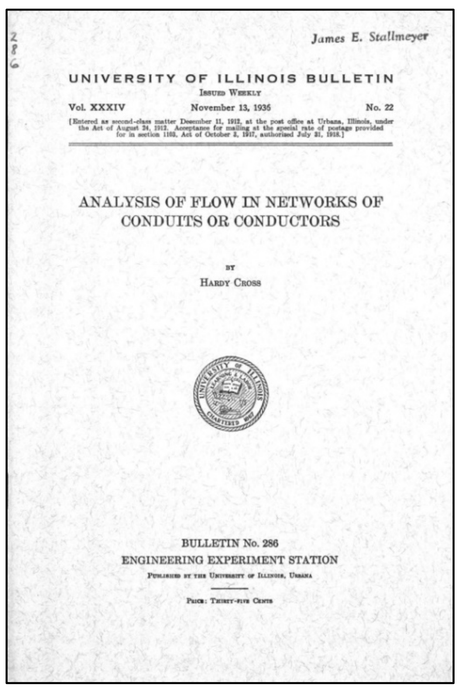
By "fixed-end moment" in a member is meant the moment which would exist at the ends of the member if its ends were fixed against rotation.

"Stiffness", as herein used, is the moment at one end of a member (which is on unyielding supports at both ends) necessary to produce unit rotation of that end when the other end is fixed.

If one end of a member which is on unyielding supports at both ends is rotated while the other end is held fixed the ratio of the moment at the fixed

Note.—Written discussion on this paper will be closed in September, 1930, *Proceedings*.
* Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

**Cover Page of Cross'1930 Paper**



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James E. Stallmeyer

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ANALYSIS OF FLOW IN NETWORKS OF CONDUITS OR CONDUCTORS

by

Hardy Cross

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**Cover Page of Cross'1936 Paper**

**Concepts and Procedures of the Hardy Cross Method**

In his 1936 paper Cross began by outlining the two major conditions his method was based on. These conditions had been realized previously so Cross was not describing anything new.

1. Continuity of flow: The total flow reaching any junction or node must equal the total flow leaving that junction or node ($\sum Q_{in} - \sum Q_{out} = 0$, where $Q$ is flow). This principal is commonly referred to as Kirchoff's First Law. Its first application was in the realm of electricity, but it can be applied to many different applications.

2. Continuity of potential: The total potential change along any closed path must equal zero ($\sum h = 0$, where $h$ is head loss). This is often referred to as Kirchoff's Second Law. An alternate way of stating this is that the head loss between two nodes must be the same regardless of the path taken.

These two conditions can be used along with some type of known relationship between flow and potential drop (or head loss) to write a system of equations with unknown variables for either the flow in each pipe/conduit/conductor or the potentials (or heads) at each junction or node. If the relationship between flow and potential is linear the system of equations developed is linear. If the relationship between flow and potential is non-linear the system of equations developed is non-linear. In the 1930s any system of non-linear equations was very difficult to solve since the calculations had to be completed by hand (at that time solving large systems of linear equations was time consuming, but not terribly difficult). Unfortunately many relationships between flow and potential drop are not linear which left engineers with what seemed like an almost intractable problem given their lack of computing power. Cross used a calculus based technique (Taylor series) to turn a system of non-linear equations into a series of linear relationships which could be iteratively solved to converge on the solution—this was the big breakthrough. Cross referred to this convergence based method as "successive corrections".

Cross outlined two different solution methods using his technique. The first, which he called the "Method of Balancing Heads" kept the continuity condition true at all times and adjusted the flow in each pipe/conduit/conductor until continuity of potential was achieved. This method is considered a *loop method*. The second, which he termed the "Method of Balancing Flows" forced the total potential change along all closed paths to remain zero at all times and adjusted the total flow into and out of each junction or node until they became equal. This type of method is called a *node method*. Generally, if the inlet and outlet flows are known the "Method of Balancing Heads" is used and if the inlet and outlet potential heads are known the "Method of Balancing Flows" is used. In Cross' paper he described head loss in general terms using the following equation:

  

$$h = rQ^n \quad (Equation\ 10)$$

Where: $h$ = Change in head for a given flow over a given length
$r$ = Head loss for a unit quantity of flow (dependent on size and roughness or resistance of the conduit or conductor)
$Q$ = Flow
$n$ = Coefficient expressing the relationship between flow and head loss

The steps for the "Method of Balancing Heads" are listed below:

1. Assume Flow Distribution: Assume any quantity and direction of flow in each pipe/conduit/conductor as long as the quantities and directions maintain continuity of flow at each junction or node.

2. Calculate Head Loss: Calculate the loss of head in each pipe/conduit/conductor using your relationship of choice. In the general case $h = rQ^n$.

3. Calculate Total Head Loss Around Each Loop: Calculate the total loss of head around each closed loop while taking into account the direction of head loss. Taking into account the direction of head loss can be accomplished by assuming flow in the clockwise direction results in positive head loss and flow in the counterclockwise direction results in negative head loss, or vice versa. In the general case $\sum h = \sum rQ^n$.

4. Calculate $R$: Calculate the $R$ value, which is approximately the resistance to change in flow, in each pipe/conduit/conductor using your relationship of choice. In the general case $R = nrQ^{(n-1)}$.

5. Calculate Total $R$ Around Each Loop: Calculate the $R$ value around each closed loop without taking into account the direction of head loss (meaning all $R$ values are positive). In the general case $\sum R = \sum nrQ^{(n-1)}$.

6. Calculate the Flow Correction: Calculate the change in flow for each closed loop for the next iteration so that total head loss around each loop gets closer to zero. In the general case $\Delta Q = \sum h / \sum R$.

7. Correct Flows: Calculate the revised flows for each pipe/conduit/conductor by using the calculated $\Delta Q$ value(s).

8. Repeat: Repeat the procedure starting at step 2 until the $\Delta Q$ term is sufficiently small for your application.

  

The step-by-step procedures for implementing the “Method of Balancing Flows” are not presented in this course. Regardless if the balancing heads or balancing flows method is used the Hardy Cross method assumes no head losses at junction points (nodes), for example minor losses due to a cross fitting, are small compared to friction losses and as a result they are neglected. This is a reasonable assumption for most water distribution systems, but may not be an appropriate assumption for some mechanical or chemical systems.

The following example from Cross’ paper is a good illustration of how to use the method for a generic situation.

EXAMPLE:

Figure 5 shows the geometry for a simple pipe network. The flow into A is 100 (no units), the flow out of D is 100, the $r$ value along path ABCD is four times the $r$ value along AD, and the distance between AB, BC, CD, and AD are all the same. For this problem $h$ is directly proportional to $Q^{1.5}$.

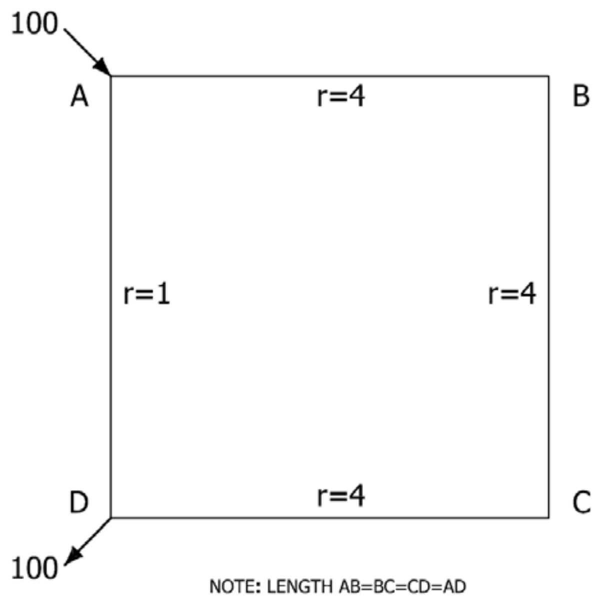


**Figure 5: Simple Pipe Network with Unitless Input Flow, Outlet Flow, and *r* Values**

SOLUTION:

Please refer to Figure 6 throughout this solution for a visual reference.

  

Step 1 - Assume Flow Distribution: Assume all flow travels along path ABCD. To keep continuity true, given this assumption, no flow can be assumed to travel along path AD. Please note this is about as bad of an initial guess as possible.

Step 2 - Calculate Head Loss: Given that $h$ is directly proportional to $Q^{1.5}$ the head loss along each path is:

$$h_{ABCD} = rQ^n = (4)(100)^{1.5} = 4{,}000$$
$$h_{AD} = rQ^n = (1)(0)^{1.5} = 0$$

Step 3 - Calculate Total Head Loss Around Each Loop: Assume flow in the clockwise direction results in positive head loss and flow in the counterclockwise direction results in negative head loss. For illustrative purposes the zero head loss in AD is shown as counterclockwise in Figure 6 and this calculation step. The total loss of head around the loop while taking into account the direction of head loss is:

$$\sum h = 4{,}000 - 0 = 4{,}000$$

Step 4 - Calculate $R$: Given that $h$ is directly proportional to $Q^{1.5}$ the $R$ value along each path is:

$$R_{ABCD} = nrQ^{(n-1)} = (1.5)(4)(100)^{(1.5-1)} = 60$$
$$R_{AD} = nrQ^{(n-1)} = (1.5)(1)(0)^{(1.5-1)} = 0$$

Step 5 - Calculate Total $R$ Around Each Loop: The total $R$ value around the loop without taking into account the direction of head loss is:

$$\sum R = 60 + 0 = 60$$

Step 6 - Calculate the Flow Correction: The change in flow for the next iteration is:

$$\Delta Q = \frac{\sum h}{\sum R} = \frac{4{,}000}{60} = 67$$

Step 7 - Correct Flows: Using the originally assumed flows and the calculated $\Delta Q$ value, the revised flows are:

  

$Q_{ABCD} = Q_{ABCD} - \Delta Q = 100 - 67 = 33$ (clockwise flow)
$Q_{AD} = Q_{ABCD} - \Delta Q = 0 - 67 = -67$ (counterclockwise flow)

The flow between A and D is counterclockwise since the *Q* value calculated is negative. The middle pipe network in Figure 6 shows the estimated flows after one correction. Repeat steps 2 through 7 given the new flows of 33 and 67.

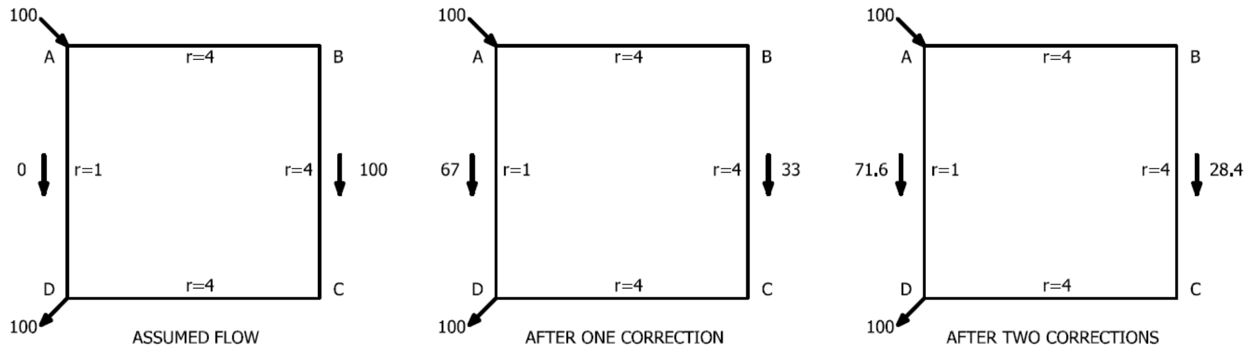


**Figure 6: Simple Pipe Network with (Left) Assumed Flows and (Middle & Right) Corrected Flows**

Step 2 - Calculate Head Loss:

$h_{ABCD} = rQ^n = (4)(33)^{1.5} = 758$
$h_{AD} = rQ^n = (1)(67)^{1.5} = 548$

Step 3 - Calculate Total Head Loss Around Each Loop:

$$\sum h = 758 - 548 = 210$$

Step 4 - Calculate *R*:

$R_{ABCD} = nrQ^{(n-1)} = (1.5)(4)(33)^{(1.5-1)} = 34$
$R_{AD} = nrQ^{(n-1)} = (1.5)(1)(67)^{(1.5-1)} = 12$

Step 5 - Calculate Total *R* Around Each Loop:

$$\sum R = 34 + 12 = 46$$

  

Step 6 - Calculate the Flow Correction:

$$\Delta Q = \frac{\sum h}{\sum R} = \frac{210}{46} = 4.6$$

Step 7 - Correct Flows:

$Q_{ABCD} = Q_{ABCD} - \Delta Q = 33 - 4.6 = 28.4$ (clockwise flow)
$Q_{AD} = Q_{ABCD} - \Delta Q = -67 - 4.6 = -71.6$ (counterclockwise flow)

The rightmost pipe network in Figure 6 shows the estimated flows after two corrections. Repeat steps 2 through 7 given the new flows of 28.4 and 71.6.

Step 2 - Calculate Head Loss:

$h_{ABCD} = rQ^n = (4)(28.4)^{1.5} = 605$
$h_{AD} = rQ^n = (1)(71.6)^{1.5} = 606$

Step 3 - Calculate Total Head Loss Around Each Loop:

$$\sum h = 605 - 606 = -1$$

Step 4 - Calculate *R*:

$R_{ABCD} = nrQ^{(n-1)} = (1.5)(4)(28.4)^{(1.5-1)} = 32$
$R_{AD} = nrQ^{(n-1)} = (1.5)(1)(71.6)^{(1.5-1)} = 13$

Step 5 - Calculate Total *R* Around Each Loop:

$$\sum R = 32 + 13 = 45$$

Step 6 - Calculate the Flow Correction:

$$\Delta Q = \frac{\sum h}{\sum R} = \frac{-1}{45} = -0.02$$

  

Since the $\Delta Q$ term is very small, further correction is not necessary. Therefore the previously calculated values of $Q_{ABCD} = 28.4$ (clockwise) and $Q_{AD} = 71.6$ (counterclockwise) are retained. As shown by this simple example even a very bad first guess for flows does not matter in terms of quickly converging on the correct answer. Of course better initial guesses result in less iteration to obtain a solution with a negligible $\Delta Q$ value. Systems with more loops are less tolerant of bad guesses in the sense that they either require many more iterations or in some instances they may diverge instead of converge.

The above steps can be used regardless if the application is in the realm of fluids or electricity. In the case of water distribution systems the form of common friction head loss formulas have been rearranged so they are in a form more convenient for use with the Hardy Cross method. Two of these forms are included below. In water distribution applications Cross' $r$ value is more commonly called $K$ instead of $r$.

In English and S.I. units the Hazen-Williams form is typically written:

$$h_f = rQ^n = rQ^{1.85} = \frac{K_c L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = KQ^{1.85} \quad (Equation\ 11)$$

Where: $h_f$ = Head loss due to friction along the length of pipe, in ft [m]
$r = K$ = Head loss for a unit quantity of flow
$Q$ = Discharge (flow), in ft$^3$/s [m$^3$/s]
$n$ = Coefficient expressing the relationship between flow and head loss
$K_c$ = Coefficient equal to 4.72 when using English units and equal to 10.67 when using S.I. units
$L$ = Length of pipe, in ft [m]
$C_{HW}$ = Hazen-Williams coefficient
$D$ = Internal pipe diameter, in ft [m]

In English and S.I. units the Darcy-Weisbach form is typically written:

$$h_f = rQ^n = rQ^2 = f\left(\frac{L}{D^5}\right)\left(\frac{8}{g\pi^2}\right)Q^2 = KQ^2 \quad (Equation\ 12)$$

Where: $h_f$ = Head loss due to friction along the length of pipe, in ft [m]
$r = K$ = Head loss for a unit quantity of flow
$Q$ = Discharge (flow), in ft$^3$/s [m$^3$/s]

  

$n$ = Coefficient expressing the relationship between flow and head loss
$f$ = Darcy-Weisbach friction factor
$L$ = Length of pipe, in ft [m]
$D$ = Internal pipe diameter, in ft [m]
$g$ = Gravity, in ft/s$^2$ [m/s$^2$]

The Hardy Cross method generally requires more rounds of corrections when multiple loops are involved. A specific example tailored to water distribution with multiple loops is shown below.

EXAMPLE:

Figure 7 shows the geometry and pipe sizes for another simple pipe network. Each node represents a separate building on the grounds of a manufacturing plant. An agreement with the municipal water supplier guarantees an inflow of 50 cubic feet per second (cfs) from the municipal water system at node A. Manufacturing processes performed in buildings B, C, D, E, and F require water flows of 10 cfs, 8 cfs, 0 cfs, 20 cfs, and 12 cfs respectively. Use the Hazen-Williams formula as the relationship between head loss and flow. Assume all pipes have a $C_{HW}$ value of 100.

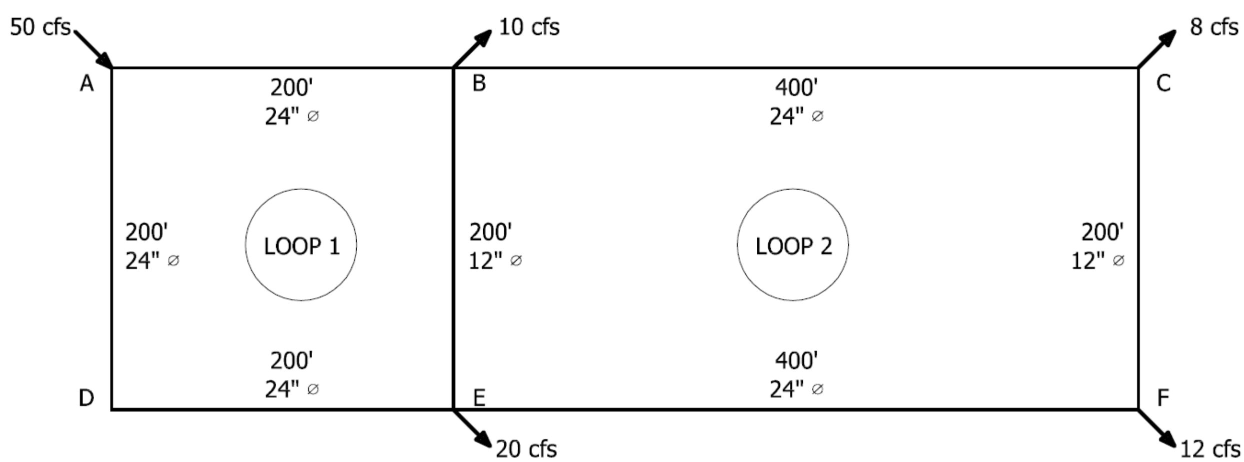


**Figure 7: Pipe Network for Manufacturing Plant Example**

SOLUTION:

Since the inlet and outlet flows are known use the "Method of Balancing Heads".

  

Step 1 - Assume Flow Distribution: Any flow quantities and directions may be assumed as long as continuity at each junction or node is maintained. The initial assumption used for this solution is shown in Figure 8.

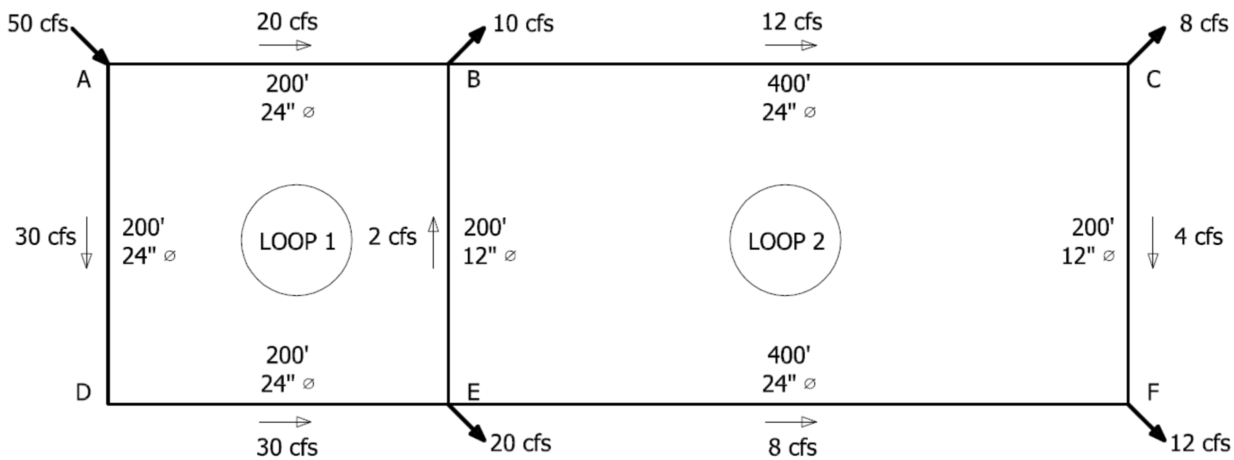


**Figure 8: Assumed Flow Distribution for Manufacturing Plant Example**

Step 2 - Calculate Head Loss: The problem statement calls for the use of the Hazen-Williams formula as the relationship between head loss and flow. Using Equation 11 the head losses in each pipe are:

$$h_{AB} = rQ^n = \frac{4.72L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(200)}{(100)^{1.85}(2)^{4.87}} (20)^{1.85} = 1.644\ ft$$

$$h_{BE} = rQ^n = \frac{4.72L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(200)}{(100)^{1.85}(1)^{4.87}} (2)^{1.85} = 0.679\ ft$$

$$h_{DE} = rQ^n = \frac{4.72L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(200)}{(100)^{1.85}(2)^{4.87}} (30)^{1.85} = 3.480\ ft$$

$$h_{AD} = rQ^n = \frac{4.72L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(200)}{(100)^{1.85}(2)^{4.87}} (30)^{1.85} = 3.480\ ft$$

$$h_{BC} = rQ^n = \frac{4.72L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(400)}{(100)^{1.85}(2)^{4.87}} (12)^{1.85} = 1.278\ ft$$

$$h_{CF} = rQ^n = \frac{4.72L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(200)}{(100)^{1.85}(1)^{4.87}} (4)^{1.85} = 2.448\ ft$$

$$h_{FE} = rQ^n = \frac{4.72L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(400)}{(100)^{1.85}(2)^{4.87}} (8)^{1.85} = 0.604\ ft$$

Step 3 - Calculate Total Head Loss Around Each Loop: Assume flow in the clockwise direction results in positive head loss and flow in the counterclockwise direction results in negative head loss. The total loss of head around each loop while taking into account the direction of head loss is:

$$\sum h_{loop\,1} = 1.644 - 0.679 - 3.480 - 3.480 = -6.00\,ft$$

$$\sum h_{loop2} = 1.278 + 2.448 - 0.604 + 0.679 = 3.80\,ft$$

Please note the head loss for pipe BE has a negative sign for the calculation involving loop 1 and a positive sign for loop 2. This is because the pipe flow direction is counterclockwise in loop 1 but clockwise relative to loop 2.

Step 4 - Calculate $R$: Given the same Hazen-Williams $r$ value formula used in step 2, the pipe AB $R$ value is:

$$R_{AB} = nrQ^{(n-1)} = n\left[\frac{4.72L}{C_{HW}^{1.85}D^{4.87}}\right]Q^{(1.85-1)} = (1.85)\left[\frac{(4.72)(200)}{(100)^{1.85}(2)^{4.87}}\right](20)^{(0.85)} = 0.152\,\frac{s}{ft^2}$$

Similar calculations can be completed for each pipe segment.

Step 5 - Calculate Total $R$ Around Each Loop: The total $R$ value around the loops without taking into account the direction of head loss (meaning all $R$ values are positive) is:

$$\sum R_{loop1} = 0.152 + 0.628 + 0.215 + 0.215 = 1.21\,\frac{s}{ft^2}$$

$$\sum R_{loop2} = 0.197 + 1.132 + 0.140 + 0.628 = 2.10\,\frac{s}{ft^2}$$

Step 6 - Calculate the Flow Correction: the changes in flows for each loop in the next iteration are:

$$\Delta Q_{loop1} = \frac{\sum h}{\sum R} = \frac{-6.00}{1.21} = -4.96$$

$$\Delta Q_{loop2} = \frac{\sum h}{\sum R} = \frac{3.80}{2.10} = 1.81$$

  

Step 7 - Correct Flows: Using the originally assumed flows and the calculated $\Delta Q$ values, the revised flow calculations are shown below. The assumed flow and corrected flow values are positive if they flow in a clockwise direction and are negative if they flow in a counterclockwise direction. Pipe BE is in both loops; as a result both $\Delta Q$ values are used in the flow correction for that pipe. Notice that flow in pipe BE has changed direction compared to the assumed flow direction.

Loop 1:

$Q_{AB} = Q_{AB} - \Delta Q_{loop1} = (20) - (-4.96) = 24.96\ cfs$ (clockwise flow)

$Q_{BE} = Q_{BE} - \Delta Q_{loop1} + \Delta Q_{loop2} = (-2) - (-4.96) + (1.81) = 4.77\ cfs$ (clockwise flow)

$Q_{DE} = Q_{DE} - \Delta Q_{loop1} = (-30) - (-4.96) = -25.04\ cfs$ (counterclockwise flow)

$Q_{AD} = Q_{AD} - \Delta Q_{loop1} = (-30) - (-4.96) = -25.04\ cfs$ (counterclockwise flow)

Loop 2:

$Q_{BC} = Q_{BC} - \Delta Q_{loop2} = (12) - (1.81) = 10.19\ cfs$ (clockwise flow)

$Q_{CF} = Q_{CF} - \Delta Q_{loop2} = (4) - (1.81) = 2.19\ cfs$ (clockwise flow)

$Q_{FE} = Q_{FE} - \Delta Q_{loop2} = (-8) - (1.81) = -9.81\ cfs$ (counterclockwise flow)

$Q_{BE} = Q_{BE} - \Delta Q_{loop2} + \Delta Q_{loop1} = (2) - (1.81) + (-4.96) = -4.77\ cfs$ (counterclockwise flow)

The corrected flow quantities and directions after this first iteration are shown in Figure 9.

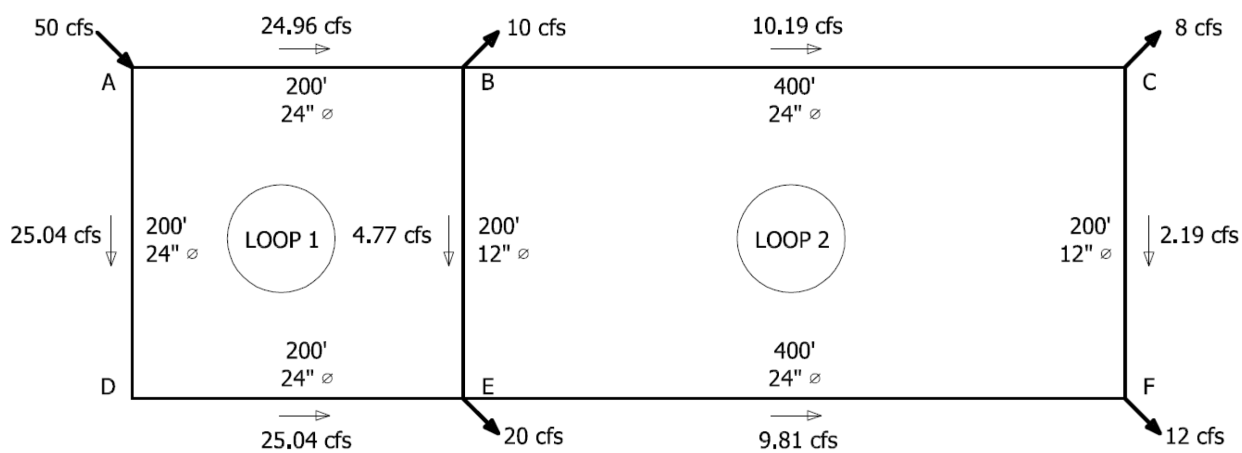


**Figure 9: First Flow Correction for Manufacturing Plant Example**

Since the $\Delta Q$ values are large relative to the assumed flows, steps 2 through 7 should be repeated. A summary of the second and third correction iterations are shown in Tables 5 and 6.

  

The corrected flow values after three corrections are all correct to the nearest whole cubic foot per second.

| SECOND CORRECTION | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Loop | Pipe | h [ft] | $h_{direction\ +/-}$ [ft] | R [s/ft²] | Σh [ft] | ΣR [s/ft²] | ΔQ [cfs] | Corrected Flow [cfs] | Direction [-] |
| 1 | AB | 2.476 | 2.476 | 0.184 | 0.88 | 1.87 | 0.47 | 24.48 | Positive |
| 1 | BE | 3.391 | 3.391 | 1.315 | | | | 3.21 | Positive |
| 1 | DE | 2.492 | -2.492 | 0.184 | | | | -25.52 | Negative |
| 1 | AD | 2.492 | -2.492 | 0.184 | | | | -25.52 | Negative |
| 2 | BC | 0.944 | 0.944 | 0.171 | -2.53 | 2.33 | -1.08 | 11.27 | Positive |
| 2 | CF | 0.801 | 0.801 | 0.678 | | | | 3.27 | Positive |
| 2 | EF | 0.881 | -0.881 | 0.166 | | | | -8.73 | Negative |
| 2 | BE | 3.391 | -3.391 | 1.315 | | | | -3.21 | Negative |

**Table 5: Second Correction Results for Manufacturing Plant Example**

| THIRD CORRECTION | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Loop | Pipe | h [ft] | $h_{direction\ +/-}$ [ft] | R [s/ft²] | Σh [ft] | ΣR [s/ft²] | ΔQ [cfs] | Corrected Flow [cfs] | Direction [-] |
| 1 | AB | 2.390 | 2.390 | 0.181 | -1.14 | 1.49 | -0.76 | 25.25 | Positive |
| 1 | BE | 1.632 | 1.632 | 0.940 | | | | 4.19 | Positive |
| 1 | DE | 2.580 | -2.580 | 0.187 | | | | -24.75 | Negative |
| 1 | AD | 2.580 | -2.580 | 0.187 | | | | -24.75 | Negative |
| 2 | BC | 1.138 | 1.138 | 0.187 | 0.48 | 2.23 | 0.22 | 11.05 | Positive |
| 2 | CF | 1.688 | 1.688 | 0.954 | | | | 3.05 | Positive |
| 2 | EF | 0.709 | -0.709 | 0.150 | | | | -8.95 | Negative |
| 2 | BE | 1.632 | -1.632 | 0.940 | | | | -4.19 | Negative |

**Table 6: Third Correction Results for Manufacturing Plant Example**

Seven corrections to the assumed flows are required to get all flow values to within a tenth of a cubic foot per second of the true theoretical flow values (see Table 7). While not shown in the table, the $\Delta Q$ values after the seventh correction are 0.02 cfs for loop 1 and negative 0.03 cfs for loop 2. As a result an eighth correction was not completed.

  

| SEVENTH CORRECTION | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Loop | Pipe | h [ft] | $h_{direction +/-}$ [ft] | R [s/ft²] | Σh [ft] | ΣR [s/ft²] | ΔQ [cfs] | Corrected Flow [cfs] | Direction [-] |
| 1 | AB | 2.527 | 2.527 | 0.185 | -0.11 | 1.64 | -0.07 | 25.3 | Positive |
| 1 | BE | 2.242 | 2.242 | 1.087 | | | | 3.9 | Positive |
| 1 | DE | 2.442 | -2.442 | 0.182 | | | | -24.7 | Negative |
| 1 | AD | 2.442 | -2.442 | 0.182 | | | | -24.7 | Negative |
| 2 | BC | 1.165 | 1.165 | 0.189 | 0.07 | 2.41 | 0.03 | 11.4 | Positive |
| 2 | CF | 1.829 | 1.829 | 0.990 | | | | 3.4 | Positive |
| 2 | EF | 0.687 | -0.687 | 0.148 | | | | -8.6 | Negative |
| 2 | BE | 2.242 | -2.242 | 1.087 | | | | -3.9 | Negative |

**Table 7: Seventh Correction Results for Manufacturing Plant Example**

Once all the flows have been calculated it is easy to calculate the associated head at any location.

EXAMPLE:

For the previous example calculate the pressure at Building C, in psi, if the pressure at Building A is 45 psi. Use the corrected flow values from Table 7 as flow inputs for the calculations. All the buildings are at the same elevation. Assume the specific weight of water is 62.4 lb/ft$^3$.

SOLUTION:

Since the Hazen-Williams formula was used in the previous example continue to use it for this example. Find the head loss due to friction between Buildings A and B and Buildings B and C using Equation 11. Any path could be chosen, but this is the shortest path possible between Buildings A and C. Between Buildings A and B the head loss is:

$$h_f = \frac{K_c L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(200)}{(100)^{1.85}(2)^{4.87}} (25.3)^{1.85} = 2.54\, ft$$

Do the same between Buildings B and C.

$$h_f = \frac{K_c L}{C_{HW}^{1.85} D^{4.87}} Q^{1.85} = \frac{(4.72)(400)}{(100)^{1.85}(2)^{4.87}} (11.4)^{1.85} = 1.16\, ft$$

Add the two head losses just calculated together and convert to pressure loss using Equation 2 (solving for the pressure loss, *P*).

  

$$P = h\gamma = (2.54 + 1.16\ ft)\left(62.4\ \frac{lb}{ft^3}\right)\left(\frac{1}{144}\ \frac{ft^2}{in^2}\right) = \ 1.6\ psi$$

The pressure at Building C is the pressure at Building A minus the pressure loss.

$$P_{Building\ C} = 45 - 1.6 = 43.4\ psi$$

If Building C was at a different elevation than the other buildings the flow in each pipe would stay the same. The only difference would be that the pressure calculated at Building C would change.

**Hardy Cross Method and the Modified or Improved Hardy Cross Method**

Some textbooks and papers refer to both a Hardy Cross method and a Modified Hardy Cross method (or Improved Hardy Cross method). The explanations provided for the differences between the two methods many times lack clarity. Often the following procedures are listed for the "original" Hardy Cross method.

1. The flow correction magnitude ($\Delta Q$) for a loop is calculated independently without taking into account the other loops.

2. The flow correction magnitude ($\Delta Q$) for a given loop is only applied to pipes within that same loop.

3. The first flow correction magnitude ($\Delta Q$) calculated is applied to its loop before calculating the flow correction magnitude ($\Delta Q$) for the next loop.

4. Only the first power term of the Taylor series approximation is used to calculate $\Delta Q$. The remaining terms are neglected since they are assumed to be small (since the derivation of the method was not presented in this course the neglected terms were never shown).

Flow correction using these procedures is considered *sequential* since the correction is completed for one loop before moving on to the next loop. The confusing part of this list is that Cross' 1936 paper does not follow the third procedure. In his examples he calculates flow correction magnitudes for each loop and then applies the flow corrections all at the same time. In essence the third procedure is replaced with the following two procedures:

1. The flow correction magnitudes ($\Delta Q$) for every loop are calculated first.

2. If a pipe is common to more than one loop each loop's flow correction magnitude ($\Delta Q$) is applied to that common pipe.

Flow correction using these two procedures is considered *simultaneous* since the correction magnitude is calculated for all loops before any correction is applied. Then all corrections are applied to all loops at the same time. A simultaneous method, not a sequential method, was used in the manufacturing plant example. When a simultaneous approach is used the method is sometimes referred to as the Modified Hardy Cross method or Improved Hardy Cross method. Other times it is still referred to as the Hardy Cross method (meaning the original method). These differences in naming convention may be due to the fact that some people believe Cross originally proposed a sequential method. Additionally, some textbooks provided alternate formulas for the Hardy Cross method which do not look exactly like those shown here. This is usually because hydraulic books tend to show application of the method using either the Hazen-Williams or Darcy-Weisbach formulas, but Cross used neither in his original paper.

**Advantages and Disadvantages of the Hardy Cross Method**

There are both advantages and disadvantages to the Hardy Cross method. They include:

Advantages

- Easy to apply
- Can be completed by hand
- Self-correcting even if small calculation errors occur

Disadvantages

- In large networks the number of iterations required for convergence increases with size
- In some instances the method may not converge if initial guesses are too far off
- In its original form it does not work well with boundary conditions, pumps, multiple reservoirs, etc. It can work with pumps, certain valves (check valves, flow control valves, pressure reducing valves), and multiple reservoirs (meaning more than one source) with alterations and the use of pseudo loops created with imaginary links. Pseudo loops will be shown in an example for a different method later in the course.

# Successors to the Hardy Cross Method

For many years the Hardy Cross method was the primary method used for pipe network analysis. Solutions were obtained by hand or with the aid of computers. However, most current hydraulic modeling software now uses alternative methods which are more efficient. Three of these methods, including the Newton-Raphson method, Linear Theory method, and Gradient method, are briefly discussed. All these methods vary from the Hardy Cross method because they take into account all loops simultaneously and therefore generally converge in fewer iterations. It should be mentioned that often a single iteration with any of these methods is more computationally intensive than a single Hardy Cross iteration.

**Newton-Raphson Method**

The Newton-Raphson method is a numerical method which can solve systems of equations simultaneously as opposed to using a convergence method as is employed by the Hardy Cross method. Kirchoff's First and Second Laws are used as the basis for writing the system of equations. The method is able to solve systems of nonlinear equations by transforming them into linear equations using the first two terms of a Taylor series (recall that, often, the first term of a Taylor series is a constant, the second term has a $x$, the third term has a $x^2$, etc.). This means partial differentiation is used to make the equations linear. The system of linear equations is then solved by a root finding method. The method is capable of solving many types of non-linear equations including those with exponent terms, logarithmic terms, trigonometric terms, and hyperbolic terms. The method can be used to solve for either unknown heads or unknown flows. The output from each iterations' solved system of equations can therefore be either $\Delta H$ or $\Delta Q$ correction values. The correction is then applied before completing the next iteration. When $\Delta H$ values are being solved for, the initial flow guesses must obey continuity of flow. The method was first proposed for analysis of water distribution systems in the 1960s. The Newton-Raphson method is not as compact or as commonly used as the Linear Theory and Gradient methods. As a result an example will not be presented here.

**Linear Theory Method**

Since friction equations have exponents, as opposed to logarithmic, trigonometric, or hyperbolic terms, a method suitable for only exponential terms can be used to solve the friction equations. The Linear Theory method is based on this idea. Kirchoff's First and Second Laws are still used as the basis for writing the system of equations. The method takes the $Q$ exponent of the chosen friction equation (1.85 for Hazen-Williams and 2.0 for Darcy-Weisbach) and merges the non-linear portion of the exponent (0.85 for Hazen-Williams and 1.0 for Darcy-Weisbach) into the pipe resistance constant, $R$. This turns the non-linear system of equations into a linear system of equations which can then be solved using any number of methods, but often matrices. The initial flow guesses do not have to adhere to continuity of flow since both flow and head loss equations are solved at the same time. The method can be used to solve for either flow or head, though most applications choose to solve for flow. As a result, the example provided will solve for flow not head. Regardless of whether flow or head is solved for initially, the other can easily be calculated (as shown in the manufacturing plant example). The method was developed for analysis of water distribution systems in the 1970s.

The equation used to write the continuity of potential equations in linearized form is shown below. All the terms before the ${}_{t+1}Q_x$ term are the modified resistance constant that has the non-linear portion of the exponent incorporated into it.

$$\sum R_x \left| {}_{t}Q_x \right|^{(n-1)} {}_{t+1}Q_x \quad (Equation\ 13)$$

Where:
- $R$ = Pipe resistance constant
- $x$ = Pipe number
- $t$ = Iteration
- $Q$ = Discharge
- $n$ = Coefficient expressing the relationship between flow and head loss

EXAMPLE:

Given the pipe network shown in Figure 10, solve the first iteration for the system and write the second iteration loop 1 equation using the Linear Theory method. Assume pipes are the same length. Use an $n$ value of 1.85. This problem is taken from the text by Bhave listed in the reference section.

  

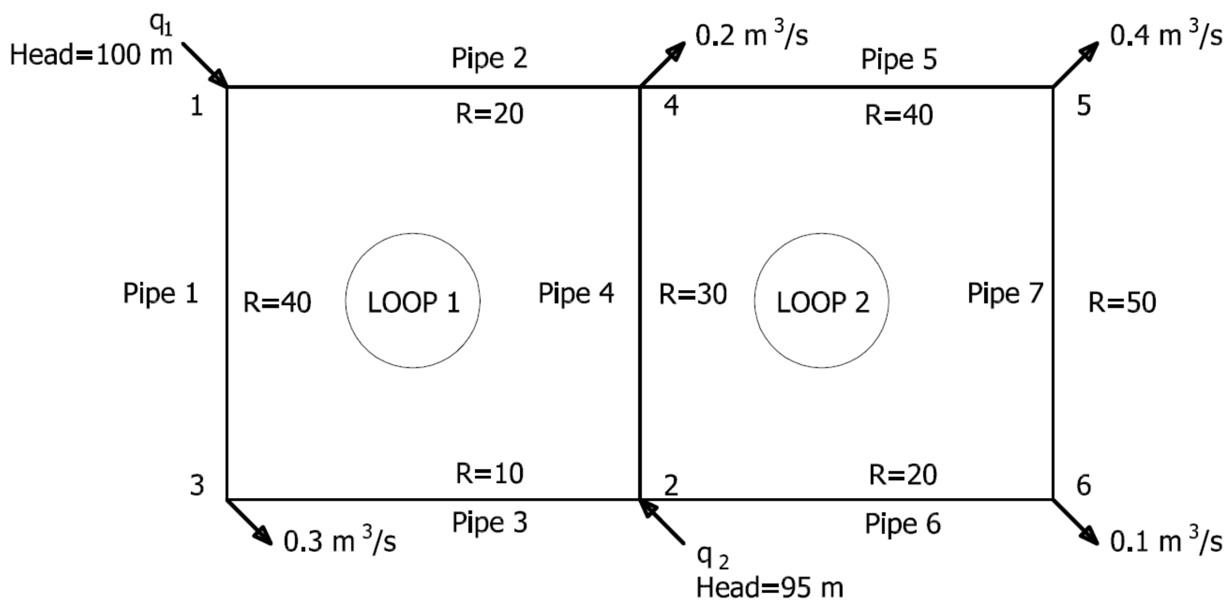


**Figure 10: Pipe Network for Liner Theory Method Example**

SOLUTION:

A pseudo loop is required for this example using the Linear Theory method. A pseudo loop is a closed loop connecting supply nodes with known, differing heads formed by imaginary pipes. The imaginary pipes do not carry flow but do allow a pressure drop between the supply nodes. Figure 11 shows the problem setup. The arrows show the assumed flow direction in each pipe.

  

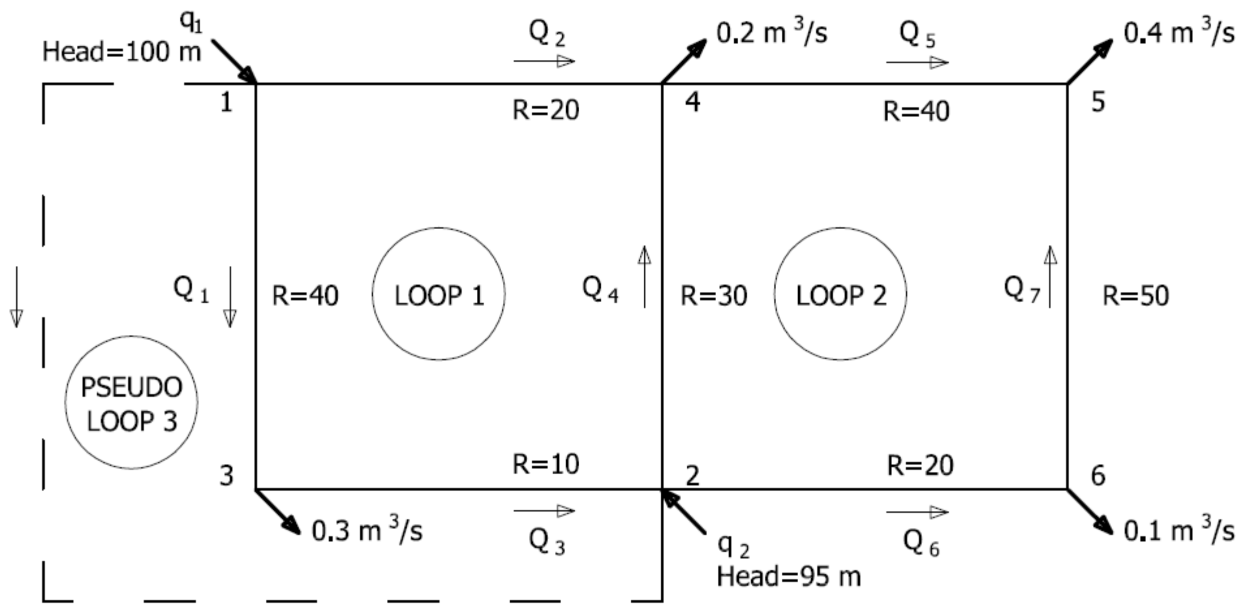


**Figure 11: Linear Theory Method Problem Setup**

The flow equations at nodes 3, 4, 5, and 6 are written using Kirchoff's First Law.

Node 3: $Q_1 - Q_3 - 0.3 = 0$
Node 4: $Q_2 + Q_4 - Q_5 - 0.2 = 0$
Node 5: $Q_5 + Q_7 - 0.4 = 0$
Node 6: $Q_6 - Q_7 - 0.1 = 0$

The head equations around loop 1, loop 2, and pseudo loop 3 are written using Kirchoff's Second Law in the form of Equation 13. Assume clockwise flow results in a positive head loss and that for the first iteration all values of $Q$ are equal to 1 m$^3$/s.

Loop 1: $\sum R_x \left| {}_tQ_x \right|^{(n-1)} {}_{t+1}Q_x$

$$(20)|1|^{(1.85-1)}Q_2 - (30)|1|^{(1.85-1)}Q_4 - (10)|1|^{(1.85-1)}Q_3 - (40)|1|^{(1.85-1)}Q_1 = 0$$

$$20Q_2 - 30Q_4 - 10Q_3 - 40Q_1 = 0$$

Loop 2: $40Q_5 - 50Q_7 - 20Q_6 + 30Q_4 = 0$

Pseudo Loop 3: $40Q_1 + 10Q_3 - (100 - 95) = 0$

$$40Q_1 + 10Q_3 - 5 = 0$$

The pseudo flow direction between node 1 and node 2 goes from high to low head which is in the counterclockwise direction; therefore the head loss between the two nodes is considered negative in the equation shown above. Recall that technically no flow occurs in the imaginary pipe between nodes 1 and 2.

Solving the four node and three loop equations simultaneously gives:

$Q_1$ = 0.16 m$^3$/s
$Q_2$ = 0.37 m$^3$/s
$Q_3$ = - 0.14 m$^3$/s
$Q_4$ = 0.08 m$^3$/s
$Q_5$ = 0.25 m$^3$/s
$Q_6$ = 0.25 m$^3$/s
$Q_7$ = 0.15 m$^3$/s

These values are then plugged back into the loop equations and improved modified resistance constants are calculated. Only the new loop 1 equation for iteration two is shown below.

Loop 1:

$$\sum R_x \left| {}_tQ_x \right|^{(n-1)} {}_{t+1}Q_x$$

$$(20)|0.37|^{(1.85-1)}Q_2 - (30)|0.08|^{(1.85-1)}Q_4 - (10)|-0.14|^{(1.85-1)}Q_3 - (40)|0.16|^{(1.85-1)}Q_1 = 0$$

$$8.59Q_2 - 3.51Q_4 - 1.88Q_3 - 8.42Q_1 = 0$$

The four original node equations and the three new loop equations are then solved simultaneously. Iterations are continued until the flow values changes are small.

**Gradient Method**

The Gradient method uses the same principles as the Newton-Raphson method, but simultaneously solves for both flow and head values. It is considered a variant of the Newton-Raphson method and is a *gradient / node-loop method*. Unlike the Newton-Raphson method the better flow and head values calculated by each Gradient method iteration are directly calculated as opposed to correction values being calculated. Kirchoff's First Laws is still used as the basis for writing the flow equations at nodes. The head equations are written using the following format.

$$ {}_{t+1}H_i - {}_{t+1}H_j - nR_x \left| {}_tQ_x \right|^{(n-1)} {}_{t+1}Q_x = (1-n)R_x \, {}_tQ_x^n \quad (Equation\ 14) $$

Where: $t$ = Iteration
$H$ = Energy head
$i$ = Node flow is leaving
$j$ = Node flow is approaching
$n$ = Coefficient expressing the relationship between flow and head loss
$R$ = Pipe resistance constant
$x$ = Pipe number
$Q$ = Discharge

The system of flow and head equations are then solved using any number of methods, but usually matrices. Similar to the Linear Theory method, the initial flow guesses do not have to adhere to continuity principles. Often all initial flow guesses are assumed equal to a value of one. The method was developed for analysis of water distribution systems in the 1980s. Advantages of the Gradient method over some other methods are that it can directly solve both looped and partly branched pipe networks and it is more numerically stable.

EXAMPLE:
Using the same pipe network shown in the Linear Theory example solve the first two iterations for the system using the Gradient method.

SOLUTION:
A pseudo loop is not required with the Gradient method, but variables for the unknown heads at each demand node are needed. Figure 12 shows the problem setup. The arrows show the assumed flow direction in each pipe.

  

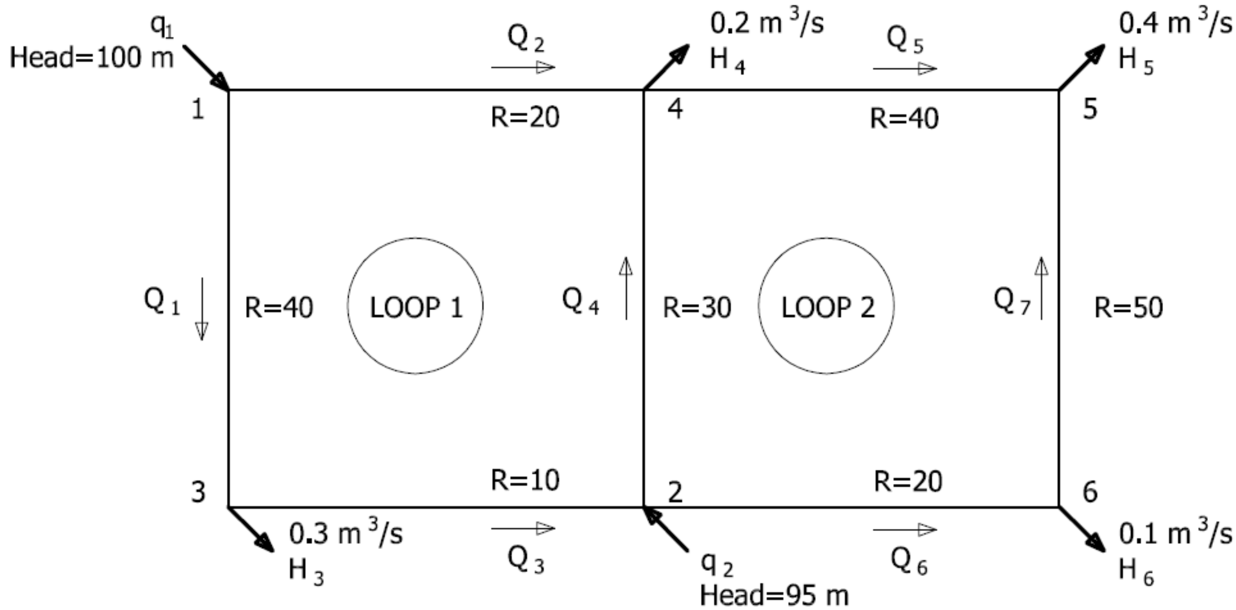


**Figure 12: Gradient Method Problem Setup**

For the first iteration ($t$=1) assume all values of $Q$ are equal to 1 m$^3$/s. Use Equation 14 to write head loss equations for pipes 1 through 7.

Pipe 1: ${}_{t+1}H_i - {}_{t+1}H_j - nR_x \left| {}_tQ_x \right|^{(n-1)} {}_{t+1}Q_x = (1-n)R_x \ {}_tQ_x^n$

$(100) - H_3 - (1.85)(40)(1)^{(1.85-1)} Q_1 = (1 - 1.85)(40)(1)^{1.85}$

$(100) - H_3 - (1.85)(40)Q_1 = (-0.85)(40)$

Pipe 2: $(100) - H_4 - (1.85)(20)Q_2 = (-0.85)(20)$

Pipe 3: $H_3 - (95) - (1.85)(10)Q_3 = (-0.85)(10)$

Pipe 4: $(95) - H_4 - (1.85)(30)Q_4 = (-0.85)(30)$

Pipe 5: $H_4 - H_5 - (1.85)(40)Q_5 = (-0.85)(40)$

Pipe 6: $(95) - H_6 - (1.85)(20)Q_6 = (-0.85)(20)$

Pipe 7: $H_6 - H_5 - (1.85)(50)Q_7 = (-0.85)(50)$

The continuity equations for nodes 3, 4, 5, and 6 are the same as in the Linear Theory method example. Solving the seven head loss equations and four continuity equations simultaneously results in:

  

$Q_1$ = 0.5735 m$^3$/s
$Q_2$ = 0.2123 m$^3$/s
$Q_3$ = 0.2735 m$^3$/s
$Q_4$ = 0.2046 m$^3$/s
$Q_5$ = 0.2169 m$^3$/s
$Q_6$ = 0.2831 m$^3$/s
$Q_7$ = 0.1831 m$^3$/s

$H_3$ = 91.56 m
$H_4$ = 109.14 m
$H_5$ = 127.09 m
$H_6$ = 101.53 m

For the second iteration ($t$=2) plug the values of $Q$ just calculated into Equation 14 to write the new head loss equations for pipes 1 through 7.

Pipe 1: ${}_{t+1}H_i - {}_{t+1}H_j - nR_x \left| {}_tQ_x \right|^{(n-1)} {}_{t+1}Q_x = (1-n)R_x \; {}_tQ_x^n$

$$(100) - H_3 - (1.85)(40)(0.5735)^{(1.85-1)} Q_1 = (1-1.85)(40)(0.5735)^{1.85}$$

$$(100) - H_3 - 46.130Q_1 = -12.155$$

Pipe 2: $(100) - H_4 - 9.911Q_2 = -0.967$
Pipe 3: $H_3 - (95) - 6.146Q_3 = -0.772$
Pipe 4: $(95) - H_4 - 14.407Q_4 = -1.354$
Pipe 5: $H_4 - H_5 - 20.186Q_5 = -2.012$
Pipe 6: $(95) - H_6 - 12.658Q_6 = -1.646$
Pipe 7: $H_6 - H_5 - 21.849Q_7 = -1.838$

Solving the seven new head loss equations and the four original continuity equations simultaneously results in:

$Q_1$ = 0.3782 m$^3$/s
$Q_2$ = 0.4697 m$^3$/s
$Q_3$ = 0.0782 m$^3$/s
$Q_4$ = 0.0029 m$^3$/s
$Q_5$ = 0.2726 m$^3$/s
$Q_6$ = 0.2274 m$^3$/s
$Q_7$ = 0.1274 m$^3$/s

$H_3$ = 94.71 m
$H_4$ = 96.31 m
$H_5$ = 92.82 m
$H_6$ = 93.77 m

Additional iterations are then completed until sufficiently small changes are present between successive $Q$ and $H$ values.

# Common Water Distribution Modeling Software Programs

**History of Water Distribution Modeling**

Prior to the 1960s, hand calculations for very small piped water systems were performed using the Hardy Cross method. Due to the availability of some digital computers in the 1960s, a small number of hydraulic models were available to engineers capable of modeling systems larger than those feasible by hand calculations. Only *steady-state analysis*, that is an analysis at an instant in time, was practical due to limited computing speed and memory.

Between 1970 and 1990, modeling software became more advanced. Extended period simulation (a series of steady-state analyses, typically covering a 24-hour period) and limited water quality modeling first became available. Graphical user interfaces (GUIs) were introduced to make constructing system models easier and more intuitive. Outputs and model results also became available to display through GUIs.

The 1990s was a period of immense growth in the hydraulic modeling arena. The U.S. Environmental Protection Agency (EPA) EPANET software and source code were made available to the public and commercial entities. Commercial software companies created improved GUIs to make software more user-friendly. Commercial software allowed integration with Computer Aided Drafting (CAD) and Geographic Information System (GIS) software, which allowed modelers to use existing information instead of building models from the ground up.

From 2000 to 2020, the growth, sophistication, and capabilities in modeling software has continued at a fast pace due to greater computing power, rapid increases in collected data available for model use, and machine learning.

**Popular Current Models**

Many different public-domain and commercially available water distribution modeling programs have been developed. These programs typically do far more than just calculate flow in each pipe segment or link and pressure at each node. A number of the most popular models used in the United States are summarized in the table below.

| Software Program | Hydraulic Balancing Method | Head Loss Methods | Developer |
|---|---|---|---|
| EPANET | Gradient | Hazen-Williams, Darcy-Weisbach, or Manning | Environmental Protection Agency |
| InfoWater, InfoWater Pro, IWLive Pro, $H_2O$ Net | Gradient | Hazen-Williams, Darcy-Weisbach, or Manning | Innovyze |
| MIKE+, MIKE OPERATIONS | Gradient | Hazen-Williams, Darcy-Weisbach, or Manning | DHI |
| Pipe Flow Expert | Linear Theory, Newton-Raphson, Newton-Raphson variant | Darcy-Weisbach | Daxesoft Ltd |
| WaterGEMS, WaterCAD, WaterOPS, WaterSight | Gradient | Hazen-Williams, Darcy-Weisbach, or Manning | Bentley Systems |

**Table 8: Summary of Common Water Distribution Modeling Software Programs**

From looking at the above table you can see that most software now uses the Gradient method for hydraulic balancing. In fact, many of the models use EPANET in the background (or code from EPANET) to perform much of the analysis. Many of the software packages can both analyze an existing system or design/optimize a system. Typically the hydraulic balancing method is used for analysis, but not design/optimization since for design/optimization a trial and error approach to potential system changes would be required. For design and optimization purposes the models often utilize alternate techniques such as genetic algorithms or what is often referred to as the Hybrid method, which uses a combination of linear programming and genetic algorithms. A brief summary of software shown in Table 8 is presented next.

**EPANET**

EPANET was developed by the EPA and is a public domain Microsoft Windows-based program. It allows for both hydraulic modeling and water quality modeling. There are multiple add-on extensions available to model chemical and biological inputs and see how they propagate through the distribution system. These extensions are useful for water security and resiliency investigation purposes.

  

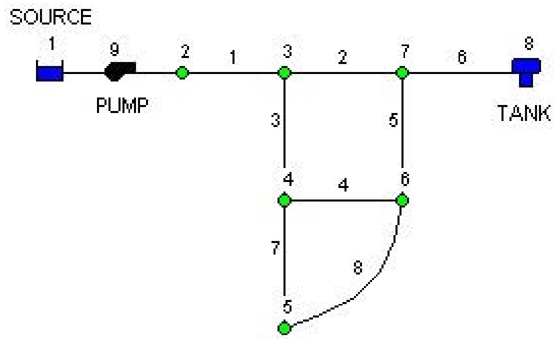


**Figure 13: An example EPANET pipe network (from EPANET 2.2 User's Manual)**

EPA software documentation summarizes the hydraulic water modeling capabilities as follows:

- Evaluates system operation using simple tank level, timer controls, or complex rule-based controls
- No network size limit
- Multiple options for calculating friction losses (Hazen-Williams, Darcy-Weisbach, or Chezy-Manning formulas)
- Takes into account minor head losses from bends, fittings, valves, etc.
- Ability to model constant or variable speed pumps
- Options to calculate energy used for pumping and the associated energy costs
- Models different valve types (shutoff, check, pressure regulating, flow control, etc.)
- Permits storage tanks to have varying area associated with height
- Can handle multiple demand flow rates and flow rate times at nodes
- Capability to use pressure dependent demands at nodes
- Allows for low and zero flow conditions

The EPANET user's manual summarizes the program's water quality modeling capabilities as:

- Ability to model the movement of both non-reactive and reactive material as it grows (e.g., a disinfection by-product) or decays (e.g., chlorine residual) over time
- Keeps track of the age of water throughout the pipe network
- Calculates the percentage of flow from a given node distributing to all other nodes with time
- Models reactions both in the bulk pipe flow and reactions at the pipe wall
- Allows input of a limiting concentration for growth or decay reactions
- Allows global reaction rate coefficients or pipe-by-pipe reaction rates
- Permits wall reaction rate coefficients to be linked to pipe roughness/pipe material
- Gives option for time-varying concentration or bulk inputs at any network node
- Allows complete mix, plug flow, or two-compartment reactors for storage tanks

  

Analysis and output from the EPANET model are diverse both for hydraulic and water quality modeling. Two examples of hydraulic modeling results are shown in Figure 14 and Figure 15.

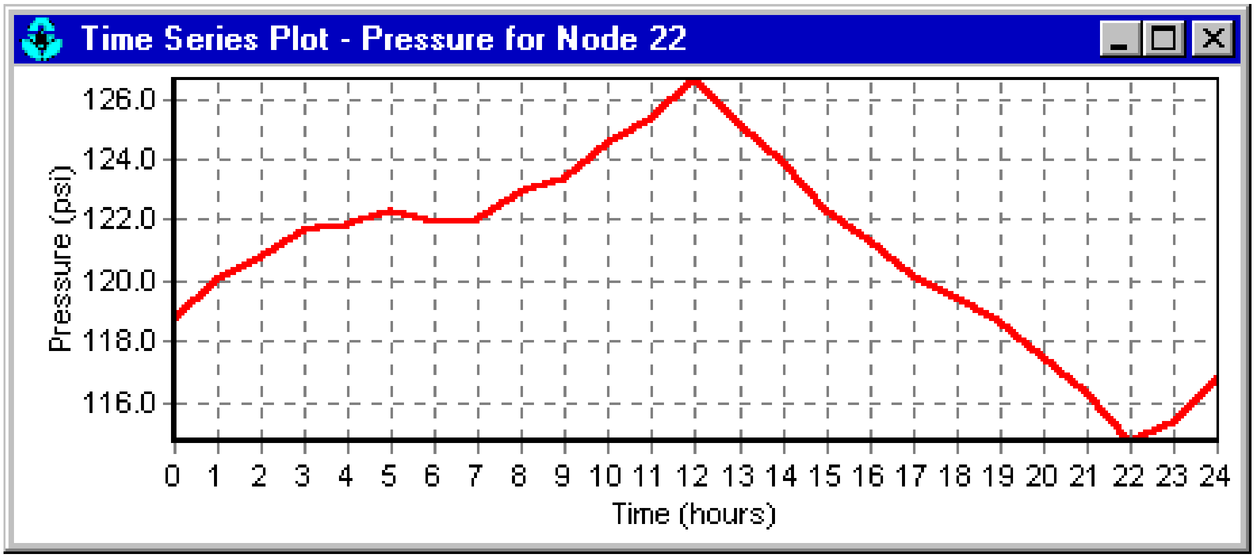


**Figure 14: An example time series plot showing the pressure at a given node over a 24-hour period (from EPANET 2.2 User's Manual)**

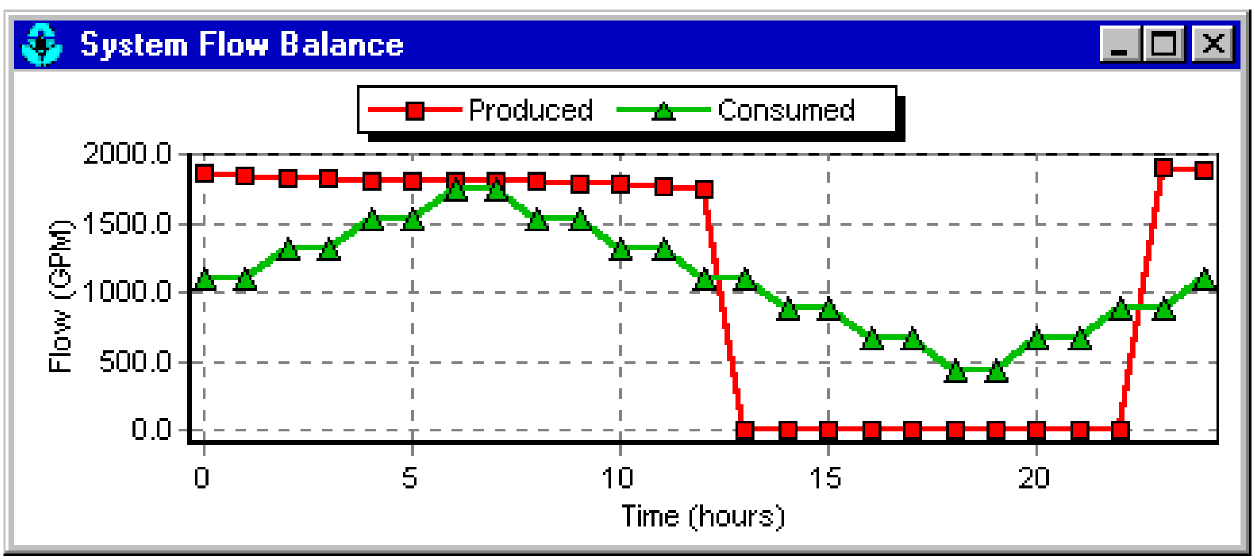


**Figure 15: An example system flow plot showing the amount of water produced and the amount of water consumed over the course of a simulated day (from EPANET 2.2 User's Manual)**

Two examples of water quality modeling outputs are shown in Figure 16 and Figure 17.

  

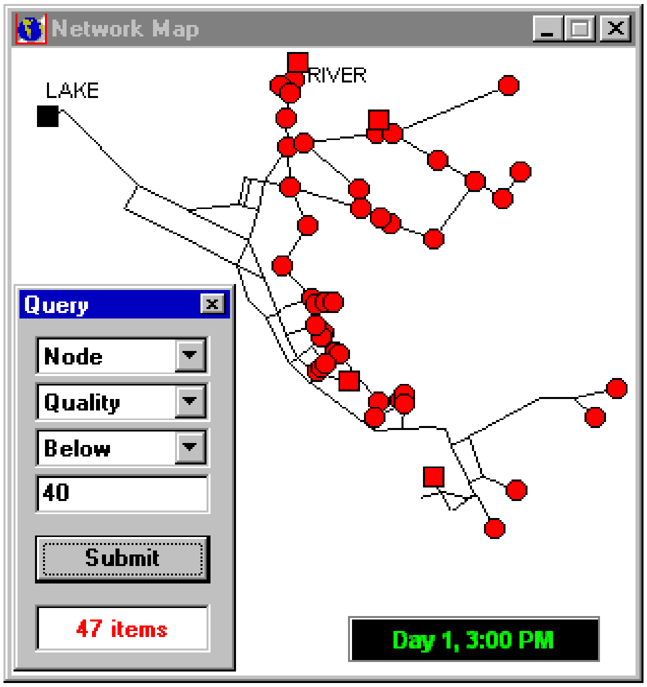


**Figure 16: An example map query showing nodes meeting the conditions of the query (from EPANET 2.2 User's Manual)**

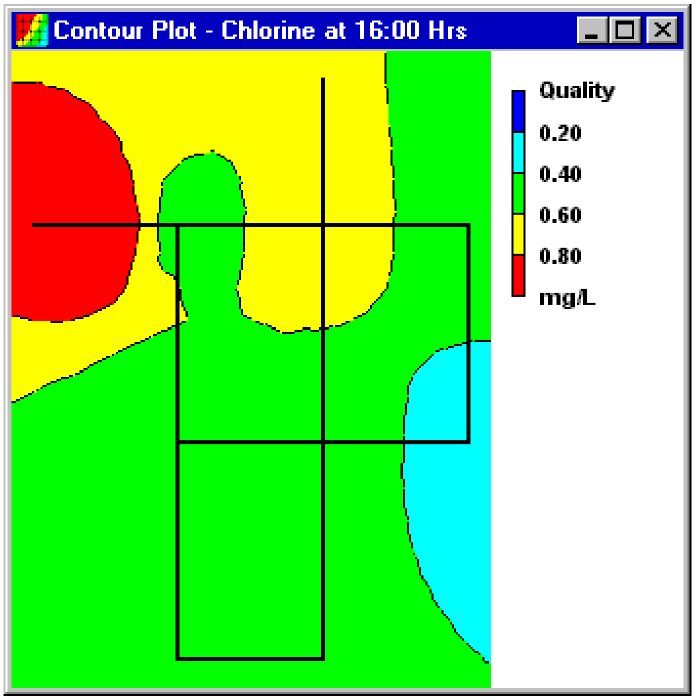


**Figure 17: An example contour plot showing chlorine concentration residuals in the distribution system at a selected time (from EPANET 2.2 User's Manual)**

**InfoWater, InfoWater Pro, IWLive Pro, and $H_2O$ Net**

The U.S. company Innovyze has a suite of water distribution modeling and management software including InfoWater, InfoWater Pro, IWLive Pro, and $H_20$ Net. Most of these software programs perform both hydraulic and water quality modeling.

InfoWater works within Esri's ArcGIS software. Geographic Information Systems (GIS) attach information to objects with spatial or geographic attributes. For example, a pipe in Main Street between 1$^{st}$ Avenue and 2$^{nd}$ Avenue could have the following information stored in a database and linked to it: pipe length, pipe material, pipe diameter, date installed, and elevation at each end. Most water utilities now have their storage and distribution systems' information in GIS. Using modeling software that can utilize this already existing information instead of manually having to draw the system and input the information is, of course, a monumental time saver. Advanced modeling applications within InfoWater include critical valve analysis and unidirectional flushing, among others. Critical valve analysis determines which valves, if they were to become non-operational, would cause problems for water system management. For instance, say a leak or water main break on a certain segment of pipe requires three valves closed to isolate the pipe for repair. If one of those valves couldn't be closed and it required an additional eight valves to be closed to isolate the leak/break, the water utility may want to identify that valve as critical. It might be considered critical for multiple reasons. First, because it takes much longer to isolate and repair the leak/break since more valves must be closed and, second, because many more customers may be temporarily affected by the shutdown during the repair work. The water utility might want to exercise (open and close) that critical valve more often than non-critical valves to ensure it operates correctly when needed. Unidirectional flushing is a valuable water quality improvement technique which helps remove sediment and biofilm from water mains. For each flush, the software identifies which valves need to be closed, what hydrant needs to be opened, and the duration of the flushing. It also produces field maps for the service crews who will perform the flushing operation. InfoWater Pro is similar to InfoWater, but integrates with Esri's ArcGIS Pro software.

Many water systems have sensors located at various key locations. Typical sensors measure things like water elevation in a storage tank, when a pump is turned on and its flowrate, and water main pressure at specific locations. SCADA, or supervisory control and data acquisition, allows information from sensors throughout the physical system to be gathered, displayed, and analyzed at a central location. Innovyze's IWLive Pro integrates with SCADA to allow for real-time modeling and forecasting based on real-time data.

$H_20$ Net software performs similar modeling and can interface with SCADA, but within the AutoCAD environment, instead of a GIS environment.

Figure 18 shows an example InfoWater Pro pipe network. This pipe network shows the delineation of two pressure zones. Pressure zones are common in service areas with significantly varied topography. Normally, reservoirs or storage tanks are located at or near the highest elevation of a service area or pressure zone. If a storage tank was located 300 feet higher than the elevation of a customer, the potential head, or elevation head, difference would equate to a 130 psi greater pressure at the customer compared to the storage tank. This is too high of pressure for most residences and businesses, so either individual pressure reducing valves (PRVs) would be installed inside each building or a large utility owned PRV station installed to reduce the pressure of the entire water main "downstream" of the PRV station. Normally this second option is selected (or a combination of a PRV station and PRVs for each individual service located at the lowest elevations of each pressure zone). A PRV station is shown in the lower right corner of "Pressure Zone 2" in Figure 18.

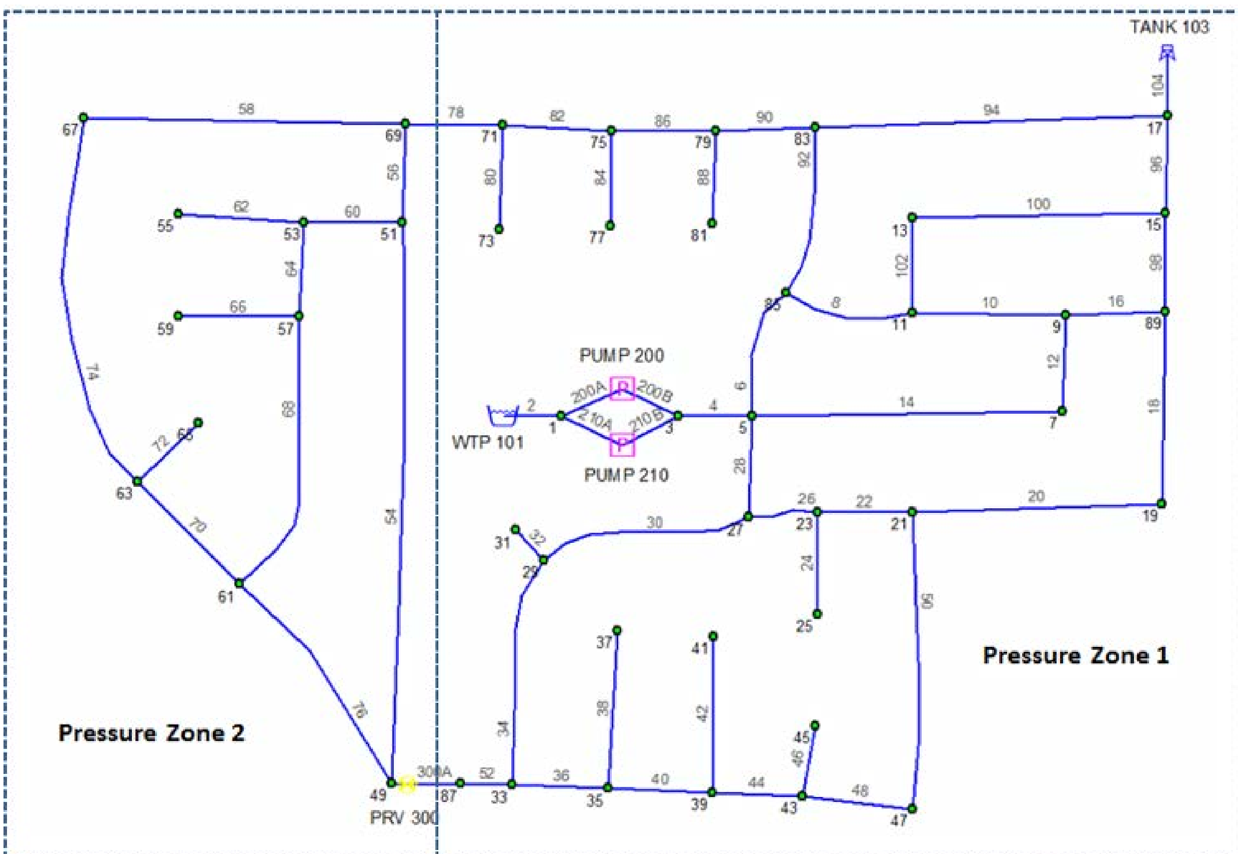


**Figure 18: An example InfoWater Pro pipe network with two different pressure zones (from InfoWater Pro Quick Start Tutorial)**

  

**MIKE+ and MIKE OPERATIONS**

Dansk Hydraulisk Institut (DHI) is a Danish based software developer that has created the MIKE suite of software. For years, its primary software for piped water systems was MIKE URBAN. MIKE URBAN modeled potable water distribution, stormwater, wastewater (sanitary sewer), and combined sewer systems. MIKE URBAN is being decommissioned. Its replacement is MIKE+. MIKE + includes four areas of analysis:

- Water Distribution – hydraulic and water quality modeling of potable water systems
- Collection Systems – hydrology and hydraulic modeling of stormwater and wastewater collection systems
- River Networks – river modeling
- Flooding – predicts flooding and mitigation risks and impacts

The idea behind combining these four modeling efforts into a single piece of software is that they are often dependent on each other. A series of intense rain storms increases water infiltration into wastewater systems components like pipes and manholes. At the same time, a river running through town rises significantly, due to the intense rain. The rising river tailwater slows the ability of the stormwater systems that outfall into that river from draining streets. The resulting flooding makes road access to critical potable water system infrastructure difficult. MIKE+ works with ArcGIS to reduce the effort required to create system models.

A separate platform called MIKE OPERATIONS provides real-time forecasts and management with similar goals as Innovyze's IWLive Pro.

**Pipe Flow Expert**

United Kingdom based Daxesoft Ltd is the developer of Pipe Flow Expert. This software is not exclusively for modeling water systems. Other liquids and gases can be modeled and the program comes with a built-in library of common fluids and their properties. User may also create their own user-defined fluids as long as the necessary fluid properties are also input. Pipe Flow Expert is widely used in the oil and gas industry due to its generic pipe system modeling capabilities. It does not have the ability to perform water quality modeling.

Pipe Flow Expert uses the Linear Theory method, Newton-Raphson method, and a Newton-Raphson variant instead of the Gradient method used by most other software discussed in this section.

**WaterGEMS, WaterCAD, WaterOPS and WaterSight**

Bentley Systems is a U.S. based software development company with a wide variety of software products both inside and outside of the water industry. Their OpenFlows WorkSuite has over a dozen software products for modeling potable water, wastewater, and stormwater systems.

OpenFlows WaterGEMS provides hydraulic and water quality modeling of potable water systems similar to InfoWater and MIKE+. It has a pipe replacement prioritization component as well. WaterGEMS runs within ArcGIS.

OpenFlows WaterCAD is similar to WaterGEMS, but runs within AutoCAD or MicroStation software. MicroStation is a CAD program developed by Bentley Systems. It is heavily used in the transportation engineering field.

Similar to Innovyze's IWLive Pro and DHI's MIKE OPERATIONS, Bentley's OpenFlows WaterOPS and OpenFlows WaterSight software integrates with SCADA, GIS, hydraulic models, and customer information to allow for real-time modeling and forecasting of water systems.

## Conclusion

Water distribution system modeling allows engineers to answer many important questions about municipal and commercial water supply systems. Most undergraduate engineering students are taught the Hardy Cross method of analysis but not the present-day methods used in most software programs. In an effort to help engineers better understand the software they use for water distribution system analysis, the Hardy Cross method was reviewed and the Newton-Raphson, Linear Theory, and Gradient methods were introduced and applied to simple examples. Finally, a brief overview of popular modeling software from five providers was presented.

**Write a Review**

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