



*A SunCam online continuing education course*

# Introduction to Electrical Theory and DC Circuits

by

James Doane, PhD, PE



## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course***Contents**

1.0	Course Overview .....	4
2.0	Fundamental Concepts.....	4
2.1	Electric Charges .....	4
2.1.1	Insulators and Conductors.....	4
2.2	Coulomb's Law .....	5
2.3	Current, Voltage, and Resistance .....	6
2.3.1	Current .....	6
2.3.2	Voltage.....	8
2.3.3	Resistance .....	8
3.0	Resistors.....	11
4.0	Ohm's Law and Power.....	13
4.1	Ohm's Law.....	13
4.2	Power.....	14
5.0	Single and Multi-Loop Circuits .....	16
5.1	Introduction .....	16
5.2	Single-Loop Circuits (Series Circuits).....	16
5.2.1	Resistors in Series .....	16
5.2.2	Kirchoff's Voltage Law .....	19
5.3	Multi-Loop Circuits (Parallel Circuits).....	19
5.3.1	Resistors in Parallel.....	19
5.3.2	Kirchoff's Current Law.....	20
6.0	DC Circuit Analysis.....	22
6.1	Introduction .....	22
6.2	Series Circuits .....	22
6.2.1	Voltage Divider Rule .....	24



Introduction to Electrical Theory and DC Circuits  
*A SunCam online continuing education course*

6.3 Parallel Circuits ..... 25  
    6.3.1 Current Divider Rule..... 28  
6.4 Series-Parallel Circuits..... 30



Introduction to Electrical Theory and DC Circuits  
*A SunCam online continuing education course*

## 1.0 Course Overview

---

This course provides a general introduction to electrical theory and DC circuits. Section 2.0 provides basic information about electrical theory. It defines charge, current, voltage, and resistance. Specific information about resistors is given in Section 3.0. Section 4.0 discusses Ohm's law and power. A general overview of circuit analysis is provided in Section 5.0, while Section 6.0 gives application examples of DC circuit analysis.

## 2.0 Fundamental Concepts

---

### 2.1 Electric Charges

Charge is a fundamental property of matter. If a neutral atom would lose an electron, it would become positively charged. A neutral atom would become negatively charged if it were to gain an extra electron. When one atom exchanges an electron with another atom, the charge gained by one will equal the charge lost by the other. Therefore, the net charge will never change for an isolated system (conservation of electric charge). Every object has electric charge, but the electric charge is hidden because of the equal amounts of positive and negative charge.

It is, however, possible to transfer electric charge. Proof of this can be seen every time you rub your shoes across a wool rug, and then you touch something (or someone) to remove that charge. This example is a static charge because charge does not move too freely, and the movement is known as static electricity.

Here are a few additional concepts of electrical charge. The SI unit for charge is the coulomb. Charges with the same electrical sign repel each other (like charges repel), and charges with opposite electrical sign attract (unlike charges attract).

#### 2.1.1 Insulators and Conductors

Materials that do not readily allow charges to move are known as **insulators**, or **nonconductors**. Glass and rubber are common examples of insulators. If electric charges move under the influence of electric forces, the material is known as a **conductor**. Copper and aluminum are examples of conductors.



## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

A third material class, known as **semiconductors**, have electrical properties between insulators and conductors. Silicon is a common example of a semiconductor. Materials that are perfect conductors, which allow charge to move without any interference, are called **superconductors**.

### 2.2 Coulomb's Law

**Electrostatic force** is a force (either repulsion or attraction) due to charge properties of objects. Coulomb's law is the equation that defines that force for charged particles. Charles-Augustin de Coulomb experimentally determined the relationship in 1785. Consider particle 1, which has a charge  $q_1$ , and particle 2, which has a charge  $q_2$ . If  $r$  defines the distance between the particles, the electrostatic force is given by

**Coulomb's Law** 
$$F = k \frac{|q_1||q_2|}{r^2}$$
 Equation 1

The constant  $k$  is known as the Coulomb constant defined by

**Coulomb Constant** 
$$k = 8.9875 \times 10^9 \frac{N \cdot m^2}{C^2}$$
 Equation 2

If you remember much about Physics, you may notice the remarkable similarity of Equation 1 to Newton's gravitational law for two masses. The gravitational force between  $m_1$  and  $m_2$  separated by a distance  $r$  is defined as

$$F = G \frac{m_1 m_2}{r^2}$$

where  $G$  is a gravitational constant.



Introduction to Electrical Theory and DC Circuits  
 A SunCam online continuing education course

*Example 1*

Two charges, each with a magnitude of 5  $\mu\text{C}$ , are 8 cm apart. What is the force of repulsion between the two charges?

**Solution:**

From Equation 1

$$F = k \frac{|q_1||q_2|}{r^2}$$

$$F = 8.9875 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \cdot \frac{|5 \times 10^{-6}| |5 \times 10^{-6}|}{(0.08 \text{ m})^2}$$

$$F = 35.1 \text{ N}$$

## 2.3 Current, Voltage, and Resistance

### 2.3.1 Current

Current exists when electric charge moves. In other words, current results from charges in motion. Current is the rate at which charge flows through a cross-sectional surface. Let  $\Delta Q$  represent the amount of charge passing through an area, and let  $\Delta t$  be the time interval. The current ( $I$ ) is then defined by

$$I = \frac{\Delta Q}{\Delta t} \quad \text{Equation 3}$$

The SI unit for current is ampere (A), which is defined by the current equivalent to 1 C of charge passing through a cross-section in the time interval of 1 second.

Introduction to Electrical Theory and DC Circuits  
A SunCam online continuing education course

---

*Example 2*

---

A charge of  $8 \times 10^{-3} \text{ C}$  passes through a wire in 2 seconds. What is the current in the wire?

**Solution:**

From Equation 3

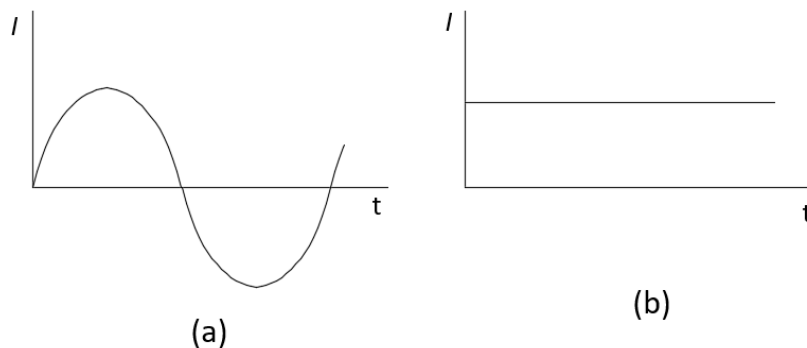
$$I = \frac{\Delta Q}{\Delta t} = \frac{8 \times 10^{-3} \text{ C}}{2 \text{ s}} = 4 \times 10^{-3} \text{ A}$$

$$I = 4 \text{ mA}$$

---

Conventional current is defined as by the direction where positive charge carriers flow through a circuit. Therefore, electric current flows from the power supply's positive terminal through the circuit and returns to the negative terminal.

If the magnitude of current changes with time, it is called **alternating current**. If the magnitude of current does not change with time, it is called **direct current**. Figure 1 illustrates both types of currents.



---

*Figure 1* (a) Alternating current and (b) Direct current



## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

### 2.3.2 Voltage

A voltage source is any device that converts one form of energy into electrical potential energy. Voltage source categories include chemical sources, solar and photovoltaic cells, and electrical conversion (such as a dc power supply that converts an alternating signal into a fixed magnitude signal).

### 2.3.3 Resistance

The ease of flow of electrons in a material will depend on the number of free electrons. If a material has few electrons per unit volume, it will have a large opposition to the flow of current. The resistance to current flow will depend on the type of material, the length, the cross-sectional area, and temperature. The resistance of a conductor is directly proportional to length, and inversely proportional to its cross-sectional area. Consider a conductor with a length  $L$  and a cross-sectional area  $A$ . The resistance is given by

**Resistance**

$$R = \rho \frac{L}{A}$$

Equation 4

where  $\rho$  is resistivity. Resistivity is an intrinsic property of the material, which depends on the electronic structure of the material. Good conductors will have a very low resistivity, while good insulators will have a high resistivity.





Introduction to Electrical Theory and DC Circuits  
 A SunCam online continuing education course

*Example 3*

Calculate the resistance of a piece of aluminum that is 15 cm long and has a cross-sectional area of  $10^{-3} \text{ m}^2$ . Use a resistivity of aluminum equal to  $2.82 \times 10^{-8} \Omega \cdot \text{m}$ .

**Solution:**

Using Equation 4 gives

$$R = \rho \frac{L}{A} = 2.82 \times 10^{-8} \Omega \cdot \text{m} \left( \frac{0.15 \text{ m}}{10^{-3} \text{ m}^2} \right)$$

$$R = 4.23 \times 10^{-6} \Omega$$

Resistivity also depends on temperature. As temperature increases, the atoms vibrate with larger amplitude. This causes, for most materials, the resistivity to increase when temperature increases. Over a limited temperature range, resistivity changes by the following.

$$\rho = \rho_0 [1 + \alpha(T - T_0)] \quad \text{Equation 5}$$

In Equation 5,  $\rho$  is the resistivity at temperature  $T$  (in  $^{\circ}\text{C}$ ). The value of  $\rho_0$  is the resistivity at the reference temperature  $T_0$ , which is typically  $20^{\circ}\text{C}$ . The coefficient  $\alpha$  is the temperature coefficient of resistivity. Because the resistance is proportional to the resistivity, a similar equation can be developed to directly calculate resistance based on temperature change.

$$R = R_0 [1 + \alpha(T - T_0)] \quad \text{Equation 6}$$

Resistivity at  $20^{\circ}\text{C}$ , along with values for the temperature coefficient of resistivity, for a few common materials are given in Table 1.



Introduction to Electrical Theory and DC Circuits  
A SunCam online continuing education course

Table 1 Resistivity (at 20°C) and temperature coefficients of resistivity for common materials

Material	Resistivity ( $\Omega\cdot\text{m}$ )	Temperature coefficient of resistivity ( $^{\circ}\text{C}$ ) <sup>-1</sup>
Copper	$1.7 \times 10^{-8}$	$3.9 \times 10^{-3}$
Aluminum	$2.82 \times 10^{-8}$	$3.9 \times 10^{-3}$
Iron	$10 \times 10^{-8}$	$5 \times 10^{-3}$
Silicon	640	$-75 \times 10^{-3}$

Conductance is defined as the reciprocal of resistance. Therefore, conductance is a measure of the ease with which current will flow. The SI unit for conductance is siemens (S) and the symbol is G.

**Conductance**

$$G = \frac{1}{R}$$

Equation 7

For example, if a conductor has a resistance of 250  $\Omega$ , the conductance would be equal to the inverse of 250 which is 0.004 S (or 4 mS). Similarly, the inverse of resistivity would give conductivity.

**Conductivity**

$$\sigma = \frac{1}{\rho}$$

Equation 8

## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

### 3.0 Resistors

---

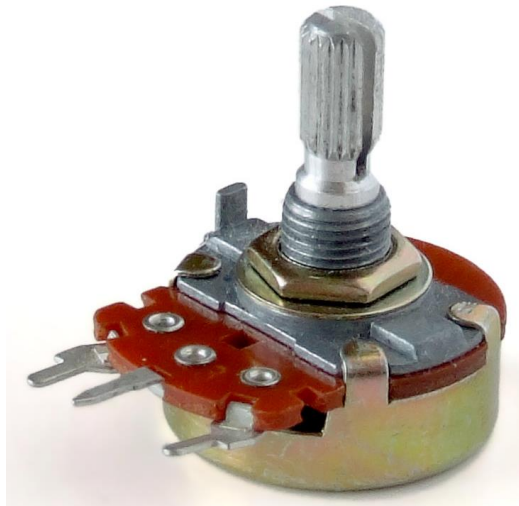
Resistors conduct electricity but also dissipate heat. There are two basic types of resistors: composition resistors and wire-wound resistors.



---

*Figure 2*      Symbol for fixed resistor

There are also variable resistors, which come in two basic types. The two types are the rheostat and the potentiometer.



---

*Figure 3*      Potentiometer



## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

Near one end of the resistor there will be color bands indicating resistance. Table 2 shows the color code for carbon composition resistors. The first three colors indicate magnitude of resistance. An added fourth band of gold or silver gives tolerance of  $\pm 5\%$  or  $\pm 10\%$  respectively (no fourth band indicates  $\pm 20\%$ ). For some resistors there will be an additional fifth band indicating reliability. The reliability factor is the percentage of failure per 1000 hours of use.

Table 2 Color code for carbon composition resistors

Color	First Band	Second Band	Third Band	Fourth Band	Fifth Band
	First significant digit	Second significant digit	Multiplier	Tolerance (%)	Reliability (%)
Black	-	0	$10^0$	-	-
Brown	1	1	$10^1$	-	1
Red	2	2	$10^2$	-	0.1
Orange	3	3	$10^3$	-	0.01
Yellow	4	4	$10^4$	-	0.001
Green	5	5	$10^5$	-	-
Blue	6	6	$10^6$	-	-
Violet	7	7	$10^7$	-	-
Gray	8	8	$10^8$	-	-
White	9	9	$10^9$	-	-
Gold	-	-	$10^{-1}$	$\pm 5$	-
Silver	-	-	$10^{-2}$	$\pm 10$	-
None	-	-	-	$\pm 20$	-



Introduction to Electrical Theory and DC Circuits  
*A SunCam online continuing education course*

As an example, consider a resistor with band 1 red, band 2 violet, band 3 orange, and band 4 is gold. The rating is determined as follows:

Red	=	2 (first significant digit)
Violet	=	7 (second significant digit)
Orange	=	$10^3$ (multiplier)
Gold	=	$\pm 5\%$ (tolerance)
Resistance	=	$27 \times 10^3 \Omega \pm 5\%$

## 4.0 Ohm's Law and Power

---

### 4.1 Ohm's Law

It can be determined, for many conductors, that current is proportional to potential difference across the conductor.

$$I \propto V$$

Also, the current in a wire is inversely proportional to the wire's resistance.

$$I \propto \frac{1}{R}$$

George Simon Ohm (1787 – 1854) noted that the current ( $I$ ) in a conductor is directly proportional to the applied voltage ( $V$ ). The proportionality constant was later determined to be the resistance ( $R$ ), which gives the following equation known as Ohm's law.

**Ohm's Law**

$$V = IR$$

Equation 9

It is important to note that Ohm's law is based on experimental observation, and it is not a fundamental law of nature. Therefore, not all materials will obey Ohm's law. Materials that obey Ohm's law are called ohmic and those that do not are called non-ohmic.



## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

### *Example 4*

An object carries a current of 6 A when connected to a 120 V source. What is the resistance of the object?

**Solution:**

Using Ohm's law from Equation 9 gives

$$V = IR$$

$$R = \frac{V}{I} = \frac{120 \text{ V}}{6 \text{ A}}$$

$$R = 20 \Omega$$

## 4.2 Power

The amount of work that can be accomplished in a specified amount of time is called power. Work has units of joules (J), and time has units of seconds (s). Therefore, power has units of joules per second. For electrical work, the unit is defined as a watt (W). One watt is equal to 1 joule of work that is done in 1 second. Without derivation, equations for electrical power are given below.

**Power**

$$P = IV$$

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

Equation 10



Introduction to Electrical Theory and DC Circuits  
 A SunCam online continuing education course

*Example 5*

What power is delivered to a DC motor when the input current is 10 A and the supply voltage is 120 V?

**Solution:**

From Equation 10

$$P = IV = 10 A(120 V)$$

$$P = 1,200 W$$

*Example 6*

How much does it cost to burn a 60 W lightbulb for 8 hours if electricity costs 12 cents per kilowatt-hour?

**Solution:**

The energy consumed will equal the power times time.

$$E = (0.06 kW)(8 hr) = 0.48 kWh$$

At 12 cents per kilowatt-hour, the cost is

$$Cost = 0.48 kWh(\$0.12/kWh)$$

$$Cost = \$0.06$$

## 5.0 Single and Multi-Loop Circuits

---

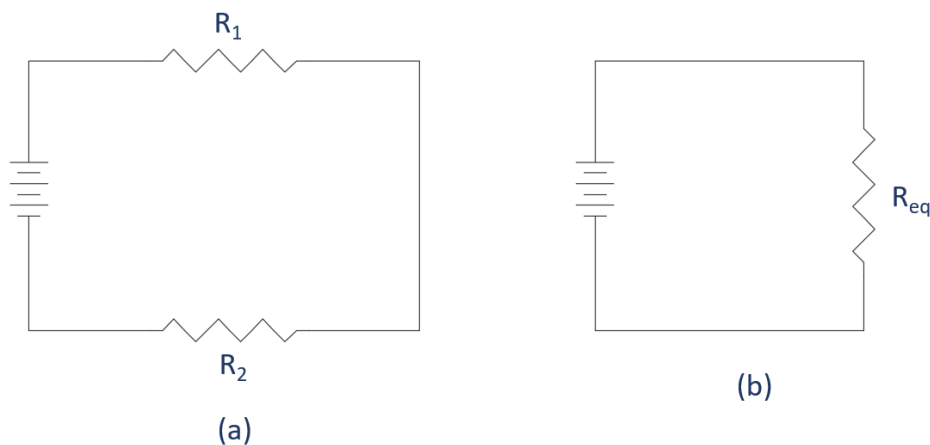
### 5.1 Introduction

This section will provide foundational information dealing with circuit analysis. Content is divided into two categories: single-loop circuits, and multi-loop circuits. Single-loop circuits have all resistors arranged in series, while multi-loop circuits contain all resistors arranged in parallel. Circuits containing a combination of series and parallel arrangements will be discussed in Section 6.4.

### 5.2 Single-Loop Circuits (Series Circuits)

#### 5.2.1 Resistors in Series

Figure 4 (a) shows a series circuit with two resistors. For a series circuit, there is only one pathway for the current.



---

Figure 4 (a) Two resistors in series (b) Circuit with equivalent resistance





## Introduction to Electrical Theory and DC Circuits

### *A SunCam online continuing education course*

Therefore, current is the same through each resistor in a series circuit. The potential drop through the first resistor is equal to  $IR_1$ , and the potential drop through the second resistor is equal to  $IR_2$ . Therefore, the potential drop across both will be

$$V = IR_1 + IR_2 \quad \text{Equation 11}$$

Figure 4 (b) shows the circuit with a single equivalent resistor with a resistance  $R_{eq}$ . Applying Ohm's law to the equivalent resistor gives

$$V = IR_{eq} \quad \text{Equation 12}$$

Equating Equation 11 and Equation 12 gives

$$\begin{aligned} IR_{eq} &= IR_1 + IR_2 \\ R_{eq} &= R_1 + R_2 \end{aligned} \quad \text{Equation 13}$$

Therefore, resistors in series can be replaced by an equivalent resistor with a resistance equal to the summation of all resistors. For N resistors in series the equivalent resistance is calculated using

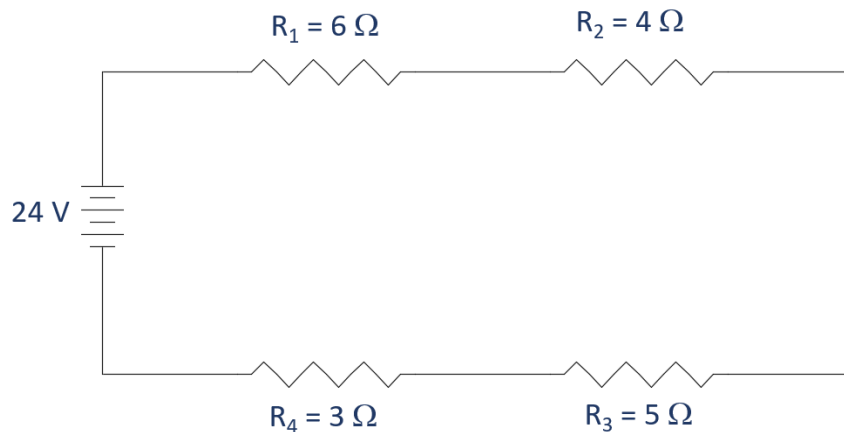
$$\begin{aligned} R_{eq} &= R_1 + R_2 + \cdots + R_N \\ R_{eq} &= \sum_{i=1}^N R_i \end{aligned} \quad \text{Equation 14}$$



Introduction to Electrical Theory and DC Circuits  
 A SunCam online continuing education course

*Example 7*

A circuit contains 4 resistors in series. What is the current in the circuit?



**Solution:**

The equivalent resistance is calculated using Equation 14.

$$R_{eq} = \sum_{i=1}^N R_i = 6\ \Omega + 4\ \Omega + 5\ \Omega + 3\ \Omega = 18\ \Omega$$

Ohm's law, from Equation 12, can now be used to calculate current.

$$V = IR_{eq}$$

$$I = \frac{V}{R_{eq}} = \frac{24\ V}{18\ \Omega}$$

$$I = 1\frac{1}{3}\ A$$

## Introduction to Electrical Theory and DC Circuits

A SunCam online continuing education course

## 5.2.2 Kirchoff's Voltage Law

In single-loop circuits (series circuits), the algebraic sum of the changes in potential encountered in the loop must equal zero. This law, which is referred to as Kirchoff's loop rule or Kirchoff's voltage law (named after German physicist Gustav Robert Kirchoff), is illustrated graphically in Figure 5.

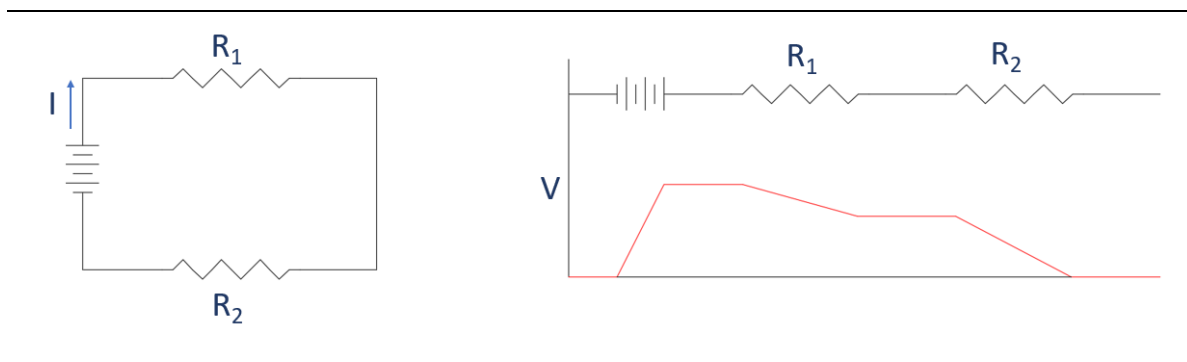


Figure 5 Voltage change through the circuit

Another way to word the voltage law is that the sum of all voltage drops in a circuit must equal the applied voltage. For a series of  $N$  resistors in a circuit, this can be expressed mathematically as

$$V_{supply} = V_{R1} + V_{R2} + V_{R3} + \dots + V_{RN} \quad \text{Equation 15}$$

### 5.3 Multi-Loop Circuits (Parallel Circuits)

#### 5.3.1 Resistors in Parallel

Consider two resistors connected in parallel, as shown in Figure 6 (a). The potential difference must be the same across each resistor when arranged in parallel. The current, in general, will not be the same in each resistor (it will only be the same if the resistors have equal resistance).

Introduction to Electrical Theory and DC Circuits  
 A SunCam online continuing education course

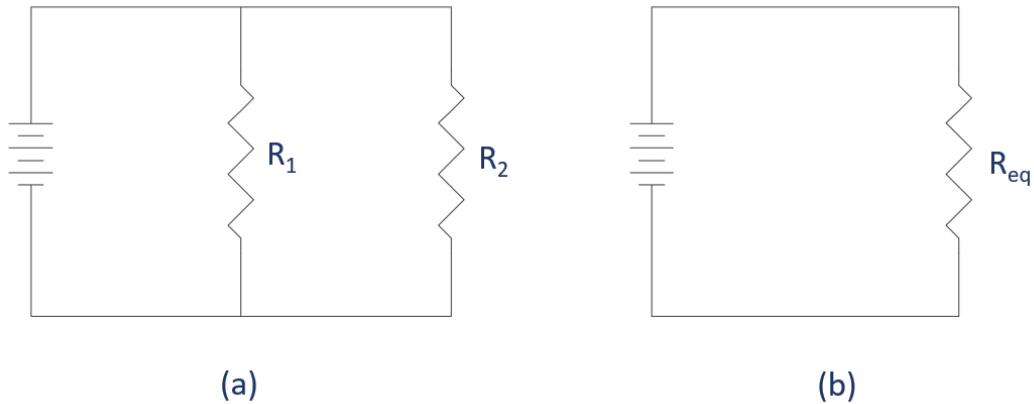


Figure 6 (a) Two resistors in parallel (b) Circuit with equivalent resistance

### 5.3.2 Kirchoff's Current Law

Kirchoff's current law states that the sum of the currents entering any junction must be equal to the sum of the currents leaving that junction. The concept is illustrated in Figure 7.

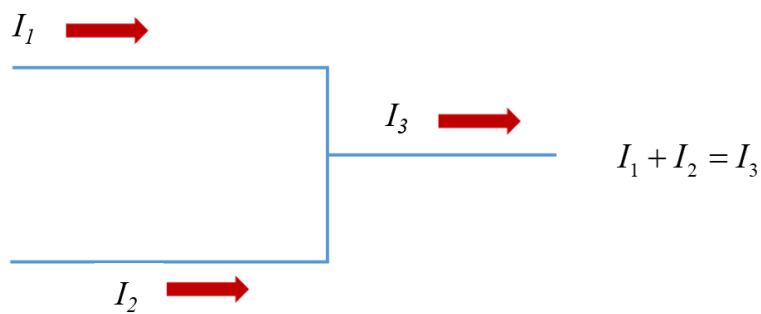


Figure 7 Kirchoff's current law



## Introduction to Electrical Theory and DC Circuits

### *A SunCam online continuing education course*

The resistors in parallel can be replaced by an equivalent resistor as shown in Figure 6 (b). Applying Ohm's law to the equivalent resistor give

$$I = \frac{V}{R_{eq}} \quad \text{Equation 16}$$

Equation 16 can be substituted into Kirchoff's current law, which is shown in Figure 7, to give

$$\begin{aligned} \frac{V}{R_{eq}} &= \frac{V}{R_1} + \frac{V}{R_2} \\ \frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} \end{aligned} \quad \text{Equation 17}$$

Therefore, the equivalent resistance for N resistors arranged in parallel equals

$$\frac{1}{R_{eq}} = \sum_{i=1}^N \frac{1}{R_i} \quad \text{Equation 18}$$

From Equation 18 it can be determined that the equivalent resistance of two or more resistors connected in parallel will always be less than the smallest resistor in the group.



Introduction to Electrical Theory and DC Circuits  
A SunCam online continuing education course

---

*Example 8*

---

Three resistors, which resistances of  $3\ \Omega$ ,  $6\ \Omega$ , and  $9\ \Omega$ , are connected in parallel. What is the equivalent resistance?

**Solution:**

From Equation 18

$$\frac{1}{R_{eq}} = \sum_{i=1}^N \frac{1}{R_i} = \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{9\ \Omega}$$

$$R_{eq} = 1.64\ \Omega$$

---

Further examples of parallel circuits will be provided in Section 6.3.

## 6.0 DC Circuit Analysis

---

### 6.1 Introduction

This section will provide application examples of series and parallel DC circuits. The concepts covered in previous sections will be used to analyze the circuits. In addition, some new concepts will be introduced to aid in the analysis of the circuits. Application examples will be separated into categories of series circuits and parallel circuits. The last section will consider problems that are a combination of series and parallel arrangements.

### 6.2 Series Circuits

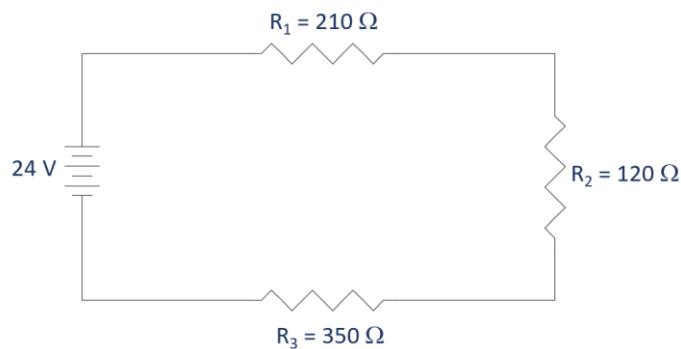
We will begin with examples involving only series circuits.



Introduction to Electrical Theory and DC Circuits  
 A SunCam online continuing education course

*Example 9*

Determine the amount of current that will flow for the circuit shown. What is voltage drop across resistor  $R_2$ ?



**Solution:**

The total equivalent resistance can be calculated using Equation 14.

$$R_{eq} = R_1 + R_2 + R_3$$

$$R_{eq} = 210 \Omega + 120 \Omega + 350 \Omega = 680 \Omega$$

Rearranging Equation 12 will give the current.

$$V = IR_{eq}$$

$$I = \frac{V}{R_{eq}} = \frac{24 V}{680 \Omega}$$

$$I = 35.29 \text{ mA}$$

The voltage drop across resistor  $R_2$  can be determined by multiplying the current and the resistance.

$$V_{R2} = IR_2 = 0.03529 A(120 \Omega)$$

$$V_{R2} = 4.24 V$$



## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

### 6.2.1 Voltage Divider Rule

The second part of the previous example asked for the voltage drop across a resistor. The method used required the current. However, another method exists for calculating voltage drops across a given resistor in a series circuit. The **voltage divider rule** states that the ratio between any two voltage drops in a series circuit is equal to the ratio of the two resistances.

Mathematically, the voltage divider rule states

$$V_{Rx} = V_T \frac{R_x}{R_T} \quad \text{Equation 19}$$

---

#### *Example 10*

---

Use the voltage divider rule to determine the voltage drop across each resistor in the previous example.

**Solution:**

Using Equation 19 for each resistor gives

$$V_{R1} = V_T \frac{R_1}{R_T} = 24 V \frac{210 \Omega}{680 \Omega}$$

$$V_{R1} = 7.41V$$

$$V_{R2} = V_T \frac{R_2}{R_T} = 24 V \frac{120 \Omega}{680 \Omega}$$

$$V_{R2} = 4.24 V$$

$$V_{R3} = V_T \frac{R_3}{R_T} = 24 V \frac{350 \Omega}{680 \Omega}$$

$$V_{R3} = 12.35 V$$

---





Introduction to Electrical Theory and DC Circuits  
*A SunCam online continuing education course*

### 6.3 Parallel Circuits

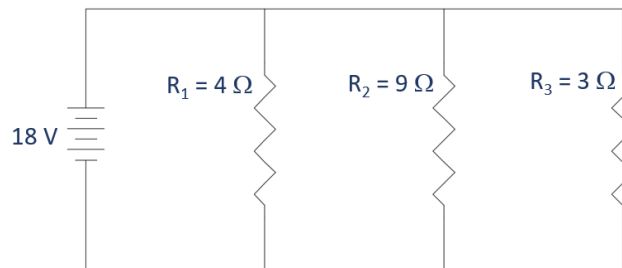
Now we can consider examples only using parallel arrangements.

---

*Example 11*

---

Three resistors are connected in parallel as shown. Determine the current in each resistor.



**Solution:**

For a parallel circuit, the voltage will be the same through each resistor.  
 Using Ohm's law gives

$$I_1 = \frac{V}{R_1} = \frac{18\ V}{4\ \Omega}$$

$$I_1 = 4.5\ A$$

$$I_2 = \frac{V}{R_2} = \frac{18\ V}{9\ \Omega}$$

$$I_2 = 2\ A$$

$$I_3 = \frac{V}{R_3} = \frac{18\ V}{3\ \Omega}$$

$$I_3 = 6\ A$$


---

## Introduction to Electrical Theory and DC Circuits

### *A SunCam online continuing education course*

The concept of parallel circuits can be treated the same as fluid flow in a branch. Fluid will follow the path of least resistance. Notice that the flow (current) followed the path of least resistance because the largest current flow was through the lowest resistor.

Next, we will consider a problem using Kirchoff's current law discussed in Section 5.3.2. The current law was briefly introduced in Section 5.3.2, but it will be expanded here. It is important to understand the sign convention. The four rules below apply, with each assuming we are moving from point *a* to point *b*.

- 1 If a resistor is traversed in the direction of the current:

$$\Delta V = V_b - V_a = -IR$$



- 2 If a resistor is traversed in the direction opposite the current:

$$\Delta V = V_b - V_a = +IR$$



- 3 If a voltage source is traversed from - to +:

$$\Delta V = V_b - V_a = +V$$



- 4 If a voltage source is traversed from + to -:

$$\Delta V = V_b - V_a = -V$$

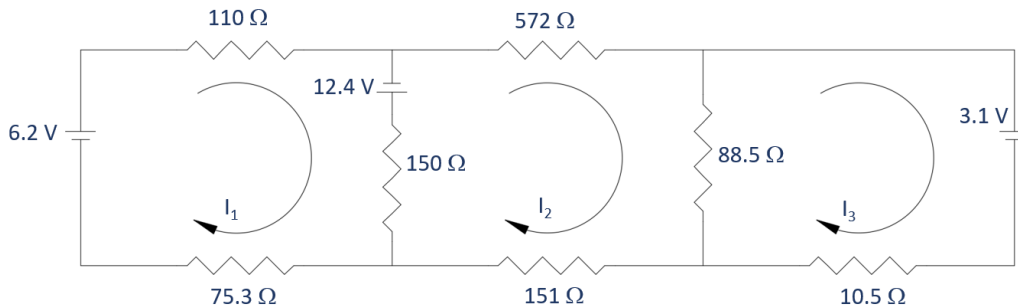


The rules, and the process of using Kirchoff's current law, are best illustrated in an example problem. For each loop in the following example use the rules above to determine the proper sign.

Introduction to Electrical Theory and DC Circuits  
A SunCam online continuing education course

*Example 12*

Use Kirchoff's law to determine the value of the three currents.



**Solution:**

Loop 1

$$6.2 - 110I_1 + 12.4 - 150I_1 + 150I_2 - 75.3I_1 = 0$$

$$-335I_1 + 150I_2 = -18.6 \quad [1]$$

Loop 2

$$-150I_2 + 150I_1 - 12.4 - 572I_2 - 88.5I_2 + 88.5I_3 - 151I_2 = 0$$

$$150I_1 - 962I_2 + 88.5I_3 = 12.4 \quad [2]$$

Loop 3

$$-88.5I_3 + 88.5I_2 - 3.1 - 10.5I_3 = 0$$

$$88.5I_2 - 99I_3 = 3.1 \quad [3]$$

Putting equations [1], [2], and [3] in matrix form gives

$$\begin{bmatrix} -335 & 150 & 0 \\ 150 & -962 & 88.5 \\ 0 & 88.5 & -99 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -18.9 \\ 12.4 \\ 3.1 \end{bmatrix}$$

Solving the system of equations for the current values gives



## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

$$I_1 = 52.7 \text{ mA}$$

$$I_2 = -8.22 \text{ mA}$$

$$I_3 = -38.7 \text{ mA}$$

---

### 6.3.1 Current Divider Rule

The current divider rule states that amount of current in one of two parallel resistances is equal to the product of total current and other resistance divided by the sum of resistance.

#### **Current Divider Rule**

$$I_1 = \frac{I_T R_2}{R_1 + R_2}$$

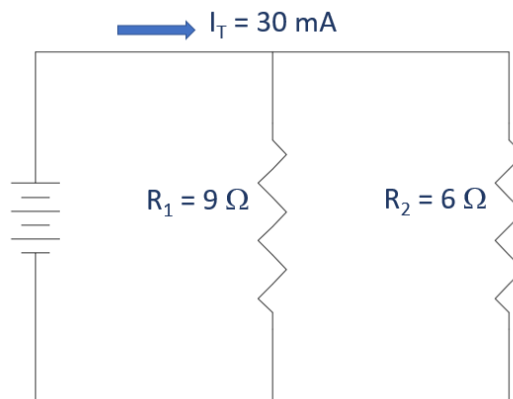
$$I_2 = \frac{I_T R_1}{R_1 + R_2}$$

Equation 20

Introduction to Electrical Theory and DC Circuits  
 A SunCam online continuing education course

*Example 13*

Use the current divider rule to determine the currents flowing through  $R_1$  and  $R_2$ .



**Solution:**

Using the current divider rule in Equation 20

$$I_1 = \frac{I_T R_2}{R_1 + R_2} = \frac{30\ \text{mA}(6\ \Omega)}{(9\ \Omega) + (6\ \Omega)}$$

$$I_1 = 12\ \text{mA}$$

$$I_2 = \frac{I_T R_1}{R_1 + R_2} = \frac{30\ \text{mA}(9\ \Omega)}{(9\ \Omega) + (6\ \Omega)}$$

$$I_2 = 18\ \text{mA}$$

## Introduction to Electrical Theory and DC Circuits

A SunCam online continuing education course

### 6.4 Series-Parallel Circuits

Circuits that contain a combination of series and parallel circuits are known as series-parallel circuits. The equations for equivalent resistance for series and parallel circuits can be applied to systems with a combination of series and parallel arrangements. Consider the arrangement shown in Figure 8. The  $5\ \Omega$  and  $9\ \Omega$  resistors are arranged in series, and they can be simplified to the  $14\ \Omega$  equivalent resistor using Equation 14. The  $6\ \Omega$  and  $3\ \Omega$  resistors are arranged in parallel, and they can be simplified to the equivalent resistance of  $2\ \Omega$  using Equation 18. The new arrangement has two resistors ( $14\ \Omega$  and  $2\ \Omega$ ) in series, which can be simplified again to the final equivalent resistance of  $16\ \Omega$ .

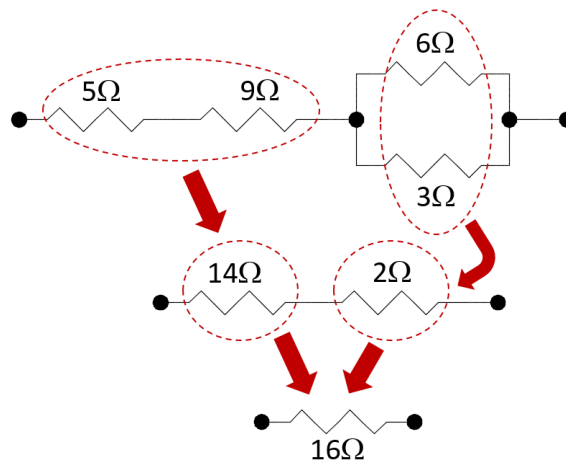


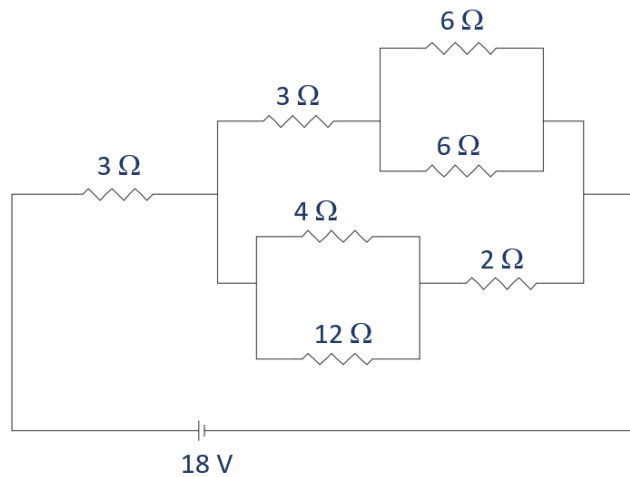
Figure 8 Equivalent resistance concept for series-parallel circuits



Introduction to Electrical Theory and DC Circuits  
*A SunCam online continuing education course*

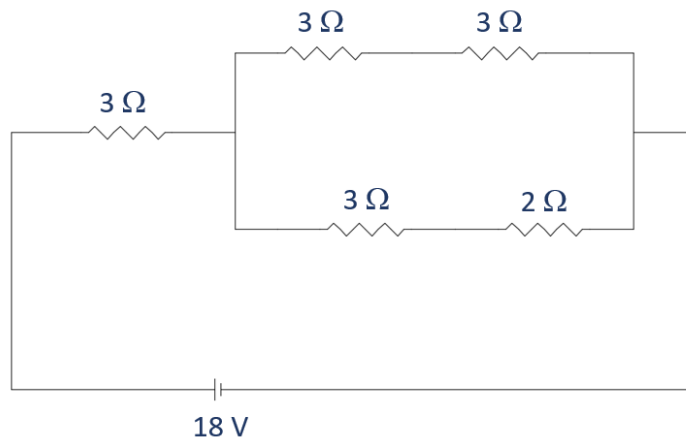
*Example 14*

Determine the current in the  $12\ \Omega$  resistor for the circuit shown.



**Solution:**

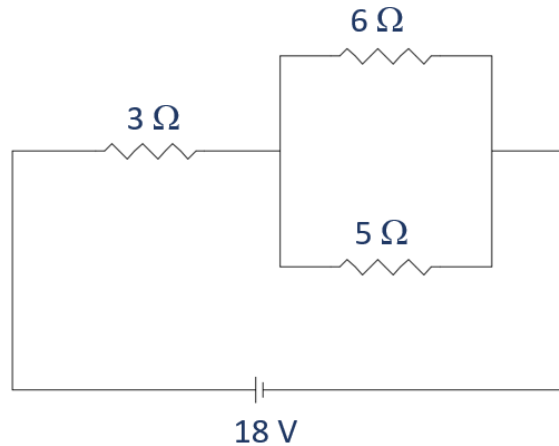
The first step is to determine the equivalent resistance for all resistors. The two  $6\ \Omega$  resistors and the  $4\ \Omega$  and  $12\ \Omega$  resistors in parallel can be reduced using Equation 18.



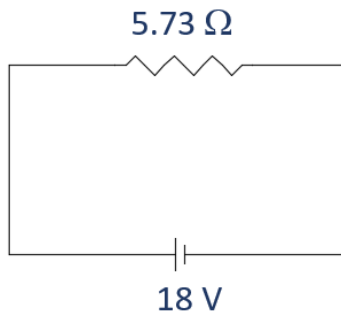
## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

The resulting parallel circuits now have resistors in series, which can be simplified using Equation 14.



The two resistors in parallel can be reduced to one 2.73 Ω, which leaves two resistors in series. Summing the two in series gives the final equivalent resistance.



Ohm's law can now be used to determine the total current.

$$I_T = \frac{V}{R_{eq}} = \frac{18\text{ V}}{5.73\ \Omega} = 3.14\text{ A}$$

Move now to the previous figure. The full 3.14 A will pass through the 3 Ω resistor and will be split between the two in parallel. The current divider rule can be used to determine the current passing through the 5 Ω resistor.





## Introduction to Electrical Theory and DC Circuits

*A SunCam online continuing education course*

$$I_{5\Omega} = \frac{I_T R_1}{R_1 + R_2} = \frac{3.14 A(6 \Omega)}{6 \Omega + 5 \Omega} = 1.71 A$$

Now reference the original figure. The 1.71 A will be split between the 4  $\Omega$  and 12  $\Omega$  resistors. Using the current divider rule to determine the current through the 12  $\Omega$  resistor gives

$$I_{12\Omega} = \frac{I_T R_1}{R_1 + R_2} = \frac{1.71 A(4\Omega)}{4 \Omega + 12 \Omega}$$

$$I_{12\Omega} = 0.429 A$$

---