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WHAT EVERY ENGINEER SHOULD KNOW ABOUT STATISTICAL PROCESS/QUALITY CONTROL I

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Introduction

The American National Standards Institute (ANSI) defines Quality Assurance (QA) as "All of those planned or systematic actions necessary to provide adequate confidence that an item will perform satisfactorily in service." A more operational definition of quality is: "Fitness for Use" This is as defined by the customer not the producer or manufacturer.

1.1 Background

There is a tendency to think of quality as a recent development or phenomenon. However, the basic idea of making a quality product with high degree of uniformity has been around for as long as man has made a product; the idea that statistics may be instrumental in assuring the quality of manufactured products goes as far back as the advent of modern production. The widespread use of statistical methods in problems of quality control is even more recent. Many problems encountered in the manufacturing of a product are very amenable to statistical treatment or analysis. Statistical quality control refers to three special techniques:

a) Process control, b) Acceptance control, c) Parameter design and the establishment of tolerances

1.2 Historical Context

The concepts of probability and statistics are the primary bases for quality control

- Probability was first used 400-500 years ago
- Statistics - around 1850
- About 1922 - Walter Shewhart developed control chart theory at Bell Labs
- 1940's -SPC used extensively during ww2 to solve ammunition problems
- 1929 Dodge-Romig developed acceptance sampling schemes. Included in the acceptance sampling schemes are:
  - MIL STD 105F - Attribute (Go, No Go)
  - MIL STD 414B - Variable (Average, Tolerance, Weight)
  - MIL STD 718B - Reliability of electronic components

1.3 The Quality System

Quality System of an organization is the organizational structure, responsibilities, procedures, processes and resources for implementing quality management as set out in the Quality Assurance and Quality Control programs. The "quality function" of a company is not so much the degree to which the company product conforms to the design or specification—rather, it is a collection of those activities through which "fitness for use" is achieved.

Quality Assurance(QA) is a process focused concept, where the processes are put in place to ensure the correct steps are done in the correct way. If the correct processes are in place, there is some assurance that the actual results will turn out as expected. Quality Assurance lays out the big picture and is more strategic. It sets out the process or set of procedures intended to ensure that a

product or service under development (before work is complete, as opposed to afterwards) meets specified requirements.

Process/Quality Control (SPC/SQC).

If the correct controls are in place, then it is certain that the desired results would be achieved because the results would be verified to ensure that they are what was intended. The control function would include visual inspections throughout the process, and reviewing the results of the various tests performed, to ensure that the desired outcome is achieved. Process/Quality control is tactical, and lays out the pathways to ensure that the big picture is achieved. It is an aggregate of the activities – such as design analysis and inspection for non-conformance or defects—designed to ensure adequate quality is achieved, especially in manufactured products or service. It is a system for verifying and maintaining a desired level of quality in a new or existing product or service through: careful planning, use of proper inspection methods, and corrective action when necessary. A quality control program is a summary of a company's quality control policies.

QA is constructed around human behavior and is focused management philosophy; whereas SPC/SQC are about scientific and technological equipment, techniques, and principles required to achieve desired controls. Quality Assurance processes are put in place to provide some comfort that the end-product is what is wanted. Process/Quality control are the physical and mechanical tests that take place throughout the process—to ensure the Quality Assurance processes have been followed and that, in fact, the resulting end-product coincides with original intent. To implement an effective SPC/SQC program, a company must first decide which specific standards the product or service must meet. Then, the extent of SPC/SQC actions must be determined (for example, the sample size for each lot and the corresponding acceptance number in the case of attributes, and the Control and Specification limits in the case of variables. Next, real-world data must be collected (for example, the percentage of units that fail), and the results reported to management personnel. After this, corrective action must be decided upon and taken (for example, nonconforming units must be repaired or rejected until the desired quality is achieved or the customer is satisfied). If too many unit failures or instances of poor service occur, a Quality Assurance plan must be devised to improve the production or service process, and then that plan must be put into action through the company's process/quality control program—**which is really a summary of the company's process/quality control policies**.

From a Quality Assurance point of view, the work of the design engineer, the machinist and the equipment engineer all contribute to enhancing the product quality characteristics. However, the work of the inspector does not contribute to the product quality characteristics even though it ensures that nonconformance is identified. **In other words, the inspector does not add to the inherent quality characteristics of a product or its integrity**.

When planning a total quality system, one key ***object***ive is to provide a way to guarantee that product integrity is consistently maintained. Some of the ways to work towards this would be by

maintaining adequate drawing and print controls, insisting on adequate and timely calibrations of manufacturing, maintenance and test equipment, and the specification of appropriate change control. However, a way to ultimately ensure and guarantee product integrity is to specifically identify and segregate nonconforming material.

1.3.1 Cost of Quality (Cost of Poor Quality)

Cost of quality (COQ)—or rather, the cost of poor quality (COPQ)—is made up of the costs associated with providing poor quality products or services. Assessing the cost of quality allows an organization to determine the extent to which its resources are used for activities that: prevent poor quality, appraise the quality of the organization's products or services, and result from internal and external failures. Having such information allows an organization to determine the potential savings to be gained by implementing process improvements. While the objectives of a quality program include the identification of the source of quality failures, the basic objective of a quality cost program is the bottom-line, namely, to improve the company's profit margins.

There are four categories associated with quality costs, namely: **prevention costs** (costs incurred to keep failure and appraisal costs to a minimum); **appraisal costs** (costs incurred to determine the degree of conformance to quality requirements); **internal quality cost** (costs associated with nonconforming products or hardware found before the customer receives the product or service); **and external failure costs** (costs associated with nonconforming products or hardware found after the customer receives the product or service). Thus, quality related activities that incur costs may be divided into prevention costs, appraisal costs, and internal/external failure costs.

i). Prevention costs

Prevention costs lead to reduction in quality problems, and hence reduction in failure costs. Such costs include engineering design work, implementation, and maintenance of the quality management system. They are planned and incurred before actual operation, and may include:

a). Product or service requirements: establishment of specifications for incoming materials, processes, finished products, as well as services.

b). Quality planning: cost of writing instructions/procedures, creation of plans to ensure effective quality, reliability, operation procedures, inspection, and testing.

c). Quality assurance: creation and maintenance of the organization's quality system

d). Capacity Development/Training: human development for the preparation, and maintenance of programs.

ii). Appraisal costs

Appraisal costs are associated with measuring and monitoring activities related to quality. These costs are associated with the suppliers', and the customers' evaluation of purchased materials, processes, products, and services—to ensure that they conform to specifications. They could include:

a). Verification of incoming material/products, as well as process setup against agreed specifications.

b). Quality audits, namely, verification and confirmation that the quality system is functioning as intended. It is not for failure costs.

c). Supplier rating, i.e., the assessment and approval of suppliers of products and services

iii). Internal failure costs

Internal failure costs are incurred (such as increase in prevention costs) to remedy nonconformance of product or service before delivery to the customer. These costs occur when the product does not meet specifications or established design standards and are identified before they are released or shipped to the customer. They could include:

a). Waste: unnecessary work or holding of stock because of errors, poor organization, or communication

b). Scrap, namely, nonconforming product, and/or material that cannot be repaired, used, or sold

c). Rework or rectification—correction of nonconforming material

d). Failure analysis, namely, activities performed to establish the causes of internal product (or service) failure

iv). External failure costs

External failure costs are incurred to remedy nonconformance discovered by customers. These happen when products or services that do not meet design standards are not detected until after they get to the customer. Thus, external failure costs include costs due to supplier analysis of non-conforming products and/or hardware. They could include:

a). Repairs and servicing of both returned products and those in the field

b). Warranty claims for failed products that are replaced or serviced under a guarantee

c). Complaints including all work and costs associated with handling and servicing customers' complaints

d). Returns included handling and investigation of rejected or recalled products, including transport costs

In summary, the costs of doing a quality job, conducting quality improvements, and achieving goals must be carefully managed so that the long-term effect of quality on the organization has a desirable effect on the organization. These costs must be a true measure of the quality effort, as determined from an analysis of the costs of quality. Such analysis provides a method of assessing the effectiveness of the management of quality by examining opportunities, savings, and action plans. For some organizations, the true cost of quality may be as high as 15 to 20 percent of sales revenue; and in some cases, this could go as high as 40 percent of operations cost. A general rule of thumb is that costs of poor quality in an organization should be no more than 10 to 15 percent of operations cost. Effective

quality improvement programs are the best avenue to substantially reduce this, and thus improve the company profit profile. The quality cost system, once established, should become dynamic and have a positive impact on the achievement of the organization's mission, goals, and objectives.

1.3.2 Quality Auditing Process

A quality audit is a process by which you review and evaluate an element of your business to ensure that you're meeting certain standards. The standards may vary. Typically, a company can set a standard, or the standard may be set by the industry in which the company exists and in certain cases, and for public safety reasons the standard may be prescribed by government. A systematic and independent examination is used to determine whether quality activities and related results comply with planned arrangements; and whether these arrangements are implemented effectively and are suitable to achieve objectives. It consists of collecting objective evidence to permit an informed judgment about the status of the systems or product being audited. An observation is a statement of fact made during an audit and substantiated by objective evidence. An Objective evidence, in this case, is a qualitative or quantitative information, records or statements of facts pertaining to the quality of an item or service or to the existence and implementation of a quality system element—which is based on observation, measurement or test and which can be verified.

Ideally, the Quality Audit function, if it is to be effective, should be an independent organizational segment in the Quality Control function or department.

There are three main types of quality audit, namely:

i). Internal (First Party or Self Audit). This type audit includes audits by company employees, consultants and contractors. This type of audit asks the question: Does the company comply to its own established or approved standards. Examples include an audit of the manufacturing process or the finished product. In the case of the finished product, the idea is to ensure that the finished products fulfil technical specifications.

In product testing—which aims to determine if product performance meets specification—products should be subjected to tests designed to approximate the conditions to be experienced in customer's application. Not doing so will render the testing meaningless.

ii). External Audit. (You want to audit your supplier or potential supplier). Supplier audit or second party is conducted by your consultant or another company on your behalf or by your own employees on your supplier of goods or services.

ii). Third Party Audit (A customer wants an audit of your company) using an independent organization. It may also be imposed on both the manufacturer or supplier by a third party—usually the government or an interested party who will use the final product. An example of this is the relationship among Pratt (aircraft engine manufacturer), Boeing (airframe manufacturer and US government. The government may impose mandatory audit of Pratt to ensure that the engine meets its design requirements

1.3.3 Supplier Quality Improvement

Due to globalization and liberalization of information technology, most companies are becoming highly dependent on suppliers and must assess and manage quality in the supply chain to reduce business risks and prevent revenue losses. An example of this is Amazon, which does not have a manufacturing plant but depends almost entirely on suppliers to fill its product needs. The excellent management of its supply chain has moved Amazon to the top as one of the highest revenue grossing companies in the US, and perhaps even in the world. In addition, very large companies with tens of thousands of suppliers have developed customized vendor rating system (VRS) or supplier Report Card (SRC) to reduce material variability and nonconformance. A realistic assessment of the supplier resources required to implement Quality Improvement, and to manage and sustain the Quality Improvement process, should be carefully conducted before engaging a supplier as a vendor.

The Supplier plan for Quality Improvement represents the supplier's overall quality improvement initiatives, and must document responsibilities, involvement, plans and criteria for implementing process controls. The plan should also indicate who is vested with the responsibility for ensuring that tooling, equipment, and processes used to demonstrate the capability of the process to consistently produce quality parts with minimum variation are secured. Supplier management, monitoring, and assessment should be initiated as soon as the supplier begins work and must be planned and calculated to support the customer's business objectives. The primary reason for evaluating and maintaining surveillance over a supplier's quality program is to motivate suppliers to improve quality—and so the need for an effective supplier quality management process is crucial.

The supplier should have an internal process for the routine audit of product and process quality, and should perform internal quality audits scheduled on a regular basis to set a benchmark for continuous improvement of the quality systems and to demonstrate compliance with established standards and requirements. Such supplier Quality Improvement Plans should represent the supplier's planned quality initiatives for improving overall service level to customers. The results of the internal audits should be distributed to the appropriate personnel and an action plan developed, tracked and documented for all areas that are in deficit. Typical information that needs to be tracked and documented include:

- Calibration and Gage Repeatability & Reproducibility (R&R)
- Notification and Control of Nonconforming Material
- Preventative / Corrective Action and Problem Resolution
- Supplier Controlled Shipping Procedures
- Manufacturing Capabilities (Process Capability Analysis) and Control of Special Process
- Tooling and Equipment Scheduling and Management
- Inventory Age Control
- Product Change Notification, and Drawing and Engineering Change Control

1.4 Quality of Design and Quality of conformance

1.4.1 Quality of design

Goods and services are produced in various grades or levels of quality. The variation in grade or level (e.g. Chevrolet Cadillac versus Chevrolet Sonic) are intentional between the types of product; including:

a) the types of materials used in construction,
b) tolerance in manufacturing and reliability

1.4.2 Quality of conformance

a) This is an indication of how well the product conforms to the tolerances and specifications required by design

b) It is influenced by the choice of manufacturing processes, operations, the extent to which QA procedures are followed, etc.

1.4.3 Differences Between Quality of Conformance and Quality of Design

Achieving quality of design requires conscious decisions during the design stage to ensure that certain functional requirements are met (usually involves high cost). Quality of conformance can be enhanced by changing certain aspects if the QA system—such as the use of statistical process control—inspection procedures (usually results in lower cost)

1.5 What is Statistical Control

Statistical Control is the application of statistical tools to process performance data, to:

a) Separate the effects of the inherent, random variability from assignable cause and
b) Control variability so that the optimum process output may be attained.

The two related tools used to implement statistical control, with respect to quality assurance, are Statistical Process Control (SPC) and Statistical Quality Control (SQC)

1.5.1 Difference between SQC and SPC

There is little difference between Statistical Quality Control (SQC) and Statistical Process Control (SPC). At one some point, there may have been what some perceived as a philosophical difference, but today, these two exist as general synonyms under the overarching theme of quality assurance (QA). Some prefer SQC because the idea of "quality" is larger and more encompassing than that of "process." Yet others counter by arguing that the term "process" is problem focused, whereas "quality" is symptomatic of the problem. In other words, poor quality is a symptom of the problem in the process. Still others look at SQC as the management version of SPC. The bottom line is that both approaches ultimately work towards the goal of conformance to specifications.

As part of the general conversation on product or system conformance, some quality professionals (including this author) argue that Six Sigma Quality (SSQ) is just another surrogate of Total Quality Management (TQM)—especially with respect to how both SQC and SPC form the backbone of TQM. To better understand the surrogate relationship, specifically about how the ideas

of SQC and SPC support the aims of SSQ, we will examine the primary and supplementary tools advanced by Dr. Ishikawa. In 1974 Dr. Kaoru Ishikawa brought together a collection of process improvement tools in his text *Guide to Quality Control*. Known around the world as the seven (7) quality control (7–QC) tools, they are:

- Cause–and–effect analysis
- Check sheets/tally sheets
- Control charts
- Graphs
- Histograms
- Pareto analysis
- Scatter analysis

In addition to the basic 7–QC tools, Dr. Ishikawa also identified some additional tools known as the seven supplemental (7–SUPP) tools:

- Data stratification
- Defect maps
- Events logs
- Process flowcharts/maps
- Progress centers
- Randomization
- Sample size determination

Statistical quality control (SQC) is the application of the 14 statistical and analytical tools (7–QC and 7–SUPP) to monitor process outputs (dependent variables). Statistical process control (SPC) is the application of the same 14 tools to control process inputs (independent variables).

1.5.2 Major Steps in Statistical Control

- Determine the inherent capability of process, distribution shape, center, spread
- Achieve stability by removing assignable causes
- Compare capability with engineering specifications
- Resolve differences (if any)
- Fix the process or fix the specifications
- Use control charts to insure continued process stability
- Implement continuous effort to reducing process variability

1.6 Role of Quality in Manufacturing

Since very early times, craftsmen/craftswomen have always taken pride in the wares they made. Such pride was exhibited in several different forms—including the imprinting of an indelible mark or stamp on the product. Because a craft person's reputation was at stake and because the

customers tended to be within the immediate vicinity or location, there was an extra care to produce the very best. With the industrial revolution—and later the introduction of factories and especially automated factories—some of the individuality have tended to disappear. Even as markets have widened and become global, and as technology has moved manufacturing out of cottages, the age old need to make products with an inherent desirable attribute have persisted. Thus, quality was not really borne out of technology and, perhaps, has very little to do with technology.

In the broadest sense, quality may be defined as the inherent ability to satisfy customer needs or requirements. The customer is the consumer of the goods or services from the production effort—which implies that a customer may be internal as well as external. A customer could be the next production sequence on the factory floor, or it could be the individual who purchases the finished product. Regardless of how one chooses to define a customer, it is obvious that it is the customer, rather than the producer, who determines what quality is and what it entails. Until very recently, it was very common practice for the producer to define quality, or rather, assume what quality level is desired in a product. Such a situation was created or fueled to a great degree by the existence of technological and information monopolies, lack of communication and technological lapses.

With the globalization of markets and advances in technology, the consumer is now able to choose from a sleuth of product lines and are not restricted solely to home grown products. In the United states, for example, the consumer market is saturated with different models and grades of automobiles from different countries with the result that a buyer has the bargaining power and opportunity to "play the market". For the manufacturers, this has resulted in the pressure to find better ways to meet customer needs, reduce cost and increase productivity. One of the clearest ways a company can hold on to its competitive advantage is to ensure the quality of its products.

The changes that have occurred because of technological and product innovations, global competitiveness, marketing strategies, and consumer expectations have led to changes in the product life cycle. In this respect, the trend in the industry today is for reduced development and prototype times, shorter and shorter time to market, rapid product obsolescence, and proliferation of information technologies. Thus, it is no longer sufficient to rest on one's laurels based on any initial successes. Continuous quality improvement has become the watch word and the key and integral component of the operating strategies of organizations that are successful in today's global scene.

1.7 Quality Improvement

Continuous improvement for quality purposes is an iterative process that does not have an end-point. When an organization buys into the continuous improvement idea, it means entering into a mode of progressive self-evaluation and re-evaluation. Since the customer is the one who defines quality, any quality improvement efforts must begin with the identification of customer current and future needs through proven scientific marketing research methodologies (Quality Function

Deployment-QFD) and translating those needs to product functional requirements. Based on the product life cycle considerations, any such effort would include:

- The initial phase of planning the production activities to meet the functional requirements
- Designing to meet the need at the design and redesign phases
- Making the product to satisfy the functional requirements
- Development of a program of sustained repair and maintenance if necessary, and
- Providing for product disposal at the end of the life-cycle.

One of the major obstacles to continuous improvement is marrying the customer needs to the functional requirements to provide a metrics for quality. Several researchers have proposed breaking down quality into its constituent dimensions, and then focusing on each dimension, to reduce the tedium developing measurable and quantifiable characteristics that represent customer needs. Several authors and researchers including (Moen et al, 2012) suggest the use of dimensions in the attempt to codify and elucidate the relationship between customer needs and the corresponding quality metrics. Some of the dimensions are:

Table 1: Dimensions of Quality

Dimension	Characteristics of Dimension
Performance	Primary operating characteristics
Features	Secondary operating characteristics
Time	Time in waiting, product cycle time, time to complete service
Reliability	The amount of failure free operation possible for given environmental conditions and specified time frame
Durability	Amount of use until replacement is preferable to repair
Uniformity	Low variation among repeated process outcomes
Maintainability	Repair and replacement capabilities
Dependability	A quantitative measure of availability
Aesthetics	Desirable characteristics related to appearance
Personal interface	Characteristics such as punctuality, courtesy and professionalism
Harmless	Safety, health and well being
Perceived Quality	Indirect measures or inferences about one or more of the dimensions

1.8 Total Quality Management (TQM)

During the past decade, there has been a growing awareness of the importance of an overall quality program for the entire organization. The work of three quality pioneers (Dr. Juran, Dr. Deming and Phil Crosby) and the success of the Japanese automotive and electronics enterprises, have been largely responsible for bringing quality issues to the fore and making it a total organization effort and experience, rather than simply a problem for manufacturing and production.

When planning a total quality system, **one key objective** is to provide a means to guarantee and maintain product integrity by the identification and segregation of nonconforming materials.

Total Quality Management (TQM) is a philosophy of operation (or a way of life) in which the concept of quality permeates all fabrics of the organizational structure and functions in the production of goods or services. Under such an environment, success is defined or measured by the degree to which the customer has been satisfied. TQM also implies the notion of continuous improvement as a way of meeting the objective of customer satisfaction. Some of the underlying principles of TQM include the following:

- A new culture or new direction of viewing the customer as the final arbiter of quality and the idea of anticipating customer needs through continuous improvement. Here-to-for the culture and thinking has been for the manufacturer to determine the quality level and the key features of a product with minimal regard or input from the customer
- A supportive environment in which there is participative management, two-way communication between management and workers, management behavior that reinforces belief in quality and the various quality processes and tools.
- The existence of well-defined change mechanisms in such areas as training, and communication, as well as management driven/supported programs relating to reward and recognition, and customer satisfaction.
- The idea that the cost of quality, and perhaps more importantly the cost of non-quality is measured by the cost of not satisfying customer requirements. In a more macro sense, Taguchi (1989) sees this as the cost to society since somehow, someone suffers (in terms of injury, loss of product use, loss of time, etc.)

The recent revolution in quality, and all that has accompanied it, has been inspired by the work of three pioneers whose influence has transcended continents and economic systems. All over the world, their combined work has led to a renaissance in how quality is viewed and how it can be put to maximum advantage. The focal point of their work is on the development of an organizational culture and mindset for quality and continuous process improvements. These pioneers, led by Dr. J. Juran, Dr. E. Deming and Mr. Philip Crosby, have proposed different but similar approaches that ultimately lead to the goal of world class quality culture. The specific concepts proposed by these three Quality gurus (Juran, Deming, Crosby) provide not only the roadmap, but also the road signs, that are vital to acquire and maintain a quality environment and culture. While the focus of their work is the manufacturing environment, the simplicity of the approach makes it clear that the application can be extended and indeed do extend to service, distribution, retail and not for profit organizations. A careful analysis of the approaches shows **six key areas** that are common. These are: Management commitment**,** Strategic planning**,** Training**,** Measurement**,** Identification and elimination of errors and their sources**,** and a culture of continuous improvement.

1.8.1 Management commitment

A breakthrough in management attitude and mindset is a must for the quality culture to take a foothold. Leading by example is the best and easiest way to get workers and subordinates to believe in management. Too often, the problem for quality control has been how to make sure that the effort to get the product to conform to specification and to meet customer requirements, does not impede the effort of the production staff to meet production quota and hence shipping deadlines. A management that emphasizes quality at the beginning of the week, but then turns around to the worker towards the end of the week and says: "*Look, I do not care how you do it, we have a shipment to make Friday at 5 pm, please do whatever it takes to get the product out the door, so our customer would have it Monday morning*", is clearly not committed to quality. To the worker on the shop floor, such a conflicting message breeds suspicions as to the true intent of management.

All three pioneers start by emphasizing the importance of having the directives about quality come from the very top of the organization in a believable form. Dr. Deming's 14 points are basically management obligations. Dr. Juran also believes that all management levels should provide leadership in quality improvement though the execution of one or more quality projects. Mr. Crosby also has a 14-Step process like Dr. Deming's that starts with management commitment. He believes that management must not only understand that quality is a definable, measurable and manageable criterion that requires constant action but that it is important to communicate such understanding and commitment to the workforce. **There is uniform agreement that over 80% of all the production and quality problems are management induced**.

Dr. W. Edwards Deming's 14 points

1. Create constancy of purpose for improvement of product and service.
2. Adopt the new philosophy of refusing to allow defects.
3. Cease dependence on mass inspection and rely only on statistical control.
4. Require suppliers to provide statistical evidence of quality.
5. Constantly and forever improve production and service.
6. Train all employees
7. Give all employees the proper tools to do the job right.
8. Encourage communication and productivity.
9. Encourage different departments to work together on problem solving.
10. Eliminate posters and slogans that do not teach specific improvement methods.
11. Use statistical methods to continuously improve quality and productivity.
12. Eliminate barriers to pride in workmanship
13. Provide ongoing retraining to keep pace with changing products, methods, etc.
14. Clearly define top management's permanent commitment to quality.

1.8.2 Strategic planning

Deming believes in an organizational structure that concentrates on implementing the 14 points. Juran proposes the development of an organizational steering arm that guides the overall planning and problems solving efforts by establishing the direction, priorities and use of resources. Crosby provides a structured approach to launching the improvement process and changing the culture. All three agree that given the right priorities and resources, most organizations would be well on their way to world class quality culture. The stumbling block, however, is the beginning planning phases, cumulating in the strategic decisions; regarding how a company positions itself early in the process to become more competitive.

1.8.3 Training

All three emphasize the importance of training and education in the rudiments of quality—either in the statistical aspects or in the problem formulation and problem-solving techniques. Mr. Crosby's focus on training is on developing a new quality culture and the instructions necessary for implementing the quality improvement process. Dr. Juran emphasis is on the development of problem solving skills and quality management practices. Dr. Deming believes that understanding the underlying statistical relationships, and what they mean for quality, is vital.

1.8.4 Measurement

In defining quality, the three promote different viewpoints but similar ideas. Deming's definition is that quality must have some predictable and measurable degree of uniformity in the product. Low cost and market needs are also the cornerstone of the definition. Crosby on the other hand defines quality as conformance to requirements. Juran looks at quality in terms of the products' fitness for use.

1.8.5 Identification and elimination of errors and their sources

One of the most significant steps in the development of world class quality culture is the identification and the eventual removal of the sources of problems that lead to nonconformance.

1.8.6 A culture of continuous improvement

One of the vital ingredients in the effort to sustain quality is continuous improvement. Ongoing improvement is the idea that today's design—regardless of how good it is in terms of meeting customer requirements—would only serve as a basis for tomorrow 's redesign. TQM implementation to be accomplished by**: a). Mission definition, b). Identification of system yield/output, c). Understanding the true customer, d). Converting customer needs into design and functional requirements.**

1.8.8 Summary of the Approaches Proposed by the three TQM Gurus

DEMING: Deming does not define quality in a single phrase. He asserts that the quality of any product or service can only be defined by the customer. Quality is a relative term that will change

in meaning depending on the customers' needs. To meet or exceed the customers' needs, managers must understand the importance of consumer research, statistical theory, statistical thinking, and the application of statistical methods to processes. *Deming takes a systems and leadership approach to quality. Concepts associated with his approach include (1) the System of Profound Knowledge, (2) "the Plan-Do-Check-Act Cycle-PDCA Cycle," (3) "Prevention by Process Improvement," (4) "the Chain Reaction for Quality Improvement, " (5) "Common Cause and Special Cause Variation, "(6) the 14 Points, " and (7) "the Deadly and Dreadful Diseases"*

CROSBY: The foundation of Crosby's approach is prevention. His approach to quality is best described by the following concepts: *(1) "Do It Right the First Time" (2) "Zero Defects" and "Zero Defects Day (A day that provides a forum for management to reaffirm its commitment to quality and allows employees to make the same commitment*) " (3) the "Four Absolutes of Quality" *(Quality is conformance to the requirements; The system of quality is prevention; The performance standard is "Zero Defects"—do it right the first time; The measurement of quality is the price of nonconformance)* " (4) the "Prevention Process" (5) "Quality Vaccine"; and (6) the Six C's (*Comprehension, commitment, competence, communication, correction, continuance).*

JURAN: JURAN proposes a strategic and structured (i.e., project-by-project) approach to achieving quality. Concepts he developed to support his philosophy include *(1) the "Spiral of Progress in Quality (The spiral shows actions necessary before a product or service can be introduced to the market)" (2) the* "Breakthrough Sequence *(Breakthrough sequence are sequence of activities if carried out properly will result in improvements in quality and will eventually unprecedented performance),*" (3) the "*Project-by-Project Approach*," (4) the "Juran Trilogy *(This trilogy states that management for quality consists of three interrelated quality-oriented processes -- quality planning, quality control, and quality improvement. Each process in the trilogy (planning, control, improvement) is "universal" (inherent in organizations focusing on quality. Relevant activities include identify customers, establishing measurements, and diagnosing causes),*" *and (5) the principle of the "Vital Few Trivial Many (the 20/80 or ABC principle)."*

Dr. Deming's approach is to remove major roadblocks to quality improvement. His 14-points initiated the renaissance in quality and the idea of management's responsibility for quality. He employs a bottoms-up approach, which stresses the use of statistics, in determining where the process is and where it is likely to go. Dr. Juran, on the other hand, stresses a project-by-project methodology and the breakthrough sequence. He believes that short cuts, from symptoms to cause without finding the causes to apply appropriate remedies, impede the journey towards world class quality. Mr. Crosby's program focuses on the development of a quality culture that requires the effort of everyone in the organization. To implement his quality improvement process Crosby delineates a 14-step approach consisting of activities that are the responsibility of top management, but also involve workers.

1.8.9 Lean and Six Sigma

Lean Six Sigma is a synergized managerial concept of Lean and Six Sigma. Lean traditionally focuses on the elimination of the seven kinds of wastes classified as defects: overproduction, transportation, waiting, inventory, motion, and over processing. Lean is the reduction of waste. All

waste can be classified as nonvalue-added. Nonvalue-added refers to some function or task the customer is not willing to pay for. Any overproduction uses labor, utilities, and space that might be used more profitably in other areas. Production that cannot be sold, builds up inventory, and defective product is scrapped or reworked—causing lost productivity. Waiting time can never be recovered. Wasted motion is one of the most overlooked types of waste. Needless walking, turning, bending, and lifting are all nonvalue-added. Transportation waste is also often overlooked. A company that doesn't use all its employees' talents and ideas, wastes potentially good ideas for improvement. Extra inventory may have to be stored until it can be used. At some point in the process, the inventory must be moved again when the next process is ready for it. To be successful in the global economy, where some countries such as China and Mexico have much lower labor rates, companies must do everything possible to cut costs and improve quality. Lean emphasizes teamwork, producing according to demand, smaller batches, quick setups, and cellular production

Six Sigma seeks to improve the quality of process outputs by identifying and removing the causes of defects (errors) and minimizing variability in (manufacturing and business) processes. Synergistically, Lean aims to achieve continuous flow by tightening the linkages between process steps; while Six Sigma focuses on reducing process variation (in all its forms) for the process steps, thereby enabling a tightening of those linkages. In short, Lean exposes sources of process variation and Six Sigma aims to reduce that variation enabling a virtuous cycle of iterative improvements towards the goal of continuous flow.

Lean Six Sigma uses the DMAIC phases like that of Six Sigma. Lean Six Sigma projects comprise aspects of Lean's waste elimination and the Six Sigma focus on reducing defects, based on critical to quality characteristics. The DMAIC toolkit of Lean Six Sigma comprises all the Lean and Six Sigma tools. The training for Lean Six Sigma is provided through the belt based training system, like that of Six Sigma. The belt personnel are designated as white belts, yellow belts, green belts, black belts and master black belts—like judo.

For each of these belt levels, skill sets are available that describe which of the overall Lean Six Sigma tools are expected to be part at a certain Belt level. These skill sets provide a detailed description of the learning elements that a participant will have acquired after completing a training program. The level upon which these learning elements may be applied is also described. The skill sets reflect elements from Six Sigma, Lean and other process improvement methods—like the theory of constraints (TOC) and Total Productive Maintenance (TPM).

As indicated earlier, the Lean Six Sigma process is encapsulated in the strategy; often referred to as the DMAIC (Design-Measure-Analyze-Improve-Control) process. Fundamentally, DMAIC is a data-driven quality strategy for improving processes, and is an integral part of a company's Six Sigma Quality Initiative. Each step in the cyclical DMAIC Process is necessary to ensure the best possible results. The process steps:

Define the Customer, their Critical to Quality (CTQ) issues, and the Core Business Process involved.

- Define who customers are, what their requirements are for products and services, and what their expectations are
- Define project boundaries the stop and start of the process
- Define the process to be improved by mapping the process flow

Measure the performance of the Core Business Process involved.

- Develop a data collection plan for the process.
- Collect data from many sources to determine types of defects and metrics.
- Compare to customer survey results to determine shortfall

Analyze the data collected and process map to determine root causes of defects and opportunities for improvement.

- Identify gaps between current performance and goal performance
- Prioritize opportunities to improve
- Identify sources of variation

Improve the target process by designing creative solutions to fix and prevent problems.

- Create innovate solutions using technology and discipline.
- Develop and deploy implementation plan

Control the improvements to keep the process on the new course.

- Prevent reverting back to the "old way"
- Require the development, documentation and implementation of an ongoing monitoring plan
- Institutionalize the improvements through the modification of systems and structures (staffing, training, incentives)

In order to understand the DMAIC process, it is important to examine several important concepts that underlie process integrity in the overall context of customer satisfaction and value creation, namely:

i). The role of design of experiments in quality/process design and improvement
ii). The guiding principles that drive sound design of experiments
iii). Different approaches and philosophies to planned experimentation
iv). How to mitigate the effects of noise in the experimental environment and the process
v). Contrasting philosophies for dealing with nuisance or noise factors

1.8.10 Role of Design of Experiments in Quality Design and Improvement

There is a need for continuous process and performance monitoring, with a view towards the identification of those areas that present opportunities for product and process improvements. This

makes a strong case for the need to push the quality issue farther and farther upstream into the engineering design arena—where the effects of the factors that are perceived to be important, with respect to product or process performance, can be properly studied by purposefully varying or changing their levels in the experimental realm. Specifically, for process control, a crucial step is the ability to diagnose or discover the root cause—the fault that is responsible for the variation in the process/product—in order to fully understand and appreciate how best to implement process and quality improvements.

Figure 1 shows the cyclical process of statistical process control.

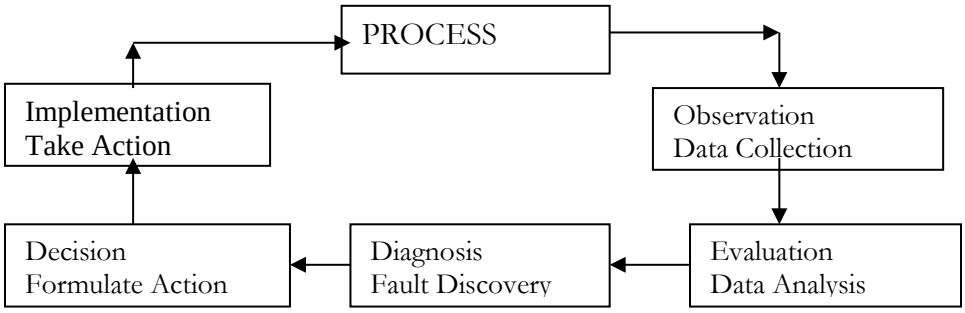


Figure 1: The Cyclical Process of Statistical Process/Quality Control

Oftentimes, to get to the root cause of the problem, we will need to experiment with the process; purposely changing certain factors with the hope of observing corresponding changes in the responses of the process. On the other hand, the problem could be a system problem in the sense that the process could be in control, but the variation happens to be too high, resulting in very large defect rates, and so on. This portends a fundamental problem that is not revealed easily without a comprehensive study of process performance across a range of conditions, large number of factors. Without an organized and systematic approach to experimentation, a costly and time-consuming "random walk" approach to looking for 'root cause' or effects of change can lead to very little and perhaps nothing in terms of an enhanced knowledge of the process. The methods of design of experiments present a systematic approach, that would result an efficient and reliable procedure, that would lead to better process understanding. It is important to note that the power of design of experiments can be greatly enhanced if the environment in which the experiments are conducted has been changed through variation reduction methods, such as statistical process control. Statistical process control ensures a more stable process. A stable process will allow the effects of small changes in the process parameters to be more readily observed. In those cases where statistical control of a process has been established, subsequent experimentation and the associated improvement actions taken are more likely to result in a stable process in future operation of the process since when the process is under statistical control, the future is more predictable. While statistical control of the process is not necessarily a prerequisite to drawing valid conclusions from the results of a designed

experiment, it can, however, greatly enhance the sensitivity of the experiment in the context of its ability to detect the effects of the variables.

Control Techniques for On-Line and Of-Line Control

Control actions are necessary to stabilize a process or to bring a process that is tethering under control. Several control actions, based on classical control theory, have been developed to aid in the control of continuous or discrete processes. These include feed-back, feed-forward and hybrid strategies that have been enhanced by advances in technology that enable some of the control actions to be rendered in the real-time domain. In the context of control strategies, two types have generally been employed, depending on the specific manufacturing context or the desired system goals. These two approaches are; **off-line and on-line**.

2.1 Off-line control

In off-line control, the idea is to be able to impact the process before the fact, after the fact, but not during the process. Thus, the off-line control strategies are implemented mostly in the realm of product or process design and not during process operation. In this sense, off-line control cannot be real time. Several techniques have been proposed for implementing off-line control for a product or process. Notable among these are statistical experimental design in the form of parameter design to determine or obtain optimal process or product settings. Most other related techniques, such as response surface methods (RSM), factorial and fractional factorial design, central composite designs, and regression analysis are imbedded in the formal techniques of parameter design. The research into some of these techniques, especially how they are incorporated into or implemented in the concurrent design and virtual manufacturing environment, are not yet mature and hence are still taking shape. Suffice it to say that the existence of powerful computing platforms and intelligent software systems have enabled major inroads into understanding some of the problems that underlie off-line control techniques.

2.2 On-line control

On-line control is a set of control actions that take place during the actual production cycle or production run. In on-line feedback control, measurements of the product's characteristics are fed back upstream (beginning) processes for adjustment, thereby reducing product deviation. One of the major impediments to process control, for discrete part manufacturing, is the difficulty of obtaining information about the process as it happens. The reason is that, in discrete part manufacturing, the time lag between the production of a part and the measurement of its critical characteristics is not insignificant. Over the years, advances in automated inspection and other forms of advanced metrology have reduced the tedium involved in obtaining useful information that is fed back in time to affect the production process. In terms of equipment, some of the methods involved range from sophisticated two or three-D machine vision and image acquisition systems to coordinate measuring machines (CMM's).

In software and algorithms, there have been considerable developments over the last few years in analytical techniques—such as artificial neural networks and expert systems. It is still believed that the major impediment is how fast the information can be made available to have a realistic impact on the process. For example, data acquisition consists of several sequential processes, starting with:

- preparing the scene (lighting considerations)
- capturing the image of the object using a 2-D or 3-D camera,
- digitizing the image
- transforming the image to the equivalent pixel values
- comparing the image to the model, and
- the decision regarding whether the image matches or does not match the model.

While the processing speeds for computers have grown exponentially over the past decade, 3-D image processing and analysis have not really improved with respect to speed. Consequently, in most operations, while 100% inspection may be the goal, the effectiveness of 100% inspection or monitoring is questionable. In processes where many parameters are not to be evaluated, then 100% inspection may be viable. To keep product characteristics close to the nominal values during the production cycle, monitoring and adjustment of the process parameters are necessary.

2.2.1 Feedback Control with one unit/measurement interval

- This is the type of feedback control where characteristic values are automatically measured immediately after processing by comparing each output to a standard or model.
- The deviations from target are communicated to upstream stations
- This type of control where the measurement interval is one-piece part has contributed to significant gains in quality improvement (Taguchi, 1989)
- In some instances, the volume of production is such that it is desirable to measure more than one unit of production.
- In such a case, it may be difficult to have an automated system capable of measuring every piece of output immediately after processing and provide such information for process control

When the measurement interval is more than one unit of production, operators are recommended. When human operators are used, the cost of measurement is significantly higher. In addition, the operator may not be able to measure all the pieces. This could result in the possibility of higher losses. When operators are used for measurement, Taguchi (1989) recommends decreasing the measurement interval to one piece during the production interval if such is economically feasible.

Control Charts

One of the important applications of statistics today is in quality assurance. The underlying principle of this application may be summarized as follows: Measured quality of manufactured product is always subject to a certain amount of variation due to inherent natural variability. Some stable

'system of chance cause' is inherent in any scheme of production and inspection. Variation within this stable pattern is inevitable. Variations outside the stable pattern is due to assignable causes and maybe corrected using control charts.

Ultimately the primary use of control charts is to detect assignable causes of variation in the process.

In the repetitive manufacturing environment, measured quality of manufactured product is always subject to a certain amount of variation because of chance. As indicated earlier, regardless of the degree of refinement of the manufacturing process, some stable "*system of chance causes"* is inherent in any scheme of production. The best possible scenario for the manufacturer is thus to manage and "helm" in the process, to the point where measurements of a sequence of output, say: $x_1, x_2, x_3,..,x_n$, from such a process would have the aimed-at mean value and very small variance.

If the manufacturing process has been managed and refined to the best possible extent, then the output from the process behave like independent and identically distributed (IID) random variables from a homogeneous population. When this is the case, the manufacturing process is said to be in statistical control. The variations outside the stable pattern is due to assignable causes. In most production situations, it is not economically feasible to measure every output from the process. Hence, the manufacturer takes a small sample from the lot and makes measurement to ascertain whether the process is in control.

A simple but effective method of making this determination for possible intervention is using control charts. In essence, control charts are used for analyzing and interpreting the fluctuations of measurements on successive random samples from a production lot to determine whether the system that generated the output was stable or not. Control charts may be used to maintain surveillance over an already stable process where the parameters of the process are approximately known, ***OR*** it may be used to understand the process behavior and to make inferences about the process parameters.

3.1 Types of Control Charts

- X-bar & R, and X-bar & s charts (**Variable chart or V-chart**)
- np chart for a common sample size (**Attribute chart or A-chart**)
- p charts (p = fraction nonconforming or rejected) for varying sample size (e.g., different production volumes/shift) (attribute chart)
- c charts (c = no. of defects/units) (attribute chart)
- Cumulative Sum (CuSum) Chart for small shifts (variable chart)
- Individual Measurement chart and Moving Range Chart (variable chart)
- EWMA charts (exponentially weighted moving average) (variable chart)

3.2 Steps for Designing Control Charts

The following outline gives the necessary steps for using these charts for any quality characteristics of manufactured product.

- Decision preparatory to the control charts, e.g., objectives of the chart, choice of variables, sub-grouping, method of measurement, etc. Starting the control charts, e.g., making measurement and calculating of parameters and estimates.
- Determining the initial or trial control limits, e.g., plotting the central lines and limits of the charts using, X-bar and R, or X-bar and s etc.
- Drawing preliminary conclusions from the charts, e.g., indication of control or lack thereof and actions suggested by the chart, etc.
- Continue the use of the charts, e.g., revision of the central line and control limits for R & s.

3.3 Variable and Attribute Charts using Variable data and Attribute data.

Attribute Charts

If, due to design requirement or technological constraints, it is not possible to obtain a measurement of the component's vital characteristics, the measurement may be performed based on: **Go-No-Go**; **Good-Bad**; **Failure-Success**, and so on.

In such a case, several characteristics or attributes are grouped/lumped together to provide a single measure and the performance adequacy with respect to meeting design or functional requirements is based on inspection. Thus, when the record shows only the number of such components conforming or failing to conform to a specified performance or design requirement, then the record is said to be one of attributes.

Variable Chart

When records are kept of an actual measured quantity, such as diameter, weight, length, and so on, then the records or the quantities are said to be variables**. Note that individual measurements or values are never plotted on an X-bar chart. It is a wrong use of the chart to do so**.

3.4 Characteristics of Control Charts

Histograms and plots summarize the performance of the process. They do not display the potential capability of the process because they do not address the problem of statistical control or show systematic patterns in process output which, if reduced, will reduce the variability. Both attribute charts (A-chart) and variable charts (V-chart) are useful in this regard. The V-chart more so because of greater power and better information relative to A-charts. To use the p-chart, the product specification must be available. However, it is possible to use X-bar and R (or s) charts to study a process without regard to the specification. Note **that only mean values (or X-bars) are plotted on an X-bar chart and R values on the R-chart, same for the s-chart**. Both X-bar and R (or s)-chart should be used simultaneously in interpreting the process on a V-chart**. Both must show statistical control for the process to be considered in control**. The reason is that because the X-bar and R

charts measure different aspects of the process, there should be no correlation between them and they should not track each other with respect to direction or magnitude.

3.4.1 Between Sample Variation Measured by the Mean (X-bar chart)

Because the time interval between samples is fairly large with respect to the time interval during which each sample is taken, one can assume that there is a much higher chance that any variation between samples would likely contain assignable cause effects—implying long-term or external influences. The X-bar chart reflects conditions external to the process and the most significant indications are those that represent assignable cause situations between subgroups, and therefore most often represents environmental influences. Also, since the statistic computed is the X-bar, it can be looked at as measuring "between sample differences", i.e., sample to sample differences.

Because the time interval between samples is large with respect to the time interval during which each individual sample was taken, one could assume that there is a much higher chance that any variation evident between samples would likely contain assignment cause effects—long term external influences

3.4.2 Within Sample Variation Measured by Variance (R or s chart)

By collecting data—or by using, say, four consecutive pieces produced under relatively stable and identical conditions—we can conclude that any variation between pieces within the sample would give an indication of natural, inherent variability of the process at the particular setting. Also, if it is assumed that because the time interval between the first and last sample piece was relatively short, the chance of an assignable cause variation within the sample will be quite small—***hence, implying short-term interval influences.***

The range chart or the s chart reflects conditions internal to the process, and the variation within the samples normally represents the processes inherent internal variability without assignable causes that are long-term in nature. A good way of thinking about range charts is that they most often show the need for machine repair: worn bearings, loose tool, unstable cutting edges etc. Because the "R" or "s" statistic is measured between individuals, it is often referred to as "within sample variation. The Range-chart or s-chart reflects short-term internal influences.

3.5 Rationale for Specifying Frequency, Sub-grouping and Subgroup Size (n)

i). Frequency

- If 100% is used for inspection for a shift or a day's production, then both sample size and frequency are interrelated.
- In general, the recommendation is to select a sampling frequency that is appropriate for the production rate. This then fixes the sample size.

ii). **Sub-grouping**

- One of the major considerations in the design of control charts is the subgroup size. The subgroup size represents the number of items or data points necessary to provide a given level of confidence in the data and the process.
- Decisions about the subgroup size is vital in balancing the cost of monitoring vis-a-vis the cost of improvement and enhanced quality of the process.
- Rational sub-grouping plays a role in determining the sampling frequency.
- As an example, if three shifts differ, then each shift could serve as a subgroup rather than pooling all output from all shifts to form a subgroup
- In forming a subgroup, we want items grouped together that are produced very close to each other.

The farther apart the time of production between the items, the more likely it is that assignable causes may exist and hence the less likely it is that the items are homogeneous

3.5.1 Sub-group Selection Scheme

There are two types of schemes, namely;

a). Selection made to permit minimum within subgroup variation and maximum variation from sample to sample. In this case, the output from the subgroups should be produced as nearly as possible at one time.

b). Selection based on representative sample over a given period

In the first case, the items may be produced as close together in time as possible. For example, an inspector measures the last five parts, say, produced before the hourly visit to the machining center. This is possible on machine parts if the parts are placed in pallets or trays in the order of production. Or the inspector may wait for 5 consecutive items to come off the machine and then measure them as they come off. In the second case, one subgroup may consist of products intended to be representative of all the production over a given period or shift. The next subgroup could consist of items intended to be representative of the production in a later period. If the products accumulate at the point of production, the inspector may simply choose a random sample from all items made since the last visit. If this is not practical, then there could be 5 visits (n=5) approximately equally spaced over a given production time, with one measurement made per visit. The five measurements then constitute one subgroup.

In general, the selection of subgroups should permit minor within subgroup variation, and maximum variation from sample to sample. In such a case, all the products in the sample should be produced as close together as possible at the same time. On the other hand, if it is desired to make a decide on the entire lot, then the selection should be based on a representative sample over the desired period. The first method of sub-grouping can be expected to provide the best estimate of s', that represents the capabilities of a process, that can be obtained if the assignable cause variation from one subgroup to another can be eliminated. It also provides a more sensitive measurement of the shift in

process average. The second method is used when the purpose is to influence decision on acceptance of entire process output.

3.5.2 Sub-group Size (n)-Variables Chart

Suppose it is desired to detect a change in the process average of say Δ, where Δ =±k%. If we want to do this with a confidence level of 95%. If we know the process variance, then we can compute the desired sample size as shown assuming the process is normally distributed. follows:

$$n = \left[\frac{Z_{\alpha/2}\sigma}{\Delta}\right]^2 = \left[\frac{Z_{.975}\sigma}{\Delta}\right]^2$$

Assume k=±10%, and given an area of (1-0.025) %= 0.975 or 97.5% (two-sided deviation) and variance σ^2 =0.26 (σ=0.51). The Z-score for an area corresponding to 97.5% is 1.96. Thus

$$n = \left[\frac{Z_{\alpha/2}\sigma}{\Delta}\right]^2 = \left[\frac{1.96(0.51)}{0.01}\right]^2 = 99.92 = 100 \text{ units}$$

3.5.3 Sub-group Size (n)-Attributes Chart and Type I and Type II Errors

To see how this can be accomplished, we will quickly discuss the upper and lower control limits for control charts for proportions as an example. Later we will discuss the fundamentals of control charts in greater details. If we want to choose a sample size for a **p-chart** so that at least we can observe one nonconforming item. If p is small, we will choose n sufficiently large to detect at least 1 nonconforming unit. Example: Suppose from historical records a has p = 0.01 and n = 8. Then the 3-sigma upper and lower control limits are as follows:

$$UCL = p + 3\sqrt{\frac{p(1-p)}{n}} = 0.01 + 3\sqrt{\frac{0.01(0.99)}{8}} = 0.1155, \quad UCL_P = 0.1155$$

$$LCL = p - 3\sqrt{\frac{p(1-p)}{n}} = 0.01 - 3\sqrt{\frac{0.01(0.99)}{8}} = 0, \qquad LCL_P = 0$$

Suppose for such a process, we find one nonconforming item? Then with n = 8 and one nonconforming item, p = 0.125. Given the established control limits earlier, this value of plots outside of the control limits hence it is reasonable to conclude that the process is out of control. However, since for any p > 0, there is a chance of producing some defective item, it is unreasonable to conclude, upon observing one defective unit, that the process is out of control. To avoid this, one could choose n so that the probability of observing at least 1 nonconforming item is at least some value k. Example: If p=0.01, we want the probability of at least one nonconforming item 'd' to be at least 0.95. The corresponding probability statement is:

Prob. (d ≥ 1) = 0.95 => 1 – Prob. (d ≤0) =1-p(d=0) = 0.95

Prob. (d = 0) =1 - 0.95 = 0.05, that is, Prob. (d = 0) =0.05

Using the Poisson approximation where the Poisson parameter λ =np

From any Poisson table with x = 0 (that is d = 0) and Prob.(d=0)=0.05, at $\lambda = 3$

But: $\lambda = np = 3$, Thus n (0.01) = 3. Hence **n = 300**

Type I (α) and Type II (β) errors for the Control Chart

The null hypothesis, in this case, is that the process is not producing more than the allowable nonconforming item p_o (H_0: $p=p_0$). The alternative Hypothesis (or Type I error α) is that the process is producing more than the allowable number when it is not (H_1: $p>p_0$). The Type II (β) in this case will be that the process is not producing too many nonconforming items when indeed it is.

Fundamentals of Control Charts

Control charts are one of the most popular SPC tools used by manufacturers. They are used to determine whether a process is in or out of control. When points on a control chart move outside the upper or lower control limit, the process is said to be "out of control." If the points are within control limits, the process is "in control." But, what does an out of control process indicate? Many believe that an out of control process produces nonconforming parts. That's not always true. A control chart can have points outside of both the lower control limit (LCL) and the upper control limit (UCL), indicating that the process is out of control. However, if the individual point remains within the specification limits, then parts are still conforming to specifications.

4.1 Variable Control Chart ($\bar{X}$, R and s Charts)

The various equations for central lines and 3-sigma control limits for the $\bar{X}$, R and s control charts are as shown. The constants used for computing the limits of the charts are shown in table 2.

$\bar{X}$ Charts: i). (σ, μ are known or assumed), Central line $CL_{\bar{X}} = \bar{X}_0 = \mu$, Control limits: $\mu \pm A\sigma$

ii). (σ, μ unknown but estimated from R and $\bar{X}$-bar respectively)

Central line: $CL_{\bar{X}} = \bar{\bar{X}}$, where $\bar{\bar{X}} = \left(\sum \bar{X}\right)/k$, Control limits: $\bar{\bar{X}} \pm A_2\bar{R}$

iii). (σ, μ unknown but estimated from $\bar{s}$ and $\bar{X}$-bar respectively)

Central line: $CL_{\bar{X}} = \bar{\bar{X}}$ ($\bar{X}$-bar), Control limits: $\bar{\bar{X}} \pm A_3\bar{s}$

R Charts i). (σ known): Central line $CL_R = R_0 = d_2\sigma$, Control limits: $UCL_R = D_2\sigma$, $LCL_R = D_1\sigma$

ii). (σ unknown but estimated from $\bar{R}$)

Central Line $CL_R = \bar{R}$, Control limits: $UCL_R = D_4\bar{R}$, $LCL_R = D_3\bar{R}$

s Chart i). (σ known) Central line $CL_s = s_0 = c_4\sigma$, Control limits: $UCL_s = B_6\sigma$, $LCL_s = B_5\sigma$

ii). (σ unknown but estimated from $\bar{s}$)

Central line $CL_s = \bar{s}$, Control limits: $UCL_s = B_4\bar{s}$, $LCL_s = B_3\bar{s}$

4.2 Computation of Parameters for Variable Control charts ($\bar{X}$, R and s chart)

The following (see table 1) represents data from 7 days of production. Each day, a sample of four is taken from the lot. Thus, for this problem, the subgroup size (n) is 4, and the number of

subgroups (k) is 7. Note that the recommendation for control purposes is to have 25 subgroups (i.e., k= 25). The reason is that this value provides enough differences in subgroups to ensure an estimate that is statistically significant. While this may be true, it is unlikely that many shops would have the volume to meet this goal. According to some estimates, over 80% of the shops in the US are job shops, hence most of these companies, for economic and technological reasons, would be hard pressed to justify 25 subgroups in their operations for establishing process. Does this then mean that the result from an analysis with k < 25 is faulty? Not in the least. While it is true that as the number of subgroups (k) increases, the precision of the estimates is enhanced thus reducing the chance of type II error, however, more than anything else, it is important to recognize the reason for control charts and related analyses in the first place. The motivation for this kind of analyses stems from the need to have some understanding about the process, so that appropriate intervention strategies may be imposed on the process as warranted. Thus, whether there are a million pieces of data or just a few pieces of data should not deter from the primary goal of trying to understand and establish the process behavior. The results would of course be more precise in the case of a lot of data than if the data were limited. Most engineers would opt for some information (resulting from more data) rather than no information whatsoever—especially when there is a clear need to understand the process. Thus, while it is good engineering practice to use as much data as possible, the nature of the manufacturing environment may limit what can realistically be accomplished. It is therefore imperative that whoever must use such data understands some of the limitations and assumptions so as to draw conclusions or make inferences accordingly. For variables control chart, the most useful data is X-bar & R (or s).

If the process is assumed to be under control, then the process std. deviation variance σ can be estimated by either: $\sigma = \bar{R}/d_2$ or $\sigma = \bar{s}/c_4$. From the data on table 2, we obtain the following statistics: $\bar{\bar{X}}$-bar = 1.677, $\bar{R}$ = 0.005, $\bar{s}$ = 0.0019

Case I: σ , μ known or assumed, also let σ = 0.001, μ = 1.680, also, n=4

For the $\bar{X}$-chart: $CL_{\bar{X}} = \bar{\bar{X}} = 1.680$

$UCL_{\bar{X}}$, $LCL_{\bar{X}} = \bar{\bar{X}} \pm A\sigma = 1.680 \pm 1.5(0.001) = [1.682, 1.679]$

For the R-chart: $CL_R = d_2\sigma = (2.059)(.001) = 0.0021$

$UCL_R = D_2\,\sigma = (4.698)(.001) = 0.0047$, $LCL_R = D_1\sigma = (0.0)\ (0.001) = 0$

$[UCL_R, LCL_R] = [0.0047, 0]$

For the s-chart: $CL_s = c_4\,\sigma = (0.9213)\ (0.001) = 0.0009$

$UCL_s = B_6\,\sigma = (2.09)\ (0.001) = 0.0021$

$LCL_s = B_5\,\sigma = (0.0)\ (0.001) = 0$, $[UCL_s, LCL_s] = [0.0021, 0]$

Case II: σ, μ unknown but estimated from R and $\bar{X}$ respectively (n = 4)

For the $\bar{X}$-chart, $CL_{\bar{X}} = \bar{\bar{X}} = 1.677$ (from table 2),

$UCL_{\bar{x}}$, $LCL_{\bar{x}} = \bar{\bar{X}} \pm A_2\,\bar{R} = 1.677 \pm (0.729)(0.005)$, $= [1.681, 1.673]$

For the R-chart; $CL_R = \bar{R} = 0.005$

$UCL_R = D_4 \bar{R} = (2.28)(.005) = 0.0146$, $LCL_R = D_3 \bar{R} = (0.00)(.005) = 0$

$[UCL_R, LCL_R] = [0.0114, 0]$

Note: If we now have the points given by: $\bar{X} = 1.66, R = 0.01$, Then point is in control

$\bar{X} = 1.68, R = 0.02$, Then point out of control

$\bar{X} = 1.69, R = 0.00$, Then point out of control

NOTE: X-bar and Spread of Individual Observations with Respect to Process Capability

For the Shewhart control chart, the spread for normal process capability for the $\bar{X}$-chart is: $\pm 3\sigma_{x\text{-bar}}$ or $6\sigma_{x\text{-bar}}$. The spread of the individual observations from a normal process is $\pm 3\sigma$ or a total of 6σ, where $\sigma = \bar{R}/d_2$ or $\sigma = \bar{s}/c_4$

Case III: σ, μ unknown but estimated from s and $\bar{X}$ respectively (n = 4)

For the $\bar{X}$-chart: $CL_{\bar{x}} = \bar{\bar{X}} = 1.677$

$UCL_{\bar{x}}, LCL_{\bar{x}} = \bar{\bar{X}} \pm A_3\bar{s} = 1.677 \pm (1.63)(0.0019), [UCL_{\bar{x}}, LCL_{\bar{x}}] = [1.680, 1.674]$

For the s-chart, $CL_s = \bar{s} = 0.0019$

$UCL_s = B_4\bar{s} = (2.28)(0.0019) = 0.004$, $LCL_s = B_3\bar{s} = (0.00)(0.0019) = 0$

$[UCL_s, LCL_s] = [0.004, 0]$

Day (k)	Sample # 1	Sample # 2	Sample # 3	Sample # 4	$\bar{X}$(in)	R	s
1	1.677	1.680	1.678	1.672	1.677	.008	.0029
2	1.673	1.679	1.675	1.674	1.675	.006	.0023
3	1.680	1.679	1.678	1.678	1.679	.002	.0008
4	1.680	1.678	1.675	1.677	1.678	.005	.0018
5	1.676	1.675	1.675	1.679	1.676	.004	.0016
6	1.678	1.681	1.679	1.676	1.679	.005	.0018
7	1.675	1.675	1.672	1.678	1.675	.006	.0021
Total					11.7380	0.036	0.0133
Average					1.6770	0.005	0.0019

Table 1: Data for control chart analysis; measurements in inches

In our effort to determine if the process is stable (in control), we look at two types or two different sources of variation which is represented by $\bar{X}$ and R- chart, or the $\bar{X}$ and s-chart. One source is the variation in subgroup averages (variation among subgroups). The other source is the variation within a subgroup.

For the subgroup variation, the range or the R-chart (or the standard deviation chart (s-chart) shows how much variation there is within each subgroup. Since the assumption is that subgroups are identical (or nearly so), we would expect this variation to be small and be consistent over time. The chart for averages ($\bar{X}$ or X-bar) presents a different type of variation from the range (or standard

deviation) chart. The $\overline{X}$ chart shows variation among the subgroups. That is how much day to day, or week to week, or even month to month variation there is in the subgroups. We would like this variation to be small and be consistent over time. Thus, while the two types of variations are focused on different aspects of the process (within the items in each subgroup, and among the subgroups) both are needed to ultimately determine process stability and capability.

	X-Charts				s-Charts				R-Charts					
n	A	A_2	A_3	C_4	B_3	B_4	B_5	B_6	d_2	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9690	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	0.428	1.572	0.421	1.544	3.472	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	0.497	1.503	0.490	1.483	3.689	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	0.523	1.477	0.516	1.459	3.778	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	0.534	1.466	0.528	1.448	3.819	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	0.545	1.455	0.539	1.438	3.858	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	0.555	1.445	0.549	1.429	3.895	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	0.565	1.435	0.559	1.420	3.931	0.708	1.806	6.056	0.459	1.541

Table 2: Control Chart Constants. Adapted from Table 27 of ASTM STP 15D ASTM Manual on Presentation of Data & Control Chart Analysis, copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken, PA 19428.

Please note that because the two charts (the mean chart and the variability chart) measure different things they do not, and should not, track each other. The $\overline{X}$ chart alone does not and cannot

not tell the whole story—neither does the R (or s) -chart. However, both must show or exhibit control for the process to be adjudged as being in control. The question that is often asked is which one, the R-chart or the s-chart, is more effective in measuring the within variation effect? From theoretical considerations, we know that while both the range (R) and standard deviation (s) are estimators of variability, the range is a better estimator when the sample size is less than five ($n<5$) while the standard deviation is a better estimator when the sample size is five or more (i.e., $n\geq5$). In the case of small samples ($n<5$), range variation estimates are less inflated by small samples.

Standard deviation estimate uses the square formula, and thus is prone to making estimates rather large—especially if outliers are found in a small sample. However, this defect is minimal when the sample is larger. Most would argue that it is simply a matter of economics, rather than statistical validity. The Range, as a measure of variability, is less sensitive to small data than standard deviation—so most chose the Range for that reason, and the fact that it involves less data cost.

In our analysis of the X-bar chart and the companion R-Chart (or s-Chart), we will only consider the combination of X-bar and R-charts. This is what is mostly obtained in practice, even though some do advocate for the use of the s-chart when the sample size is greater than 5. Going forward for the rest of our discussion on the use of the X-bar chart, we will only consider the tandem of X-bar and R-charts.

4.3 Analysis of the Control Chart Plot

Based on the plots on the X-bars chart, it can be observed that all the points (the X-bars) plot inside the control limits. More importantly, the plot does not show any discernable trend. At this point, we cannot conclude that the process is under statistical control until we examine the plot on the R-chart. Upon close examination, that chart also shows that all the points plot within the control limits and there is no discernable trend. Additionally, there is no relationship between the two charts by way of tracking. In other words, one plot does not resemble the other nor do they track each other. Given these conditions, we can now say that the process is under control.

It is important to realize that we carry the analysis of the control chart as way mean to understand the process and its behavior. So, even after we have shown that the process is in control, we now have the burden of showing that the process is producing with specifications. This is an important fact that eludes many young engineers. Specification limits are different from control limits. For starters, one is a single measure or individual value (in the case of spec limits) while the other is a mean value. Thus, cannot and should not use the control chart to measure or determine if the process meets specifications. So, the question is how do we determine if the process is producing within specifications? We begin by making sure that the process is stable or under control based on the control chart (X-bar and R charts together). Given that the process is stable, then we can estimate the process variability σ^2 by suing the relationship. $\sigma = \frac{\overline{R}}{d_2}$. Also note that once we verify that the process

under control, the analysis on the Spec limits uses the estimate of the process variability. Also, the control limits remain the same are not changed for all future analysis until the process reevaluated due to new information. The specification limits may be adjusted based on the realities of the process as evidenced on the control charts.

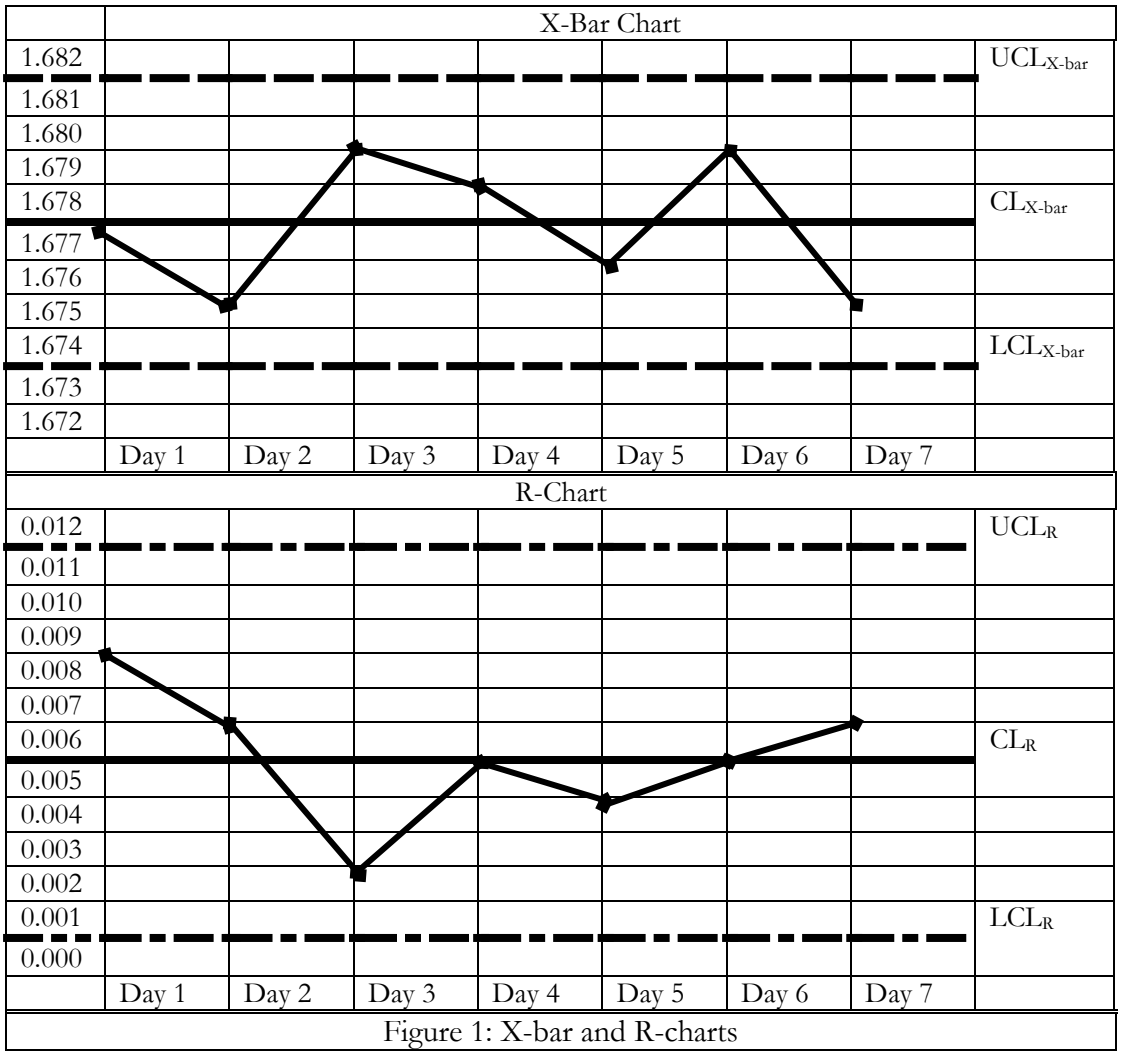


Figure 1: X-bar and R-charts

Example: A certain machining operation puts out finished bolts whose diameters have a mean $\mu = 0.440$ in. and $\sigma = 0.02$ inches.

a) compute the 3-sigma control limits for the **X-bar and R** charts based on a subgroup size of 16
b) what is the probability of not detecting a shift in **σ to 0.05** inches on the X-bar chart on the first subgroup sampled after the shift (assume the mean stays the same),

c) what is the average number of samples (i.e., the expected number) required to detect the shift in above in part (b) above. Give your answers to the nearest whole number.

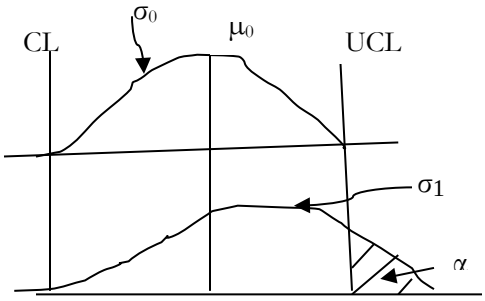
Solution: Given σ, μ are known or assumed

a). For X-Bar chart; Central line $CL_{\bar{X}} = \bar{\bar{X}}_0 = \mu$, Control limits: $\mu \pm A\sigma$, where A(for n=16)=0.75

$CL_{\bar{X}}$ =0.440, $UCL_{\bar{X}}$ =0.44+(0.75)(0.2)=0.455, $LCL_{\bar{X}}$ =0.44 - (0.75)(0.2)=0.425

For R-chart; Central line $CL_R = R_0 = d_2\sigma$, Control limits: $UCL_R = D_2\sigma$, $LCL_R = D_1\sigma$

b). σ_0=0.02, now due to the shift, $\sigma=\sigma_1$=0.05, [UCL, LCL]=[0.455,0.425] unchanged.



$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.05}{\sqrt{16}} = 0.125$$

Probability point fall within UCL $= \dfrac{UCL-\mu}{\sigma_{\bar{X}}} = \dfrac{0.455-440}{0.0125} = 1.2 = Z_1$

Probability point fall outside LCL $= \dfrac{LCL-\mu}{\sigma_{\bar{X}}} = \dfrac{0.425-0.440}{0.0125} = -1.2 = Z_2$

Probability points are within control limits=$\beta = \Phi(Z_1) - \Phi(Z_2) \Rightarrow \Phi(1.2) - \Phi(-1.2)$

$\Phi(-1.2) = 1 - \Phi(1.2)$;

From the standard normal table $\Phi(1.2) = 0.8849,\ \Phi(-1.2) = 0.1151$

Thus $\beta = 0.8849 - 0.1151 = 0.7698$

Probability detect a shift $= (1-\beta) = \alpha = 0.2302$

c). Given prob. of detecting a shift $= (1-\beta)$, then ARL= $\dfrac{1}{(1-\beta)} \Rightarrow ARL = \dfrac{1}{0.2302} \approx 5\, samples$

Example: A manufacturer of a certain subassembly item used in robot control, is having difficulty with a specific dimension on the unit. Two automatic screw machines produce the parts at a rate of 100 per hour each. Items from both machines are discharged into a single tote from which a **subgroup of 4** is selected every half-hour. The process appears to be in statistical control with the **3-sigma** control limits for the **X-bar** chart equal to **140 ± 5** units. The R-chart also showed control. The process follows the normal probability distribution.

a) Assuming no points are out of control and no runs are apparent, if the buyer of the subassembly components has design specification limits of **139 ± 10 units,** and 10,000 parts undergo this process, how many parts will be out of specification.

b) Suppose a sudden shift in the process average increases the mean to **0.25σ** but the process variance remains unchanged, what % of the product will be out of control on the manufacturer's control chart. What is the probability of detecting this shift by the 4th sample taken after the shift?

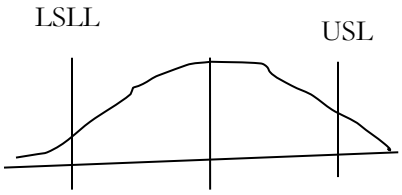
a). For the X-bar chart we have: μ±3σ=140 ± 5, [UCL, LCL] =[145, 135].

$$[UCL, LCL] = \overline{X} \pm A_2\overline{R} \Rightarrow A_2\overline{R} = 5$$

$$\overline{R} = \frac{5}{A_2(n=4)} = \frac{5}{0.729} \Rightarrow \overline{R} = 6.86$$

If the process is in control, then $\sigma = \dfrac{\overline{R}}{d_2} \Rightarrow \dfrac{6.86}{2.326} = 2.95$

Upper Spec Limit (USL)= 149 units, Lower Spec Limit (LSL)=129



Probability point fall within USL $= \dfrac{USL - \overline{X}}{\sigma} = \dfrac{149-139}{2.95} = 3.39 = Z_1$

Probability point fall outside LSL $= \dfrac{LSL - \overline{X}}{\sigma} = \dfrac{129-139}{2.95} = -3.39 = Z_2$

Probability points are within spec limits=β= $\Phi(Z_1) - \Phi(Z_2) \Rightarrow \Phi(3.39) - \Phi(-3.39)$

From the standard normal table $\Phi(3.39) = 0.9997,\ \Phi(-3.39) = 0.0003$

Thus $\beta = (0.9997 - 0.0003) = 0.9994$

Probability detect a shift $= (1-\beta) = \alpha = 0.0006$

Number of part out-of-control out of 10,000 parts = 10,000(0.0006)=6 (or 6 parts in 10,000)

b). $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} \Rightarrow \sigma = \sqrt{n}\sigma_{\overline{X}}$

$$0.25\sigma = 0.25\sigma_{\overline{X}}\sqrt{n}$$

UCL: $\dfrac{3\sigma_{\overline{X}} - \overline{X}}{\sigma_{\overline{X}}} = \dfrac{3\sigma_{\overline{X}} - 0.25\sigma_{\overline{X}}\sqrt{n}}{\sigma_{\overline{X}}} = \dfrac{[3 - 0.25(2)]\sigma_{\overline{X}}}{\sigma_{\overline{X}}} = 2.5 = Z_1$

LCL: $\frac{-3\sigma_{\bar{X}} - \bar{\bar{X}}}{\sigma_{\bar{X}}} = \frac{-3\sigma_{\bar{X}} - 0.25\sigma_{\bar{X}}\sqrt{n}}{\sigma_{\bar{X}}} = \frac{[-3-0.25(2)]\sigma_{\bar{X}}}{\sigma_{\bar{X}}} = -3.5 = Z_2$

Probability points are within control limits=$\beta = \Phi(Z_1) - \Phi(Z_2) \Rightarrow \Phi(2.5) - \Phi(-3.5)$

$\Phi(2.5) = 0.9938,\ \Phi(-3.5) = 0.0002$

Thus $\beta = (0.9938 - 0.0002) = 0.9936$

Probability detect a shift $= (1-\beta) = \alpha = 0.0064$

Let x = number of samples before detection, then x is a Geometric distribution given as:

$P(X = x) = p(1-p)^{x-1}$

Probability that the shift will be detected by the 4th sample can be explained as follows:
(Either detect at 1st sample) OR (not detect at 1st sample and detect at 2nd sample) OR (not detect at 1st sample and not detect at 2nd sample and detect at 3rd sample) OR (not detect at 1st sample and not detect at 2nd sample and not detect at 3rd sample and detect at 4th sample)

$(either\ 1st) \cup (not\ 1st \cap 2nd) \cup (not\ 1st \cap not\ 2nd \cap 3rd) \cup (not\ 1st \cup not\ 2nd \cap not\ 3rd \cap 4th)$

$P(X = x) = p(1-p)^{x-1}, \quad P(X \le 4) = \sum_{i=1}^{4} p(1-p)^{i-1}$

$0.0064 + (0.9936)(0.0064) + (0.9936)^2(0.0064) + (0.9936)^3(0.0064) = 0.0253$

$ARL = E(X) = \frac{1}{p} = \frac{1}{0.0064} = 156.25 \approx 157$

Probability the shift is detected by 4th sample taken after the shift is **0.0253**, and **ARL=157**

Attributes Control Chart

Defective (nonconformance). In a restricted technical sense, this refers (nonconformance) to an item that does not conform to specifications in some respect. The value of nonconformance is measured either as number nonconforming np, or fraction nonconforming, p.

Defect (nonconformities). A defect is nonconformity to some specification (a defective item has one or more defects). Depending on the nature and severity of the defect, it is quite possible for a unit to contain several nonconformities and still be classified as conforming. For example, the count of the number of defective welds in each length of a pipeline may render the pipeline conforming or nonconforming depending on whether the number of defective welds found exceed a critical value say c. The count of the number of discolorations on a roll of carpet of a given area could render the roll as conforming or nonconforming depending on the value of c. The number of defects (or nonconformities) in a unit is well modeled by the Poisson distribution since the interest is really on the occurrences rather than non-occurrences of events. Also, when the inspection unit is the same for each sample then there is an equal area of opportunity for the occurrence of nonconformities. The same assumption would hold for the average number of nonconformities per unit namely, u.

N: number of items in a lot or batch, n: number of items in a sample

m: number of defective pieces (i.e., pieces not conforming to specifications) in each sample (m < n)

p: fraction nonconforming; in a sample p = m/n

p': true (population) process average fraction nonconforming of a product submitted for inspection

c: number of nonconformities in a unit.

u: average number of nonconformities per unit

5.1 np-charts (constant sample size n)

The control limits are: $UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}, \quad CL = n\bar{p}, \quad LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

Example: Consider the following for an np chart. In 20 lots inspected, the number of nonconforming items found was 60. If each sample, n, taken out of the 20 lots contains 200 units, construct an np chart and estimate the process fraction nonconforming p'. Based on this value of p', what is the probability that the 21st lot inspected will have no more than 6 nonconforming items.

No. of lots = K = 20, no. of nonconforming items in 20 lots = d = 60, n = sample size=200

$n\bar{p}$= 60/20 = 3, $\bar{p}$ = 3/200 = 0.015. $[UCL, LCL] = 3 \pm 3\sqrt{3(1-0.015)} = 3 \pm 5.157 = [8.517, 0]$

(a) 3σ limits for the np chart, CL=3, UCL, LCL= [8.517,0]

(b) estimate for p' =$\bar{p}$ = 0.015 or 15%

(c) if p' = 15% what is the probability that the 21th lot will contain no more than 6 nonconforming items?

Based on the Poisson approximation, p'= np = (200) (0.015) = 3.

From the table of cumulative Poisson probability, **$P(c \leq 6 \mid \lambda=3) = 0.966$**

After establishing the control limits suppose now we take data from the lots of size 200 units for the next seven days (days 22 through 28) with the data as shown on table 3. Plot the np control chart and indicate whether the process is under control.

Day	Sample (n)	Number nonconforming (np)
22	200	6
23	200	2
24	200	4
25	200	3
26	200	5
27	200	1
28	200	4

Table 3: No. of nonconforming items for given lots

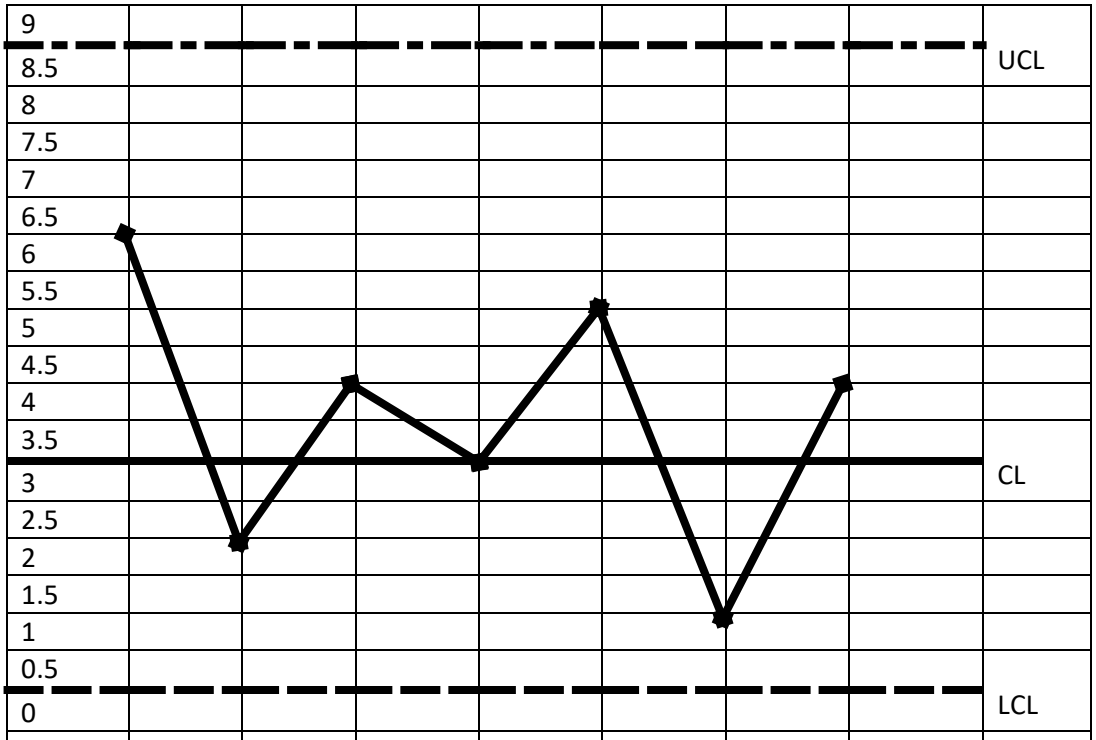


Figure 2: np chart based on lots for days 22-28

A look at the control chart shows that all the points fall within the control limit, indicating that the process is still under control

5.2 p-Chart (for varying sample size n)

Suppose in the previous example, it was decided to set the proportion nonconforming at p' = 0.015. Also, assume that because of differences in shift output, the sample size for each subgroup is now different going forward. This means that since the control limit for the p-chart depends on the sample size, it cannot be known until after the sample has been taken. This would result in different, but dynamic, control limits depending on the size of n. Let the data in table 4 represent the nonconforming items for days 29 through 31.

Day	Sample (n)	Number nonconforming (np)	Proportion (p)
29	250	5	0.020
30	300	3	0.010
31	220	4	0.018

Table 4: Number of nonconformance for a given production period

For p-charts: **Central line = CL= p',** *Control limits* $= p' \pm 3\sqrt{\frac{(p')(1-p')}{n}}$

In this scenario, the control limits for p chart are given by:

$$UCL_p = p' + \frac{3\sqrt{p'(1-p')}}{\sqrt{n}} = 0.015 + \frac{0.365}{\sqrt{n}}$$

$$LCL_p = p' - \frac{3\sqrt{p'(1-p')}}{\sqrt{n}} = 0.015 - \frac{0.365}{\sqrt{n}}$$

i. for day 29, control limits are: $0.015 \pm 0.365/\sqrt{250} = 0.015 \pm 0.023$, $p_{29}=0.020$

ii. for day 30, control limits are: $0.015 \pm 0.365/\sqrt{300} = 0.015 \pm 0.021$, $p_{30}=0.010$

iii. for day 31, control limits are: $0.015 \pm 0.365/\sqrt{220} = 0.015 \pm 0.025$, $p_{31}=0.018$

Based on the control limits and the proportion nonconforming for each day, we can conclude that no out of control condition exists for all three days since the proportion nonconforming for each day fell within each day's control limits.

Note an important difference between the np and the p chart. For the np chart, the sample size is constant, but they vary for the p chart. **This means that the control limits for the p-chart is dynamic while those for the np-chart are static.**

5.3.1 Problems introduced by variable subgroup size

The larger the subgroup, the more likely it is that the proportion nonconforming for each subgroup will be close to the universe or population value; hence, p varies inversely with $\sqrt{n}$. The practical difference between an X-bar and a p-chart is that because most measurement used for the X-bar chart is for control purposes, it is easy to keep the sample size constant. Many p charts however are taken for purposes other than control chart and hence the subgroups tend to be vary. Thus, when subgroup size varies, a decision must be made about the best way to show control limits on the p-chart.

5.3.2 How to handle varying subgroup size for the p-chart

i) Compute new control limits for every subgroup and show fluctuating limits on the chart.

ii) Estimate the expected subgroup size for future use. Compute one set of limits based on this average. However, whenever a point from a small subgroup falls outside of the control limits, recomputed the control limits based on the subgroups sample size to see whether it is indeed outside of the limits.

iii) Draw several sets of control limits on the chart based on the different subgroups. One approach will be to use the following three sets of limits:

a. A set based on the expected average subgroup size
b. A set based on the maximum expected subgroup size
c. A set based on the minimum expected subgroup size

5.4 c-charts: (constant unit of production, Poisson approximation)

The X-bar and R charts are typically applied to any quality characteristic that is measurable. The control chart for p may be applied to any results of inspection that accepts or rejects individual items or products. The control chart for nonconformities, the c-chart, usually has a restricted usage. Hence, it is important to carefully determine whether the conditions are right for the use of the c-chart. A nonconforming product is one that does not meet spec in some way. Each instance of the product's lack of conformity to spec is a nonconformity or defect. Hence, every product has one or more nonconformities or defects. In those instances where it is appropriate to talk about the count of the number of nonconformities (defects) in each article or product or in each group of an equal number of similar articles, it may be reasonable to use a c-chart or a u-chart. For a c-Chart, the subgroup could be two or more articles. For c charts, the area of opportunity to detect defects should be the same from unit to unit, i.e., equal opportunity for the occurrence of nonconformities

When the area of opportunity changes from subgroup to subgroup, then it is important to use a different type of chart that takes into consideration the differences in the area of opportunity.
In this case the u-chart is used where u represents the nonconformities per unit

When the area of opportunity changes from subgroup to subgroup, then it is important to use a different type of chart that takes into consideration the differences in the area of opportunity.
In this case the u-chart is used where u represents the nonconformities per unit. Ku-charts are used in lieu of the u chart when there are nonconformities per multiple units. E.g. production unit varies per day and the nonconformities is measured per a given unit produced (per 100 units produced).

$$\underline{Control\ Limits}: [UCL,\ LCL] = c \pm 3\sqrt{c},\ \ CL = c$$

$$note: The\ mean\ \mu_c = c,\ \ \text{var}iance\ \sigma_c^2 = c\ (by\ reason\ of\ the\ Poissson\ distribution)$$

Note that for the Poisson distribution, the mean and the variance are the same
Find the control limits for c chart for which c = 12. Also find the 0.995 and 0005 probability limits for a c-chart when c=12.0

a. $c \pm 3\sqrt{c}$

$c = 12.0$

$$12 \pm 3\sqrt{12} = 12 \pm 10.4 \begin{cases} 12.4 \Rightarrow ULC \\ 1.600 \Rightarrow LCL \end{cases}$$

b. Using Poisson tables and $\lambda = c' = 12$

for $P_a = 0.005$, c = 4.00, LCL = 4

for $P_a = 0.995$, c = 21.3, UCL= 21.3

c. What is the probability of points falling outside the limits above?

$P(x < 4) = 0.005$, $P(x > 21.3) = 1 - P(x \leq 21.3) = 0.005$

$\therefore$ P (points outside) = 0.005 + 0.005 = 0.01

Other Charts

6.1 Cumulative Sum Control (CuSum Charts)

The traditional Shewhart control charts are designed to control the Type I error. If the process is in control, then rarely will the points fall outside the 3-sigma limits. In practice, the Type II error could be a major concern since a poor-quality product could result. Some of the major differences between the Shewhart and the CuSum chart include the following:

The Shewhart control chart uses only information about the process contained in the last plotted point—ignoring information given by the entire sequence of points. While other control mechanism—such as, test of runs and warning control chart—limits attempt to incorporate information from the entire set of data points into the decision process, they have the tendency to reduce the simplicity and ease of interpretation of the Shewhart chart. On the other hand, the CuSum chart has been proposed as an alternative to the Shewhart chart for the following reasons:

a) The CuSum chart can protect against both errors of Type I and Type II.

b) It directly incorporates all the information in the sequence of sample values by using the cumulative sum of deviations from the target.

Example: Suppose n > 1 samples are collected, and $\bar{X}_m$ is the average of m^{th} sample. If μ_0 is the target for the process, then the CuSum control chart is developed by plotting the quantity S_m (or S'_m) against the sample number j, where S_m is the cumulative sum up to and including m^{th} sample and is defined as follows: $S_m = \sum_{j=1}^{m}\left(\overline{X_j} - \hat{\mu}_0\right)$ *or* $S'_m = \frac{1}{\sigma_{\bar{x}}}\sum_{j=1}^{m}\left(\overline{X_j} - \hat{\mu}_0\right)$. Also $\hat{\mu}_0$ is the estimate of the in-control mean and $\sigma_{\bar{x}}$ is the known (or estimated) standard deviation of the sample means. The choice of which of these two quantities (S_m or S'_m) is usually determined based on ease of implementation or application. In any case, if the process remains in control and centered at $\hat{\mu}_0$, the CuSum plot will show variation in a random pattern centered about the mean. If the process mean shifts upward, the charted CuSum points will eventually drift upwards, and if the mean shifts downwards, the drift will be downwards.

c) Because CuSum charts combine information from several samples, they are more effective in detecting small process shifts. Typically, CuSum charts can detect shifts of about 1-sigma or less than can the traditional Shewhart charts.

d) They are equally effective when n=1. This makes CuSum charts a useful tool for the control of futuristic manufacturing cell where the optimum lot size is anticipated to be 1 (theoretically).

In CuSum charts, starting from a given point, all subsequent plots contain information from the whole of the observations. The statistic to be summed may be individual measurements, sample means, sample ranges, sample fraction nonconforming, sample number of defects, etc. In general, a cumulative statistic is more efficient than single-sample data points in control chart applications. The

idea of cumulative sum control is based on the method of sequential likelihood ratio test developed by Wald (1947); whereby, as each new sample point (in this cumulative sample point) becomes available, a test is conducted to determine whether the parameter of interest deviates by at least a specified amount from the target value $\hat{\mu}_0$.

For the CuSum chart to be useful, some objective criterion must be developed to determine when a trend can be assumed to occur because of a shift in the process. For processes that are normally distributed, this is accomplished by placing a "V mask" of internal angle 2θ a distance "d" in front of the last plotted point on the CuSum chart. If a previously plotted point falls beyond the arms of the V-mask, the hypothesis that the process $\mu = \mu_0$ is rejected. The V-mask is like the V produced by the famous Wald's two-sided sequential sampling plan developed by Wald (1947).

Because the computation of the parameters of the CuSum chart are somewhat involved and required more in-depth and advanced knowledge of process control, we will limit our discussion only to the benefits of such a chart at this time. Information about the construction is available in advanced test in SPC/SQC some of which ae included in the reference.

6.1.2 Construction CuSum Charts

When the process is in control at the target value μ_0, the cumulative sum s_i should vary randomly around zero. However, if the mean shifts, say $\mu_1 > \mu_0$, then an upward drift (or positive drift) in s_i would be observed. Conversely, if $\mu_1 < \mu_0$, then a downward or negative drift in s_i will develop. Thus, if a trend up or down develops, then this is evidence of a shift in μ.

The v-mask is used as a formal decision procedure for determining whether the process is in or out of control. The parameters of the V-mask are

1). the lead distance 'd';
2). the angle 'θ' between the upper and lower control limits. Total angle of the V-mask =2θ;
3). α: the probability of a false alarm, i.e., the probability of incorrectly concluding that a shift has occurred (type I error) when it did not;
4). β: the probability of failing to detect a shift in the process mean μ– (type II error) when in fact the shift occurred;
5). δ(delta): the amount of shift in the process mean that we wish to detect, expressed as a multiple of the standard deviation of the data points (in this case are the sample means).
6). k: the scale factor relating the vertical scale unit to the horizontal unit. For a process with sigma $=\sigma_x$, then it is recommended that $\sigma_x \le k \le 2\sigma_x$.
Note that ($\sigma_x \le k \le 2.5\sigma_x$) is also acceptable, with $k = 2\sigma_x$ being preferred.
7). ARL, the number of sample points that must be plotted before an out-of-control is detected. ARL = 1/p, where p is the probability of detection.

Note that S_m and $-S_m$ represent the upper (UCL), and the lower (LCL) control limits respectively. Often, for reasons of compactness or if we wanted to shrink the vertical axis, then one can choose an arbitrary scaling factor $k = 2\sigma_x$. This means plotting (m versus S'_m). The appropriate V-mask is then of lead distance "d", angle 2θ', where d is as previously defined.

$\sigma = \bar{R}/d_2$: $S_m = +\left\{\frac{\sigma^2}{\delta}\ln\left[\frac{\beta}{\alpha/2}\right]+\frac{\delta}{2}m\right\}$, $S'm = \frac{S_m}{\sigma_{\bar{X}}}$

$$d = \left(\frac{2\sigma^2}{\delta^2}\right)\ln\left(\frac{\beta}{\alpha/2}\right) = \frac{2}{\delta^2}\ln\left(\frac{1-\beta}{\alpha}\right) \quad , \quad \tan\theta = \left(\frac{\frac{\delta\sigma}{\sqrt{n}}}{2K}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{\delta\sigma}{2K\sqrt{n}}\right)$$

Note that θ is typically assumed to lie between 30˚ and 60˚. If we decide we want to quickly detect a shift as large as one-sigma then we will set δ =1

6.1.3 Procedures for Developing CuSum Chart (V-Mask)

1. Specify μ_0, n (subgroup size), α, β, and number of subgroups;
2. Compute $\bar{X}_i$ for each subgroup;
3. compute S_m; 4. Compute d; 5. Compute k ;6. Compute; 7. Compute θ;
8. indicate d, UCL, LCL on the chart;
9. Construct the angle θ, noting that θ radiates from the last subgroup value;
10. Plot S_m versus number of subgroup m. Note that each arm subtends and angle θ. Thus, the angle between the two arms is 2θ.

Figure 3 is a typical CuSum chart with V-masks. The V-lines are [S_m, $-S_m$], where S_m is the m^{th} cumulative difference between the observation x_i and the mean. Note that θ is typically assumed to lie between 30˚ and 60˚. Where α and βare the type I and type II errors respectively. Note: If β is small (i.e., β<<<1), then the lead distance 'd' is: $d = \left(\frac{2\sigma^2}{\delta^2}\right)\ln\frac{\alpha}{2}$

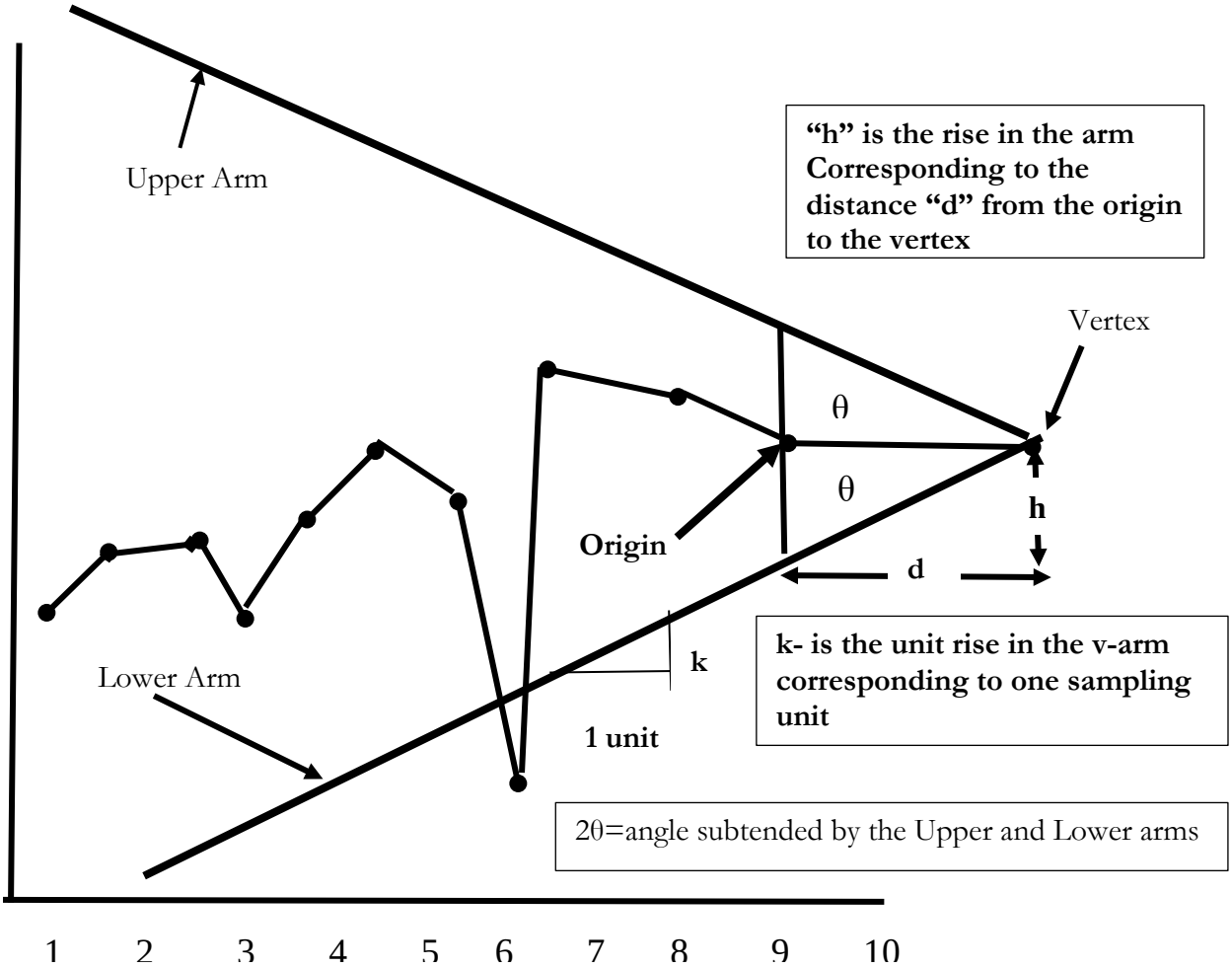


Figure 3: Typical V-mask for a CuSum Chart

6.2 Individual Measurements and Moving Range Control (I and mR) Chart

In statistical quality control, the individual/moving-range chart (I-mR chart) is a type of control **chart** used to monitor variables data from a process for which it is impractical to use rational subgroups. The individual measurement and the moving range (I-mR) control charts arise in situations where the process output is limited to one output per production period. In this type of situation, it is more difficult to justify the normality assumption because the sample size n=1, unless of course it is known a-priori that the underlying process is normally distributed. If the normality assumption can be made, then the I-chart is a plot of individual measurements. However, the range chart is a plot of the moving range values obtained by the difference between consecutive individual parameter measurements. Before interpreting the individual chart (I chart) to determine if the process is in

control, it is important to first examine the moving range chart (mR chart) to determine if the process variability is stable. If the mR chart is not in control (or stable), then the control limits on the I chart are not accurate. The moving range chart plots the moving ranges which measures process variability. Recall that for a process to be considered in control the X-bar or (I chart) and the variability chart must show control. Table 5 represents data about a critical shaft diameter (unit is inches) that is under investigation. The shaft production cycle is one week. Figure 4 shows a plot of the individual and moving range charts. The plots show the process is in control.

Week #	X (in)	R(in)
1	0.877	-
2	0.876	0.001
3	0.876	0
4	0.874	0.002
5	0.88	0.006
6	0.878	0.002
7	0.875	0.003
Σ	6.136	0.014
AVG	0.87657143	0.002333

Table 5: Data about the critical parameter of a shaft diameter

$\bar{X} = (\Sigma X)/7 = 6.136/7 = 0.8766$ inches

$\bar{R} = (\Sigma R)/6 = 0.014/6 = 0.0023$ inches, n = 2

I-chart: CL = $\bar{X} = 0.8766$

$UCL_X, LCL_X = \bar{X} \pm A_2 \bar{R}$ (for A_2, n = 2)

$= 0.8766 \pm (1.88)(0.0023) =$ **0.8809, 0.8723**

mR-chart: CL = $\bar{R} = 0.0023$

$UCL_R = D_4 \bar{R} = (3.267)(0.0023) =$ **0.0075**

$LCL_R = D_3 \bar{R} = (0.000)(0.0023) =$ **0**

Figure 4 shows the control charts (I chart and mR chart) of the data on table 3. The plot shows that the process is under control. However, the data point for week 5 seems a bit out of place and monitoring of the chart should continue until it shown through detailed analyses that it not due to assignable causes.

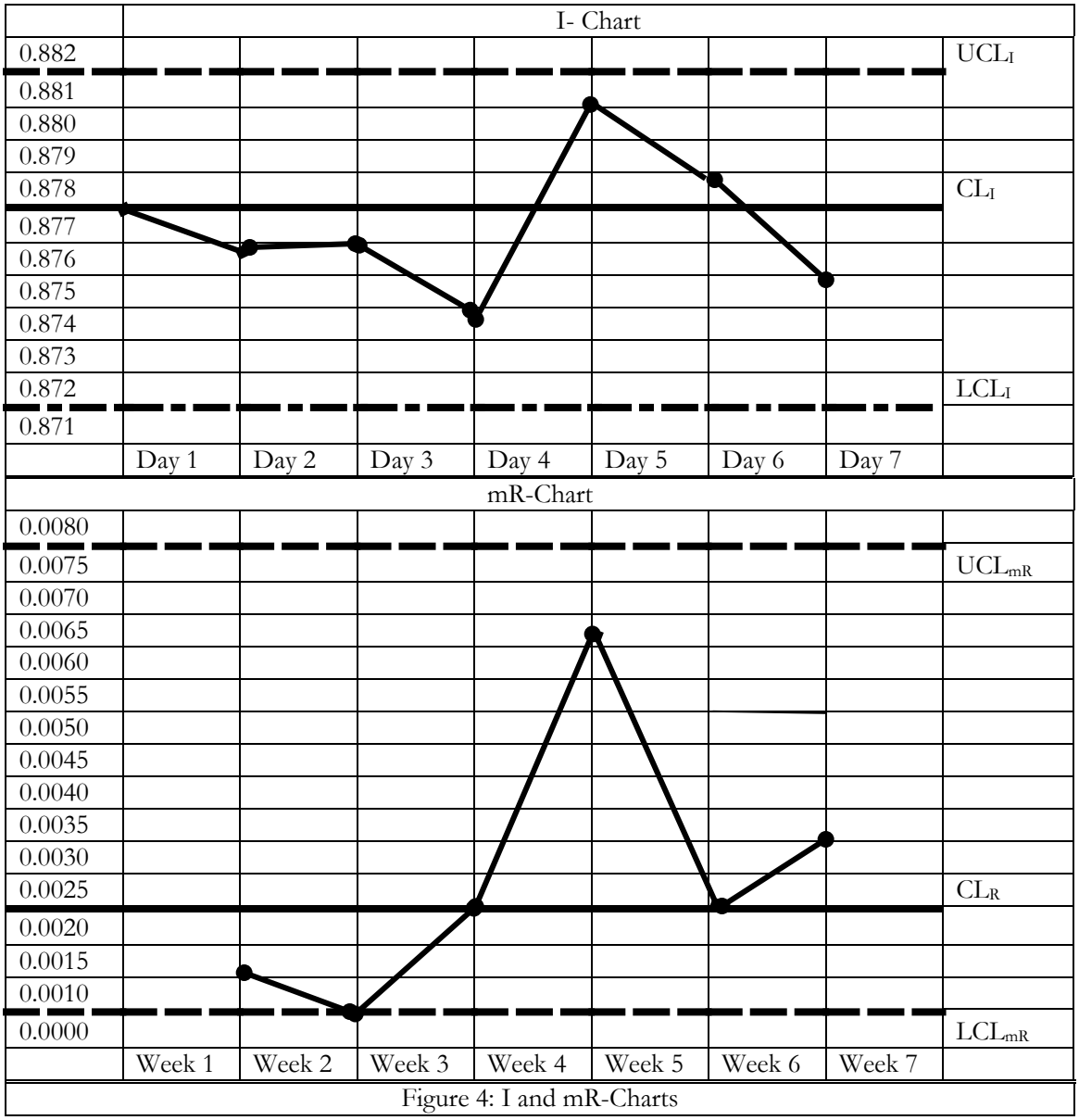


Figure 4: I and mR-Charts

Interpreting Shewhart Control Charts

An out of control process indicates the presence of non-random variation. Non-random variation is caused by definite, specific causes that are called assignable causes. These assignable causes make the process go out of control or become statistically unstable. The presence of an out of control condition should prompt further investigation. It is important to find the assignable causes and act to remove them. Once assignable causes are removed, and only random variation due to common causes

remains, the process will become stable and return to an in-control condition. A very useful purpose of a control chart is to judge the impact of a process improvement program initiated by management through the process improvement team. If the process changes work, then new control limits are calculated, and the process can be monitored into the future for the appearance of any special causes. As long as the points are within control limits, the process is "in control." But, what does an out of control process indicate? Many believe that an out of control process produces defective parts. That's not always true. A control chart may show points outside of both the lower control limit (LCL) and the upper control limit (UCL), indicating that the process is out of control. However, the points may remain within the specification limits. If the measurements of such points indicate that they are within specifications, then parts are not nonconforming and are said to conform to specifications. Some of the common rules of thumb used to monitor and interpret shifts on control charts are shown

RULE #	**Description**	**Possible Problem Indicated**
Rule 1	One point is more than 3-standard deviations from the mean on either side.	Plotted point out of control
Rule 2	Nine (or more) points in a row are on the same side of the mean.	Some prolonged bias exists
Rule 3	Six (or more) points in a row are continually increasing (or decreasing).	A trend exists
Rule 4	Fourteen (or more) points in a row alternate in direction, increasing then decreasing	This much oscillation is beyond noise.
Rule 5	Two (or three) out of three points in a row in the same direction.	A trend exists
Rule 6	Four (or five) out of five points in a row in the same direction.	A strong tendency for samples to be out of control. A trend exists.
Rule 7	Fifteen points in a row are all on either side of the mean	A trend exists
Rule 8	Eight points in a row exist and the points are in both directions from the mean.	Jumping from above to below in this fashion is rarely random

Table 4: Common Rules of Thumb for Interpreting out-of-control Conditions on Shewhart 3σ charts

Process Capability Evaluation

In any discussion about process optimization, a major issue of interest is how to determine the ability of the process to conform to the imposed tolerance spread. It is difficult to optimize a process simply based on the performance of the process because process performance represents the actual behavior of the process, including both the random (natural process variability) and assignable causes. On the other hand, the capability of the process is the behavior of the process when the only effects present are random effects resulting from the inherent natural variability of the process commonly known non-assignable causes. The information about process performance is important

only to the extent that such information is useful for understanding the performance envelop, or the overall performance range. Thus, while the two metrics (performance and capability) are related, they are different in several important respects.

Process Performance is the day-to-day behavior of the process which includes all the random (natural variability) as well as the assignable cause effects. Thus, any information about process performance would yield suboptimal results because such information could be based on a local optimum rather than the global optimum. In other words, process performance is the totality of everything the process can do. Process capability, on the other hand, is an estimate of the best a process can do after all the effects of assignable causes and controllable variation have been removed. Most experts believe that the performance surface of any given process is convex and, as such, it is possible to determine the global optimum in terms of the process decision variables. The difference between process performance and process capability is that with process capability analysis, there is a huge potential for significant process improvement that can be realized through better trouble shooting, better understanding of the process behavior, better training of operators and personal, use of proper evaluation tools, and so on. One of the major reasons many firms have great difficulty satisfying even their own in-house tolerance requirements is that it is sometimes difficult to see the long-term payoff of taking a disciplined approach to process design and optimization which is the essence of process capability. Some of the compelling reasons for determining the capability of the process include; i). Cost trade-offs, ii). Elimination of Restrictive Tolerances, Targeting Information for Process Improvement.

i) Cost trade-off

Process capability is important in order to make intelligent trade-off decisions between scrap, rework, and the cost for process improvements especially if design requirements exceed process capabilities.

ii) Elimination of Restrictive Tolerances

One of the major problems between design and manufacturing is that design furnishes the tightest tolerance possible so that in the event manufacturing "messes up", the product would still be within specification. This problem of specification or tolerance "over padding" is unwarranted and introduces delays and waste especially with the trend towards shorter time to market.

iii) Targeting Information for Process Improvement

One of the off-shoots of embarking on a process capability study is the potential for maximum payoff by providing targeting information on decision variables and parameters to could lead to process improvement. Process Capability Index $\{C_p\}$

C_p =Tolerance width (sigma) / Process capability (sigma)

=Tolerance/ {6x (standard deviation)}

= $2\Delta/(6\sigma)$:(2Δ difference between upper spec limit and lower spec limit)

= {USL-LSL}/(6σ): **USL= Upper Spec Limits, LSL = Lower Spec limits**

The problem with the C_p index is that it is difficult to tell whether the process is properly centered. To overcome this problem, a different metric has been proposed, namely, the C_{pk} index. The C_{pk} index is based on one side of the tolerance interval. In addition, in those instances where bilateral tolerance limits are specified (i.e., plus, minus), it is possible that the desired variation may not be equal in both directions with the target value. **Note that $\mu = \overline{\overline{X}}$ when the process is in control**

C_{pu} $=(USL-\mu)/3\sigma$

C_{pl} $= (\mu- LSL)/3\sigma$

C_{pk} $=\min \{C_{pu}, C_{pl}\}$

Use of the Process Capability Index provides an objective means of comparing processes, for judging the effectiveness of improvement programs, and for selecting appropriate processes where alternatives are possible. Based on the experiences of many organizations who have successfully implemented Statistical Process Control, the following rule of thumb/criteria have been established.

C_p	C_{pk}	Usage	Decision
>1.67	>1.33	Critical Characteristic.	Good. Operator acceptance.
1.33-1.67	1.00-1.33	Major Characteristic	Ok. But requires process audit and inspection
1.0-1.33	<1.00	Minor Characteristic.	Unacceptable. May use on-machine control chart
Below 1.00	-	Inadequate	Do not use

Table 5: C_p and C_{pk}: Uses and Decisions

The decision to specify use of on-machine control charts for C(p) between 1.00 and 1.33 does not preclude their use on more capable processes. It merely underscores the need for close control as near to the generation of the characteristics as possible, to provide maximum assurance that process drift or other assignable cause situations do not "creep in" unannounced and begin to generate nonconforming product.

Summary

Assuring process and product quality is very important to every organization that is seeking continuous improvement with respect to service and product conformance. Quality control includes service quality given to customer, company management leadership, commitment of management, continuous improvement, fast response, actions based on facts, employee participation and a quality driven culture. The main objectives of the product and process assurance are the control of material supply chain, internal and external rejections, warranty claims, action based on facts with strong employee participation. The development and use of performance indicators and metrics are vital and lead—directly or indirectly—to customer requirements and satisfaction. Some of benefits of SPC or SQC include

1). It provides a means of detecting error at inspection. 2). More uniform quality of production. 3). Improved relationship with the customer.4). Reduction in inspection costs. 5). Reduces the number of rejects and saves the cost of material. 6). It provides a basis for attainable specifications. 7). It points out the bottlenecks and trouble spots. 8). Helps in establishing the capability of the manufacturing process.

Reference

1. ASTM Manual on Quality Control of Materials. Prepared by ASTM Committee E-11 on Quality Control of Materials. Special Tech. Publ. 15-C. Am. Soc. Testing Materials, Philadelphia, PA 1951.
2. Deming, W E (1982) Out of the Crisis: Quality, Productivity and Competitive Position, MIT Press, Cambridge, Mass.
3. Deming, W E (1975) "On probability as a basis for action", Am. Statistician, 29(4), 146–52
4. Duffy, G. L., (2013), The ASQ Quality Improvement Pocket Guide: Basic History, Concepts, Tools, and Relationships, ASQ Quality Press, pages 62–65.
5. Engineering Statistics Handbook 6.3.2, NIST/SEMATECH e-Handbook of Statistical Methods National Institute of Standards and Technology, Dec 2006
6. Grant, E. L., Leavenworth, R. S. (1996). Statistical Quality Control, 7th ed. McGraw Hill, NY.
7. Grant, E.L (1952). Statistical Quality Control. McGraw-Hill, Inc., New York, NY.
8. Kaoru Ishikawa (1976) Guide to Quality Control, Asian Productivity Organization, Hong Kong: Nordica International.
9. Moen, R Nolan, W., Provost, L (2012) Quality improvement through planned experimentation, 3rd ed, McGraw-Hill Education, New York, NY
10. Montgomery, D.C. (2012). Statistical Quality Control 7th ed. Wiley and Sons, NJ
11. Nelson, L. S. (1984). The Shewhart control chart - tests for special causes. "Technical Aids," Journal of Quality Technology, 14, 238-239.
12. Oakland, J (2003) Statistical Process Control, 5th ed., Butterworth and Heinemann, NY.
13. Shewhart, W.A. (1931) Economic Control of Quality of Manufactured Product. D. Van Nostrand Company, Inc., New York, NY.
14. Shewhart, W A (1939) Statistical Method from the Viewpoint of Quality Control, Dover Publications, Mineola, NY.
15. Suarez J. Geráld (1992) "Three Experts on Quality Management: Philip B. Crosby, W. Edwards Deming, Joseph M. Juran" Department of the Navy Office of the Under Secretary of the Navy, Total Quality Leadership Office, TQLO Publication No. 92-02.
16. Taguchi, G., Elsayed, E., Hsiang, T (1989) Quality Engineering in Production Systems. McGraw-Hill, NY
17. Wald, A. (1947) Sequential Analysis. Wiley and Sons, New York, NY.
18. Wheeler, Donald J. (2000). Understanding Variation. SPC Press, Inc., Knoxville Tennessee
19. Wheeler, D. J. (2000) Normality & the Process-Behavior Chart, SPC Press, Inc., Knoxville Tennessee.
20. Wise, Stephen A. & Fair, Douglas C (1998). Innovative Control Charting: Practical SPC Solutions for Today's Manufacturing Environment. ASQ Quality Press, Milwaukee, WI

