A SunCam online continuing education course

WHAT EVERY ENGINEER SHOULD KNOW ABOUT STATISTICAL PROCESS/QUALITY CONTROL II

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Introduction

The American National Standards Institute (ANSI) defines Quality Assurance (QA) as "All of those planned or systematic actions necessary to provide adequate confidence that an item will perform satisfactorily in service." A more operational definition of quality is: "Fitness for Use" This is as defined by the customer not the producer or manufacturer. Quality assurance is anchored by the following four pillars of techniques and strategies, namely, Design for Robustness, Statistical Quality/Process, and Acceptance Control.

The first course in the sequence focused on Process/Quality control. This second course focuses on Design for Robustness and Acceptance Control. Robust Design encompasses all areas related to system design, parameter design and tolerance design.

Acceptance control is the idea of using statistical sampling strategies to determine if a production process is operating within its designed limits as measured by its output performance. It is implemented to maintain the integrity of the product and its supply chain both within and outside of a company. The techniques have been used by industries worldwide (especially since World War II) for assuring the quality of incoming and outgoing goods. Simply put, it allows a company to measure the quality of a batch of products by selecting a specified number of products for testing. Acceptance sampling plans determine the sample size and criteria for accepting or rejecting a batch based on the quality of a sample, using statistical principles. In addition, we will also examine some of the Military Standards used for Acceptance Control more specifically MIL- STD- 1916(released for public use April 1, 1996 and replaced MIL STD 105E) and its companion MIL-HDBK-1919 (released February 10, 1999) as well as other relevant Government publications and documents that have been developed to standardize quality plans. Publications such as MIL-STD-414 and ISO 9000 while important will not be specifically addressed as part of the materials in this course sequence.

1.1 Definitions of Terms

The following basic definitions were culled from the MIL-STD-1916 and its companion MIL-HDBK-1916 document and are consistent with current industry usage of those terms. They will have the same meaning and implication throughout the materials presented in this document.

Inspection. Examining and testing supplies or services (including, when appropriate, raw materials, components, and intermediate assemblies) to determine conformance to contract requirements.

Major characteristic. A characteristic, other than critical, that must be met to avoid failure or material reduction of usability of the unit of product for intended purpose.

Major nonconforming unit. A unit of product that fails to conform to specified requirements for one or more major characteristics but conforms to all critical characteristics.

Minor characteristic. A characteristic, other than critical or major, whose departure from specification is not likely to materially reduce its usability for its intended purpose or whose departure from established standards has little bearing on the effective use or operation of the unit.

Minor nonconforming unit. A unit of product that fails to conform to specified requirements of one or more minor characteristics but conforms to all critical and major characteristics.

Production interval. A period of production under continuous sampling assumed to consist of essentially homogeneous quality. It is normally a single shift. It can be a day if it is reasonably certain that shift changes do not affect quality of product but shall not be longer than a day.

Nonconformance. A departure from a specified requirement for any characteristic.
Nonconforming unit. A unit of product that has one or more nonconformances.

Quality Assurance. A planned and systematic pattern of all actions necessary to provide adequate confidence that adequate technical requirements are established; products and services conform to established technical requirements; and satisfactory performance is achieved.

Quality Audit. A systematic examination of the acts and decisions with respect to quality in order to independently verify or evaluate the operational requirements of the quality program or the specification or contract requirements of the product or service.

Quality Program. A program which is developed, planned, and managed to carry out cost effectively all efforts to affect the quality of materials and services from concept through validation, full-scale development, production, deployment, and disposal.

Screening inspection. An inspection process whereby every unit is checked, and all nonconforming units are removed; also referred to as 100 percent inspection.

Traceability. The ability to trace the history, application or location of an item or activity, or similar items or activities, by means of recorded identification.

1.2 Background

To better understand the implications of quality, we want to consider quality as a system. A quality system is one which aims to achieve controlled production of products with good quality. When a quality system fails to provide a controlled production environment that results in quality products, we have quality loss or loss in quality. Quality loss refers to losses incurred by a system or organisation (society) from the time a product is released for shipment to the time of its ultimate use and disposal. Such losses are as a result of noise (or factors) that cause **deviation from the target or**

the nominal of a specific product's functional characteristics which ultimately prevent the product from functioning as previously planned or designed. Two types of noise (factors) are responsible for deviation from targets, namely a). External noise, and b). Internal noise

a). External Noise Factors consist of:

- Operating Environment
- Temperature
- Humidity
- Operators
- Suppliers

b). Internal Noise Factors consist of:

- Tool wear
- Machine setting
- Raw material variation

In general, a good (carefully designed) quality System will make products that are robust (insensitive) to variations due to the noise factors. This fact is not true of just any quality system.

1.3 Designing for Robustness

Robustness is needed from the design phase, through the production engineering phase and the actual production operation phase. Designing for robustness requires **three** specific design steps, namely; system, parameter, and tolerance.

1.3.1 System Design

The System Design phase consists of two sub-phases, namely; i). Development and Research phase and ii). Product Design phase.

i). Development and Research

The development and research phase (or the planning phase) involves the development of the prototype as well as the parts, components, and the assembly system.

ii). Product Design phase

This phase consists of the determination of manufacturing technology and the attendant processes.

System design is the determination of manufacturing processes/technologies that can provide the desired tolerance at the lowest cost. It involves the development of the basic prototype that can perform the desired functions and comes about by the selection of the appropriate:

- material
- parts and components
- assembly

1.3.2 Parameter Design

In the parameter design phase, the levels of controllable factors (design parameters) are selected or identified for the purpose of reducing the noise factors. Upon the establishment of the optimal design (or desirable system design), then the optimal levels of the parameters identified in the system design are concretized. In other words, what are the optimal levels of the parameters (factors) that would result in optimal product as prescribed by the system design. These are the operating levels of the factors that minimize variations in the product characteristics.

1.3.3 Tolerance Design

This design phase represents the "honing" in of the process. This includes the identification of the suitable limits within which the factors should be set or should operate to ensure the functional requirements are met.

Tolerance design is concerned with limits of each factor (parameter) based on the quality and resulting cost. It makes allowances for changes in operating conditions. The narrower the limits (range of deviations), the costlier the product.

Design engineers have primary responsibility for tolerance design. The design engineers emphasize performance, appearance, and reliability whereas the production and manufacturing engineers produce whatever has been designed.

In tolerance design, there is need to specify both the functional limits and the design tolerance of a given design as part of the product characteristics.

a) Functional limits

The functional limits of a product characteristic are the threshold values beyond which the characteristic will not function. It is determined either by in-depth engineering knowledge of the product characteristics or through experimentation. When the product characteristics do not conform to the functional limits, then losses will inevitably occur. The losses are both i). tangible such as warranty cost (if in the field) and rework (if found inhouse), and ii). intangible such as loss of goodwill and loss of market share. If a characteristic exceeds its functional limits or deviates from the nominal, then there is going be failure or loss. To reduce the incidence of failure, careful engineering analyses is required to determine the tolerance of the characteristics. Thus, tolerance of the product characteristics is vital in minimizing loss. design which specifies the tolerance limits is a way to militate against the loss.

b) Tolerance limits/design.

The tolerance limits are the set of values that are offset from the nominal that form the limits of the useful operating region of the characteristics which ultimately ensures the functional integrity of the characteristic and hence the product. The tolerance of the product characteristic significantly affects the quality loss. The optimal tolerances are those that minimize total quality loss. Several factors prevent product characteristics from meeting the functional tolerance or staying at the nominal. These factors (also called noise) include

- Environmental factors
- Deterioration factors
- Imperfections in manufacturing processes

Both environmental factors and deterioration factors are controlled by the design engineer in the parameter design phase by developing counter measures to minimize variation due to these factors.

1.4 Process Capability (Cp, Cpk) and Process Performance (Pp, Ppk)

It is important that we establish some basic understanding of process capability and process spread and how those affect the quality loss. Process capability is defined as the ability of the process to operate within the process spread or within the desired tolerance as specified either by the customer or the producer and it is sometimes referred to as the process sigma. The higher the process sigma, the better the process. In other words, most of the items produced falls within specifications. The fallout (or the percent of items that do not conform) is minimal for higher process sigma values.
For example, (see in table 1), in a 3-sigma (3σ) process over 99.73% of the items produced will fall within specifications while 0. 27% will fall out of specifications. In terms of PPM/DPMO (Parts Per million or Defects Per Million Opportunities) this means that 2,700 nonconforming parts out one million parts produced by the process will fall out of specs. On the other hand, a 5-sigma (5σ) process will contain only have one (1) nonconforming part out of **one million** parts produced by the process.

A	B	C	D	E
C_{pk}	**The sigma σ level or the Z-Score(Φ) corresponding to the process spread (i.e., area under the curve)**	**Probability associated with the area under the curve**	**Process Yield**	**Process fallout (PPM/DPMO) =(1-C) *10⁶ =100(1-D) *10⁶**
0.33	1σ	0.6826894921	68.27%	317311
0.67	2σ	0.9544997361	95.45%	45500
1.00	3σ	0.9973002039	99.73%	2700
1.33	4σ	0.9999366575	99.99%	63
1.67	5σ	0.9999994267	99.9999%	1
2.00	6σ	0.9999999980	99.9999998%	0.002

Table 1: Process capability index and corresponding process yield and process fallout

Please note that for completeness we will define the following
USL=Upper Specification Limit (or Upper Spec Limit
LSL=Lower Specification Limit (or Lower Spec Limit)
SL =Specification limit (Spec Limit)

$$C_{pk} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right\} \rightarrow C_{pk} = \left(\frac{|SL-\mu|}{3\sigma}\right) = \frac{\Delta}{3\sigma}$$

$$If\ \ C_{pk} = 1 \rightarrow \frac{\Delta}{3\sigma} = 1 \rightarrow \frac{\Delta}{\sigma} = 3, \rightarrow \left(Z_{score} = \frac{\Delta}{\sigma} = 3\right)$$

The probability of the tail area for a Z-score value of 3, i.e., {1-Φ (3)} =0.00135 for the USL or LSL
For both the USL and LSL, we have total tail area=2[1-Φ (3)] =2(0.00135) =0.0027
∴ The area within spec limits =1-total tail area outside the spec limits= (1-0.0027) =0.9973
Note that there are two different but related measures of process capability, namely C_p and C_{pk} are defined as follows:

$$C_p = \left(\frac{USL-LSL}{6\sigma}\right)\ \ while\ \ C_{pk} = \min\left\{\left(\frac{USL-\mu}{3\sigma}\right), \left(\frac{\mu-LSL}{3\sigma}\right)\right\},\ \ \sigma_{Cp} = \frac{\bar{R}}{d_2}$$

C_{pk} measures how close the process is to the specified target and how consistent the process is towards achieving average performance. A process may be performing with minimum variation but can still be away from the specification limits, which will show up lower C_{pk}, whereas C_p will be high. On the other hand, the process may on the average be exactly at the target, but the variability is high (but still lower than the tolerance band or limits). In such case also C_{pk} will be lower, but C_p will be high. C_{pk} will be higher only when the process is meeting the target consistently with minimum variation. Based on these conditions, it is obvious that between the two measures of capability, C_{pk} provides a more consistent measure of the process capability. There is however one condition under which C_p is equal to the C_{pk} and that is when the tolerance specification is symmetric. In that scenario, (USL-μ) = (μ- LSL). So, while C_p gives an idea of process capability, C_{pk} (Process Capability Index) is the adjustment of C_p to account for the effect of non-symmetric tolerances.

1.5 Definition of Process Performance (P_p, P_{pk})

Pp= Process Performance. A simple and straightforward indicator of the process performance.
Ppk= Process Performance Index. Adjustment of Pp due to the effect of non-symmetric distribution, that is, when the process is not centered appropriately.

$$P_p = \left(\frac{USL-LSL}{6\sigma_x}\right)\ \ while\ \ P_{pk} = \min\left\{\left(\frac{USL-\mu}{3\sigma_x}\right), \left(\frac{\mu-LSL}{3\sigma_x}\right)\right\},\ \ \sigma_x = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(X-\bar{\bar{X}}\right)^2}{(nm-1)}}$$

C_{pk} gives an indication of what the process is capable of doing in the future, assuming it remains in statistical control. P_{pk} on the other hand gives an indication of how the process has performed in the past. It cannot be used to predict the future, like with C_{pk}, because the process is not in a state of control. The values for C_{pk} and P_{pk} will converge to almost the same value when the process is in statistical control. This is because σ_{Cp} (=R-bar/d_2) and σ_x (sample standard deviation) are identical (can be verified with an F-test) when the process is stable. However, when the process is not stable or is out of control, then the values will be different. Both C_p and C_{pk} are useful computing

capability index when there is subgrouping, that is (different shifts, machines, operators, etc.), while in the case of P_p and P_{pk} data is lumped and considered as one stream. It is more realistic to use P_p and P_{pk} than C_p or C_{pk} as the process variation cannot be tampered with by inappropriate subgrouping. However, C_p and C_{pk} can be very useful in order to know if, under the best conditions, the process is capable of fitting within the designed specifications. Table 2 shows diametral pitch measurement (in inches) for certain gear spur made over 25 shifts or subgroups. Each subgroup consists of 20 spur gears but only four were randomly selected for measurement. Data was coded for ease of computation.

Subgroup	**1**	**2**	**3**	**4**	X-bar	Range
1	6	8	9	-5	4.5	14
2	8	-8	-8	6	-0.5	16
3	-10	8	13	1	3	23
4	5	10	1	-8	2	18
5	-9	-4	-4	-12	-7.25	8
6	-4	0	-5	-12	-5.25	12
7	-10	12	4	10	4	22
8	-1	-5	-3	3	-1.5	8
9	-10	6	-8	-1	-3.25	16
10	10	14	3	7	8.5	11
11	-8	-1	-8	7	-2.5	15
12	9	-6	-3	7	1.75	15
13	5	7	-3	0	2.25	10
14	-2	-9	-3	9	-1.25	18
15	10	-9	12	9	5.5	21
16	10	5	7	-8	3.5	18
17	7	-1	-1	-4	0.25	11
18	-6	5	1	10	2.5	16
19	3	6	3	6	4.5	3
20	-9	14	-5	10	2.5	23
21	1	-8	11	0	1	19
22	10	-2	7	8	5.75	12
23	-10	12	7	-3	1.5	22
24	12	2	3	-11	1.5	23
25	7	-4	12	-5	2.5	17
				AVG	1.42	15.64

Table 2: Diametric pitch measurements for Gear Spurs

$$for\ C_p\ and\ C_{pk}: \sigma_{c_p} = \frac{\bar{R}}{d_2}\left(\ for\ n=4,\ d_2=2.059\right) = \frac{15.64}{2.059} \Rightarrow \sigma_{c_p} = 7.5960,\ \bar{\bar{X}} = 1.42$$

$$for\ P_p\ and\ P_{pk}: \sigma_x = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(X-\bar{\bar{X}}\right)^2}{(nm-1)}} = \sqrt{\frac{\sum_{i=1}^{4}\sum_{j=1}^{25}\left(X-\bar{\bar{X}}\right)^2}{((4*25)-1)}} = \sqrt{\frac{5350.36}{99}} = 7.351$$

From the values of the two measures of variability σ_{cp} and σ_x) the process capability (C_{pk}) and process performance (P_{pk}) identical which an indication that the process is stable. As indicated earlier that if σ_{Cp} (=R-bar/d_2) and σ_x (sample standard deviation) are identical then the process is likely stable.

2. Process Error and Quality Loss

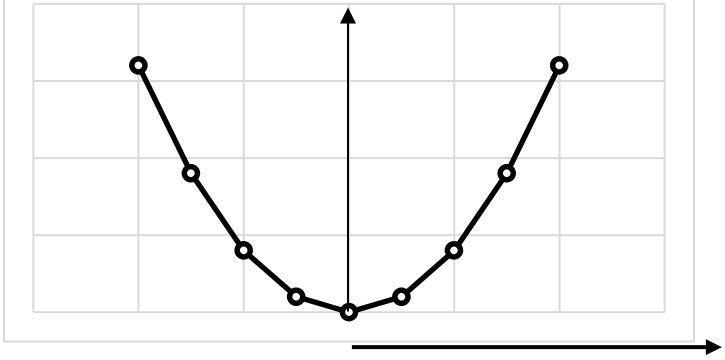
To quantify quality losses, we develop a relationship between a product's functional quality characteristic *x*, and the nominal/target value (or aimed at value) denoted as m which results in what we denote as the loss function, namely, *H(x)*. To properly construction a loss function it is important to understand the nature of such function. That understanding helps in the development of a functional relationship that is both logical and realistic. We begin by examining manufacturing errors and the resultant cost or loss.

In mathematical optimization, statistics, econometrics, decision theory, machine learning and computational neuroscience, a **loss function** or **cost function** is a function that maps an event or values of one or more variables onto a real number intuitively representing some "cost" associated with such event. A well-known and convenient **loss function** used in applications like quality loss due to manufacturing errors (including external & internal noise factors, and lack of design robustness), linear regression, mathematical optimization, etc. is the **squared** loss function or the mean squared loss function, sometimes also called the mean squared deviation. The function calculates the square of the difference between the actual value and desired design specification/target value. The resulting function penalizes the residual (i.e. the difference between the nominal and the output) in a quadratic way. For example, if the difference between the nominal and the output has a residual value of 2 units, then the loss will be 4 units. In a manufacturing system, the error, which is what causes the loss, is calculated as the difference between the nominal and the actual output. Similarly, in machine learning networks, mathematical optimization, regression analyses, and classification problems, error is calculated as the difference between the actual output and the predicted output. The function that is used to assess this error is known as Loss Function H(x), where x is the variable under study. Given an input and a target, the loss is calculated as the difference between the output variable and the target (the nominal) variable. Different loss functions will give different errors for the same analyses of actual output versus the desired or predicted outcome, and thus have a considerable effect on the performance of the model used. Any typical loss function is convex in nature which guarantees that its optimum point is the minimum point and in its simplest form can be graphically can be represented by a parabola (See figure 1).

The generalized squared loss function is a real, symmetric convex function whose optimal can be determined through mathematical optimization. Squared loss functions are computationally

feasible functions and represent the price paid for inaccuracy of predictions or in the case of manufacturing the inability to meet the target. This makes them especially useful in evaluating the cost of quality of manufactured products

Figure 1: A simple Parabola



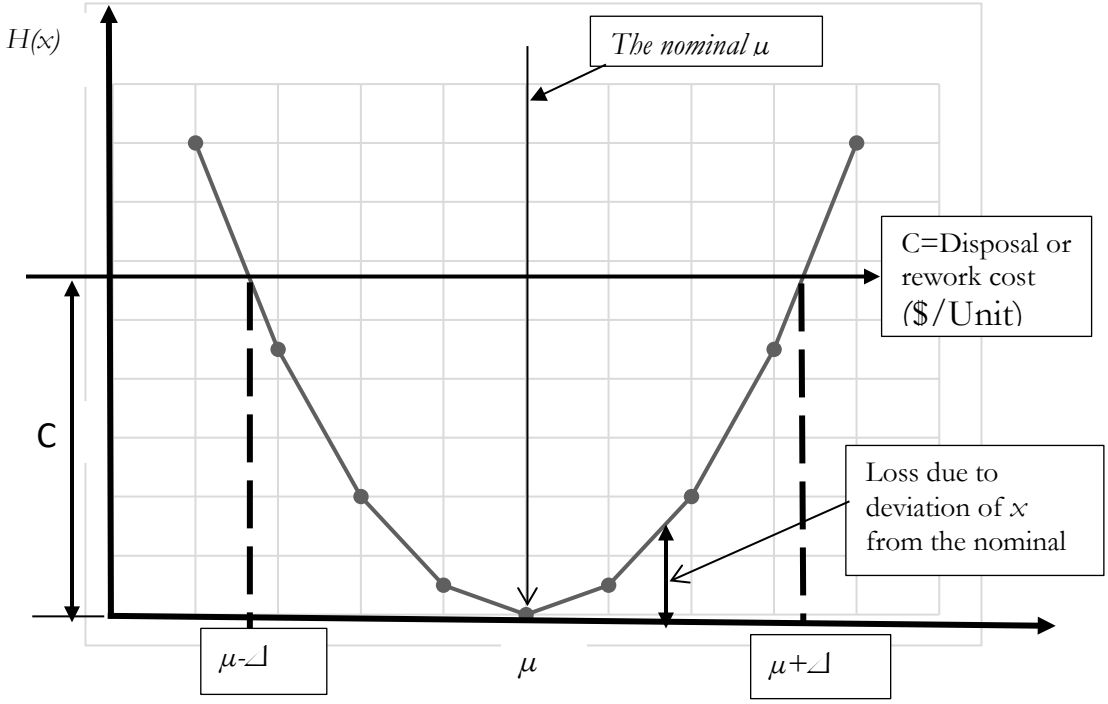


Figure 2: Loss Function H(x)

Our aim is to find a convex polynomial that provides a good approximation to the proposed loss function *H(x)* that we desire. We can determine the polynomial approximation of said loss

function by using the **Taylor Series or the Maclaurin expansion.** With the Taylor series expansion, if we want a good approximation to the function H(x) in the region near the nominal *m*, that is *x=m*, then we need to find the first, second, third (and so on) derivatives of the function and substitute the value of *m*. We then multiply those values (the derivatives) by the corresponding powers of (*x-m*) to give us the **Taylor Series expansion** of the function H(*x*) centered at the nominal μ. When the nominal μ=0, we have the Maclaurin series expansion.

Let H(*x*) = the loss function for the characteristic *x*. Expanding *H*(*x*) about the mean or nominal *m* using the Taylor series expansion yields the following:

$$H(x) = H(\mu + x - \mu) = H(\mu) + \frac{H'(\mu)}{1!}(x-\mu) + \frac{H''(\mu)}{2!}(x-\mu)^2 ..... + \frac{H'^{p}}{p!}(x-\mu)^p$$

Note the following points about this function:

1. When x = μ, i.e., no deviation, $\rightarrow$ H(μ) = H(x) = 0 minimum of the function
2. $H'(\mu)=0 \Rightarrow 1^{st}$ *derivative is zero* $(at\ Min\ or\ Max)$

Note: If we ignore the higher order functions, then we are only left with the following:

$$H(x) = \frac{H''(\mu)}{2!}(x-\mu)^2$$

$$\text{Since: } \frac{H''(\mu)}{2!} \text{ is a constant, Let } = \frac{H''(\mu)}{2!} = D$$

$$Then: H(x) = D(x-\mu)^2$$

$$E[H(x)] = D\left\{E\left[(x-\mu)^2\right]\right\}$$

$$\text{Recall that by definition } \sigma_x^2 = E\left[(x-\mu)^2\right], \text{ where: } E(x) = \mu_x$$

$$Hence: E[H(x)] = D\left\{E(x-\mu)^2\right\} = D\sigma_x^2 (\$/unit)$$

If for example we consider the cost of rework or disposal C as a loss, then

H(x) = C=D(x-μ)^2

but *(x-μ)* = deviation from nominal = Δ, note that tolerance spread = *2Δ*

Hence *C = DΔ^2*

Rewriting, we have *D = C/Δ^2*

Recall that: $C_p = \frac{2\Delta}{6\sigma}$, *why* (*see figure 1, two-sided tolerance limit*)

If the process is a 3-sigma process and is normally distributed, then C_{pk}=1 (see table 1)

$$If\ C_p = 1,\ then\ \frac{2\Delta}{6\sigma} = 1 \Rightarrow \sigma = \frac{2\Delta}{6}$$

$$Hence: \sigma = \frac{2\Delta}{6} \Rightarrow H(x) = H\sigma_y^2$$

Example: The cost regrinding the spur gear $25/unit. The spec limits are ±10 with a Δ=10. Find Loss in $/unit. Assume a 3-sigma process.

$$For\ a\ 3-sigma\ process\ \sigma = \frac{2\Delta}{6}$$

$$C = D\Delta^2 \Rightarrow D = \frac{C}{\Delta^2} = \frac{10}{(10)^2} = \frac{10}{100} \Rightarrow D = 0.10$$

$$H(x) = D\sigma^2 \Rightarrow H(x) = 0.10\left(\frac{20}{6}\right)^2 = 0.10\left(\frac{400}{36}\right) = \$1.111/unit$$

2. Suppose we have another plant operating under a 5-sigma process. Assuming the cost of regrinding and the tolerance limits are the same, find the quality loss

$$For\ a\ 5-sigma(5\sigma)\ process, C_p = 1.67,\ \sigma = \frac{2\Delta}{1.67(6)} = \frac{2\Delta}{10.02}$$

$$C = D\Delta^2 \Rightarrow D = \frac{C}{\Delta^2} = \frac{10}{(10)^2} \Rightarrow D = \frac{10}{100} = 0.10$$

$$H(x) = D\sigma^2 \Rightarrow H(x) = 0.10\left(\frac{20}{10.02}\right)^2 = 0.10(3.984) = \$0.3984/unit$$

Assuredly the loss is less because the var*iability is less*

3. Finally suppose yet another plant operates a 2-sigma process and with the same parameters as the other two plants

$$For\ a\ 2-sigma\ process(2\sigma), C_p = 0.67,\ \sigma = \frac{2\Delta}{0.67(6)} = \frac{2\Delta}{4.02}$$

$$C = D\Delta^2 \Rightarrow D = \frac{C}{\Delta^2} = \frac{10}{(10)^2} \Rightarrow D = \frac{10}{100} = 0.10,\ 2\Delta = 2(10)$$

$$H(x) = D\sigma^2 \Rightarrow H(x) = 0.10\left(\frac{20}{4.02}\right)^2 = 0.10(24.7519) = \$2.47519/unit$$

The loss is the highest of the three because the var*iability is highest*

3. Acceptance Control

Manufactured goods are produced and/or shipped in lots ranging from a few to a thousand items. Ideally each lot should have zero nonconformance items but realistically, it is almost impossible to meet such a goal. Recognizing that some nonconforming items could result even if the lot was inspected 100%, the consumer requires evidence based on careful analyses that the proportion of nonconforming items in each

lot is not excessive. In statistical sampling inspection methods, some items from the lot are selected for inspection. If that sample of items conforms to be desired quality requirements then the whole lot is accepted, if it does not, the whole lot is rejected. This method of acceptance or rejection of a sample is called Acceptance Sampling. In general acceptance sampling methods are economical and are used under the assumption that the quality characteristics of the item under consideration are under control and are relatively homogeneous. We have acceptance sampling for attributes (go-no-go) and acceptance sampling for variable (measured characteristics. Our focus here will be limited to acceptance sampling for attributes only.

3.1 Lot Acceptance Sampling Plans (*LASP*)

Lot Acceptance Sampling Plans (*LASP*) is a frequently used method to accomplish this goal. *LASP* is a sampling scheme with associated set of rules for making decisions regarding whether to accept or not accept a lot (reject or not to reject is more appropriate). It is important to realize that this decision about the lot is based on information from the sample only. Hence such decision has some risk because it is only based on a sample rather than the entire lot. As a result, in modern Lean Sigma terminology, we talk about reject or not reject (rather than accept or not accept) since accept assumes that we have all the information which we do not.

The decision, based on the count of the number of nonconformance in a sample, can be to accept the lot, reject the lot, and for multiple or sequential sampling schemes, take another sample and then repeat the decision process. The following are some of the different categories and elements *LASP*

3.1.1 Single sampling plans (*SSP*)

This is simply a specification of the sample size and the acceptance number *"c"* to be used. The choice of **"c"** is dependent on designed *AQL* and *LTPD* in association with *α, β*, and sample size *n*. One sample of items is selected at random from a lot and the disposition of the lot is determined from the resulting information. These plans are usually denoted as (*n, c*) plans for a sample size, where the lot is rejected if there are more than *c* nonconforming items. These are the most common (and easiest) plans to implement although not the most efficient in terms of average number of samples needed.

3.1.2 Double sampling plans (*DSP*)

After the first sample has been examined or screened, there are **three** possible outcomes, namely, **i) Accept the lot, ii) Reject the lot, iii) No decision**. If the outcome after the first sample has been inspected or screened is no decision (i.e., outcome # iii), then a second sample is examined and the results from both samples combined in arriving at the disposition decision based on that information. Let's look a simple numerical. Suppose a double sampling plan is specified as follows: n_1=20, c_1=0, r_1=2; n_2=20, c_2=1, r_2=2, where n is the sample size, c is the acceptance number and r the rejection number. This works as follows. Take a sample of size 20, if zero nonconforming items is found, accept and if 2 or more is found reject. If one nonconforming item is found in the first sample,

then take a second sample. If in the combined sample size of (n1+n2=20+20=40) 1 nonconforming item is found, then accept. If 2 or more found, reject. We will exam a numerical example later.

3.1.3 Multiple sampling plans (*MSP*)

This is an extension of the double sampling plans where more than two samples are needed to reach a conclusion. The advantage of multiple sampling is **smaller sample sizes**.

3.1.4 Sequential Sampling Plans (*SSP*)

This is the ultimate extension of multiple sampling where items are selected from a lot one at a time and after each is inspected a decision is made to accept/reject the lot or select another unit.

3.1.5 Skip Lot Sampling Plans (*SLSP*)

Skip lot sampling means that only a fraction of the submitted lots is inspected.

3.2 Development of Specific Plans

To develop a specific plan within one of the categories listed above depends on the properties desired in a plan. These are described as follows:

- Acceptable Quality Level (*AQL*): The *AQL* is a percent nonconformance that is the base line requirement for the quality of the producer's product. The producer would like to design a sampling plan such that there is a high probability of accepting a lot that has a nonconformance level less than or equal to the *AQL*.

- Lot Tolerance Percent Defective (*LTPD*): The *LTPD* is a designated high nonconformance level that would be unacceptable to the consumer. The consumer would like the sampling plan to have a low probability of accepting a lot with a nonconformance level as high as the *LTPD*.

- *Type I Error* (Producer's Risk): This is the probability, for a given (*n, c*) sampling plan, of rejecting a lot that has a nonconformance level equal to the *AQL*. The producer suffers when this occurs, because a lot with acceptable quality was rejected. The symbol α is commonly used for the Type I error and typical values for range from 0.2 to 0.01.

- *Type II Error* (Consumer's Risk): This is the probability, for a given (*n, c*) sampling plan, of accepting a lot with a nonconformance level equal to the *LTPD*. The consumer suffers when this occurs, because a lot with unacceptable quality was accepted or not rejected. The symbol β is commonly used for the Type II error and typical values range from 0.2 to 0.01.

- Operating Characteristic (*OC*) Curve: A sampling plan may also be described by an *OC* curve. This is a description or demonstration of the degree of protection offered by a sampling plan against incoming lots of various qualities. Given *N, n, c, p*, we can compute P_a. This curve plots the probability of accepting the lot (Y-axis) versus the fraction nonconforming level (X-axis). The *OC* curve is the primary tool for displaying and investigating the properties of a *LASP*.

- Average Outgoing Quality (*AOQ*): A common procedure, when sampling and/or testing is non-destructive, is to inspect rejected lots 100 % and replace all nonconforming units with good units. In this case, all rejected lots are made perfect and the only nonconforming items left are those in lots that were accepted or not rejected). *AOQs* refer to the long-term nonconformance level for this combined *LASP* and 100 % inspection of rejected lots process. If all lots come in with a nonconformance level of exactly *p*, and the *OC* curve for the chosen (*n, c*) *LASP* indicates a probability p_a of accepting such a lot, then over the long run the AOQ can easily be shown to be:

  $AOQ = \frac{p_a p(N-n)}{N}$, where *N* is the lot size. If N is assumed to be much greater than n, that **is N>>>>n, then AOQ=P_ap**

- Average Outgoing Quality Level (*AOQL*): A plot of the *AOQ* (Y-axis) versus the incoming lot (X-axis) will start at 0 for *p*=0 and return to 0 for *p*=1 (where every lot is 100 % inspected and rectified). In between, it will rise to a maximum. This maximum, which is the worst possible long term *AOQ*, is called the *AOQL*.

- Average Total Inspection (*ATI*): When rejected lots are 100 % inspected, it is easy to calculate the *ATI* if lots come consistently with a defect level of *p*. For a *LASP* (*n, c*) with a probability p_0 of accepting a lot with defect level p, we have

  $ATI = n + (1 - p_0)(N - n)$, where N is the lot size.

- Average Sample Number (*ASN*): The average sample number (ASN) is the expected number of observations or the sample size required to make a lot disposition decision and represents the number of samples inspected per lot of a given quality p. ASN is usually in terms of more than one sample drawn from a lot with quality p. A plot of the *ASN*, versus the incoming defect level, describes the sampling efficiency of a given LASP scheme.

The final choice in a sampling plan is a tradeoff decision. The final choice between single or multiple sampling plans that have acceptable properties is a matter of whether the average sampling savings gained by the various multiple sampling plans justifies the additional complexity of these plans and the uncertainty of not knowing how much sampling and inspection will be done on a day-by-day basis.

3.3 *LASP* Basic Procedure for Single Plans

Items (size n) are selected before shipment of lot size N or an incoming lot of size N and a decision made as to accept or not accept the lot (reject or not reject is a better way of expressing this situation. Why?). This procedure is equivalent to a test of hypothesis: Where: H_0: $p = p_0$, H_1: $p < p_0$, *where* ($p_1 > p_0$).

3.3.1 Problems Associated with Sampling

Two types of errors are associated with sampling, namely, *Type I error or a* and *Type II error or* β. The associated lot quality for the *Type II error* α is p_0 which represents the target or aimed at value of the producer also known as Acceptable Quality Level (*AQL*). Similarly, p_1 denotes the Lot Tolerance Percent Defective (*LTPD*) and represents the percent nonconforming beyond which the consumer would be unwilling to accept. *Type I error* α is the upper limit to the proportion of good lots ($p \leq p_0$) that would be rejected and represents the Producer's Risk (*P.R*). *Type II error* β is the upper limit to the proportion of bad lots (Lots with $p > p_1$) that would be accepted and represents the Consumer Risk (*C.R.*).

True State of Nature		
Decision	$p < p_0$	$p > p_0$
Accept	No error	*Type II error*(β)
Reject	*Type I error (a)*	No error

Table 3. *Type I and Type II errors*

Please note that 100% human inspection is not effective as a sampling strategy because of its fatigue effects on the inspection personnel. Additionally, placing inspectors in series to enhance efficacy is not effective due to the human psychology of social loafing that occurs in a group. For example, when more than one person is assigned a tasked, the others in the team would assume that the work has been done by the worker downstream and so they pay little to no attention to the work due to their assumption that their colleague must have done the work. Analogy to this phenomenon is the final reliability of system of components in series. The system reliability is smaller than that the worst component in the configuration.

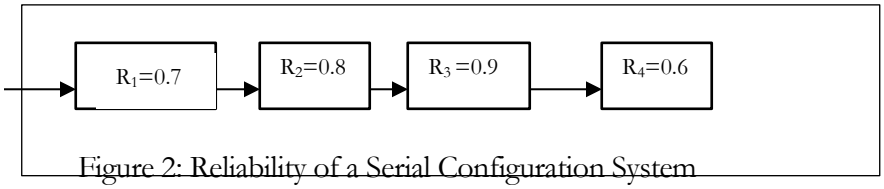


Figure 2: Reliability of a Serial Configuration System

$$R_s = \prod_{i=1}^{4} R_i = R_1 x R_2 x R_3 x R_4 = (0.7)(0.8)(0.9)(0.6) = 0.3024$$

The example shows that the reliability of this series system is less than the reliability of the worst component in the system. This is further demonstration that placing several inspectors in series with goal of improving output quality is an illusion and thus is not a judicious avenue to utilize resources.

3.3.2 Operating Characteristics Curves (*OC Curves*)

A sampling plan may also be described by an *OC* curve. This is a description or demonstration of the degree of protection offered by a sampling plan against incoming lots of various qualities. Given *N (Lot size), n (sample size), c (acceptance number), p (lot quality)*, α (and the corresponding AQL), β (and the corresponding LTPD), then we can compute P_a *(the probability of acceptance) and* P_r *(the probability of rejection).*

The OC Curve gives the probability of acceptance for each value that can be assumed by the lot in terms of proportion nonconforming *p*.

Type- A OC curves are for Finite Universe or lots—hence use the Hypergeometric Distribution for computing the probability of occurrence. Type- B OC curves are for infinite Universe or lots—hence use the Binomial or Poisson Distributions for probability computation.

Usual practice is to specify α = 0.05, where α is the probability of rejection at p=*AQL*, that is α = P_r(at *AQL*) . with α = P_r(at *AQL*), then P_r (*AQL*)= 1- P_r(at *AQL*)=(1-0.05)=0.95. Also, β is typically specified as = 0.1 (10%) = P_a (at *LTPD*). When N (the lot size) is infinite, we can employ the Poisson distribution to compute the parameters of the plan. The cumulative Poisson Distribution is given as:

$$F\left[\left(\lambda = np\right), x\right] = \sum_{i=0}^{x} \frac{\left(\lambda\right)^{x} e^{-\lambda}}{x!}$$

The sample cumulative Poisson table based on the Cumulative Poisson distribution was developed using Microsoft EXCEL and is as shown in Table 4. For values of $c'=np=\lambda$, not shown on the table, one can approximate the values from the table or better yet use the Poisson formula, whichever is more appropriate or convenient.

Example:

For a given plan, let α =0.05, α= 1-P_a (at *AQL*) =0.05, then P_a (at *AQL*) = 1-0.05=0.95, Let *N* be infinite *n* =100, c=1. Find AQL or p at P_a=0.95. Also, for *LTPD* if β=10%

Given c (or x) =1, P_a *=0.95, then from the Poisson table and* $np=\lambda$ *=0.35*

$$\lambda = np = 0.35 \Rightarrow p = \frac{\lambda}{n} = \frac{0.35}{100} = 0.0035 \Rightarrow AQL = 0.0035$$

For β=10%, c (x)=1

Given β=P_a (at *LTPD*). For P_a =0.1, and c(or x) =1, $np=\lambda$ *=3.90*

$$\lambda = np = 3.9 \Rightarrow p = \frac{\lambda}{n} = \frac{3.9}{100} = 0.039 \Rightarrow LTPD = 0.0035$$

					$\lambda t = np$					
x	**0.01**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0	0.9950	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
1	1.0000	1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**0.10**	**0.20**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.80**	**0.90**	**1.00**
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**1.10**	**1.20**	**1.30**	**1.40**	**1.50**	**1.60**	**1.70**	**1.80**	**1.90**	**2.00**
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
2	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
3	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
4	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
5	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868	0.9834
6	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
7	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
x	**2.10**	**2.20**	**2.30**	**2.40**	**2.50**	**2.60**	**2.70**	**2.80**	**2.90**	**3.00**
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146	0.1991
2	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460	0.4232
3	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696	0.6472
4	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318	0.8153
5	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258	0.9161
6	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713	0.9665
7	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901	0.9881
8	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969	0.9962
9	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991	0.9989
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4: Cumulative Poisson Table Developed from Microsoft EXCEL

In diametral wheel grinding process rectified inspection is used for its single-sampling plan. Calculate the average outgoing quality limit for a plan with n=100, c=1, N=1500 using the Poisson probability approximation assuming N is large. The proportion nonconforming is in the range from 0.01 to 0.10 in steps of 0.01. Estimate the probabilities of acceptance for values of the proportion nonconforming from 0.01 to 0.10 in steps of 0.01. Use those steps to estimate the AOQL for this sampling plan

Proportion nonconforming (p)	np	Probability of Acceptance P_a (%)
0	0	100%
0.01 (AQL)	0.5	90.97959896 % (α=0.09)
0.02	1	73.57588823 %
0.03	1.5	55.78254004 %
0.04	2	40.60058497 %
0.05	2.5	28.72974952 %
0.06	3	19.91482735 %
0.07	3.5	13.58882254 %
0.08 (LTPD)	4	9.157819444 % (β≈0.1)
0.09	4.5	6.109948096 %
0.1	5	4.042768199 %

Table 5: Data for OC Curve (p=0 to 0.1 in steps of 0.01, n=50, c=1, N=1500)

Figure 4: OC CURVE

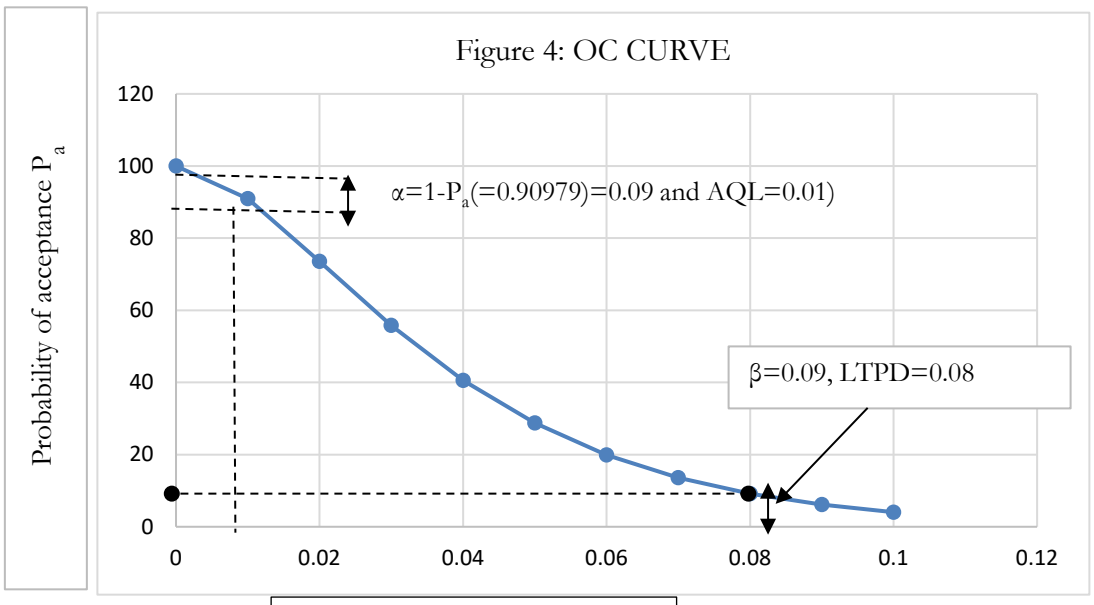


Fig 4. OC Curve: P_a versus p

3.3.4 AOQ CURVE

Average Outgoing quality (*AQL*) is the expected proportion of nonconforming items that the plan will allow to pass. We assume that all nonconforming items in the lot will be replaced with good items if the lot is rejected and that any nonconforming items in the sample will be replaced if the lot is accepted, a process usually referred to as rectified inspection or rectification. Average outgoing quality curve describes the degree of protection offered by the plan by showing the average quality of outgoing lots corresponding to each quality of incoming lots.

Computation of AOQ – (2 possibilities)

a). Lot passes inspection, or

b). Lot is rejected

For a), i.e., Lot passes inspection

If lot passed inspection based on *c* or fewer nonconforming items, the nonconforming items are replaced with good items.

For b), i.e., Lot is rejected

If the lot is rejected, it undergoes 100% inspection where all bad (nonconforming) items are replaced with good ones. Assuming the inspection scheme is perfect, then theoretically the rejected lot will end up with no nonconforming or defectives items. The actual number found *D* is denoted as follows: $c < D \le N$, where c is the acceptance number. If the inspection scheme is perfect, then theoretically the rejected lot will not contain any nonconforming items after rectification. Thus, under this condition the lot has zero nonconforming items and thus is never rejected. In the latter case of 100% nonconforming) it is always rejected to force complete rectification.

When during inspection the lot falls in-between the two extremes of (not acceptable or rejected), then the lot quality may be evaluated as follows:

Note: $E(x) = \sum xp(x)$, namely, the expected value of the random variable x with the probability mass function p(x). Let

N = Lot size

p = the actual proportion nonconforming of the lot N

P_a = Probability of accepting the lot

After the sample of size n has been taken out of the lot, there is now left *(N-n)* items in the original lot of size N. Therefore, the number of nonconforming items out of a total of (N-n) is given by: *(N-n) p*

Hence, the expected number of nonconforming items = P_a *(N-n) p*.

$$AQL = \frac{P_a(N-n)p}{N}, if\ \ n <<< N \Rightarrow \frac{(N-n)}{n} \approx 1 \Rightarrow AQL = P_a p$$

AOQL OR THE Average Outgoing Quality Limit is the limiting value of AOQ and represents the worst AOQ possible from rectifying inspection, regardless of incoming quality. AOQ is a measure of the quality level resulting from an acceptance plan when rejected lots are subjected to 100% screening and the nonconforming items found in the acceptance samples are replaced. It is a function of *n, c, p*.

AOQL=Max {AOQ_i}, that is the maximum value of the AOQ's

OC CURVES for AOQ Curve and AOQL

Use the Poisson approximation if $n/N \leq 1/10$ also if $p \leq 0.1$ or $np \leq 5$. Suppose $n = 100$, $c = 1$, $0 \leq p \leq 0.05$. Construct an OC curve. If $\alpha = 0.05$, what is AQL. If $\beta = 10\%$, what is LTPD

Given: $n = 100$, $c = 1$. For $\alpha = 0.05$, $P_a = 0.95$

From the Poisson table (table 4) with c=1, and Pa=0.95, **λ=np** is interpolated to be 0.35

np=0.350 ==>p=np/n =0.35/100 = 0.0035. Hence AQL = 0.0035

for β= 10%, $\beta = P_a$ (at LTPD), for $P_a = 0.1$, $c = 1$.

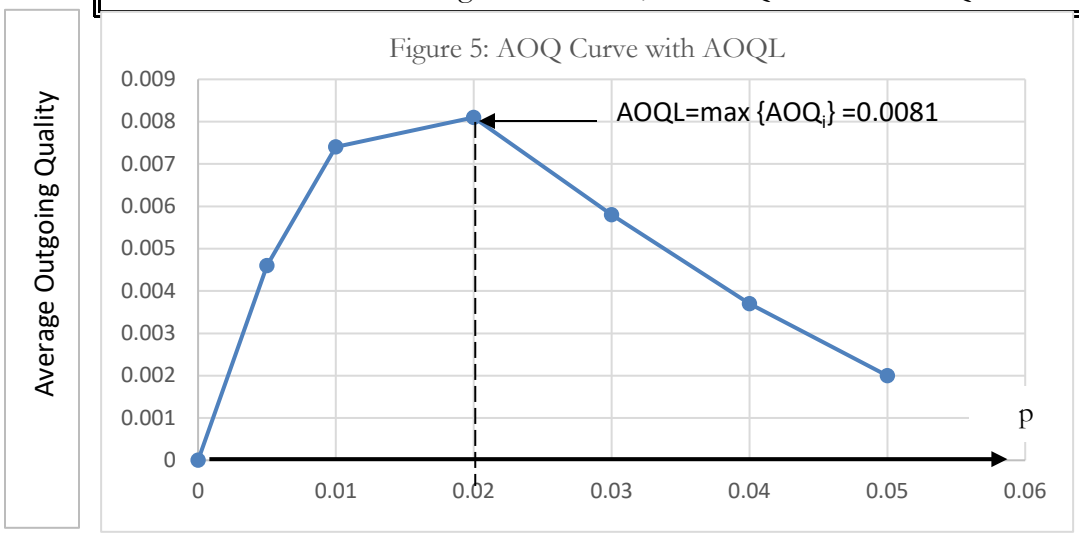
From Table 4 or by direct computation using the Poisson Distribution, $np \approx 3.9$,

hence p=np/n=3.9/100 = 0.039 ==> LTPD = 0.039

p	λ=np	P_a	AOQ=P_ap
0	0	1.0	0
0.005	0.5	0.910	0.0046
0.01	1	0.736	0.0074
0.02	2	0.406	0.0081 <--AOQL
0.03	3	0.199	0.0058
0.04	4	0.092	0.0037
0.05	5	0.040	0.0020

Table 6: Data for constructing the OC curve, the AOQ curve and AOQL

Figure 5: AOQ Curve with AOQL



Determination of a Sampling Plan

A proposed sampling plan calls for the following: If material is 1% (AQL), want to accept process 95% of the time. If material is only 7% nonconforming, want to accept only 10% of the time. Is such a plan possible if so, what are the parameters? If so, find the sample size ***n*** and the acceptance number ***c*** that would satisfy the plan.

Find: n, c

Solution: Use trial & error and the Poisson table

Let c = 1

If c=1, np= 0.35 (Given P_a=0.95, and c=1)

λ= np = 0.35 ==> n =0.35/0.01 ==> n = 35

Check whether this plan satisfies other requirements

np=λ= (35) (0.07) = 2.45

with np = 2.45 and c = 1, P_a = 0.308 which is greater 10% <--- specified

This plan of n=35, AQL=1% does not meet the requirements because the β=0.308 >10% (Specified)

Let c = 2

If c=2, np = 0.82 (Given P_a = 0.95, c= 2)

Hence 0.82= n(0.01) ==> n = 82

Check whether this plan satisfies other requirements

np=(82)(0.07)=5.86

with np=5.86, c=2, P_a=0.07

0.07<0.1. Plan is Okay.

Hence: Desired plan is given by: n=82, c=2 ***n =82, c=2***

Double Sampling Plan

Example 1: A double sampling plan with AQL =1% is given as follows

$n_1 = 20, c_1 = 0,\ r_1 = 2$

$n_2 = 20, c_2 = 1,\ r_2 = 2$

$n_1 p = n_2 p = 0.2$

Figure 6: Acceptance Scheme for Double Sampling

1st Sample	2nd Sample
(0)	
	(0)
1	

Note The item in parenthesis indicate the decision point for acceptance. As an example, for the 1st sample, acceptance occurs when the number of nonconforming items is zero. If one nonconforming item is found in the 1st sample, then the 2nd sample is taken and if zero nonconforming items is found, then acceptance occurs. It is for this reason that the schematic is called the acceptance scheme. It only displays how acceptance takes place. We will compute the probability of the occurrences using the Poisson distribution utility in Microsoft EXCEL. In using the Poisson distribution in EXCEL, the default is the cumulative distribution such as $P(c \le x_0)$. In this case the entry for the default is the word "true". If the probability desired is the exact probability such as $P(c = x_0)$, then the entry is "false". We will see how this is done in the following examples

$$P(c=0|np=0.2)=0.8187 \Rightarrow \{[POISSON.DIST[0,0.2,\ False]\}$$

$$P(c=1|np=0.2)=0.1637 \Rightarrow \{[POISSON.DIST[1,0.2,\ False]\}$$

$$P_a = P_a(n_1) + P_a(n_2)$$

$$Note: P_c^{(n)} = prob\ of\ \ c\ items\ in\ the\ n^{th}\ sample$$

$$P_a(n_1) = P_0^{(1)} = \Pr ob\ of\ \ c=0\ in\ 1^{st}\ \ sample = 0.8187$$

$$P_a(n_2) = P_1^{(1)} P_0^{(2)} = (0.8187)(0.1637) = 0.1341$$

$$\therefore P_a = P_a(n_1) + P_a(n_2) = 0.8187 + 0.1341 = 0.9528$$

Example 1: A double sampling plan with AQL =1% is given as follows

$$n_1 = 20, c_1 = 0,\ r_1 = 3$$

$$n_2 = 20, c_2 = 2,\ r_2 = 3$$

$$n_1 p = n_2 p = 0.2$$

Figure 7: Acceptance Scheme for Double Sampling

1st Sample	2nd Sample
(0)	
	(0,1)
1	
	(0)
2	

$$P(c=0|np=0.2)=0.8187 \Rightarrow \{[POISSON.DIST[0,0.2,\ False]\}$$

$$P(c=1|np=0.2)=0.1637 \Rightarrow \{[POISSON.DIST[1,0.2,\ False]\}$$

$$P(c\le 1|np=0.2)=0.9825 \Rightarrow \{[POISSON.DIST[1,0.2,\ True]\}$$

$$P(c=2|np=0.2)=0.01637 \Rightarrow \{[POISSON.DIST[2,0.2,\ False]\}$$

$$P_a = P_a(n_1)+P_a(n_2)$$

$$P_a(n_1)=P_0^{(1)}=0.8187$$

$$P_a(n_2)=P_1^{(1)}P_{0,1}^{(2)}+P_2^{(1)}P_0^{(2)}=0.1637(0.9825)+0.01637(0.8187)$$

$$P_a(n_2)=0.1608+0.0134=0.17428$$

$$\therefore P_a = P_a(n_1)+P_a(n_2)=0.8187+0.17429=0.9930$$

Multiple Sampling Plan
A multiple plan is given as follows:

Table 7. Multiple Sampling Plan

Sample #	Sample size (n_i)	Cumulative Sample Size	Acceptance No.	Rejection No.
1	50	50	*	3
2	50	400	1	3
3	50	450	1	3
4	50	200	3	5
5	50	250	4	5

Assume that the lot size N very large compared to the sample size n, (N>>>>n) and the acceptable quality p=3%. Compute the probability of acceptance for this plan.
Note $n_1p=n_2p=n_3p=n_4p=n_5p=(0.03)50=1.5$

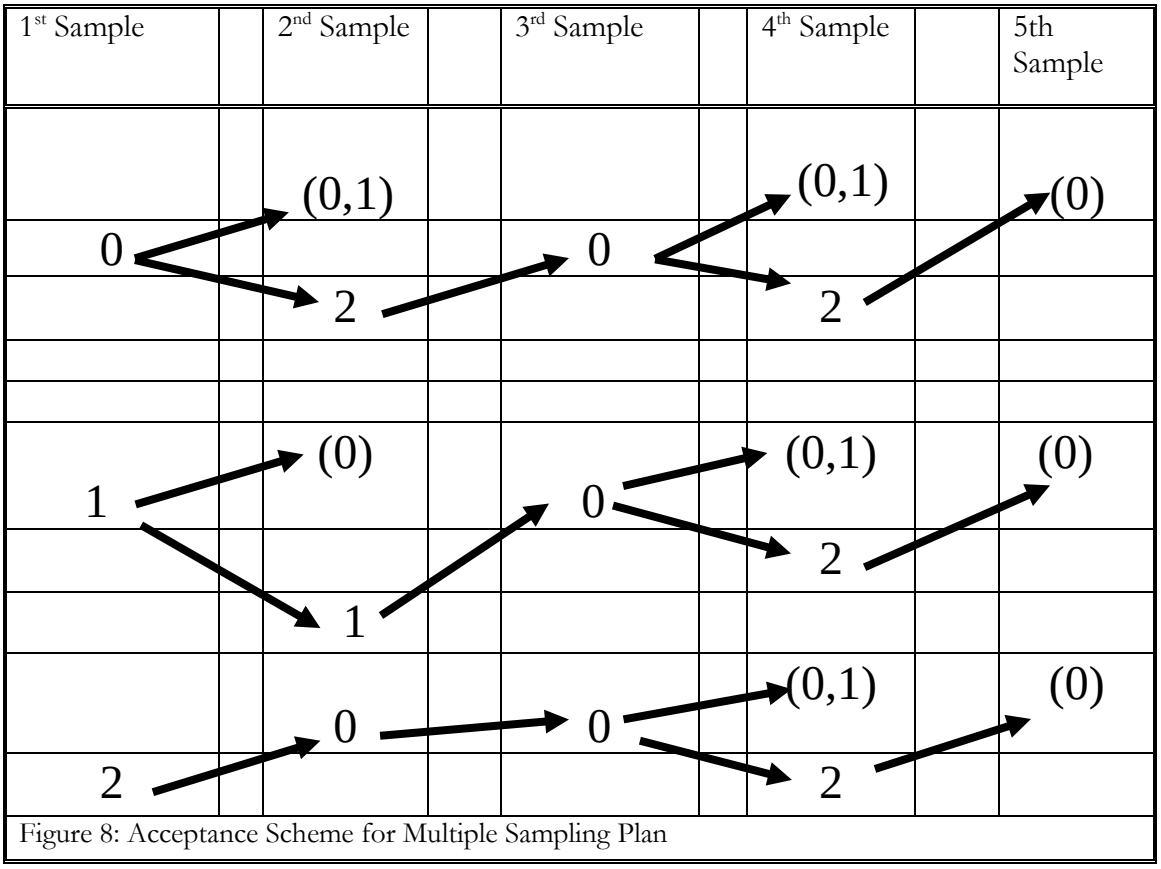


Figure 8: Acceptance Scheme for Multiple Sampling Plan

$$P\left(c=0\middle|np=1.5\right)=0.223\Rightarrow\left\{\left[POISSON.DIST[0,1.5,\ False\right]\right\}$$

$$P\left(c=1\middle|np=1.5\right)=0.335\Rightarrow\left\{\left[POISSON.DIST[1,1.5,\ False\right]\right\}$$

$$P\left(c\le1\middle|np=1.5\right)=0.558\Rightarrow\left\{\left[POISSON.DIST[1,1.5,\ True\right]\right\}$$

$$P\left(c=2\middle|np=1.5\right)=0.251\Rightarrow\left\{\left[POISSON.DIST[2,1.5,\ False\right]\right\}$$

$$P\left(c\le2\middle|np=1.5\right)=0.809\Rightarrow\left\{\left[POISSON.DIST[2,1.5,\ True\right]\right\}$$

$$P_a=P_a\left(n_1\right)+P_a\left(n_2\right)+P_a\left(n_3\right)+P_a\left(n_4\right)+P_a\left(n_5\right)$$

$$P_a = P_a(n_1) + P_a(n_2) + P_a(n_3) + P_a(n_4) + P_a(n_5)$$

$$P_a(n_1) = 0 \,(accep\tan ce\ not\ allowed\ on\ \ first\ \ sample)$$

$$P_a(n_2) = P_0^{(1)}P_{0,1}^{(2)} + P_1^{(1)}P_0^{(2)} = 0.223(0.558) + 0.335(0.223) = 0.1991$$

$$P_a(n_3) = 0\left[\,No\ accep\tan ce\ \ possible\ on\ 3^{rd}\ sample\right]$$

$$P_a(n_4) = P_0^{(1)}P_2^{(2)}P_0^{(3)}P_{0,1}^{(4)} + P_1^{(1)}P_1^{(2)}P_0^{(3)}P_{0,1}^{(4)} + P_2^{(1)}P_0^{(2)}P_0^{(3)}P_{0,1}^{(4)}$$

$$P_a(n_5) = P_0^{(1)}P_2^{(2)}P_0^{(3)}P_2^{(4)}P_0^{(5)} + P_1^{(1)}P_1^{(2)}P_0^{(3)}P_2^{(4)}P_0^{(5)} + P_2^{(1)}P_0^{(2)}P_0^{(3)}P_2^{(4)}P_0^{(5)}$$

$$P_a(n_2) = (0.223)(0.558) + (0.335)(0.223) = 0.1991$$

$$P_a(n_3) = 0$$

$$P_a(n_4) = (0.223)(0.251)(0.223)(0.558) + (0.335)(0.335)(0.223)(0.558)$$

$$+ (0.251)(0.223)^2(0.558) = 0.0278$$

$$P_a(n_5) = (0.223)(0.251)(0.223)(0.251)(0.223) + (0.335)^2(0.223)^2(0.251) + (0.251)^2(0.223)^3$$

$$= 0.000699 + 0.0014 + 0.00699 = 0.002798$$

$$P_a = 0.19914 + 0.002789 + 0.002798 = 0.2298$$

Additional Questions on Multiple Sampling Example

For this multiple plan, a). what is the probability of **acceptance and rejection** based on the first two samples (i.e., sample n_1 and n_2)? b). What is the probability of taking a third sample?

a). Probability of acceptance on sample n_1 & $n_2 = P(c \le 1)$ for both samples as shown on Table 10.

The probability of rejection on the first two samples (n_1 and n_2) = $P(c \le 1)$ for both samples as shown on Table 10.

$For\ both\ samples(n_1 \& n_2);$

$$P_a(n_1 \& n_2) = P(c \le 1) = P\left(c = 0 \middle| 1^{st}\ sample\right) P\left(c \le 0 \middle| 2^{st}\ sample\right)$$

$$+ P\left(c = 1 \middle| 1^{st}\ sample\right) P\left(c = 0 \middle| 2^{st}\ sample\right)$$

$$= P_0^{(1)}P_{0,1}^{(2)} + P_1^{(1)}P_0^{(2)} = 0.223(0.558) + 0.335(0.223) = 0.19913$$

$$P_r(n_1 \& n_2) = P(c \ge 3) = P_{\ge 3}^{(1)} + P_0^{(1)}P_{\ge 3}^{(2)} + P_1^{(1)}P_{\ge 2}^{(2)} + P_2^{(1)}P_{\ge 1}^{(2)}$$

$$Note: P_{\ge c}^{k} = \left(1 - P_{\le (c-1)}^{k}\right) \Rightarrow\ P_{\ge 3}^{(1)} = \left(1 - P_{\le 2}^{(1)}\right)$$

$$P_{\ge 3}^{(1)} + P_0^{(1)}P_{\ge 3}^{(2)} + P_1^{(1)}P_{\ge 2}^{(2)} + P_2^{(1)}P_{\ge 1}^{(2)} = \left(1 - P_{\le 2}^{(1)}\right) + P_0^{(1)}\left(1 - P_{\le 2}^{(2)}\right) + P_1^{(1)}\left(1 - P_{\le 1}^{(2)}\right) + P_2^{(1)}\left(1 - P_0^{(2)}\right)$$

$$P_{\geq 3}^{(1)} + P_0^{(1)} P_{\geq 3}^{(2)} + P_1^{(1)} P_{\geq 2}^{(2)} + P_2^{(1)} P_{\geq 1}^{(2)}$$
$$= \left(1 - P_{\leq 2}^{(1)}\right) + P_0^{(1)} \left(1 - P_{\leq 2}^{(2)}\right) + P_1^{(1)} \left(1 - P_{\leq 1}^{(2)}\right) + P_2^{(1)} \left(1 - P_0^{(2)}\right)$$
$$= (1 - 0.809) + (0.223)(1 - 0.809) + (0.335)(1 - 0.558) + (0.251)(1 - 0.223)$$
$$= 0.191 + 0.0426 + 0.148 + 0.195 = 0.5766$$

Probability of acceptance on the 1st and 2nd samples =0.19913

Probability of rejection on the 1st and 2nd samples =0.5766

To answer the question regarding the probability of taking a 3rd sample, we will look at the reverse of the question, namely, how do we **not** take a 3rd sample. We will not take a 3rd sample if either of the following happens. If we accept the plan based on the 1st and 2nd samples or if we reject the plan based on the 1st and 2nd samples. In other words, we will not need to take a 3rd sample if we accept based on the 1st and 2nd samples or if we reject based on the 1st and 2nd samples

$$P_a(n_1 \& n_2) = 0.19913,\ P_r(n_1 \& n_2) = 0.5766$$
$$\therefore P(\textit{Taking a } 3^{rd} \textit{ sample}) = 1 - [0.19913 + 0.5766] = 0.22427$$

Average Total Inspection (*ATI*):

When rejected lots are 100 % inspected, it is easy to calculate the *ATI* if lots come consistently with a nonconforming level of *p*. For a *LASP* (*N*, *n*, *c*) with a probability P_a of accepting a lot with nonconformance level *p*.

For a **Single plan**:

$$ATI = n + (1 - P_a)(N - n) = n + \{N - n - NP_a + nP_a\}$$
$$= n - n + N - NP_a + nP_a$$
$$= nP_a + N(1 - P_a)$$

Suppose we have a single plan as follows: N=1000, n=50, c=0, p=3%

Based on the Poisson table, P_a (n=50, c=0, np=1.5) =0.2231

ATI for this plan is: 50(0.2231) +1000(1-0.2231) =50(0.2231) +1000(0.7769) =788.05=789

For a **Double plan**, where the parameters are: {*N, (n_1, c_1; n_2, c_2)*}

Where: $ATI = n_1 P_a(n_1) + (n_1 + n_2) P_a(n_2) + N(1 - P_a)$

Let a double plan be given by n_1=50, c_1 =0, n_2=50, c_2=2, N=1000

$$P_a(n_1) = P_0^{(1)} = 0.2231$$
$$P_a(n_2) = P_1^{(1)} P_{0,1}^{(2)} + P^{(1)}_2 P_0^{(2)} = (0.3347)(0.5578) + (0.251)(0.2231) = 0.2427$$
$$P_a = P_a(n_1) + P_a(n_2) = P_0^{(1)} + \left[P_1^{(1)} P_{0,1}^{(2)} + P^{(1)}_2 P_0^{(2)}\right] = 0.4658$$

ATI=50(0.2231) +(50+50)(0.2427)+1000(0.4658)=501.225=502

For the previous Multiple Plan example

$$ATI = n_1 P_a(n_1) + (n_1+n_2) P_a(n_2) + (n_1+n_2+n_3) P_a(n_3) + (n_1+n_2+n_3+n_4) P_a(n_4)$$
$$+ (n_1+n_2+n_3+n_4+n_5) P_a(n_5) + N(1-P_a)$$
$$P_a(n_1) = 0, P_a(n_2) = 0.1991,\ P_a(n_3) = 0, P_a(n_4) = 0.0278, P_a(n_5) = 0.00278,\ \ P_a = 0.2298$$
$$ATI = 50(0) + 100(0.1991) + 150(0) + 200(0.0278) + 250(0.00278) + 1000(1-0.2298)$$
$$ATI = 0 + 19.91 + 0 + 5.56 + 0.695 + 770.2 = 796.3 = 797$$

Average Sample Number (*ASN*): For a single sampling LASP (*n, c*) with a sample of size *n* we can calculate ASN assuming all lots come in with a nonconforming level of *p*. A plot of the *ASN*, versus the incoming nonconforming level, describes the sampling efficiency of a given LASP scheme. ASN is the average number of sample units inspected per lot in reaching decisions to accept or reject.

$$Double:\ ASN = n_1 P_1 + (n_1+n_2)(1-P_1) = n_1 + n_2(1-P_1)$$
$$= n_1 + n_2(\Pr ob\ of\ \ taking\ \ 2nd\ \ sample)$$
$$P_1 = \Pr ob.\ \ of\ \ decision\ on\ 1st\ \ sample = P_r^{(1)} + P_a^{(1)} \Rightarrow r\,e\ ject\ \ or\ \ accept$$
$$(1-P_1) = \Pr ob\ take\ \sec ond\ \ sample$$
$$P_i = \Pr ob.\ of\ \ decision\ on\ \ sample\ \ i$$
$$Multiple:\ n_i,\ i = 5$$
$$ASN = n_1 P_1 + (n_1+n_2) P_2 + (n_1+n_2+n_3) P_3 + (n_1+n_2+n_3+n_4) P_4 +$$
$$+ (n_1+n_2+n_3+n_4+n_5)(1-P_4)$$
$$Multiple:\ n_i,\ i = 6$$
$$ASN = n_1 P_1 + (n_1+n_2) P_2 + (n_1+n_2+n_3) P_3 + (n_1+n_2+n_3+n_4) P_4 +$$
$$+ (n_1+n_2+n_3+n_4+n_5) P_5 + (n_1+n_2+n_3+n_4+n_5+n_6)(1-P_5)$$
$$Multiple:\ n_i,\ i = 7$$
$$ASN = n_1 P_1 + (n_1+n_2) P_2 + (n_1+n_2+n_3) P_3 + (n_1+n_2+n_3+n_4) P_4 +$$
$$+ (n_1+n_2+n_3+n_4+n_5) P_5 + (n_1+n_2+n_3+n_4+n_5+n_6) P_6$$
$$(n_1+n_2+n_3+n_4+n_5+n_6+n_7)(1-P_6)$$

Example: Let a Double plan be given by n_1=50, c_1 =0, n_2=50, c_2=2, N=1000, p=3%

$$P_1 = P_0^1 + P_{>2}^1 = P_0^1 + (1-P_{\le 2}^1) = 0.22313 + 0.19115 = 0.41428 = \text{Prob. of a decision on } 1^{st} \text{ sample}$$
$$ASN = n_1 + n_2(1-P_1) = 50 + 50(1-0.41428) = 79.286 \approx 80$$
$$n_1 P_1 + n_1 + n_2(1-P_1) = 50(0.41428) + 100(0.58572) \approx 80$$

4. MIL- STD-1916 and MIL-HDBK-1916

In 1996, the DoD released a new a version of MILS-STD-105E, namely MIL-STD-1916, entitled: Department of Defense Test Method Standard: DOD Preferred Methods for Acceptance of Product. Prior to 1996, we had MIL-STD-105E. A companion document MIL-HDBK-1916 was developed and released for public use on February 10, 1999.

The MIL-HDBK-1916 is approved for use by all Departments and agencies of the Department of Defense and provides guidance on the use of MIL-STD-1916 to defense contractors and other commercial organizations supplying goods and services to the U.S. Government. The most significant difference between MIL-STD-1916 and previous product acceptance Military Standards is its emphasis on process-oriented improvement efforts. MIL-STD-1916 promotes the use of alternate methods of acceptance to sampling, and specifically endorses the implementation and use of a statistical process control (SPC) program. Note any reference to MIL-STD-1916 includes MIL-HDBK-1916

The MIL STD 105 E provided AQL plans by attributes. The plans that fell into this category included those that ensured lot quality, and those that ensured average quality. The AQL concept was devised in connection with the development of the statistical acceptance sampling for the ordinance Department of the US Army in 1942 by a group of engineers from Bell Labs. In 1945, a statistics group at Columbia University developed statistical sampling tables for the US Navy. The Department of Defense adopted the Navy Tables in 1949 as the JAN (Joint Army Navy) standard 105 or the JAN-STD-105. The tables were made available for public use through the publication of the volume "Sampling Inspection. "MIL-STD-105A" which superseded the JAN-STD-105 in 1950 with several important changes. The second edition of 105A included the master tables from 105 A. In 1958 with minor changes from 105 A, the Department of Defense Adopted MIL-STD-105B. In 1961, MIL-STD-105C was adopted. From 1961 through 1962 a working group known as the ABC working group consisting of military agencies of the US, Great Britain, and Canada worked together to develop a common standard for acceptance sampling by attributes. The standards were adopted in 1963 as ABC-STD-105D or the MIL -STD-105D in the US. It was adopted for commercial purposes by ANSI in 1971. In 1974 the international standards organization changed it to ISO 2859. In 1989, there was another revision that resulted in yet another standard, the MIL-STD-105E with a few substantial changes from 105D.

The purpose of the new standard is to encourage defense contractors and other commercial organizations supplying goods and services to the U.S. Government to submit efficient and effective process control (prevention) procedures in place of prescribed sampling requirements. The goal is to support the movement away from an AQL-based inspection (detection) strategy to implementation of an effective prevention-based strategy including a comprehensive quality system, continuous improvement and a partnership with the Government. The underlying theme is a partnership between

DoD and the defense supplier, with the requisite competence of both parties, and a clear mutual benefit from processes capable of consistently high-quality products and services. The objective is to create an atmosphere where every noncompliance is an opportunity for corrective action and improvement rather than one where acceptable quality levels are the contractually sufficient goals.

4.1 Requirements and Applicability of MIL-STD-1916 and MIL-SHDBK-1916

This standard, when referenced in the contract, specification, or purchase order, is applicable to the prime contractor, and should be extended to subcontractors or vendor facilities. The quality plans are to be applied as specified in the contract documents, and deliverables may be submitted for acceptance if the requirements of this standard have been met. Note: "product" throughout this standard also refers to services and other deliverables.

Quality plans and procedures in this standard may be used when appropriate to assess conformance to requirements of the following:

- End items
- Components or basic materials
- Operations or services
- Materials in process
- Supplies in storage
- Maintenance operations
- Data or records
- Administrative procedures

The contractor is required to submit product that meets all contract and specification requirements. The application of the quality plans or procedures of this standard does not relieve the contractor of responsibility for meeting all contract product requirements. The contractor's quality system, including manufacturing processes and quality control measures, will be established and operated to consistently produce products that meet all requirements. Absence of any inspection or process control requirement in the contract does not relieve the contractor of responsibility for assuring that all products or supplies submitted to the Government for acceptance conform to all requirements of the contract.

The sampling plans and procedures of MIL-STD-1916 are not intended for use with destructive tests or where product screening is not feasible or desirable. In such cases, the sampling plans to be used will be specified in the contract or product specifications. Some of the critical elements of the Mil STD 1916 document can be summarized as follows:

- *There is an evolving industrial product quality philosophy that recognizes the need for quality policy changes that will provide defense contractors with opportunities and incentives toward improvement of product quality and cooperative relationships between the contractor and the Government.*
- *Process controls and statistical control methods are the preferable means of preventing nonconformances, controlling quality, and generating information for improvement. An effective process control system may also be used to provide information to assess the quality of deliverables submitted for acceptance. Suppliers are encouraged to use process control and statistical control procedures for their internal control and to submit effective process control procedures in lieu of prescribed sampling requirements to the Government for approval.*
- *Sampling inspection by itself is an inefficient industrial practice for demonstrating conformance to the requirements of a contract and its technical data package. The application of sampling plans for acceptance involves both consumer and producer risks; and increased sampling is one way of reducing these risks, but it also increases costs. Suppliers can reduce risks by employing efficient processes with appropriate process controls. To the extent that such practices are employed and are effective, risk is controlled and, consequently, inspection and testing can be reduced.*
- *The following points provide the basis for this standard:*

  a. Contractors are required to submit deliverables that conform to requirements and to generate and maintain sufficient evidence of conformance.

  b. Contractors are responsible for establishing their own manufacturing and process controls to produce results in accordance with requirements.

  c. Contractors are expected to use recognized prevention practices such as process controls and statistical techniques.
- *This standard also provides a set of sampling plans and procedures for planning and conducting inspections to assess quality and conformance to contract requirements. This standard complies with the DoD policy of eliminating acceptable quality levels (AQL's) and associated practices within specifications.*

4.1.1 Preferred sampling plans.

This standard establishes three sets of matched sampling plans for the sampling inspection of product submitted to the Government for acceptance. These sampling plans provide for inspecting the samples from lots or batches by attributes or variables measurement and for continuous sampling by attributes measurement. The three sets of matched sampling plans are indexed by seven specified verification levels (VL) and five code letters (CL) A-E, which are determined by the lot or production interval size. The sampling plans are matched between corresponding VL and CL combinations to result in essentially similar protection. The contractor has the option to utilize the type of plan, at the same verification level, that best complements the production process.

4.2 Formation, Identification of Lots or Batches and Corrective Actions

The product shall be assembled into identifiable lots, sublots, or batches, or in such other manner as may be prescribed. Each lot or batch shall, as far as practicable, consist of units of product of a single type, grade, class, size, and composition, manufactured under essentially the same conditions, and at essentially the same time. The lots or batches shall be identified by the contractor and shall be kept intact in adequate and suitable storage space. Although lot or batch size is not used to select a continuous sampling plan, the formation of lots or batches may remain desirable for reasons of homogeneity, shipping convenience, and facilitation of payment.

4.2.1 Determination of sampling plan.

According to the MIL STD 1916 document, a sampling plan is determined by:

a) Verification level (VL) as specified.
b) Type of sampling (attributes, variables, or continuous).
c) Lot or production interval size code letter (CL) from the table.
d) Switching procedure /Inspection severity (Normal, Tightened, Reduced).

For lot acceptance situations (attributes or variables), the occurrence of one or more nonconformances shall result in withholding acceptance of the product submitted and initiation of corrective action. When continuous sampling is in effect, the occurrence of a nonconforming unit while in a sampling phase results in withholding acceptance of that unit, a return to screening, and initiation of corrective action. If a nonconforming unit is found while in a screening phase, acceptance is withheld for that unit and screening is continued until the requirements are satisfied.

4.2.2 Sampling of lots or batches.

Based on the MIL-STD-1916, sampling of lots or batches consists of the following:

a) Selection of units. Units of product drawn from a lot for a sample shall be selected at random from the lot without regard to their quality. Random sampling requires that each unit in the lot, batch, or production interval has the same probability of being selected for the sample.
b) Representative (stratified) sampling.
   When appropriate, the number of units in the sample shall be selected in proportion to the size of sublots or sub-batches, or parts of the lot or batch, identified by some rational criterion. When representative sampling is used, the units from each sublot, sub-batch, or part shall be selected at random.
c) Process of sampling.
   A sample may be drawn after all units comprising the lot or batch have been assembled, or sample units may be drawn during assembly of the lot or batch, in which case the size of the lot or batch shall be determined before samples are drawn. When the lot or batch passes the sampling plan, such lots or batches are acceptable and may be submitted to the Government.

d) Nonconforming product.
When sample units are drawn during lot or batch assembly and nonconforming units are found, the contractor shall withhold from acceptance that portion of the lot completed and all additional production occurring prior to the initiation and verification of corrective action. For lots or batches withheld from acceptance, the contractor shall take the following actions:
   i) Screen the lots or batches and dispose of all nonconforming units in accordance with specified disposition of the product
   ii) Determine the cause of the nonconformances and implement appropriate process changes.
   iii). Initiate the switching requirements based on the inspection severity.
   iv) Advise management or the Government representative of actions taken and resubmit the screened lot or batches for evaluation/consideration.

4.2.3 Disposition of nonconforming product

All units of product found to be nonconforming by the contractor shall be removed and kept apart from the flow of production or otherwise identified or segregated to preclude submission to the Government. The contractor may rework or repair these units unless the contract excludes such activities. Corrected product shall be screened by the contractor and resubmitted to the Government apart from the regular flow of the product.

4.2.4 Critical characteristics

Unless otherwise specified in the contract or product specifications, the contractor is required for each critical characteristic to implement an automated screening or a fail-safe manufacturing operation and apply appropriate sampling plan to verify the performance of the screening operation. The occurrence of one or more critical nonconformances requires corrective action as specified in section 4.2.5 below.

4.2.5 Special reservations for critical nonconformance

When a critical nonconformance is discovered at any phase of production or during any inspection, the following immediate actions are required:

a) Prevent delivery of critical nonconforming units to the Government.
b) **N**otify the Government representative.
c) Identify the cause.
d) Take corrective action.
e) Screen all available units

Records of corrective actions shall be maintained and made available to the Government representative.

4.3 Verification Level (VL)

Seven (7) verification levels (VL) are included in MIL-STD-1916 and may be clearly called out in the contract or as part of the product specification. VL may be prescribed for individual characteristics (e.g. diameter, weight, length) in the case of variable plans, for a group of characteristics in the case of attribute plans, or for subgroup of characteristics within the group. The levels are {I, II, III} and {IV, V, VI, VII). The levels determine the relationship between the lot size and sample size. A code letter is used to designate the appropriate sample size.

Please note that in any sampling process, the larger the sample size, the more discriminatory the plan. In the case of MIL-STD-1916, level I provides less discrimination than level II (40% less than level II) and level III provides more discrimination [(2.4-2.7) x level II] than level II. Level III provides a great deal more discrimination than level I (about 6.4-6.7) x level I. When dealing with relatively small sample sizes, levels I, II, III are used especially in situations where there is destructive testing and it is desired to minimize overall cost. In general, the ***seven*** levels of inspection permit the balancing of the cost of inspection against the amount of inspection

4.4 Severity of Inspection

The MIL-STD-1916 provides for a change in the severity of inspection based on the experience gained from the inspection of immediately preceding lots. there are three degrees of severity or switching procedures, namely: Normal, Tightened, and Reduced.

4.5 Acceptance Sampling based MIL-STD-1916

The following considerations are important when acceptance sampling is to be accomplished using MIL STD 1916 handbook or document.

The VL's are specified in the contract or product specifications. A VL may be specified for individual characteristics, for a group of characteristics, or for subgroups of characteristics within the group. **The Verification Levels (VL) and code letter (CL) determine the sampling plan required to assess product compliance to contract and specification requirements**. Contractors are expected to produce and submit product in full conformance to all requirements. Lots, batches, or production intervals of product that consistently meet or exceed all requirements will be accepted by the sampling plans of this standard and will result in qualifying for reduced sampling levels.

Sampling is performed at one of three severity stages called normal, tightened, and reduced. Unless otherwise specified, the verification limit (VL) stated in the contract shall be considered the normal stage of inspection and shall be used at the start of inspection. The tightened and the reduced stages are then defined as the stages to the immediate left and right, respectively, of the initial stage. The sampling inspection stage in effect shall continue unchanged for each group of characteristics or individual characteristic except where the switching procedures given require change. The switching procedures shall be applied to each group of characteristics or to individual characteristics.

4.6 Switching procedures.

Switching Possibility Summary**: i) N→T, ii) T→ N, iii) N→R, iv) R→N**

The procedures for switching among normal, tightened, and reduced inspection are as follows:

4.6.1 Normal to Tightened

When normal inspection is in effect, tightened inspection shall be instituted when one of the following conditions occurs, depending on the type of sampling plan being used:

i) For Lot or batch sampling the decision to switch is as follows:
If 2 lots or batches have been withheld from acceptance within the last 5 or fewer lots or batches. See figure 9.

ii) For Continuous sampling the decision to switch is as follows:
If 2 nonconforming units are found within a period of inspections (whether on sampling or screening) totaling no more than 5 times the sample size based on Normal Inspection.

4.6.2 Tightened to Normal

When tightened inspection is in effect, normal inspection may be instituted as follows:satisfied:

i) The cause for producing the nonconformances is corrected.

ii) For Lot or batch sampling, 5 consecutive lots/batches have been accepted.

iii) For Continuous sampling, no nonconforming units have been found within a period of inspections (whether on sampling or screening) totaling at least 5 times the sample size based on Tightened inspection.

4.6.3 Normal to Reduced

When normal inspection is in effect, reduced inspection may be instituted when the following conditions are all satisfied:

i) If Lot or batch sampling is being used, (see Tables II and III of Mil STD 1916), the condition is as follows: 10 consecutive lots/batches have been accepted while on normal inspection.

ii) If Continuous sampling is being used (see Table IV of Mil STD 1916), the condition is as follows: No nonconforming units have been found within a period of inspections (whether on sampling or screening) totaling at least 10 times the sample size prescribed by Normal Inspection.

iii) Production is at a steady rate.

iv) The contractor's quality system is considered satisfactory by the client or the Government as the case may be.

v) Reduced inspection is considered desirable by the client or the Government.

4.6.4 Reduced to Normal

When reduced inspection is in effect, normal inspection shall be instituted when one of the following conditions occur.

i) For Lot or batch sampling (see Tables II and III of Mil STD 1916), a lot/batch is withheld from acceptance.

ii) For Continuous sampling (see Tables IV of MIL-STD-1916 for the appropriate sample size, Code Letter (CL), and Verification Level (VL):

iii) A nonconforming unit is found.

iv) Production becomes irregular or delayed.

v) The contractor's quality system is unsatisfactory.

vi) Other conditions warrant that normal inspection be re-instituted.

Switching Possibility Summary: i) N→T, ii) T→ N, iii) N→R, iv) R→N

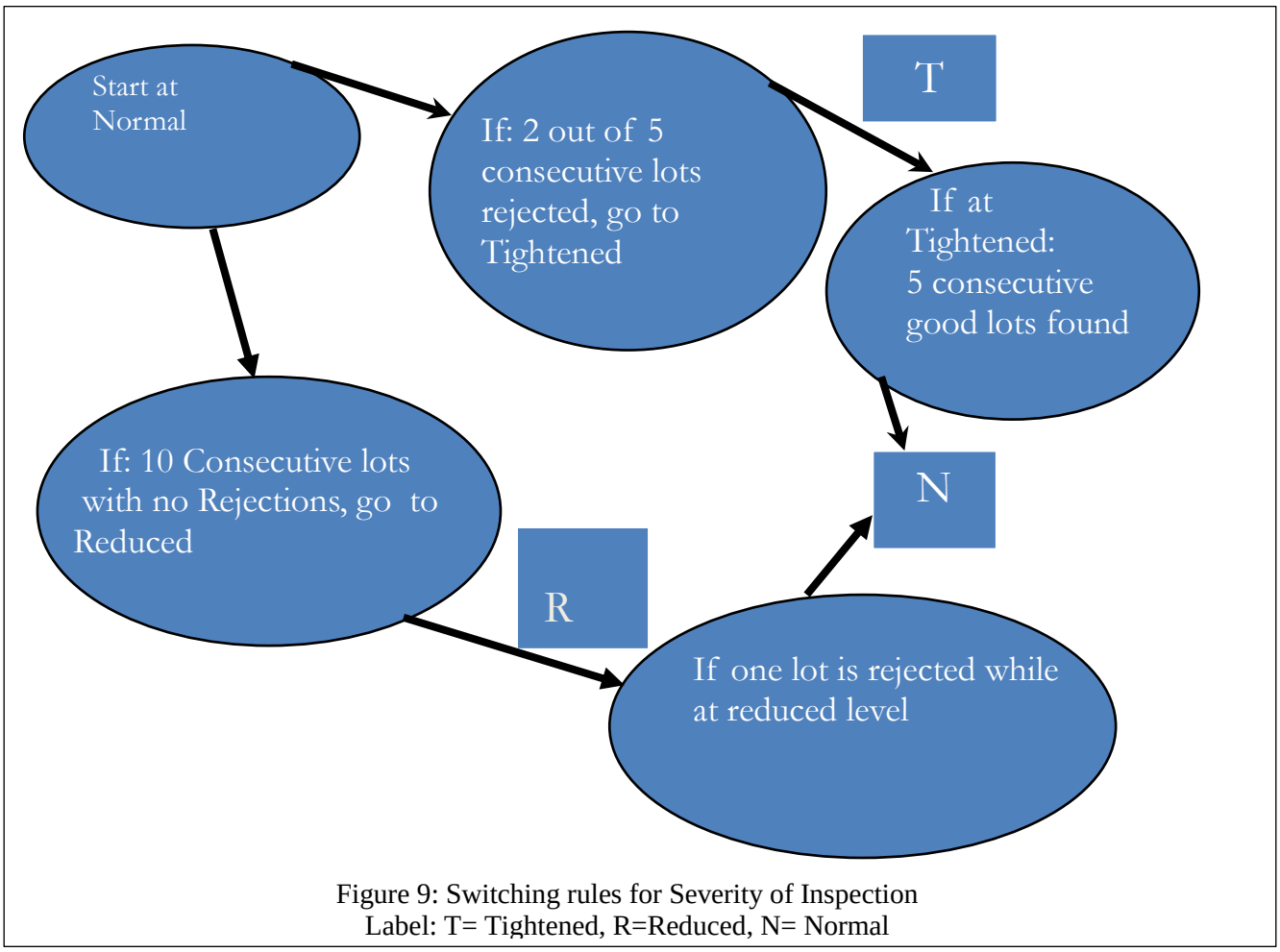


Figure 9: Switching rules for Severity of Inspection
Label: T= Tightened, R=Reduced, N= Normal

4.6.5 Zero Based Acceptance (ZBA) for MIL-STD_1916

ZBA plans are sampling plans in which the acceptance number is zero for any sample taken. They are also referred to as c=0 and Accept on Zero (AoZ) sampling plans

Why c=0? The question of why the Table II plans of MIL-STD-1916 all have an accept number (c) of zero is often asked. The accept-on-zero (AoZ) plans in this standard were formulated with a clear understanding of the following points: a), observance of zero nonconformances in a sample does not imply that the population is perfect; b), expectation of no nonconformances in an entire population of product can be unreasonable; and c), AoZ plans may not be as discriminating as non-zero-plans. It is nevertheless desirable to use sampling plans that accept on zero nonconformances for the reasons given below.

i) Customers generally do not like to give the perception that some small level of percent nonconforming product is acceptable or even tolerable. Whether intended or not, when a sampling plan of, for example, n=100, accept-on-one is used, how could a supplier not get the impression that the customer would be perfectly satisfied, if not elated, to receive product that is, say, 1% nonconforming?

ii) If the user really expects product whose quality level is nearly perfect (say, for example, 0 to 20 ppm nonconforming), then allowing one or two nonconformances in a sample of n=50 or n=100 would be inconsistent with the user's wishes. It would seem to contradict the claim that the population is of a low ppm nonconforming level.

iii) Use of non-zero-one plans of themselves do not foster a desire on the supplier's part to continuously improve. If the supplier knows that a certain percent nonconforming is acceptable, there is little incentive to continuously improve.

It should be understood that if AoZ plans are used as a "stand-alone act", one could expect that the only possible way higher quality levels would be attained would be because of the supplier's fear of excessive lot rejection. The big picture is that process controls are what is needed to stabilize, monitor, and improve processes. Process controls are then accompanied by AoZ plans, if necessary, to verify or "spot check" that the process controls are indeed working. AoZ plans are also needed when the process controls are not yet in place or have not yet reached a mature level. The intent of MIL-STD-1916 and MIL-HDBK-1916 is that process control is of primary importance; AoZ plans are secondary.

4.6.5 Discontinuation of Acceptance Sampling

If sampling inspection of lots or batches remains in tightened inspection due to discovery of nonconformances or when, on continuous sampling plans, there are long periods of screening due to discovery of nonconformances, the Government reserves the right to discontinue acceptance of the product until the causes of nonconformances are eliminated or other means acceptable to the

procuring agency have been instituted. When sampling inspection is restarted after discontinuation of acceptance, it shall be at the tightened inspection stage.

4.6.6 Switching Probabilities

There are risks associated with switching from one operating level to another.
For example, it is possible to switch from:

- ✓ Tightened to Normal, or
- ✓ from Normal to Reduced and
- ✓ from Reduced to Normal and then to Tightened

i) For example, the probability to switch from Normal to Tightened is computed based on the switching rules as follows: The Probability of rejecting 2 or more lots out of 5 consecutive lots. Assuming the binomial distribution, we will have:

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - P(X \leq 1) = 1 - \sum_{i=0}^{1} \binom{5}{i} P^i (1-P)^{5-i}, \text{ where; } \binom{n}{i} = \frac{n!}{i!(n-i)!}$$

ii) Probability that Normal Inspection will be reinstated given Tightened Inspection: This is the probability that five consecutive lots in a row are accepted at tightened Inspection

Let P_A=Probability of Acceptance

Probability (consecutive lots accepted) = $(P_A)^5$

iii) Probability of switching from Reduced Inspection to Normal Inspection involves three (3) possibilities, namely;

a) Probability of (accept and stay on Reduced Inspection)
b) Probability of accept and return to Normal Inspection
c) Reject and return to Normal inspection and correct/fix the problem

Since MIL-STD-1916 is zero based acceptance, the lot will be rejected if there is a nonconforming item in the sample. However, the fact that there is zero nonconforming item in the sample does not guarantee that there are zero nonconforming items in the population. If there is only one nonconforming item in the population, the lot still has a non-zero probability of acceptance P_a. If the number of nonconformance in the lot is a, then proportion nonconforming in the lot is p=a/N. Assuming the lot is finite which means that we can assume that the Hypergeometric distribution is acceptable, then the probability of acceptance P_a can be computed by using the probability density function of the Hypergeometric distribution given by:

$P(x)=\dfrac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}, 0\le x\le \min\{D,n\},$. For any sampling plan, the plan is specified as (n, c), where n is the sample size and c is the acceptance number. In such a case the probability of P_a is given as $P_a=\sum_{x=0}^{c}\dfrac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}$.

For MIL-STD-1916, c is zero so

$$P_a=\frac{\binom{a}{0}\binom{N-a}{n-0}}{\binom{N}{n}}=\frac{\binom{N-a}{n}}{\binom{N}{n}}=\frac{(N-a)!}{n!(N-a-n)!}\div\left[\frac{N!}{n!(N-n)!}\right],\ note:\binom{a}{0}=1$$

$$=\frac{(N-a)!(N-n)!n!}{n!(N-a-n)!N!}=\frac{(N-a)!(N-n)!}{(N-a-n)!N!}$$

If the number of nonconforming items 'a' in the lot is 1, then the probability of acceptance of the plan is given by:

$$P_a=\frac{(N-1)!(N-n)!}{(N-1-n)!N!}=\left(\frac{(N-1)!}{N!}\right)\left(\frac{(N-n)!}{(N-1-n)!}\right)$$

$$Note:\frac{(N-1)!}{N!}=\frac{(N-1)!}{N(N-1)!}=\frac{1}{N},\ and\ \frac{(N-n)!}{(N-1-n)!}=\frac{(N-n)(N-1-n)!}{(N-1-n)!}=(N-n)\Rightarrow\therefore P_a=\frac{(N-n)}{N}$$

4.6.7 Sample Computation of Sampling Plan for MIL-STD-1916 and MIL-HDBK-1916

A contract calls for the following

1. Verification Level VL=IV
2. Lot acceptance is by attributes
3. Assume lot size is 500 so, from Table I of MIL STD-1916 or Table 8 in this document, Code Letter CL is A. With Code Letter A VL=IV, Table II gives a sample size of 80.
   Given the ZBA nature of MIL-STD-1916, we accept on 0, reject on 1
4. Assume Severity Level (switching State) is Normal level

MIL-HDBK-1916 which a companion document to MIL-STD-1916, has the OC curves and tabulates performance values for the sampling plans. Table D-XXVII of MIL-HDBK-1916 specifies the performance results of this plan including acceptance Pa, and the process (%) nonconformance rate p. From that table we have the following information about the plan:

Probability of Acceptance	Process Nonconformance Rate
95%	0.0641%
50%	0.8627%
10%	2.2372%

Based on this we can interpret the results as follows:

Given VL= IV and code letter B. The plan is to sample n=80 units in normal sampling mode and accept the lot if zero nonconformances are found. If the true percent nonconforming of the lot is 0.0641%, the probability of lot acceptance will be Pa= 0.95. If the true percent nonconforming happens to be 0.8627% and 2.8372%, the probabilities of lot acceptance Pa will be 0.50 and 0.10 respectively.

Suppose instead of Normal Inspection, we instead have Tightened and Reduced Inspections. How will the results change?

Note that according to Table II of MIL-STD-1916 (Table 9 in this document), for Tightened Inspection, VI will move to the next VI level to the left of the VI for Normal Inspection. For Reduced Inspection it will move to the next VI level to the right of the VI level specified for Normal Inspection.

In our example the new sampling plan for Tightened Inspection from Table II of MIL-STD-916(table 9 in this document) is:

VL=V, Code letter =A, sample size =192 with lot size still 500. Notice that because this is Tightened Inspection, the sample size is larger than that for Normal Inspection

From Table D-XXVII of MIL-HDBK-1916, we find that if the true percent nonconforming of the lot is 0.0267%, the probability of lot acceptance will be Pa= 0.95. If the true percent nonconforming happens to be 0.3604% and 1.1921%, the probabilities of lot acceptance Pa will be 0.50 and 0.10 respectively.

For Reduced Inspection from Table II of MIL-STD-916(table 9 in this document) the sampling plan is as follows: VL=III, Code letter =A, sample size =32 with lot size still 500. Notice that because this is Reduced inspection, the sample size is smaller than that for Normal Inspection.

From Table D-XXVII of MIL-HDBK-1916, we find that if the true percent nonconforming of the lot is 0.1602%, the probability of lot acceptance will be Pa= 0.95. If the true percent nonconforming happens to be 2.1428% and 6.9428%, the probabilities of lot acceptance Pa will be 0.50 and 0.10 respectively.

NOTE: Availability of MIL-STD Documents-- MIL-STD-916 and MIL-HDBK-916

Copies of DoD adopted non-Government Standards are available to Military activities through the DoD Single Stock Point, Standardization Documents Order Desk, Bldg. 4D, 700Robbins Avenue, Philadelphia, PA 19111-5094. Military activities may obtain copies of non-DoD adopted documents from the sponsoring Industry Association. Non-military activities may obtain copies of non-Government standards and publications from the American Society for Quality Control, PO Box 3066, Milwaukee, WI 53201-3066 and the American National Standards Institute, 1430 Broadway, New York, NY 10018, as appropriate.

TABLE I. CODE Letters (CL) for entry into the sampling tables from MIL-STD-1916							
Lot or Production Interval Size	Verification Levels						
	VII	VI	V	IV	III	II	I
2-170	A	A	A	A	A	A	A
171-288	A	A	A	A	A	A	B
289-544	A	A	A	A	A	B	C
545-960	A	A	A	A	B	C	D
96-1632	A	A	A	B	C	D	E
1633-3072	A	A	B	C	D	E	E
3073-5440	A	B	C	D	E	E	E
5441-9216	B	C	D	E	E	E	E
9217-17408	C	D	E	E	E	E	E
17409-30720	D	E	E	E	E	E	E
30721 and larger	E	E	E	E	E	E	E

Table 8. Verification Levels, Code Letter and Lot sizes from MIL-STD-1916

TABLE II. Attributes Sampling Plans from MIL-STD-1916									
Code Letter	Verification Levels								
	T	VII	VI	V	IV	III	II	I	R
	Sample Size (na)								
A	3071	1280	512	192	80	32	12	5	3
B	4096	1536	640	256	96	40	16	6	3
C	5120	2048	768	320	128	48	20	8	3
D	6144	2560	1024	384	160	64	24	10	4
E	8192	3072	1280	512	192	80	32	12	5

NOTES

(1) When the lot size is less than or equal to the sample size, 100 percent attributes inspection is required

(2) One verification level (VL) to the left/right of the specified normal VL is the respective tightened/reduced plan. Tightened inspection of VL-VII is T, reduced inspection of VL-I is R

Table 9. Code Letter and Verification Levels for Attribute Sampling Plans

Summary

Assuring process and product integrity are important considerations for any organization seeking continuous improvement with respect to service and product performance in the global economy. Acceptance Control and Robust Design are the two major pillars that enhance and sustain quality performance. A quality system is one which aims to achieve controlled production of products with good quality. When a quality system fails to provide a controlled production environment which ultimately results in poor quality products or service, then we have quality loss or loss in quality.

Quality loss refers to losses incurred by a system or organisation (and ultimately the society) from the time a product is released for shipment to the time of its ultimate use and disposal. Such losses are as a result of noise (or factors) that cause deviation from the target or the nominal_which ultimately prevent the product from functioning as previously planned or designed.

The main objectives Acceptance Control methodologies including MIL-STD's and other publication is to ensure that adequate processes are in place to monitor and disposition products, processes and service. The focus of Robust Design is to minimize noise that lead to quality loss by using the principles of system design, parameter design and tolerance design as part of the overall Quality Assurance strategy. These methodologies are implemented to maintain the integrity of the product and its supply chain both within and outside of a company and have been used by industries worldwide (especially since World War II) for assuring the quality of incoming and outgoing goods as part of the overarching goal of Quality Assurance.

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