A SunCam online continuing education course

External Ballistics Primer for Engineers

Part I: Aerodynamics & Projectile Motion

by

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Introduction

This primer covers basic aerodynamics, fluid mechanics, and flight-path modifying factors as they relate to ballistic projectiles. To the unacquainted, this could sound like a very focused or unlikely topic for most professional engineers, and might even conjure ideas of guns, explosions, ICBM's and so on. But, ballistics is both *the science of the motion of projectiles in flight, and the flight characteristics of a projectile* (1). A slightly deeper dive reveals the physics behind it are the same that engineers of many disciplines deal with regularly. This course presents a conceptual description of the associated mechanics, augmented by simplified algebraic equations to clarify understanding of the topics. Mentions are made of some governing equations with focus given to a few specialized cases. This is not an instructional tool for determining rocket flight paths, or a guide for long-range shooting, and it does not offer detailed information on astrodynamics or orbital mechanics. The primer has been broken into two modules or parts. This first part deals with various aerodynamic effects, earth's planetary effects, some stabilization methods, and projectile motion as they each affect a projectile's flight path. In part II some of the more elementary measurement tools a research engineer may use are addressed and a chapter is included on the ballistic pendulum, *just for fun*. Because this is introductory course some of the sections are laconic/abridged touches on the matter; however, as mentioned, they carry application to a broad spectrum of engineering work.

Preliminary Miscellany

For the purposes of this primer, firearm related pun intended, let's make a distinction between rockets and missiles. Hereafter the word "missile" will be avoided while describing ballistic objects because generally speaking, a missile is a guided munition whereas a rocket is generally unguided. The word rocket will refer to an unguided, ballistic-path-following object, powered by a rocket motor. Ballistic missile, at least in my opinion, is a misnomer since most of them can make short course corrections. Rockets would include some rocket artillery, hobby rockets (e.g. Estes brand rockets), toy water rockets, pneumatic stomp rocket toys, and so on.

Ballistics is broken up into three comprehensive categories: internal, external and terminal. Following is a conceptual description of internal and terminal ballistics in preparation for the rest of this course on external.

Internal ballistics, sometimes called interior, refers to the propulsive period in the flight of an object. One may incorrectly assume that because the word internal or interior is used that some sort of barrel, tube or launch silo is needed around that object. This definition would exclude many types of rockets and apply only to guns, or canons and other artillery. For rockets internal ballistics applies to the powered portion of their flight.

Terminal ballistics, although a highly complex subject, may simply be described as the study of the interaction between the projectile and the target/object(s) it makes impact with.

Projectile Motion[(2)]

Projectile motion is the free-flight motion of a projectile affected only by acceleration due to gravity. Every engineer has spent time learning the basics of projectile motion in a physics course but may have forgotten some of the details, so we'll review again here. Be forewarned, projectile motion only concerns itself with a constant acceleration caused by gravity, thus in many cases it cannot accurately characterize the path of a truly ballistic projectile. The reasons why it cannot are the crux of this primer, but this section forms the foundation upon which the balance of external ballistics can be built.

Below are some rules which apply to idealized projectile motion with a level target [(3)]:

- The trajectory is perfectly parabolic

- The initial and impact velocities are equal (except with non-level targets)
- The launch and impact angles are equal (except with non-level targets)
- The range is maximum at a 45˚ angle
- There are no horizontal accelerations (drag and its ilk are all ignored)
- All motion can be described within the confines of a planar, or two-dimensional, coordinate system using simple algebraic equations

In the spreadsheet accompanying this primer the *Projectile Motion* worksheet offers the user a chance to review and roughly estimate the flight of a projectile in a drag free (read evacuated or vacuum) environment subject only to Earth's idealized gravitational force. The equations used may be found in any introductory level dynamics course text or easily found online and thus not given herein. The worksheet may be used to determine the flight path and duration of a relatively low-speed, low-drag object with fair accuracy. So, it may generate a "good enough" estimate for the engineer in cases usually well outside of most ballistic trajectory work. Eons ago, some of the Boy Scouts in my oldest son's former scout troop joined those of another troop and built a large trebuchet for a "Pumpkin Chunkin'" competition at a local pumpkin farm and this would have been an appropriate estimation tool for them to use to gauge the maximum height, flight time and so on of the pumpkin, for their machine. To use the tool there are just a few simple inputs (the light green filled cells) needed. There are five points (A-E) shown on the corresponding picture, and shown here in **figure 1**, with six applicable flight paths based on those points. The explanation of those points is as follows:

- A to D: Used when the object is thrown at some angle to horizontal (Θ_1=0<>90°), beginning at "ground" level and ending at a higher level, k some distance, d away, and at an initial horizontal velocity of v_{0x}.
- A to E: Used when the object is thrown at some angle above horizontal (Θ_1=0<>90°), beginning and ending flight at the same horizontal level, separated by some distance, d, and at an initial horizontal velocity of v_{0x}.
- B to E: Used when the object is thrown at some angle above horizontal (Θ_2=0<>90°), beginning at one height, j and ending at a lower relative height some distance, d away, and at an initial horizontal velocity of v_{0x}.
- C to E: Used when the object is thrown horizontally (Θ_3=0) from some height, h and ends its path at a lower height some distance, d away, and at an initial horizontal velocity of v_{0x}.

- D to E: This is a special case which is typically not well covered in high school physics. For flight paths beginning at A and B the indicated throwing angle must be positive or, as in the case beginning at C, horizontal. "D to E" applies to a thrown projectile with an initial velocity v_{0x}, at an angle below the horizontal, (Θ_4=-90<>0°). The object begins its path some height, k above "ground" and ends some horizontal distance, d away.
- Vertical Throw/Drop: The case where there is no horizontal velocity (x direction) component in the throw/drop, i.e. $v_{0x}= v_{fx}=0$. The initial vertical velocity, v_{0y} is either upward (+), downward (-) or zero. A directly upward throw would be set to +90° (Θ_v=90°) and a directly downward throw/drop would be set to -90° (Θ_v=-90°).

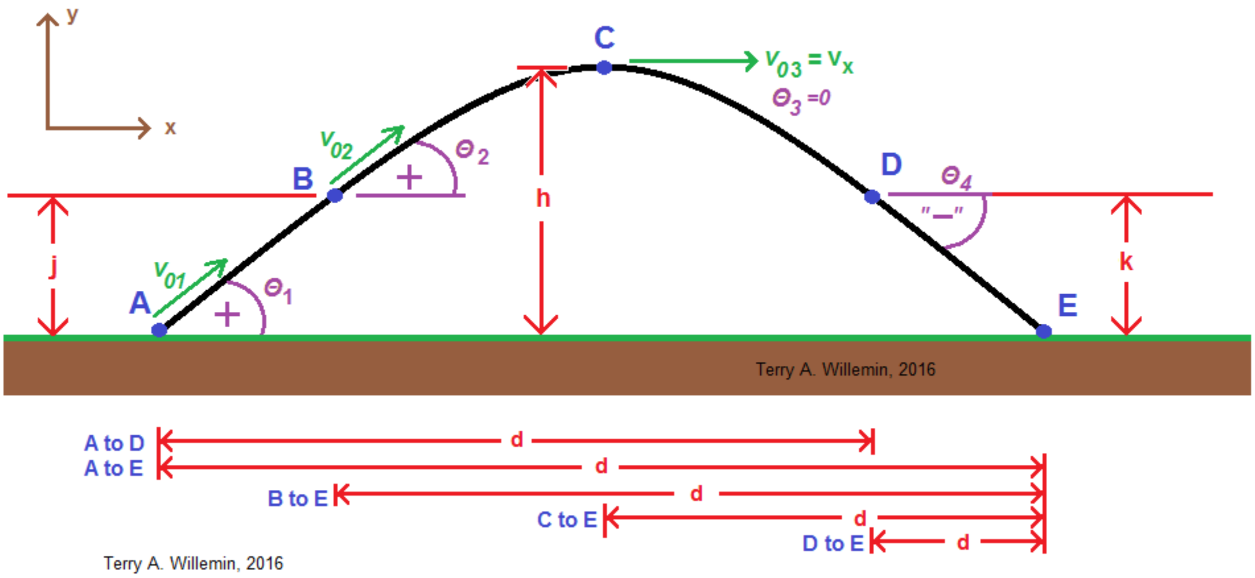

Figure 1, Projectile Motion Flightpath Graphic Included with the Projectile Motion worksheet.

An additional "check-time" (t_{chk}), input cell has been added to determine the velocity, position and angle of the trajectory at any time during the flight. If t_{chk} is set greater than the total flight time, t_{Total} of the projectile, an error will be generated which is displayed by turning the cell fill red. The time for the projectile to reach maximum height in its flight, $t_{h, max}$ has also been included.

Because ballistics deals with real conditions, the basic rules above and the projectile motion flight paths described in the worksheet can never perfectly describe the path of a real projectile through a fluid such as our atmosphere. As an example, we know that the horizontal velocity component of a bullet through air will always be less at point E than at point A in **figure 1** because of drag. And, as-is, this model cannot deal with anything beyond simplified gravitational effects, and earth's rotational effects. The remainder of part one of the primer will describe several of the forces which affect the flight of a real projectile.

Real Forces Acting on a Ballistic Projectile

In the atmosphere of the third planet from our sun, there are nearly always at least two main forces to consider as acting on a ballistic projectile, they are: gravity and drag. And, as anyone might suspect, there are other more minor forces and effects at work to throw projectiles off course. The chapter above dealt with simplified effects of gravity on a projectile in the absence of air. Now, let's dig a little deeper on how our planet and its atmosphere affect flight paths.

Gravity

Since engineers typically treat gravitational "force" as being constant when analyzing the trajectories of most projectiles, gravitational effects can generally be analyzed simply. Standard Gravitational Acceleration on earth is defined as 9.80665 m/s^2, or about 32.174 ft/s^2. At any given point on the surface of earth, from the bottom of the Mariana Trench to the top of Mount Everest, gravitational acceleration only varies by about 0.7% (9.7639 m/s^2 to 9.8337 m/s^2) [(4)]. For some high-altitude flights, using a constant acceleration may not be adequate for the needs of the engineer and a more exact solution could be de rigueur. Gravitational acceleration does not vary perfectly smoothly with distance from earth's center so for increased accuracy, the exacting *or obsessed* engineer may wish to turn to more accurate models derived from precise empirical data. However, a slightly less exact way to convey how gravitational acceleration varies with altitude above earth's surface is found in the following:

$$g_h = \frac{G * m_X}{\left(r_X + d_{agl}\right)^2}$$

Equation 1

Where G is the gravitational constant, $6.67384*10^{-11}$, m_X the mass of the planet or body in kg, earth is $5.9722*10^{24}$kg, r_X is the mean radius of the planet or body in meters, earth's mean radius is 6.375*106 m (6,375 km), d_{agl} is the vertical distance above the surface in meters and g_h is the resulting gravitational acceleration in m/s^2 at the distance, d_{agl} above the body's surface.

Equation 1 is a simplified model presented in any introductory physics course. The equation is presented for the purpose of conveying the relationship of gravitational strength decay to distance. What are some things the model can tell us? At roughly 3,000 km (1900 mi) the magnitude of acceleration is approximately half that at the surface of the earth and at about 12,000 km (7,500 mi) the value drops to about 1 m/s^2. With respect to ballistic projectiles, the engineer may only ever deal with altitudes of no more than 10-15 miles. At maximum altitudes such as that, resorting to the added complexity of more accurate gravitational models is usually unwarranted. This is the case with the work we'll be doing here too. The *Gravity* worksheet in the included file allows the input of distance to estimate the gravitational acceleration, and estimate what an object would weigh at that distance. To use, simply input the distance from earth's surface, d_{agl} in miles or kilometers in cells D22 & E22 and the mass in pounds or kilograms in cells D23 & E23. The calculator will estimate the gravitational acceleration at the indicated distance in ft/s^2 or m/s^2 and weight in pounds-force or Newtons.

A Slight Sidetrack: There are those (non-engineers of course) who mistakenly believe once an astronaut enters "space" that suddenly earth's gravity somehow loses its pull and that the astronaut becomes weightless. The supposition may be that earth's gravitational field imaginatively only extends a short distance through the atmosphere and either tapers off quickly or possibly somehow ends abruptly. Though that leaves no explanation for how planets and satellites (natural and man-made) are bound to each other by that same gravitational pull. As an undergraduate student I had the opportunity to fly an experiment with a group of other students (and other groups of students) from several universities around the US in, what NASA at the time called, the Reduced Gravity Student Flight Opportunities Program, or RGSFOP. The program offered a chance for undergraduate students to devise and subsequently perform experiments in a "microgravity" (an approximate "zero-g" condition) environment aboard an airplane. At the time, NASA used a modified KC-135A Stratotanker. To produce a microgravity condition inside a plane it is flown in a parabolic flight path, and while on the "down-side" of the parabola the plane's vertical component of acceleration is at the same rate the occupants of the plane are falling toward the ground. The plane and its occupants are accelerating toward the ground at approximately 32 ft/s^2. The students, who have no visual external cues in the nearly windowless, padded interior of the plane, cannot sense they are accelerating toward earth. They are floating inside the plane, experiencing a sense of weightlessness which lasts for about 25 seconds, followed by stomaching a nearly 2g pullup to regain altitude for a subsequent parabola. In case anyone is wondering, the transition into the 2g pull-ups are why the planes are given the nickname vomit comet. During the zero-g *freefalls*, if a student were to somehow devise a way to "stand" on a scale, i.e. attach it to his/her feet, it would read zero because he or she is floating inside the plane along with the scale. If the plane were somehow able to stop during the dive at some point in mid-air without falling toward earth similar to a hovering helicopter, then standing on that same scale would register his or her full weight, less a few ounces for the altitude. The same holds true for astronauts in orbit; they are in constant freefall accelerating toward/around the earth and thus are apparently weightless, but if their vehicle were somehow able to stop orbiting and maintain its distance relative to the earth they would also be able to stand on a scale and register a weight. Their weight would obviously be a little less than it would be on the surface, but not nearly as much as one might expect. An astronaut weighing 200 lb_f before flight would still weigh over 180 lb_f at 200 miles out! Talk about an instant weight-loss program; too bad it does nothing to reduce mass….

Aerodynamic Forces/Drag

Air friction can be such a drag (sorry, there was no way to avoid that pun). Determining aerodynamic forces on an object is one of the most complex and perplexing subjects the engineer may ever get to work with. Nowadays, other than for back-of-the-napkin purposes, analyzing fluid flows is almost strictly consigned to analysis by Computational Fluid Dynamics software, CFD, because of that complexity and because of its accuracy and extended capabilities. Of course, CFD is only as good as the information it is fed (GIGO) and in many, if not most, cases is followed by physical model testing in wind tunnels for verification. With the space following we'll ATTEMPT a crash course (that pun belongs with terminal ballistics) in aerodynamics as it relates to ballistic projectiles. This is where the general engineering fun begins.

There are Two Forms of Drag

Aerodynamic forces on the surface of a body in motion in a fluid are comprised of only two causes, they are surface pressure and surface shear stresses [(5)]. These forces create lift and several forms of drag including skin friction drag and pressure drag (or form drag for our purposes), among others. For external ballistics drag and lift comprise the total aerodynamic force.

While the types of drag can be subdivided, skin friction drag and form drag are sufficient for our purposes. Skin friction drag is a fluid frictional force that acts against the direction of motion of an object passing through it. It is a consequence of the viscosity and relative speed of the object through that fluid or, conversely, the speed of the fluid around the object acting on its surface. For reference, angle of attack is the angle between the streamlines (blue lines in **figure 2**) of the fluid flow and the longitudinal axis of the projectile.

Drag is dependent on the relative speed, shape and orientation of the projectile and various properties of the fluid through which it moves. As an example, a flat plate facing a flow of air will have some inertial force imparted to it by the air as the air has to abruptly change direction to pass around the sides of the plate, as shown in A., of **figure 2** below. This is analogous to holding an open hand out the window of a moving car, palm facing forward. The faster the car moves the more force is applied to the hand by the air. Most of the drag on the hand and practically all the drag on the plate is caused by what is referred to as form drag which is a subtype of pressure drag. Turning the plate as shown in B., replaces the form drag with skin friction drag similar to turning one's hand so that it is palm-down in the same airflow. Note that the streamlines in **figure 2** can vary significantly from what is shown, e.g. flow over a flat plate with a low Reynold's number can affect a larger area of the fluid around the plate and even upstream of it because of viscous effects. Also, the flow near the surface (referred to as the boundary layer) could turn turbulent somewhere along its path. The paths of the fluid around and downstream of the object can also vary significantly from the streamlines shown.

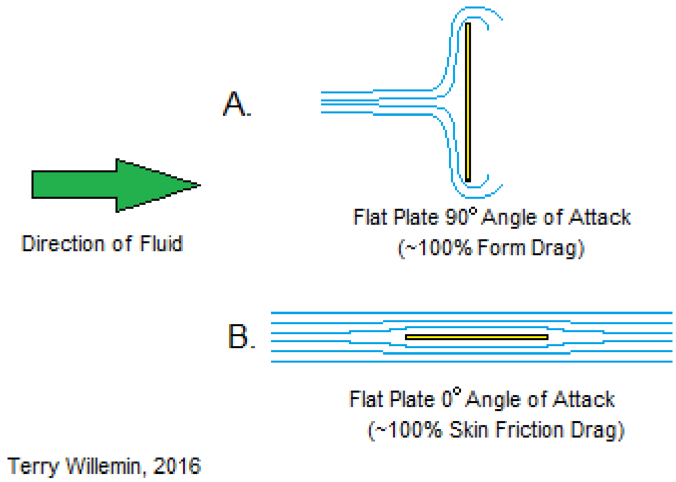

Figure 2, Skin Friction and Form Drag on a Flat Plate

So that's it, just two types of drag being discussed here. Simple right? If only it were. That's as deep as we'll get though. As a note, it generally makes no difference whether the object or the fluid is assumed to be moving relative to the other. This is partly what makes wind tunnel testing capable of verifying CFD results.

The force imparted on an object at fully subsonic speeds by drag (well below the speed of sound traveling through the fluid) can be estimated with the following equation:

$$\mathbf{F}_D = \frac{\rho v^2 C_D A}{2}$$

Equation 2

Where F_D is the drag force, ρ is the fluid density, v is the fluid to object's relative velocity, C_D is the drag coefficient and A is the projected or frontal area. When using customary U.S. units you'll also need to divide this value by the gravitational constant, g_c, (32.174 lbm-ft/lbf-s^2) to yield units of pounds-force.

By simple inspection of **equation 2** you will recognize several ways to reduce drag on a projectile to extend its flight, namely reduction of any value in the numerator on the right hand side of the equation. Most engineers are generally acquainted with these components except possibly drag coefficient, so what is it?

Drag Coefficient & Reynold's Number

Drag coefficient, C_D, is partly a function of the "Reynolds number", Re which is a ratio of the inertial to viscous forces of the fluid through which the object is traveling. The Reynolds number is a dimensionless number whose value is often used to predict at what point in the path of a flowing fluid the flow will transition between laminar and turbulent, with Re less than about 2,100 being laminar flow and Re greater than about 4,000 being fully turbulent. Unfortunately, drag coefficient curves are best determined by experimentation and not by simple geometry based calculations. Drag coefficients are also not constant values over a wide range of velocities. Using a constant coefficient of drag is only appropriate at low subsonic Mach numbers and low Reynold's numbers where Mach number and drag act independently of each other. At low Re, fluid behavior depends mostly on its viscosity. **Figure 3** lists some approximate drag coefficients for various objects. Of course, these values should not be relied upon for most engineering purposes, but may be used to generate drag force estimates for various objects under exclusive circumstances. **Figure 3** is included in the spreadsheet on the worksheet titled "Drag", to be used with the Drag Force Calculator on that sheet. Entering data in the light green filled cells will yield a drag force. This calculated value is only valid for sea level standard pressure (14.696 psi), dry air. The purpose of its inclusion in the spreadsheet is to convey a general feel for the effects of manipulating the variables. Where drag coefficient is concerned on the worksheet there are two main options: 1. select a drag coefficient from the list based on the figure, or 2. select "Enter a Value" to enter an arbitrary value in the cell below. For reference, drag coefficients should always be positive until someone invents an aerodynamically driven perpetual motion machine….

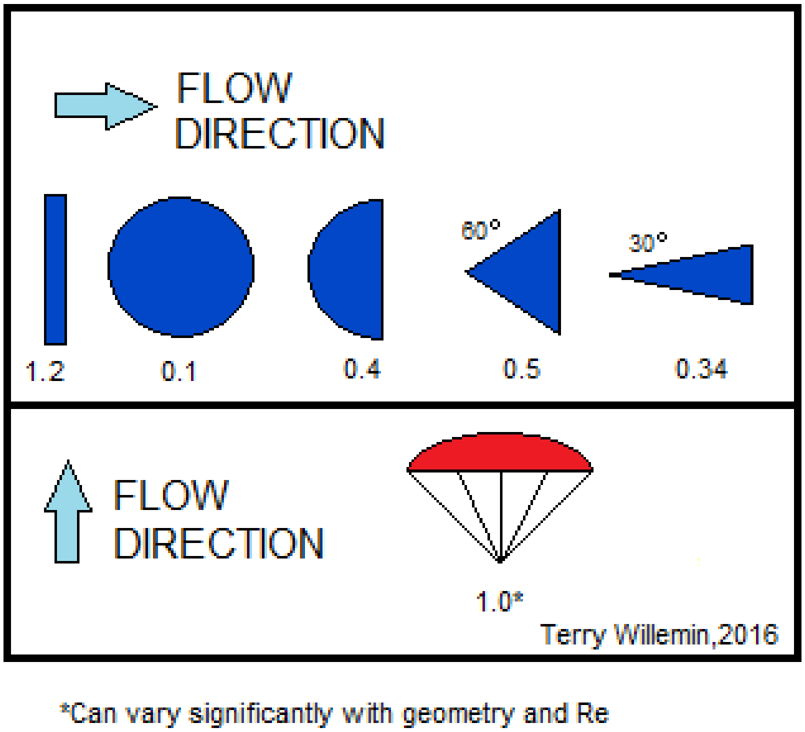

Figure 3, Drag Coefficient Approximations for Various Shapes (6)

Altitude/Air Density Variation

Air pressure and density are driven by altitude, temperature and humidity. Although initially it may seem counterintuitive, increasing humidity in air decreases its density. This is because we think of liquid water when we think of humidity rather than the light gas constituents of hydrogen and oxygen. Dry air averages about 1.27 g/l (.0793 lb_m/ft^3) while water vapor has a density of 0.8 g/l (.0499 lb_m/ft^3) at standard temperature and pressure (11). Air density has a direct effect on shear stress over the surface of the projectile. As expected, the higher the pressure, the denser the air. Also, the higher the altitude, the lower the pressure. The above statements are generalities and exceptions exist. The higher the air density the more drag is generated because the air is more viscous. "Standard atmosphere" models have been created for referencing against and they offer the user the ability to "correct" actual data to standard conditions including to differing altitudes and to estimate the rate of change with altitude. As mentioned before, part II of this course covers correcting to standard conditions. **Figure 4** below, from NASA-GRC gives equations for variation of temperature and pressure with altitude (using metric units). The figure breaks the change rates up for various layers of the atmosphere denoted, h.

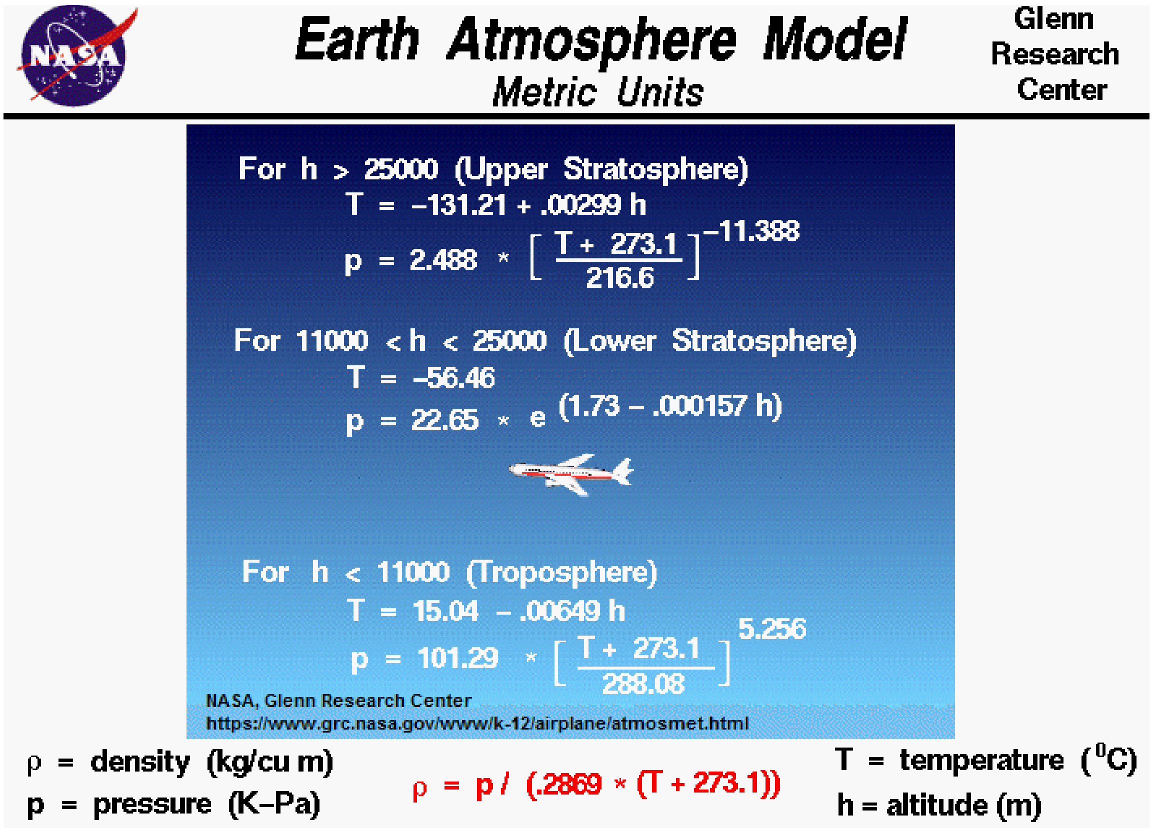

Figure 4, Standard Atmospheric Temperature and Pressure Models at Altitudes in Meters

Ballistic Coefficient

For bullets the concept of a ballistic coefficient, B.C., was devised to simplify drag calculations. At writing, it is rare for a bullet manufacturer to give a drag coefficient for their bullets to aid the ballistician or engineer in calculating drag on the bullet. Rather, what is given is called ballistic coefficient. Hornady Manufacturing, a major manufacturer of reloading supplies including bullets, describes ballistic coefficient as follows [(7)]:

A ballistic coefficient is the measure of a bullet's relative ability to overcome air resistance. Each bullet can be assigned a numerical value expressing this efficiency. The basis of this value is a ratio comparing the performance characteristics of a particular bullet against the known trajectory characteristics of a standard projectile. The ratio compares *the drag of a bullet (loss of velocity caused by air resistance encountered in flight) to the drag of the standard projectile. Expressed as a formula,*

$$B.C. = \frac{Drag\ of\ Standard\ Projectile}{Drag\ of\ Test\ Projectile}$$

Equation 3

Observe that ballistic coefficients… are, with only one exception, less than unity [1.0], indicating that these test projectiles - bullets for small arms - encountered more resistance than the standard….

The standard projectile on which all Hornady Bullets were compared was the G1 Model, based on work begun in France and refined at the U. S. Army Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland. Ballistic coefficients for all Hornady Bullets were determined by computer calculations using data from test firing research performed in our 200-yard underground test range.

Ballistic coefficient calculations combine both shape and sectional density factors. As a practical matter, most shooters understand that bullets with a pointed shape more easily retain their velocity than round nose or flat point bullets. This can be directly observed in the amount of drop bullets of the same weight using identical powders and powder weights but different shapes produce at the same target range. Expressed another way, round nose and pointed bullets will require different sight adjustments to attain the same zero over the same range. If more streamlined bullets maintain their velocity better, heavier streamlined bullets of the same shape will outperform lighter bullets at the same muzzle velocity.

A ready observation from the **equation 3** is that the less the drag on the test projectile, the higher the BC will be, for this simple reason many people who reload their own ammunition rely on BC when selecting bullets.

As mentioned in Hornady's description, the G1 drag model is used for the standard projectile's drag. The G1 projectile shape is shown on the left-hand side of **figure 5**. This model is by far the most commonly used by bullet manufacturers even though the bullet being tested may not closely resemble it in shape or flight characteristics. As a matter of fact, for long range shooting, other shapes far outperform it such as the boat-tailed G7 model shown on the right-hand side of **figure 5**. Of course, using the G7 model will generate lower BC values and for the unwitting who refers only the BC and not which model it is based on, the preferred selection of bullet would seem to be the one with the higher BC, despite that likely not being true.

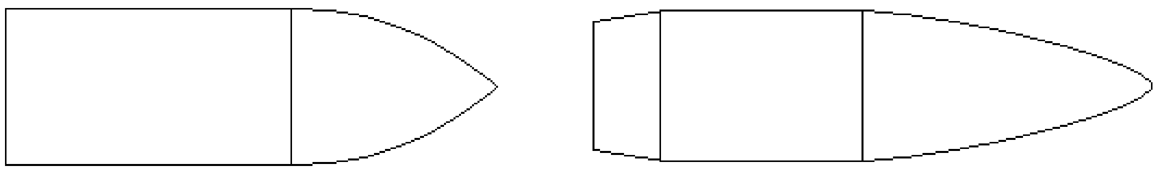

Figure 5, G1 (left) and G7 (right) Approximate Models of Bullets for Comparison

The different "G-models" on which to base a BC calculation are described below [(8)]:

- G1 or Ingalls (flat base with 2 caliber (blunt) nose ogive - by far the most popular)
- G2 (Aberdeen J projectile)
- G5 (short 7.5° boat-tail, 6.19 calibers long tangent ogive)
- G6 (flat base, 6 calibers long secant ogive)
- G7 (long 7.5° boat-tail, 10 calibers tangent ogive, very-low-drag bullet)
- G8 (flat base, 10 calibers long secant ogive)
- GL (blunt lead nose)

For the uninitiated, ogive is pronounced like oh'-jive sounding like an American colloquialism-slash-portmanteau hailing from the 1970's. It basically means a curved surface that comes to a point as the noses of some bullets do.

Calculating Ballistic Coefficients

Although BC's may be offered by bullet manufacturers, they may also be calculated. To calculate a BC a fair amount of information is required such as: temperature, pressure, altitude, wind direction & speed, humidity, an initial and final velocity and the distance between them (alternatively, the initial or final velocity and time of flight, t for the known distance). Other more minor effects are usually neglected for computational and data gathering simplicity. For reference the greater the distance between which the two velocities are measured, the more accurate the BC calculation can be. Calculating BC's is beyond the scope of this paper, but for the interested a simple internet search will yield several online ballistic coefficient calculators.

Side Note: many atmospheric condition standards have been generated by a jillion governing bodies for various uses, but for BC's in the US, data should be corrected to US Army Standard Metro. That is, 59 °F (15 °C), 14.696 psi (101.325 kPa), air density of 0.075126 lb/ft^3 (1.203403 kg/m^3) and 78% relative humidity. See Part II of this course for an example of correcting pressure data.

Mach Number and Wave Drag

As the relative speed of the fluid to projectile becomes supersonic, meaning it reaches or exceeds the speed that sound takes to travel through the fluid, a new type of pressure drag appears called wave drag. This drag is not a function of fluid viscosity, but rather a drop in total pressure and increased entropy when crossing shock waves caused by the projectile[(5)], see **figure 6** for an example. A shock wave causes very abrupt irreversible changes to the fluid, e.g. the upstream (before going through the shock wave) total pressure is always higher than the downstream total pressure even though no work is done.

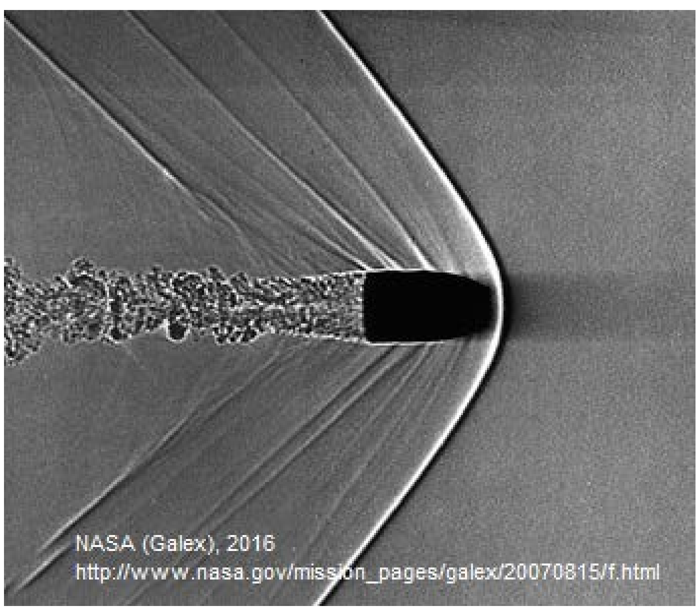

Figure 6, Shock Waves on a Bullet in Supersonic Flight.

With wave drag, the flow is considered inviscid, i.e. the viscosity of the fluid has no effect on drag. Calculating the Mach number, M can be used in turn to yield a relative idea of the increased drag force. Mach number, M is defined as the ratio of the velocity of an object, V, in some medium to the speed, a, at which sound travels in that medium. Mathematically this is expressed as:

$$M \equiv \frac{V}{a}$$

Equation 4

As in the subsonic case, drag coefficients for supersonic inviscid flows are not easily calculated except on the simplest of shapes. Suffice it to say that as Mach number increases, so does drag on the object; as intuitively expected. One other item of mention, the region of $0.8 \le M \le 1.2$ is called transonic. Within this region, flow over a projectile may have areas of subsonic and supersonic flow simultaneously. This adds complexity and can cause erratic behavior in the flight of a projectile. Where possible, velocities should be kept either fully subsonic or fully supersonic for the entire ballistic flight path.

Calculating Mach Number

To know whether or not the flow or the projectile is moving at supersonic speeds, $M \ge 1$, one usually will either: 1. Measure fluid temperature and the velocity of the projectile or, 2. Measure and compare static and total pressures at certain points in the fluid flow such as is done in most wind tunnels. Part II of this course gives more extensive details regarding these probes and pressure measuring devices.

The next few sections address example cases for calculating Mach number based on the selection from the above listed information.

Calculating Mach Number When Temperature and Projectile Velocity are Known

For an estimation of the speed of sound in a gas, a, one may use the following:

$$a = \sqrt{\gamma RT}$$

Equation 5

Where γ is a unitless value defined as the ratio of specific heats (c_p/c_v) for the gas/gaseous mixture. Air has a value of 1.4, R is the gas constant of the gas/gaseous mixture, and T is its absolute temperature. To determine the speed of sound in the gas requires only knowing its temperature and a gas constant.

Note: air has a gas constant, R of approximately 53.35 ft-lbf/lbm-˚R in customary U.S. units and 287.03 J/kg-K (287.03 m^2/s^2-K) in SI units. As with the drag force calculation when working in U.S. customary units, multiply γRT inside the radical by g_c, (32.174 lbm-ft/lbf-s^2).

Determining Mach Number When Pressures are Known

Another way to determine Mach number of a fluid flow (or of the projectile in that flow) is by use of pressure probes in the flow. The next two subsections treat two flow regime examples.

Calculating a Fully Subsonic Mach Number

If the flow is:

- Isentropic - fully subsonic or traveling no more than about M=0.3 so compressibility effect can be ignored, i.e. there is no change in entropy. For **rough estimation** purposes that Mach number can be increased, but be wary that the effect of compressibility will increase.
- Adiabatic - no heat is transferring into or out of the fluid
- Frictionless – an assumption that may or may not be accurate

Then Bernoulli's equation may be used to determine the projectile velocity. Bernoulli's equation simply states the total energy in a fluid under the conditions listed above, is equal to the sum of the pressure, kinetic energy and potential energy. With a little magic, some hand-waving, and then solving for velocity we can boil Bernoulli's down to give:

$$V = \sqrt{\frac{2(p_t - p_s)}{\rho}}$$

Equation 6

Where p_t is the total (or stagnation) pressure which is obtained from a pitot tube or other total pressure probe, p_s is the static pressure which comes from one or more static taps/ports oriented normal to the flow and ρ is the mass density of the fluid. Note, pressures must either be both absolute or gauge to have any meaning. Per the norm, when working with US customary units g_c should be added as a multiplier to the other parts of the numerator on the right-hand side of the equation.

Another example:
An object is placed in a wind tunnel at sea level. The 60 °F (288.7K) air flowing through the tunnel has a density of .0765 lb_m/ft^3 (1.2254 kg/m^3), a pitot tube in the tunnel is giving a dynamic pressure (dynamic pressure is the difference between total pressure and static pressure, or p_t-p_s) of 1 lb_f/in^2 (6.895 kPa). Remember the basic units of Pa are $kg/m\text{-}s^2$. How are the velocity and Mach number of the air flow in the tunnel calculated?

1. We will assume the flow is subsonic and begin by using **equation 6**. which gives an air speed of about 106m/s.
2. Determine the speed of sound for the indicated air temperature using **equation 5** (340.6 m/s)
3. Finally calculate the Mach number using **equation 4**, which gives us 0.31.
4. Since the Mach number is just a hair over the upper bound of the usefulness of the Bernoulli equation, we can probably say our assumption of the flow being fully subsonic will yield a sufficiently satisfactory calculation of the velocity.

Calculating a Fully Supersonic Mach Number

Although there are several methods for determining Mach number of a supersonic flow based on pressure measurements, in the interest of space only a single one will be discussed here. This method makes use of normal-shock (meaning the shock wave is a flat plane oriented perpendicular to the direction of flow), relations-based, lookup tables requiring the user to obtain a static pressure measurement just upstream of a normal shock wave, p_1 and a total pressure measurement just downstream of it, p_{t2}. Generally speaking, oblique or bow shockwaves are found in front of projectiles rather than normal shockwaves but for simplicity herein we'll only touch on normal shock relations. Where possible in a supersonic flow and in the vicinity of a shock wave we'll place the total pressure probe immediately downstream of the shockwave of the projectile since the flow cannot be brought to rest without a change in entropy across the wave. This is also because a shock wave will set up in front of the probe if it is upstream of the shockwave. It is possible to deal with bow shocks set up on the probe, and that is addressed to a small degree in part II of this course. The lookup table shown in **figure 7** below is an excerpt of NACA Report 1135, Table II [(9)].

Example: Suppose you are given a static upstream to total downstream pressure ratio of .155 across a normal shockwave. What is the approximate corresponding Mach number? Per the far-right column of the **figure 7** below (p_1/p_{t2}), .1553 corresponds to a Mach number (far left column) of 2.15. So, the given ratio of .155 should be pretty close to Mach 2.15.

M or M_1	$\frac{p}{p_t}$	$\frac{\rho}{\rho_t}$	$\frac{T}{T_t}$	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{t_2}}{p_{t_1}}$	$\frac{p_1}{p_{t_2}}$
2.15	.1011	.1946	.5196	5.226	2.882	1.813	.6511	.1553
2.16	.9956 −1	.1925	.5173	5.277	2.896	1.822	.6464	.1540
2.17	.9802 −1	.1903	.5150	5.327	2.910	1.831	.6419	.1527
2.18	.9649 −1	.1882	.5127	5.378	2.924	1.839	.6373	.1514
2.19	.9500 −1	.1861	.5104	5.429	2.938	1.848	.6327	.1502
2.20	.9352 −1	.1841	.5081	5.480	2.951	1.857	.6281	.1489
2.21	.9207 −1	.1820	.5059	5.531	2.965	1.866	.6236	.1476
2.22	.9064 −1	.1800	.5036	5.583	2.978	1.875	.6191	.1464
2.23	.8923 −1	.1780	.5014	5.636	2.992	1.883	.6145	.1452
2.24	.8785 −1	.1760	.4991	5.687	3.005	1.892	.6100	.1440
2.25	.8648 −1	.1740	.4969	5.740	3.019	1.901	.6055	.1428
2.26	.8514 −1	.1721	.4947	5.792	3.032	1.910	.6011	.1417
2.27	.8382 −1	.1702	.4925	5.845	3.045	1.919	.5966	.1405
2.28	.8251 −1	.1683	.4903	5.898	3.058	1.929	.5921	.1394
2.29	.8123 −1	.1664	.4881	5.951	3.071	1.938	.5877	.1382
2.30	.7997 −1	.1646	.4859	6.005	3.085	1.947	.5833	.1371
2.31	.7873 −1	.1628	.4837	6.059	3.098	1.956	.5789	.1360
2.32	.7751 −1	.1609	.4816	6.113	3.110	1.965	.5745	.1349
2.33	.7631 −1	.1592	.4794	6.167	3.123	1.974	.5702	.1338
2.34	.7512 −1	.1574	.4773	6.222	3.136	1.984	.5658	.1328

Figure 7, Excerpt of NACA Report 1135, Table II

Generating Drag Coefficients

Since the use of constant values for drag coefficients applies only to low subsonic Mach number trajectories and since many ballistic projectiles travel at supersonic or even hypersonic velocities, several methods have been devised to generate drag coefficient curves for the range of velocities a projectile might encounter. Several of these methods can be very involved or complex because the ballistic projectile transitions from one flow regime to another (laminar to turbulent) and/or may transition from supersonic to subsonic in flight. Additionally, verifying drag coefficients still boils down to analyzing real data from actual test firings. Generating the data and modifying it such that it may be compared from one projectile to another in varying fluid conditions requires the use of several types of measurement equipment which are described in Part II of this course. The next two subsections contain a conceptual smattering of how drag is affected by Mach number in specific flow regimes.

Drag Coefficient in the Supersonic Range of Mach 2.5-5 [(10)]

Besides the low subsonic case mentioned above, another simple case exists in the range of Mach 2.5-5. For many projectiles their velocity varies linearly with distance traveled, i.e. the drag force varies directly with velocity between Mach 2.5 to 5. That can simplify the characterization of drag coefficients. Reference 10, Projectile Supersonic Drag Characteristics, is a short read and a very enlightening article on the subject.

Above Mach 5, Hypersonic

Above about Mach 5 curious things happen to the gasses flowing around the projectile. At supersonic speeds shock waves form on the body as shown in **figure 6** above, and these shockwaves bend toward the body as Mach number increases. The fluid crossing the shockwave experiences a very abrupt change in density, temperature and pressure. Around Mach 5 and higher in a gas, the effect is so great that the gas molecules can dissociate or even ionize. When this happens characterizing or analyzing the flow becomes much more difficult and so to differentiate this flow regime from supersonic it is called hypersonic.

Stability

Knowing it is possible to calculate and/or measure the effects of gravity and drag we can now hit any intended target, right? Not unless it's relatively close. There is still much more affecting the ballistic trajectory. It is known that drag can have a huge effect on some projectiles, e.g. spheres, or cubes, and that is part of why lower drag coefficient bullets were designed. The problem with a general bullet shape though, is that it tends to become unstable and tumble in flight when fired from a smooth bore barrel. As shown in **figure 8** below, the optimal condition for stability of a projectile is to have the center of pressure aft (downstream in the flow direction) of the center of gravity/mass; think arrows, darts and, if you're old enough (and lucky enough to have survived them) lawn darts. An arrow or dart is typically stable since the fletchings are placed in the aft portion of the arrow well behind the cg. Center of pressure is similar in concept to center of gravity. It is the calculated point where all aerodynamic forces may be considered as acting through; it is the averaged location of pressure variation. Locating the cp typically requires rigorous calculation performed by CFD software. Creating this condition can be challenging for some objects including bullets where the opposite is typically true. For this reason longer bullets tend to be less stable than shorter ones since lengthening them moves the center of gravity farther back along the bullet and is also the reason why stabilization is so necessary. **Figure 9**, shows the typical relationship between cp and cg on a bullet and many other projectiles that are spin stabilized see next section). For reference, the cg of the bullet always follows the flight path (blue) even though its longitudinal axis (red) is typically not pointed exactly into the flight path. When the bullet is in flight, its nose is typically at some angle of attack (green) to stream lines of the air. In flight this angle is normally no more than 0.5 degrees.

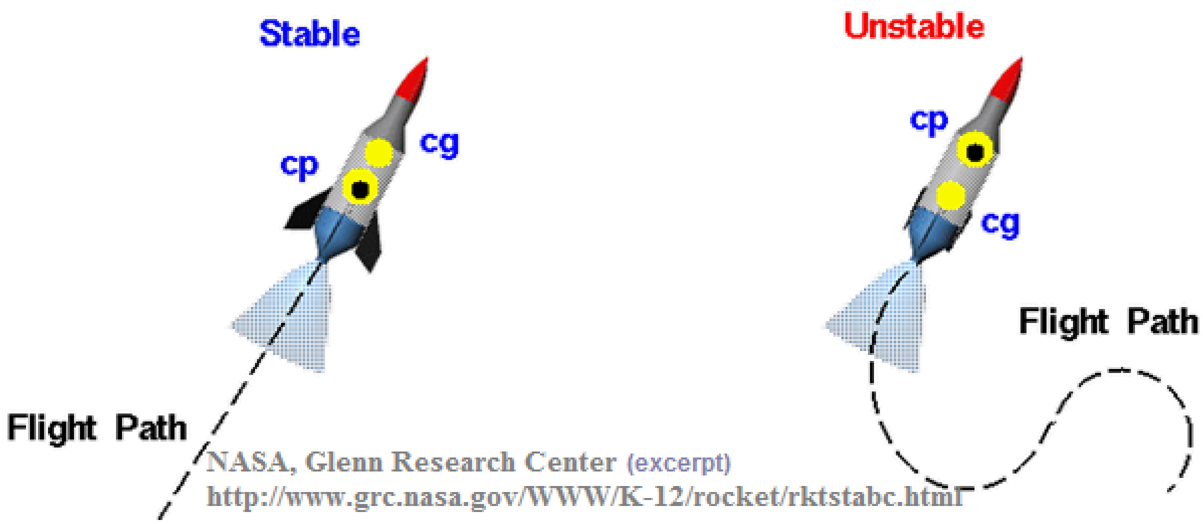

Figure 8, Model Rocket Stability. Note that the fins are at the rear of the rocket so they bring the cp behind the cg and increase stability.

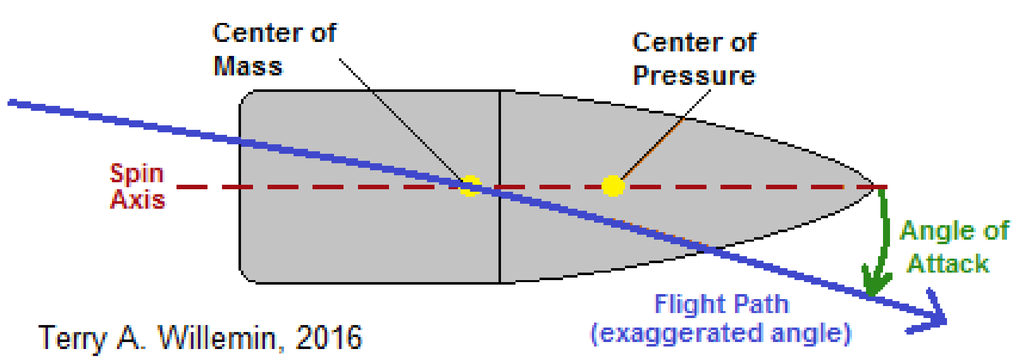

Figure 9, Center of pressure and center of gravity/mass on an example bullet.

Spin Stabilization (Gyroscopic Stabilization)

A very commonly used method for improving stability of bullets and other projectiles is spin stabilization. This is used on basically all modern rifles, some arrows, and solid fuel rockets, which includes many model or hobbyist rockets. Spin stabilization is even used to stabilize a football (no, not a soccer ball) when thrown properly. For rockets the axis along which they spin is called the roll axis as shown in **figure 10**. With bullets this is usually referred to as the spin axis. **Figure 10** additionally gives the six degrees of freedom (pitch, roll, and yaw and their three axes) along which the projectile may translate and/or rotate in 3D Euclidean space. There are many ways in which spin can be induced in a projectile including:

- Helical fluting or grooves cut into the exit of the engine nozzle (single engine only) to create a “swirl” in the flow and induce spin. The Hydra 70 rocket uses this method to impart initial spin during launch before its wrap around fins deploy.
- Multiple Canted engines (think model or hobbyist rockets). Two or more engines are oriented to induce spin. This can be a bit tricky to employ.
- Canted or partially angled/bent Fins. This is the typical method used to impart spin to an arrow when spin is desired. Modern full-sized liquid fuel rockets typically rely on thrust vectoring through gimbaled engines or engine nozzles for control and stability rather than spin stabilization, whereas some solid fuel rockets still rely on spin stabilization.
- Barrel Rifling – Practically all rifles and handguns (and even some shotguns) have internal helical grooves or ridges in their barrels. A tight-fitting bullet is caused to spin as it travels down the barrel because of the rifling. A spinning bullet may exceed 300,000 RPM in flight!

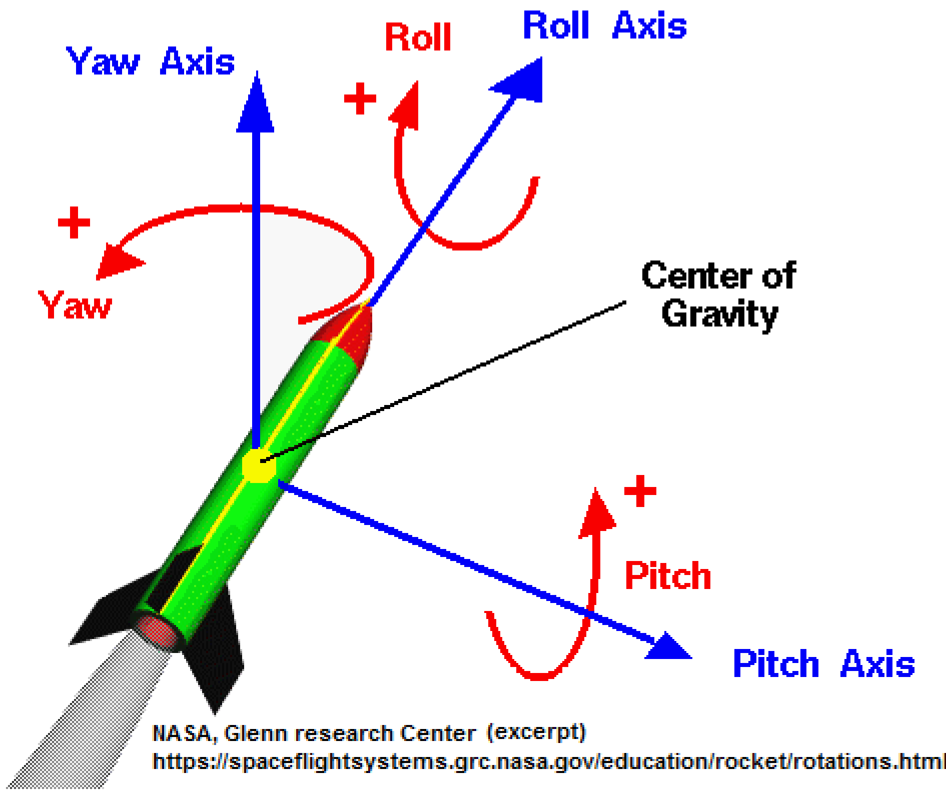

Figure 10, Rocket Body Axes.

For the purposes of brevity relating to spin stabilization, let's look at a simplified model for spin stabilization optimization as used for bullets in rifles, handguns and artillery barrels. Don Miller created the following formulas[(12)] based on experimental observational data:

$$s = \frac{30m}{t^2 d^2 l(1 + l^2)} f_v f_\rho$$

Equation 7

Where the two correction factors, f_v and f_ρ are:

$$f_v = \left(\frac{v}{2800}\right)^{1/3}, \quad f_\rho = \frac{(T + 459.67)}{518.67}\frac{P_{std}}{P_T} = \frac{\rho_{std}}{\rho_T}$$

And, s is the Miller Stability Factor, m is the bullet mass in grains, t is the twist rate in calibers per turn (barrel twist in inches per turn divided by bullet diameter), d is the bullet diameter in inches, l is the bullet length in diameters (length/diameter), f_v is a velocity correction factor, f_ρ is a density correction factor, v is the muzzle velocity in ft/s, T is the air temperature in °F, P_{std} is US Army Standard Metro air pressure at 14.696 psi (101.325 kPa), P_T is the ambient air pressure at the test site always in the same units as P_{std}, ρ_{std} is Army Standard Metro air density of 0.075126 lb/ft^3 (1.203403 kg/m^3) and ρ_T is the air density at the test site in the matching units.

Miller suggests that for optimum stability, s should never be less than 1.3 at standard conditions. He also states the US military uses a range of 1.5-2.0. However, values as high as 3.5 may be acceptable. The above equations for stability have been included in the *Stability Factor* worksheet presented with inputs and outputs in the same format as the other worksheets.

Other Forces and Effects

If only spin stabilization could remove all the remaining problems of hitting a target. There are several other "forces" at work to throw our projectile off. And, spin stabilization creates new issues.

Wind Drift

Drift of general projectiles in flight due to wind is generally not easily determined or calculated with simplified equations. A research engineer or ballistician will likely use CFD to characterize a projectile's susceptibility to winds at various angles and subsequent real-world testing can be devised to verify. For bullets there are myriad ballistics calculators from manufacturers who have done exhaustive testing to characterize a particular bullet and which will estimate wind drift for the shooter based on wind speed and direction. Since we'll not touch on any calculations in this section, below are some generalizations which are hopefully mostly intuitive:

- The smaller the BC, the greater the effect wind will have on a bullet
- Generally, the larger the C_D, the greater the effect wind will have on a projectile's flight path
- The longer the time of flight, the greater the effect wind will have on the flight path
- The higher the wind speed, the more the projectile will drift for a given air density
- The denser (cooler, dryer and/or higher pressure) the air, the greater effect wind will have

- A headwind will increase drag on the projectile with an accompanying increase in drop. A tailwind will have just the opposite effect.
- A side wind can cause a spinning projectile to rise or drop through Kutta-Joukowski Lift/Magnus Effect.

Wind speed can be measured or estimated in several ways. Anemometers, windsocks, and its effects on surroundings (e.g. the Beaufort Wind Force Scale), are just some of the tools that can provide useful wind speed estimates. Each of those methods, and others, have their merit and value. Part II of this course covers anemometers and additionally includes information on windsocks and the Beaufort Wind Force Scale.

Precession, Spin Drift, Yaw of Repose…

A spinning projectile will experience a slight turn to either the right or left depending on its direction of rotation. This effect is exacerbated by factors such as increases in: rotation rate, distance between the center of pressure and center of mass, time of flight and air density. If looking at the tail end of bullets flying away from the observer spinning in a clockwise direction (right spin) the bullet will drift slightly to the right and the counterclockwise spinning bullet (left spin) will drift left. Many refer to this as spin drift. It is a result of the geometry of the spinning bullet moving through air being acted on by torque induced precession (gyroscopic precession), this phenomenon is also referred to as yaw of repose. As shown in **figure 11**, when the center of mass is aft of the center of pressure in a spinning projectile traveling through a fluid such as air, there is a slight torque on the bullet to turn it nose upward. Using the right-hand rule gives a torque vector pointing to the right of the bullet in the figure. The slight nose up attitude relative to trajectory is caused by the cg being aft of the center of pressure. With respect to bullets, this is practically always the case. This torque on the spinning bullet causes a precession. Using the right-hand rule again for the spin direction we see the angular momentum vector pointing out the front of the bullet for a CW spin, as shown in **figure 11**, and out of the rear for a left spin. The bullet will yaw or precess to the right if spinning in the clockwise direction as shown in the figure. Flipping the spin direction will turn the nose to yaw to the left.

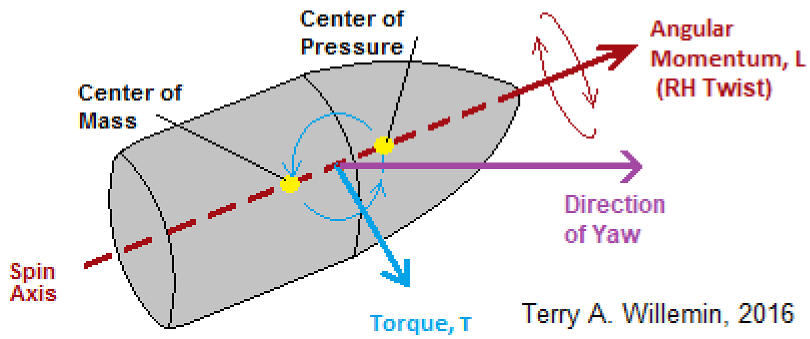

Figure 11, Using the right-hand rule it is seen that the drift of a right spinning bullet will be to the right due to torque induced precession or yaw of repose.

Typical drift is 8-9 inches at 1000 yards for small arms trajectories [(13)]. Rifling in the barrel causes the spinning of the bullet. It is referred to as either right or left twist. Ballistics author Bryan Litz proposed the following simple calculator for spin drift based on torque-induced precession:

$$D = 1.25(s + 1.2)t^{1.83}$$

Equation 8

Where D is the drift in inches, s is the Miller stability factor (**equation 7**) and t is the bullet's flight time in seconds. **Equation 8** is included on the *Spin, Drift & Drop* worksheet; try it out.

Coriolis

Coriolis Effect is composed of two components, vertical and horizontal. It always results in a drift to the right when north of the equator and to the left when south of the equator. Note that Coriolis Effect is sometimes called Coriolis force, but is not a true force. The alteration to intended course of the projectile is an effect of being in a rotating reference frame, such as standing on the surface of the earth. A rotating reference frame is called non-inertial since there is an inward acceleration toward the axis of rotation.

In the Northern hemisphere a projectile will drift to the right of its intended target and in the southern hemisphere it will drift left in addition to any spin drift mentioned previously. As in the previous section, Coriolis effect does little to alter the path of a projectile unless the time of flight is lengthy, i.e. the distance traveled is great. As an example, for about a

1,000 yard shot the drift might only be about 4 inches due to Coriolis effect. For most target shooting it can be ignored, but for rockets (and missiles) the effect can make miles of difference. To some, the causes of this effect can be conceptually challenging to grasp, and to which most long-distance shooters relegate themselves to simplified equations without a conceptual understanding of the physics behind it. Entering values into simplified equations meets their needs, but we're engineers, so we should at least examine the topic to gain a cursory understanding.

Coriolis' Horizontal Effect

Because earth is rotating about a fixed axis, and because it more or less has a spherical shape (technically an oblate spheroid); the tangential velocity on its surface varies with latitude. Latitude is the distance north or south of the equator or poles, see **figure 12** below. Lines of latitude form circles concentric with the equator. Earth has an angular velocity of close to 360˚/24hr, so someone living in Quito, Ecuador near the equator has the same 24-hour day as Santa at the North Pole (on the clock at least). Many people never consider that someone living near the equator is moving much faster relative to a non-rotating observer somewhere out in space than someone at one of the poles because we tend to think in terms of the angular velocity of the earth rather than tangential velocity. So…, besides temperatures, a huge difference exists between the two latitudes. As latitude increases from zero at the equator to 90˚ at the North or South Poles, the tangential velocity of earth decreases because the radial distance from the axis of rotation decreases. At the equator where earth's radius is nearly 3,960 miles its tangential velocity is roughly 1,040 miles per hour, and near the poles it is practically zero! This is a huge driver for launching rockets at latitudes closer to the equator. Someone in the most northern part of Alaska is moving about 330mph, while someone in Miami, Florida is moving about 940mph! Now, let's suppose someone in the northern hemisphere shoots a bullet in a southerly direction and we ignore all other external trajectory affecting factors such as those listed earlier in this paper. The bullet will have some initial trajectory, but we now have a moving target problem since the target at which the gun was fired will have moved some distance with a higher tangential velocity than the higher latitude position from which it was fired. This translates to the bullet impacting a slight distance to the right of the intended target. Just the opposite occurs in a northerly shot and results in a hit to the right of the target once again since the bullet begins its path from a higher tangential velocity region. Because of the short distances involved and/or amount of time traveled this effect is normally not recognized and for most projectiles it's ignored. As an exaggerated example though, someone launching a rocket in Houston (who compensates for every other expected trajectory modifying factor, including drag from winds), intended for somewhere in Kansas might have it hit ground in Missouri. And the person in Kansas trying to hit a target in Houston may need to aim toward somewhere in Louisiana. A *missile*, of course, can be made to compensate for that in flight unlike the *rocket*.

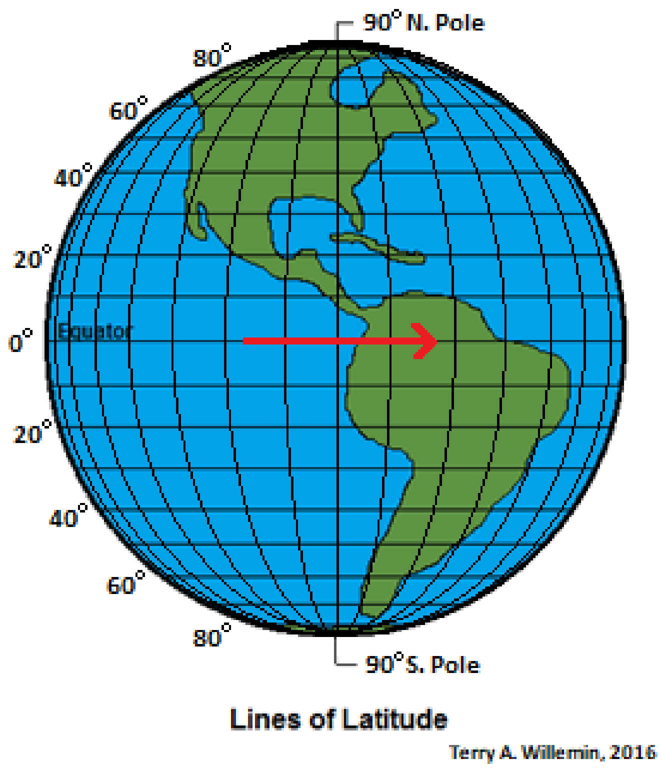

Figure 12, The horizontal lines in the picture represent lines of latitude on earth spaced at 10-degree intervals and marked at every 20 degrees & the poles. Earth's spin direction is indicated by the red arrow.

The above explains the Coriolis Effect on an object with some relatively northern or southern velocity, but what about a more easterly or westerly shot, where lines of latitude are relatively constant, i.e. constant tangential velocity? If you were to shoot a projectile due East or West, there would still be a drift of the projectile to the right of the observer. To understand the second part of what causes that rightward miss of the target let's look at centripetal force. Centripetal force is the force that pulls a rotating object in towards its axis of rotation. Referring to **figure 13**, imagine spinning a rock, which is tied to a string by hand above your head. The string is producing that centripetal force. As the angular velocity of the rock increases from zero, the radius of rotation increases until the string is pulled horizontal. Any increase in angular velocity increases tension in the string stretching it until it eventually breaks. On earth we experience centripetal force as a result of gravitational “force” and earth’s rotation. If our projectile’s kinetic energy exceeds earth’s gravitational potential energy however, the gravity “string” breaks, or stated another way we reach earth’s escape velocity and coast off into space. For earth that velocity is just over 25,000 mph! The equation for centripetal force moving with some tangential speed v, is:

$$F_c = \frac{mv_t^2}{r}$$

Equation 9

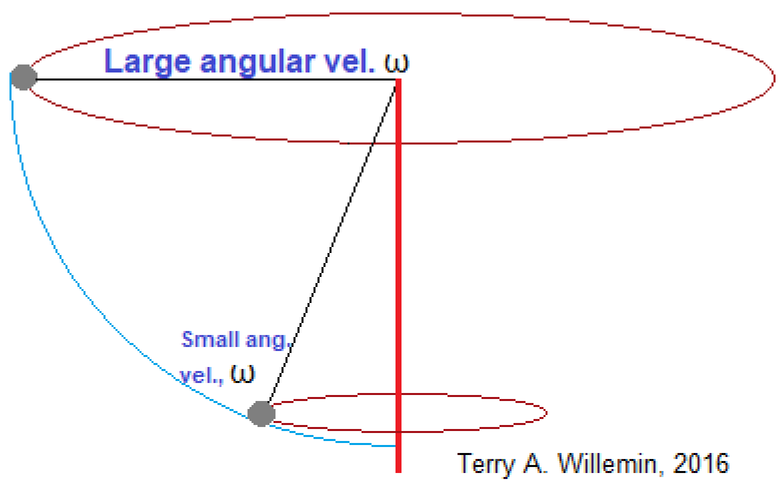

Figure 13, Increasing angular velocity on a rock tied to a string increases the tension in the string and the radius of the path it follows until the string is horizontal.

The force, F acting on our projectile is gravity, which we’ll consider to be constant. **Equation 9** tells us an increase in velocity must translate to an increase in radius if the force and mass are constant. For the purposes of this analogy now, let’s look at the component of the velocity of the object in earth’s East and West directions. Earth rotates counterclockwise (to the East) when viewed by an observer in space above the North Pole. What this means to the projectile is that with a more easterly velocity component (think of it as tangential velocity) is it is traveling faster than the earth’s rotation and will thus seek a larger radius, r; in other words, move in a southerly direction (or to the right of the intended target). Conversely, a projectile with a more westerly tangential velocity component is traveling slower around earth's axis than the earth is and will thus seek a smaller radius, i.e. move in a northerly direction (once again, to the right of the intended target). By now you may be asking, couldn't the projectile just as well move toward or away from the earth also? The answer is yes and if you read on into the next section, Eötvös Effect or Drop Factor, we'll once again conceptually explore that part of the projectile's movement.

At the beginning of this section, we recognized that as latitude increases the tangential velocity on earth's surface decreases. This means that the difference in horizontal/tangential velocity between earth and the projectile is greater in magnitude resulting in greater deflection with increasing distance from the equator. Theoretically, at the equator there is no deflection. Also, as the velocity of the projectile increases, so does the magnitude of the deflection.

For some long distance shooters a commonly used equation to determine drift due to Coriolis Effect is:

$$d_{\text{right}} = \Omega_E * r^2 * \frac{\sin(l)}{\frac{r}{t}}$$

Equation 10

Where r is the range to the target and is usually given in feet, t is the flight time in seconds, l is the latitude in radians from which the projectile is fired, Ω is the angular velocity of the earth in radians per second ($7.292*10^{-5}$) and d_{right} is the deflection of the projectile to the right of the target (has the same units as r above) in the Northern hemisphere (left in the southern hemisphere).

Coriolis' Vertical Effect: Eötvös Effect or Drop Factor

Along with the abovementioned rightward drift of a projectile there is also an upward or downward drift relative to azimuth angle. Physicist Baron Roland von Eötvös (1848-1919) noticed that while taking gravitational measurements at sea that differences existed with respect to direction of travel, and velocity; which were most pronounced when moving with an eastward or westward azimuth angle. We'll refer to this as the Eötvös Effect. This effect can easily be understood by referring back to **equation 9** and the concepts from the centripetal force section. Since earth rotates in an eastward direction (or counterclockwise as viewed from the north pole) and an object moving toward the West has a lower angular (and tangential) velocity to the stationary observer in space, not only will it experience a rightward or northerly turn to reduce distance from the axis of rotation, but it will also experience a turn toward the center of earth to reduce its distance. Just the opposite occurs with an Eastward traveling object. If an object travels in an easterly direction its angular velocity is in addition to that of the earth and hence its radius must increase if gravity and mass remain constant. It is probably obvious that the two mutually perpendicular ways to decrease distance (radius) from earth's axis of rotation are to:

1. travel toward the ground (center of the earth), i.e. inwardly normal to the surface
2. move on the surface tangentially away from the equator.

The difference between Eötvös Effect and Coriolis Effect is that Eötvös Effect is dependent not only on latitude but also on azimuthal angle (the angles we use with 2D maps, but relative to true north rather than magnetic north) with maximum and minimum in the easterly and westerly directions respectively for any given latitude.

Markswomen/marksmen and other enthusiasts call this up or down displacement drop factor, DF. For estimations a common equation used to determine DF is the following:

$$\mathrm{DF} = 1 + \frac{2 * \Omega * \mathrm{v} * \cos(\mathrm{l}) \sin(\mathrm{a})}{\mathrm{g}}$$

Equation 11

Where Ω is the angular velocity of earth in radians per second ($7.292*10^{-5}$), v is the tangential velocity of the projectile (which for our simplification could be the muzzle velocity instead of an average velocity), l is the latitude in radians, a is the azimuth angle converted to radians also and g the acceleration due to gravity. One need only multiply the expected drop due to gravity by the drop factor to get the total amount of drop the trajectory must account for.

The accompanying spreadsheet includes **equations 8, 10, & 11** in the *Spin, Drift & Drop* worksheet to give an idea of how small the effect the "two" forces are on a projectile. The target gives a visual representation of the effects separately and combined. Per the norm, light green cells are for inputs and light blue are outputs on the worksheet. As an example, suppose a bullet is shot in a vacuum near the earth's surface from a horizontal position traveling 2500 ft/s with a distance to target of 1,000 yards (3,000 ft and has a flight time of about 1.2 seconds) fired at 45 degrees latitude from west (270 degrees azimuth) with an expected drop of about -278 inches (negative for downward) before DF will have an additional drop of about -2.2 inches to deal with. In other words, the markswoman/marksman will have to deal with a total of about -280.2 inches of drop. Not such a big difference, right? There will also be an accompanying drift of about 2.24 inches to the right in the Northern Hemisphere for Coriolis Effect.

Even More Effects

There are other potential minor effects that can cause deviations to the flight path of a projectile beyond what has been discussed in preceding sections. Their effects can vary in severity with shape, time of flight, transition from internal to external ballistics and so on, however they still tend to have only very minor effects, similar to gyroscopic precession, or Coriolis effect, and for brevity have been omitted from this primer. One of these effects worth noting, and to me a very intriguing subject, is Kutta-Joukowski Lift/Magnus Effect. Along with that effect be sure to look into Flettner rotors if you're not already familiar with them.

Summary and Conclusion

In this part of the full course several factors were introduced which affect the flight of a ballistic projectile. Some basic equations were also presented that are useful to a great many engineers, outside of ballistics for making approximations with a fair bit of accuracy. Several specialized flow regimes that a ballistic projectile may pass through were also treated, though there are cases that were omitted for either lack of space or their complexity. External ballistics is such a broad-spectrum study that it has applicability or similarity to much of the work of many types of professional engineers. As such and because of the vastness of related information no one taking this introductory course need ever assume that there is not much more that may be learned about this interesting subject.

As mentioned previously, Part II of this course analyzes measurement tools at the disposal of an aerodynamicist, ballistician, or ballistics engineer. There are several new equations introduced related to the equipment's use and an additional "just for fun" chapter included on the ballistic pendulum if you ever thought to build your own.

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Further Reading

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