A SunCam online continuing education course

WHAT EVERY ENGINEER SHOULD KNOW ABOUT RELIABILITY ENGINEERING I

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Contents

Introduction

Reliability consideration is playing an increasing role in virtually all human endeavors more specifically in engineering designs. As the demand for systems that perform better and cost less increase, there is a concomitant need and perhaps even requirement to minimize the probability of component and/or system failures. Such failures, if not properly mitigated, could lead to increased cost and inconvenience, or could threaten individual and public safety.

1.1 Definition of Reliability

Reliability is defined as: the probability that when operating under stated operating conditions, the system (facility or device or component) will perform its intended function adequately for a specified period of time. In actual practical considerations, reliability may be viewed or defined differently for a given system or components etc. However, the system or unit of interest typically determines what is being studied and there is usually no ambiguity. Based on this definition, we can surmise the following about reliability:

- It is a Probability (conditional probability)
- It is a design parameter (you can specify its value just as you can strength, weight)
- It is time dependent (it changes value with time and age)
- It is dependent on the operating conditions and the environment

In defining reliability, no distinction is made between failure and failure types. There is a great deal of concern not only with the probability of failure but also the potential consequences of the different modes of failures. In reliability analysis, attention is focused not just on economic losses or inconvenience but also on the impact of failures on public safety and well being. For example, a home appliance manufacturer must be concerned not only with frequent failures and the cost of maintenance, but the fact that such failures could become a safety hazard due to shock or electrocution. For a system such as an aircraft, there is less distinction between reliability and safety considerations. Overall, safety and reliability go hand in hand.

1.1.1 Performance and Reliability

The tradeoffs between performance and reliability are often subtle involving loading, complexity, etc. While performance is frequently improved through overdesign and overloading, high reliability requirement is often achieved sometimes by worst case design and most assuredly by determining the interference region between stress and strength. In other words, reliability is the probability that load (stress) is less than strength (capacity), i.e., **P(c>s).**

1.1.2 Trade-offs: Reliability versus Cost

In designing a race car, performance is the overriding goal. The designer must tolerate high probability of breakdown with high probability of winning the race. In the case of a commercial airline, safety and reliability are paramount, so performance and speed are sacrificed. For military aircraft, both performance and reliability are equally useful. Performance must be high or the number of losses during combat mission would be high. In situations other than life and death, reliability is viewed in terms of economics. While management is concerned about reliability, management is less concerned about the technical jargon surrounding reliability. So the best way to communicate the importance of reliability to management is in terms of dollars and cents.

1.1.3 Time Element of Reliability

The way in which time is specified can also vary with the nature of the system under consideration.

a). In an intermittent system, one must specify whether calendar time or number of hours of operation is the metric to be used (car, shoes, etc)

b). If the system operation is cyclic (switch, etc), then time is likely to be specified in terms of number of operations

c). If reliability is to be specified in calendar time, it may also be necessary to indicate the number of frequency of system stops and go's.

1.1.4 Operating Condition

a) Principal design loads(weight, electrical load)

b) Environmental conditions (Dust, salt, vibrations) and Temperature extremes

1.1.5 Other Performability Measures

In addition to reliability, other quantities used to characterize the performability of a system include:

- MTTF and Failure rate for repairable system
- System Safety
- Availability and
- Maintainability.

1.2 Definition of Failure

A system or unit is commonly referred to as having failed when it ceases to perform its intended function. When there is total cessation of function e.g., engine stops running, structure collapses etc, then the system has clearly failed. However, a system can also be considered to be in a failed state when its deterioration function is within certain critical region or boundary. Such subtle form of failure makes it necessary to define or determine, quantitatively, what is meant by failure. Typical failure types include: creep, degradation, catastrophic, intermittent, drift, fracture, crack, shock, etc.

As a result, the mathematical model of reliability can be quite complex because of the: different component probability distributions, complexity of the interference between stress and strength, environmental conditions and stresses, as well as variations in equipment use conditions.

Reliability Models

2.1 Parametric and Nonparametric Relationships

Define "**t**" **as** random variable representing the time to failure, and define "**T**" as the age of the system. If the failure density function is given by **f(t)**, then Prob**(t ≤ T)** is the probability of failure and is represented as **F(t)**. **F(t)** is also known as the distribution function of failure process. The nonparametric relationship between **f(t)** and **F(t)** is given as: $F(t) = \int_0^t f(s)ds$

$$\textit{The Reliability function is given by}: R(t) = 1 - F(t) = 1 - \int_0^t f(s)ds = \int_t^\infty f(s)ds$$

If 't' is a negative exponential random variable with a constant parameter θ (The Mean Time to Failure or MTTF), we can use probability to show the relation between **f(t)**, **F(f)**, and **R(t)**, that is:

$$f(t) = \frac{1}{\theta}\exp\left(\frac{t}{\theta}\right)$$

$$F(t) = \frac{1}{\theta}\int_0^t \exp\left(-\frac{s}{\theta}\right) = -\frac{\theta}{\theta}\left[\exp\left(-\frac{s}{\theta}\right)\right]_0^t = -1\left[\exp\left(-\frac{s}{\theta}\right) - 1\right] = 1 - \exp\left(-\frac{t}{\theta}\right)$$

$$R(t) = 1 - F(t) = \exp\left(-\frac{t}{\theta}\right)$$

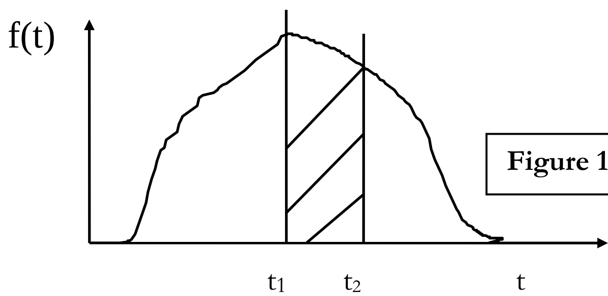


Figure 1: The failure Density Function

2.2 Failure Density Function

This gives a relative frequency of failure from the viewpoint of initial operation at time t =0. The failure distribution function F(t) is the special case when t_1 = 0 and t_2 =t, i.e. $F(t_2)$= F(t)

2.2.1 Failure Probability in the interval (t_1,t_2)

$$\int_{t_1}^{t_2} f(t)dt = \int_0^{t_2} f(t)dt - \int_0^{t_1} f(t)dt = F(t_2) - F(t_1) = [1 - R(t_2)] - [(1 - R(t_1)] = R(t_1) - R(t_2)$$

2.3 Reliability of Component of age t

The reliability (or survival probability) of a fresh unit with mission duration x is by definition:

$R(x) = \overline{F}(x) = 1 - F(x)$,

where F(x) is the life distribution of the unit. The corresponding conditional reliability of the unit of age t for an additional time duration x is given by:

$$\overline{F}(x/t) = \frac{\overline{F}(t \cap x)}{\overline{F}(t)}; \; where \; \overline{F}(x) > 0, \; but \; \overline{F}(t \cap x) = \overline{F}(t+x),$$

that is, the total life of the unit up to time (t+x) $\therefore \overline{F}(x/t) = \dfrac{\overline{F}(t+x)}{\overline{F}(t)}$

Similarly, the conditional probability of failure during the interval of duration x is F(x/t) where: F(x/t) = 1 - $\overline{F}$ (x/t) by definition, But $\overline{F}(x/t) = \dfrac{\overline{F}(t+x)}{\overline{F}(t)}$

$$\text{Hence: } F(x/t) = 1 - \frac{\overline{F}(t+x)}{\overline{F}(t)} = \frac{\overline{F}(t) - \overline{F}(t+x)}{\overline{F}(t)}$$

2.4 Conditional Failure Rate (Hazard Function)

The conditional failure probability is given by F(x/t). Hence the conditional failure rate is given by:

$\dfrac{F(x/t)}{x}$, That is, $\dfrac{F(x/t)}{x} = \dfrac{1}{x}\left[\dfrac{\overline{F}(t) - \overline{F}(t+x)}{\overline{F}(t)}\right] = \dfrac{1}{x}\left[\dfrac{R(t) - R(t+x)}{R(t)}\right]$

The hazard function is the limit of the failure rate as the interval (x in this case) approaches zero. The hazard function is also referred to as instantaneous failure rate because the interval in question is very small. The hazard rate is a function that describes the conditional probability of failure in the next instant x (or Δt) given survival up to a point in time, t.

$$h(t) = \underset{x \to 0}{Limit}\left[\frac{R(t) - R(t+x)}{x}\right]\left[\frac{1}{R(t)}\right] \Rightarrow \underset{x \to 0}{Limit}\left[\frac{R(t+x) - R(t)}{x}\right]\left[-\frac{1}{R(t)}\right]$$

$h(t) = -\dfrac{1}{R(t)}\left[\dfrac{d}{dt}R(t)\right]$. **Note: If f(t) is the exponential then and only then is h(t)=λ or 1/θ**

Note: $R(t) = 1 - F(t) \Rightarrow \dfrac{d}{dt}R(t) = -\dfrac{d}{dt}F(t) = -f(t)$

$$\therefore h(t) = -\frac{1}{R(t)}\left[\frac{d}{dt}R(t)\right] = -\frac{1}{R(t)}\left[-\frac{d}{dt}F(t)\right] = -\frac{1}{R(t)}\left[-f(t)\right] = \frac{f(t)}{R(t)} \Rightarrow f(t) = h(t)R(t)$$

This parametric relationship between the hazard function, the reliability function and the density function is perhaps the most important relationship in reliability work. We can explore this further to establish a more robust relationship among these functions to make it easy to determine the reliability function or the density function once the hazard function is known or given.

$$h(t) = \frac{f(t)}{R(t)} = -\frac{1}{R(t)}\frac{d}{dt}R(t) = -\frac{d}{dt}\left[\ln R(t)\right]$$

$$R(t) = \exp\left[-\int_0^t h(\tau)d\tau\right] \Rightarrow f(t) = h(t)\exp\left[-\int_0^t h(\tau)d\tau\right]$$

2.5 Mean Time To Failure (MTTF and MTBF)

The mean time to failure is the expected value of the time to failure. By definition, the expected value of a density function 'y' is the following:

$$E(y) = \int_{-\infty}^{\infty} xf(x)dx, \quad -\infty < x < \infty$$

For the mean time to failure or expected time to failure or the average life of the system we have;

$E(T) = \displaystyle\int_0^{\infty} R(s)ds, \quad 0 \le T \le \infty$ = MTTF=expected life of the system

By proper transformation and integration (integration by parts), the mean time to failure is:

$E(T)=\int_0^\infty R(s)ds=\int_0^\infty sf(s)ds$, How? By Integration by parts.

$$E(T)=\int_0^\infty R(s)ds, 0\le T\le\infty,\ \ Let: \int udv = uv-\int vdu$$

Let u = R(s), dv = ds ⇒ then v = s, and du = d (R(s))ds = - f(s)ds

$$\Rightarrow \int_0^\infty R(s)ds = \mathrm{sR(s)}\Big|_0^\infty + \int_0^\infty sf(s)ds$$

When s =0, R(0) =1, sR(s)=0, at s= ∞, R(∞)=0, hence:∞(0)=0

$$\therefore\ E(T)=MTTF=\int_0^\infty R(s)ds=\int sf(s)ds$$

Example with exponential density function

$$f(t)=\frac{1}{\theta}\exp(-t/\theta) \qquad Note: if\ \ f(t)=\lambda e^{-\lambda t};\ then\ and\ only\ then\ h(t)=\frac{f(t)}{R(t)}=\lambda=\frac{1}{MTTF}$$

$$E(T)=MTTF=\int_0^\infty t\left(\frac{1}{\theta}\right)\exp(-t/\theta)dt=\left(\frac{1}{\theta}\right)t\exp(-t/\theta)dt$$

$$\text{Using Integration by parts}: \int udv=uv-\int vdu$$

$$Let\ \ s=u, \qquad Let: \exp(-s/\theta)ds=dv\Rightarrow v=-\frac{\theta}{\theta}\exp(-s/\theta),$$

$$(-1)\frac{\theta}{\theta}s\exp(-s/\theta)\Big|_0^\infty+\int_0^\infty \exp(-s/\theta)ds$$

$$(-1)s\exp(-s/\theta)\Big|_0^\infty-\theta[\exp(-s/\theta)]\Big|_0^\infty$$

$$When\ \ s=\infty,\ s\exp(-s/\theta)=0,\ when\ s=0,\ s\exp(-s/\theta)=0$$

$$When\ s=\infty,\ \exp(-s/\theta)=0,\ when\ s=0,\ \exp(-s/\theta)=1$$

$$\Rightarrow E(T)=0-\theta[0-1]=\theta=MTTF=\frac{1}{\lambda}$$

$$R(t=\theta)=R(MTTF)=\exp\left(-\frac{t=\theta}{\theta}\right)=\exp\left(-\frac{\theta}{\theta}\right)=\exp(-1)=0.3679,\ F(MTTF)=1-R(MTTF)$$

For the Normal Density:

$$P(t<\mu)=F(t)=\frac{t-\mu}{\sigma}=\frac{t-MTTF}{\sigma}=Z_0,$$

$$When\ t=MTTF\Rightarrow Z_0=\frac{MTTF-MTTF}{\sigma}=0\Rightarrow\Phi(0)=0.5\Rightarrow F(MTTF)=0.5$$

$$R(MTTF)=1-F(MTTF)=0.5$$

Thus, even if MTTF is the same and known, reliability could change depending on the distribution or density function associated with failure. Please note that for non-repairable system, we have MTTF, namely mean time to failure. For repairable systems it is mean time to first failure (MTFF).

2.6 Hazard Functions for Common Distributions

It is important to note that not all hazard functions are legitimate probability functions. Only legitimate hazard probability functions can produce reliability and probability density functions.

2.6.1 Exponential

Given: $f(t) = 1/\theta\ e^{-t/\theta}$, $R(t) = e^{-t/\theta}$, or $f(t)= \lambda \exp(-\lambda\ t)$, $R(t)= \exp(-\lambda t)$, where: $\lambda=1/\theta$

$$h(t) = \frac{f(t)}{R(t)} = \frac{1}{\theta} = \lambda$$

<u>Note</u>: This is true only when **f(t)** is the exponential**.** Some properties of the exponential distribution include: memoryless property; the occurrences follow the poison process; and constant failure rate.

2.6.2 Normal Distribution (Standard Normal Distribution)

$$f(t) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) = \frac{\phi(z)}{\sigma}$$

$$R(t) = 1 - \int_{-\infty}^{Z} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\tau^2}{2}\right) d\tau = 1 - Ft) = 1 - \Phi(z)$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\phi(z)/\sigma}{R(t)} = \frac{\phi(z)}{\sigma(1-\Phi(z))}$$

where $\phi(z)$ = pdf for standard normal variable, and Φ (z) = cdf for standard normal variable

2.6.3 Log Normal Distribution

$$f(t) = \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln t - \mu}{\sigma}\right)^2\right]$$

$$F(t) = \int_0^t \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln \tau - \mu}{\sigma}\right)^2\right] d\tau = \Phi\left[\frac{\ln t - \mu}{\sigma}\right]$$

$$R(t) = 1 - \int_0^t \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln \tau - \mu}{\sigma}\right)^2\right] d\tau$$

$$R(t) = 1 - F(t) = 1 - P(T \le t) = 1 - P\left[z \le \frac{\ln t - \mu}{\sigma}\right] = 1 - \Phi(z)$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\phi\left(\frac{\ln t - \mu}{\sigma}\right) / \sigma t}{1 - F(t)} = \frac{\phi\left(\frac{\ln t - \mu)}{\sigma}\right) / (\sigma t)}{t\sigma(1 - \Phi(z))}$$

2.6.4 Weibull Distribution

$$f(t) = \frac{\beta(t-\delta)^{\beta-1}}{(\theta-\delta)^\beta} \exp\left[\left(\frac{t-\delta}{\theta-\delta}\right)^\beta\right],\ t \ge \delta \ge 0$$

$$R(t) = 1 - F(t) = 1 - \int_0^t f(\tau) d\tau = \exp\left[\left(\frac{t-\delta}{\theta-\delta}\right)^\beta\right],\ h(t) = \frac{\beta(t-\delta)^{\beta-1}}{(\theta-\delta)^\beta}$$

2.7 Estimating R(t), h(t), f(t) Using Empirical Data

2.7.1 Small sample size (n ≤ 10)

Median estimator using order statistic

Consider the following ordered failure times:

$_OT_1, {}_OT_2, {}_OT_3, {}_OT_4, \ldots\ldots, {}_OT_n$

Where: $_OT_1 \le {}_OT_2 \le {}_OT_3 \le \ldots\ldots \le {}_OT_n$

Let: $_nP_j = \hat{F}({}_OT_J)$, that is:

$_nP_j$ is the fraction of the population failing prior to the j[th] observation in a sample of size n.

The **best** estimate for $_nP_j$ is the median value, i.e.

$$_nP_j = \hat{F}({}_OT_J) = \frac{j-0.3}{n+0.4}$$

Hence the cumulative distribution at the j[th] ordered failure time t_j is estimated as:

$$\hat{F}({}_OT_J) = \frac{j-0.3}{n+0.4}$$

$$\hat{R}({}_OT_J) = 1-\hat{F}({}_OT_J) = 1-\left(\frac{j-0.3}{n+0.4}\right) = \frac{n+0.4-j+0.3}{n+0.4} = \frac{n-j+0.7}{n+0.4}$$

$$\hat{h}({}_OT_j) = \frac{\hat{R}({}_OT_j)-\hat{R}({}_OT_{j+1})}{({}_OT_{j+1}-{}_OT_j)\hat{R}({}_OT_j)} = \frac{1}{({}_OT_{j+1}-{}_OT_j)(n-j+0.7)}$$

$$\hat{f}({}_OT_j) = \frac{\hat{R}({}_OT_j)-\hat{R}({}_OT_{j+1})}{({}_OT_{j+1}-{}_OT_j)\hat{R}({}_OT_j)} = \frac{1}{(n+0.4)({}_OT_{j+1}-{}_OT_j)}$$

2.7.1 Large Sample size (n >10)

$$R(t) = \frac{\overline{N}(t)}{N}, \quad f(t) = \frac{\overline{N}(t)-\overline{N}(t+x)}{N.x}$$

$$h(t) = \frac{f(t)}{R(t)} = \frac{\overline{N}(t)-\overline{N}(t+x)}{\overline{N}(t)x}, \quad where\ x = \Delta t$$

<u>**Estimation Using Empirical Data**</u>

$$f_e(t) = \frac{n_f(t)}{n_0\Delta t}$$

$$h_e(t) = \frac{n_f(t)}{n_s\Delta t}$$

$$R_e(t) = \frac{f_e(t)}{h_e(t)}, \quad and\ \ F_e(t) = 1-R_e(t)$$

These expressions are good for empirical data

- $n_f(t)$= the number that failed during any interval
- $n_0(t)$ = original number of items that was put on the test
- $n_s(t)$ = number that survived at any given instance

Example: 300 units of electronic circuit boards were cycled for 6000 hours as shown in table 1. The units that failed and those that survived in their corresponding intervals are as shown. The numerical values of the parameters are computed using the formulas shown:

Table 1a: Failure Data for Electronic Circuit Board		
t	n_f	n_s
0<t<1000	100	200
1000<t<2000	80	120
2000<t<3000	60	60
3000<t<4000	40	20
4000<t<5000	10	10
5000<t<6000	8	2
t >6000	2	0

Table 1b: Different Distributions for the Failure Data for Electronic Circuit Board Table 1a							
t	n_f	$n_S = \overline{N}(t)$	$N = \overline{N}(t)$	$\hat{R}(t)$	$\hat{F}(t)$	$\hat{f}(t)$	$\hat{h}(t)$
1000	100	200	100	0.6667	0.3333	0.000333	0.00033
2000	80	120	180	0.4000	0.6000	0.000266	0.00040
3000	60	60	240	0.2000	0.8000	0.00020	0.00050
4000	40	20	280	0.0667	0.9330	0.000133	0.000667
5000	10	10	290	0.03333	0.9667	0.000033	0.0005
6000	8	2	298	0.00667	0.9937	0.000023	0.0008
>6000	2	0	300	0.000	1.000	0.000006	0.001

$$\hat{R}(t) = \frac{\overline{N}(t)}{N} = \frac{n_S}{N}$$

$$f(t) = \frac{\overline{N}(t) - \overline{N}(t+\Delta t)}{N(\Delta t)}$$

$$h(t) = \frac{\overline{N}(t) - \overline{N}(t+\Delta t)}{\overline{N}(t)(\Delta t)}$$

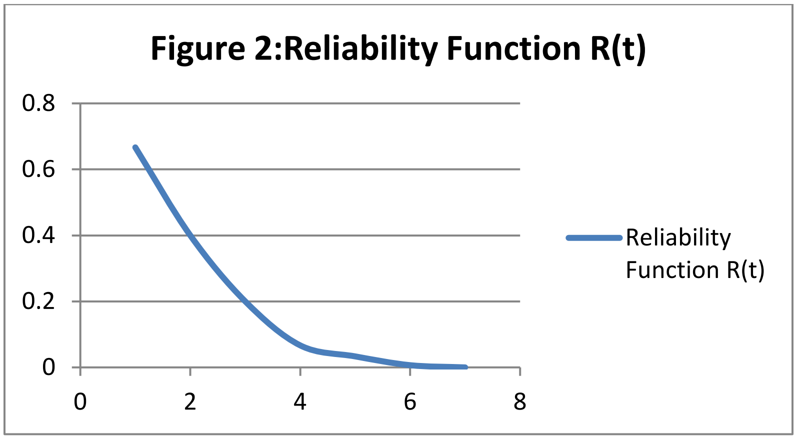


Figure 2:Reliability Function R(t)
0.8
0.6
0.4
0.2
0
0
2
4
6
8
Reliability Function R(t)

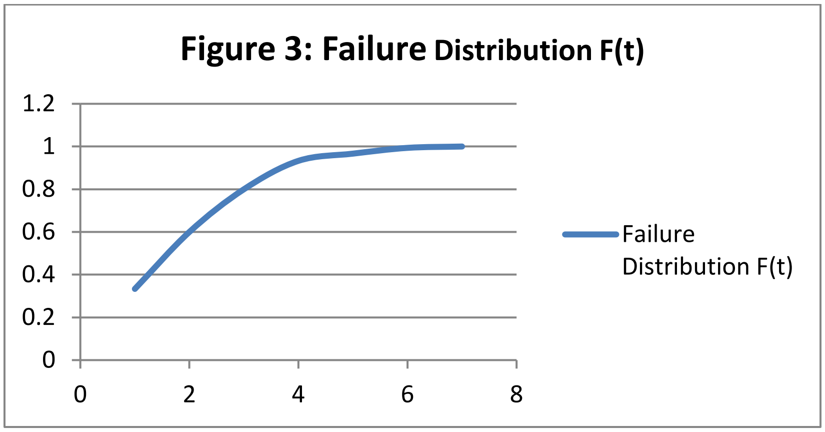


Figure 3: Failure Distribution F(t)
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
Failure Distribution F(t)

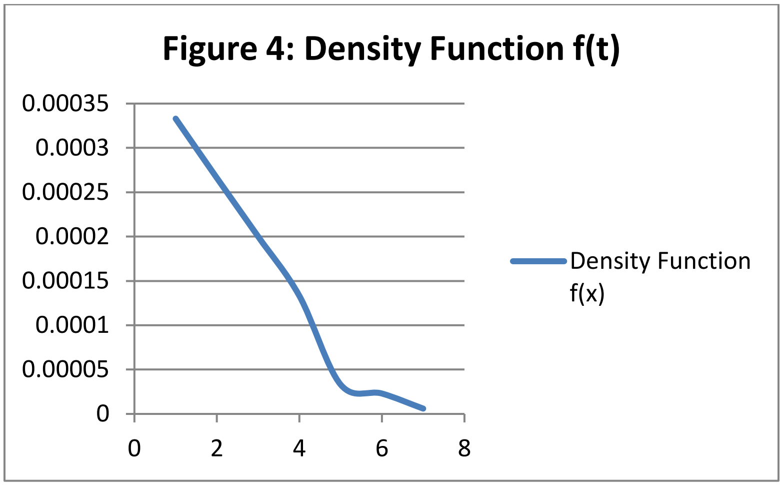
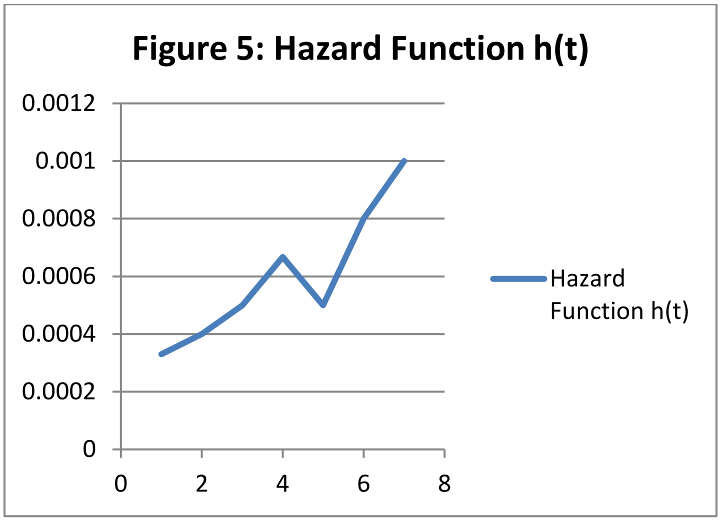


Figure 4: Density Function f(t)
0.00035
0.0003
0.00025
0.0002
0.00015
0.0001
0.00005
0
0
2
4
6
8
Density Function f(x)



Example: Table 2 represents the failure data for a small sample. We will show how to compute the various statistics such as the failure density function, the reliability function, the failure distribution function and the hazard function:

Table 2: Failure Data for Eight Springs	
Failure Number	**KILOCYCLES TO FAILURE**
1	**190**
2	**245**
3	**265**
4	**300**
5	**320**
6	**325**
7	**370**
8	**400**

For the data in Table 2, n <10, so we will use the following formula to compute f(t), R(t), and h(t).

$$\hat{F}({}_OT_J) = \frac{j-0.3}{n+0.4}$$

$$\hat{R}({}_OT_J) = 1 - \hat{F}({}_OT_J) = 1 - \left(\frac{j-0.3}{n+0.4}\right) = \frac{n-j+0.7}{n+0.4}$$

$$\hat{h}({}_OT_j) = \frac{\hat{R}({}_OT_j) - \hat{R}({}_OT_{j+1})}{({}_OT_{j+1} - {}_OT_j)\hat{R}({}_OT_j)} = \frac{\frac{(n-j+0.7)}{n+0.4} - \frac{(n-(j+1)+0.7)}{n+0.4}}{({}_OT_{j+1} - {}_OT_j)\frac{(n-j+1+0.7)}{n+0.4}} = \frac{(n-j+0.7-n+j+1-0.7)(n+0.4)}{(n-j+1+0.7)({}_OT_{j+1} - {}_OT_j)(n+0.4)}$$

$$\hat{h}({}_O T_j) = \frac{1}{({}_O T_{j+1} - {}_O T_j)(n - j + 0.7)}$$

$$\hat{f}({}_O T_j) = \frac{\hat{R}({}_O T_j) - \hat{R}({}_O T_{j+1})}{({}_O T_{j+1} - {}_O T_j)\hat{R}({}_O T_j)} = \frac{1}{(n+0.4)({}_O T_{j+1} - {}_O T_j)}$$

Table 3: Computation of Reliability Measures for the Spring Test Data

Failure Number	t	$t_{i+1} - t_1$	$\hat{F}(t)$	$\hat{R}(t)$	$\hat{f}(t)$	$\hat{h}(t)$
1	190	55	0.083	0.917	0.0022	0.0024
2	245	20	0.202	0.798	0.0060	0.0075
3	265	35	0.321	0.679	0.0034	0.0050
4	300	20	0.440	0.560	0.0059	0.0171*
5	320	5	0.560	0.440	0.0248	
6	325	45	0.679	0.321	0.0025	0.0082
7	370	30	0.798	0.202	0.0040	0.0198
8	400	-	0.917	0.083	-	-

***This value of the hazard rate was obtained by combining intervals four and five together and thus considering it as a single interval of 20+5=25 kilocycles**

Static Reliability

In performing the reliability analysis of a complex system, it is almost impossible to treat the system in its entirety. A logical approach is to decompose the system into functional entities composed of units, subsystems or components. Each entity is assumed to have either of two states – good or bad (success or failure). System block diagrams (SBD), where necessary, are generated to show desirable system operation. Models are then formulated to fit the logical structure.

After the system block diagram has been completed, the system reliability diagram is then developed. The system reliability diagram (RBD) is a logical diagram or graph whose edges represent the system components and indicates how the system will successfully operate. A reliability block diagram is a graphical representation of the components of the system and how they are related or connected in terms of their reliability. It provides a success oriented view of the system and facilitates the computation of system reliability from component reliabilities. It should be noted that RBD may differ from the system block diagram. SBD shows how the components are physically or functionally connected while the RBD shows how the system will successfully operate (or not).

The unit or subsystem reliabilities are computed, and subsequently used, to derive the overall system reliability. Most systems can be decomposed into series; parallel or hybrid structures. In many cases when the structure is of a more complicated or complex nature, more general techniques are used. In the series and parallel models, the assumption is that each unit or entity is independent of the others. In a series structure, the functional operation of the system depends on the proper operation of all system components. Parallel paths are redundant, meaning that all of the parallel paths must fail for the parallel network to fail. By contrast, any failure along a series path causes the entire series path to fail.

3.1 Series System

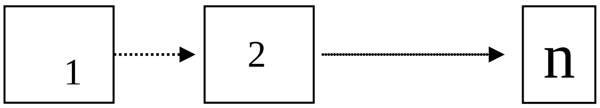


Fig 6. Series Configuration System

The block diagram shows that a single path, from cause to effect, is created. Failure of any component is represented by removal of the component which interrupts the path, thereby causing system failure. Define:

E_i = event that subsystem i will operate successfully

R_i=Subsystem survival probability

R_s = system reliability

Then R_s(series) = $P[E_1 \cap E_2 \cap E_3 \ldots . \cap E_n]$
But for any two independent events A and B

$$P(A \cap B) = P(A) \text{x } P(B) \Rightarrow R_s = P(E_1)P(E_2)P(E_3) \ldots P(E_n) = \prod_{i=1}^{n} R_i$$

Let R_1=0.90, R_2=0.85, R_3=0.99, R_s=(0.90)(0.85)(0.99)=0.7574

<u>Note:</u> The reliability of a series system is no better than the reliability of the worst component.

$$R_s = \begin{cases} e^{-(\lambda_1+\lambda_2+\ldots+\lambda_n)t}, & \text{if the } \lambda_i\text{'s are different} \\ e^{-n\lambda t}, & \text{if the } \lambda_i\text{'s are same} \end{cases}$$

For a series structure, the system reliability decreases rapidly as the number of series components increases; thus the reliability of a series system is, at most, as good as the poorest or least reliable component.
Let q_i = probability that a subsystem or component 'i' will fail.

$$Rs = (1- q_1)(1 - q_2)(1 - q_3) \ldots (1 - q_n) = \prod_{i=1}^{n}(1-q_i)$$

$$\approx 1 - nq; \text{ if } q\text{'s are identical}$$

$$\approx 1 - \prod_{i=1}^{n} q_i \text{ if } q\text{'s are different}$$

For series configuration, $R_S(t)= R_1(t) \text{ x } R_2(t) \ldots . R_n(t)$. If the components have exponential life

$$\boldsymbol{R(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} + \ldots + e^{-\lambda_n t} = e^{-t\Sigma\lambda_i}}$$

The system failure rate $\lambda_S = \Sigma\lambda_i$, i=1, ..n, with system MTBF $(\mu)=1/\lambda_S$

$$\int_0^\infty R_s(t)\,dt = \int_0^\infty e^{-t\Sigma\lambda_i} = \mu_s$$

$$\mu_s = \frac{1}{\Sigma\lambda_i} = \frac{1}{\frac{1}{\mu_1}+\frac{1}{\mu_2}+\frac{1}{\mu_3}\ldots+\frac{1}{\mu_n}}$$

If the Components are identical, i.e., $\lambda_1=\lambda_2=\lambda_3=\ldots=\lambda_n$, then $\mu_s = \frac{1}{n\lambda} = \frac{\mu}{n} = \frac{\theta}{n}$

3.2 Parallel Systems

In many systems, several signal paths perform the same operation. If the system configuration is such that failure of one or more paths still allows the remaining path or paths to perform properly, the system can be represented by a parallel model. A parallel system is one that is not considered to have failed, unless all components have failed. The reliability block diagram is represented as

Define: Qs = unreliability of the system

$$Let\ E_i = event\ compoent\ 'i'\ works\ , R_i = P(E)$$

$$Let\ \overline{E}_i = event\ component\ 'i'\ does\ not\ work,\ Q_i = P(\overline{E})$$

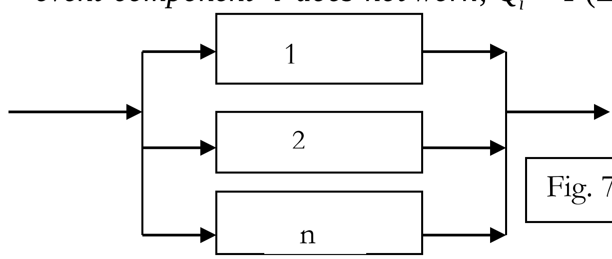


Fig. 7. Parallel Configuration

$$Q_s = P(\overline{E}_1)P(\overline{E}_2)\ldots.P(\overline{E}_n) \Rightarrow Q_s = (1-P(E_1))(1-P(E_2))\ldots\ldots(1-P(E_n)) = \prod_{i=1}^{n}(1-P(E_i))$$

$$R_s = (1-Q_s) = \left[1-\prod_{i=1}^{n}(1-P(E_i))\right] = \left[1-\prod_{i=1}^{n}(1-R_i)\right]$$

$$if\ \ R_1 = 0.9, R_2 = 0.85, R_3 = 0.99,\ Q_s = 0.1(0.15)(0.01) = 0.00015,\ R_s = (1-Q_s) = 0.99985$$

Consider a three unit redundant system (three components in parallel fig 7).

$$Let\ E_i = event\ compoent\ 'i'\ works\ , R_i = P(E),$$

$$Let\ \overline{E}_i = event\ component\ 'i'\ does\ not\ work,\ Q_i = P(\overline{E})$$

The probability that the system fails is:

$$Q_s = P(\overline{E}_1)P(\overline{E}_2) = (1-R_1)(1-R_2)(1-R_3)$$

$$R_s = 1-Q_s = 1-[(1-R_1)(1-R_2)(1-R_3)]$$

$$R(t) = \left\{1-\prod_{i=1}^{3}(1-R_i) = 1-[(1-R_3)(1-R_1-R_2+R_1R_2)]\right\}$$

$$= R_1 + R_2 + R_3 - R_1R_2 - R_1R_3 - R_2R_3 + R_1R_2R_3$$

If we assume that the failure rate h(t) is constant, then the failure density function is the exponential distribution. We can show this by using the non-parametric relationship between R(t), h(t), and f(t).

$$Given:\ \ h(t) = c, \quad \Rightarrow R(t) = \exp\left(-\int_0^t h(\tau)d\tau\right) = \exp\left(-c\int_0^t d\tau\right) = e^{-ct}$$

$$f(t) = h(t)R(t) = ce^{-ct} = \lambda e^{-\lambda t} \Rightarrow Which\ is\ the\ exponential\ distribution$$

$$R(t)=e^{-\lambda t},\quad Taking \log s, -\ln(R(t)=\lambda t \Rightarrow t=\frac{1}{\lambda}\ln\left(\frac{1}{R(t)}\right)=MTTF\ln\left(\frac{1}{R(t)}\right)$$

$$R_s(t)=e^{-\lambda_1 t}+e^{-\lambda_2 t}+e^{-\lambda_3 t}-e^{-\lambda_1 t}e^{-\lambda_2 t}-e^{-\lambda_1 t}e^{-\lambda_3 t}-e^{-\lambda_2 t}e^{-\lambda_3 t}+e^{-\lambda_1 t}e^{-\lambda_2 t}e^{-\lambda_3 t}$$

$$if\ \lambda_1=\lambda_2=\lambda_3=\lambda \Rightarrow R(t)=3e^{-\lambda t}-3e^{-2\lambda t}+e^{-3\lambda t}$$

$$MTTF=E(T)=\int_0^\infty R_s(t)=\int_0^\infty 3e^{-\lambda t}-\int_0^\infty 3e^{-2\lambda t}+\int_0^\infty e^{-3\lambda t}$$

$$=\frac{-3}{\lambda}\left[e^{-\lambda t}\right]_0^\infty+\frac{3}{2\lambda}\left[e^{-2\lambda t}\right]_0^\infty+\frac{1}{\lambda}\left[e^{-3\lambda}\right]_0^\infty$$

$$=\frac{3}{\lambda}-\frac{3}{2\lambda}+\frac{1}{3\lambda}=\frac{1}{\lambda}\left[3-\frac{3}{2}+\frac{1}{3}\right]=\frac{1}{\lambda}\left[1+\frac{1}{2}+\frac{1}{3}\right]$$

$$\therefore MTTF=\frac{1}{\lambda}\sum_{i=1}^{n}\frac{1}{i},\ if\ \ components\ are\ identical$$

The results are true for active parallel systems in which all the components are active in the system, starting from time zero. In a different type of redundant system, namely the standby system, the second unit is turned only after the first unit fails. In that scenario, the failures rates are no longer independent but depend on the failure of the main or primary unit.

There is a relationship between the design life of a system or component, and the end of life reliability. In practice, the engineer will set the design life so as to achieve a desired end-of-life reliability goal. For example, if the design life is 5 years, we want the reliability at the end-of-life of the system to be some value of reliability. Please also note that the MTTF is a single but important time point in the time domain and thus is a time measured value.

Example: Through predictive analytics, the MTTF of systems with constant failure rate has been determined and it is equal to $MTTF_0$ or 15 years. The engineer wants to set the design life in consonant with the predetermined MTTF so that the end-of-life reliability is 85%.

a). Determine the design life T with respect to MTTF

b). To enhance system performance, two identical units with same failure rate are utilized as part of the active parallel system configuration to increase the design life. How does this new configuration affect the design life given that the end-of-life reliability remains the same?

Solution part (a).

$$Let\ the\ \ failure\ rate\, be\ \lambda=1/MTTF$$

$$R(T)=e^{-\lambda T}\Rightarrow Taking\ \log s:-\lambda T=\ln(R(T)\Rightarrow T=\left(\frac{1}{\lambda}\right)\ln\left(\frac{1}{R(T)}\right)$$

$$T=\ln\left(\frac{1}{0.85}\right)MTTF_0=0.1625MTTF_0=2.44yrs$$

Solution part (b).

$$R(T)=2e^{-\lambda T}-e^{-2\lambda T};\ \ let\ x=e^{-\lambda T}\Rightarrow e^{-2\lambda T}-2e^{-\lambda T}+R=0$$

$$\Rightarrow x^2-2x+R=0$$

$$x=\frac{+2\pm\sqrt{4-4R}}{2}\Rightarrow x=\begin{bmatrix}1-\sqrt{1-R}\\1+\sqrt{1-R}\end{bmatrix}=\left(1-\sqrt{1-R}\right)$$

$$x=1+\sqrt{1-R}\ \ is\ \ not\ \ permisible\ because\ x>1\Rightarrow x=e^{-\lambda T}=R(t)\ will\ be>1$$

$$T=\ln\left(\frac{1}{R}\right)MTTF_0=\ln\left[\frac{1}{\left(1-\sqrt{1-R}\right)}\right]MTTF_0=0.4899MTTF_0=0.4899(15)=7.34yrs$$

The redundant arrangement has more than three times the design life of the single unit.

Reliability Improvement

One approach to reliability improvement is to alter the system structure so as to obtain higher reliability while maintaining the basic system function. This is generally accomplished by creating additional parallel paths in the system structure and is usually termed REDUNDANCY. The straightforward approach is to take existing system and connect a duplicate one in parallel. This results in two separate systems. Such an approach, which involves paralleling the entire system or unit, is called system or unit redundancy. A different approach is to parallel each component resulting in component redundancy. The hybrid model, resulting from a mix of both system and component redundancies, is called the compromise redundancy.

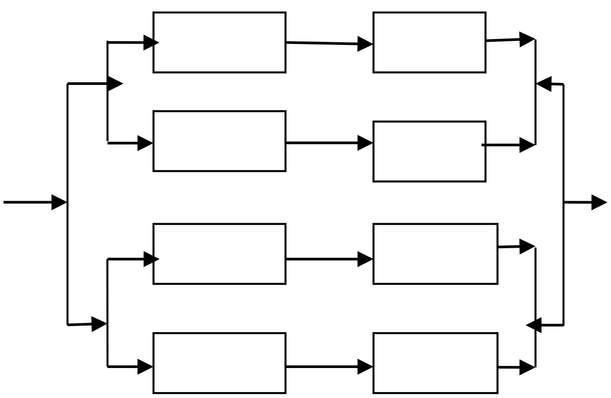


Fig 8.System Redundancy

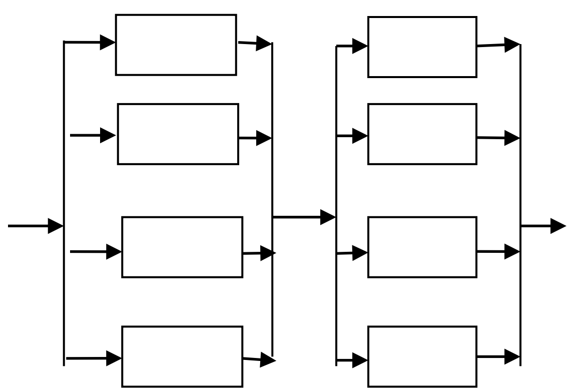


Fig 9.Component Redundancy

3.1 Redundancy-High level

High level redundancy is based on the system or subsystem (See Fig 8). Each subsystem consists of individual units in series. The resulting serial configuration is placed in parallel with other subsystems to form a bank. Several such banks are placed in parallel to form a **High Level Redundant** system. Assuming there are 'm' identical components per serial configuration subsystem which form a bank and 'n' banks in parallel, then the system reliability assuming identical components is given by:

$$R_{series} = R^m$$
$$Q_{series} = (1-R^m)^n$$
$$R_{subsys} = 1-(1-R^m)^n$$
$$Q_s = \left[1-\left(1-(1-R^m)^n\right)\right]^n \Rightarrow R_s = 1-Q_s = 1-\left[1-\left(1-(1-R^m)^n\right)\right]^n$$

Under certain conditions, low level redundancies give higher reliability values than high level redundancies, namely; the reliabilities of the component cannot depend on the configuration in which they are located, the failure process must be truly independent for both configurations, and the component reliabilities must be same for both configurations.

Example: From Fig 8, Let R_i=0.9 for all components. Use n=2 and m=3. Note that in fig 8, m=2.

$$R_{series} = R^m = (0.9)^3 = 0.729$$
$$Q_{series} = (1-R^3)^2 = (0.271)^2 = 0.0734$$
$$R_{subsys} = 1-(1-R^m)^n = 0.92656$$
$$Q_s = \left[1-\left(1-(1-R^3)^2\right)\right]^2 = (0.0734)^2 = 0.005394$$
$$R_s = 1-Q_s = 1-\left[1-\left(1-(1-R^m)^n\right)\right]^n = 1-005394 = 0.994606$$

3.2 Redundancy-Low level

Low level redundancy is redundancy based on the component (See Fig 9). Thus, components are placed in parallel in banks, where each bank consists of individual units in parallel. Assuming there are 'm' components per bank and 'n' banks in series in the system then the system reliability assuming identical components is given by:

$$For\ each\ bank,\ R_{sub} = 1-(1-R)^m$$
$$For\ n\ banks,\ R_{sys} = \left[1-(1-R)^m\right]^n$$

From Fig 9, assuming R_i=0.9 for all components and m=4, n=2

$$R_{sub} = 1-(1-R)^m = 1-(1-0..9)^4 = 1-(0.1)^4 = 1-0.0001 = 0.9999$$
$$R_{sys} = \left[1-(1-R)^m\right]^n = (0.9999)(0.9999) = 0.9998$$

3.3 Active and Standby Redundancy

3.3.1 Active or Parallel System Models

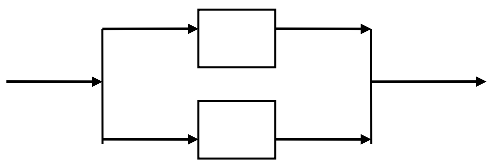


Fig 10: Active Parallel System

For this two-unit system, we can compute the unreliability for the system, and based on the complementary nature of R(t) and F(t), we can compute the reliability. The unreliability of the system is given by the product of the probability of failure of both components, i.e.,

$Q_s(t) = P[t_1<t \cap t_2<t \cap t_3<t \ldots \cap t_n<t)$

$Q_s(t) = P(t_1<t)P(t_2<t)P(t_3<t)\ldots P(t_n<t)$

but $P(t_1<t) = 1-P(t_1>t) = 1-R(t)$

If the failure mechanism for the components is independent, then:

$$Q_s(t) = \prod_{i=1}^{n}[1-R_i(t)] \Rightarrow\Rightarrow R(t) = 1-\prod_{i=1}^{n}[1-R_i(t)]$$

$$If\ R(t) = e^{-\lambda_i t}\ then\ R(t) = 1-\prod_{i=1}^{n}\left(1-e^{-\lambda_i t}\right)$$

For two identical units, $R(t) = 1-\left[\left(1-e^{-\lambda_i t}\right)\left(1-e^{-\lambda_i t}\right)\right] = 2e^{-\lambda t} + e^{-2\lambda t}$

Note: Failure rates in the exponential case are summed to combine independent series elements in reliability analysis. In general the exponents are summed when a product of elements of the exponential are desired.

3.4 Passive or Standby Configuration with Switching

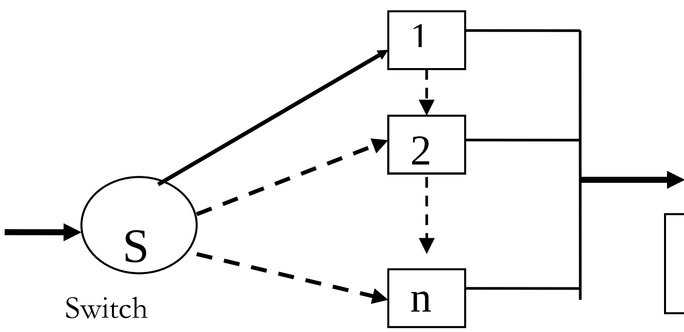


Fig. 11 Passive Standby with switching

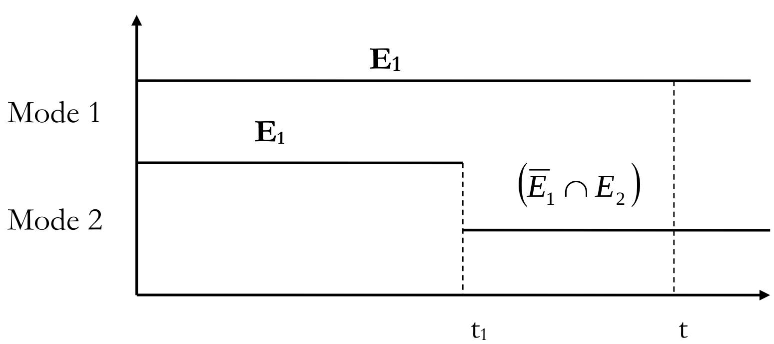


Fig. 12 Success Mode for 2-unit standby with switching

The system operation is as follows: First the primary unit is switched on, with the other unit in standby. Should the primary unit fail, then the switching mechanism (perfect switch) switches over to the standby unit which then works till time t. This results in two success modes as depicted in fig 12.

Mode 1: Primary unit works till end of life- t.

Mode 2: Primary unit fails at t_1 and the standby unit takes over and works till t.

$R_s(t) = P[(\mathbf{t_1 > t}) \cup (t_1 \le t \cap t_2 > t - t_1)]$

Assuming that the success modes are mutually exclusive, then

$R^2_s(t) = P[(t_1 > t)] + P[(t_1 \le t \cap t_2 > t - t_1)]$

$$R_s^2(t) = R_1(t) + \int_0^t f_1(t_1) R_2(t - t_1) dt_1, \ \ if \ \ \lambda_1 = \lambda_2 = \lambda$$

$$R_s^2(t) = e^{-\lambda t} + \int_0^t \lambda e^{-\lambda t_1} e^{-\lambda(t - t_1)} dt_1 = e^{-\lambda t} + \lambda t e^{-\lambda t} = e^{-\lambda t}(1 + \lambda t)$$

In general, for an (n) unit standby system with identical components with one primary and (n-1) standby units, the system reliability is given by $R_s^n = e^{-\lambda t} \sum_{i=0}^{n-1} \frac{(\lambda t)^i}{i!}$

3.4 Imperfect Switching

There are several ways in which a switch can fail. The failure modes depend on the switching mechanism and the system.

Case I: When the switch fails to operate when called upon.

In case I, the probability that the switch performs when called upon to do so is p_s.

For the two-unit standby system, the system reliability is given as:

$$R_s(t) = R_1(t) + p_s \int_0^t f_1(t_1) R_2(t - t_1) dt_1 = e^{-\lambda t}[1 + p_s \lambda t]$$ for constant failure rate system.

Example: Let p=0.99, λ=0.02 /hr for a two unit standby system with constant failure rate. Find the reliability for a mission time of 50 hours

Solution: $R_s(t) = e^{-\lambda t}[1 + p_s \lambda t] = e^{-0.02(50)}[1 + 0.99(50)(0.02)] = 0.5018$

Case II: When the switch is a complex piece of equipment with a constant failure rate equal to λ_{sw}.

In case II, $R_{sw}(t) = e^{-\lambda_{sw}}$ =The reliability of the switching mechanism

For the two-unit standby system: $R_s(t) = P[(t_1 < t) \cup (t_1 \le t \cap t_{sw} > t_1 \cap t_2 > t - t_1)]$

$$R_s(t) = R_1(t) + \int_0^t f_1(t_1) R_{sw}(t_1) R_2(t - t_1) dt_1$$

$$= R_1(t) + \int_0^t f_1(t_1) e^{-\lambda_{sw} t_1} R_2(t - t_1) dt \Rightarrow R_s(t) = e^{-\lambda t}\left[1 + \frac{\lambda}{\lambda_{sw}}(1 - e^{-\lambda_{sw} t})\right], t \ge 0$$

Example: A two-unit standby system with switch with constant failure rate equal to λ_{sw} =0.001/hr, The two units have identical constant failure rate of λ=0.04/hr, Find R(60) hr

$$R_s(t) = e^{-\lambda t}\left[1 + \frac{\lambda}{\lambda_{sw}}(1 - e^{-\lambda_{sw} t})\right] = e^{-0.04(60)}\left[1 + \frac{0.04}{0.001}\left(1 - e^{-0.001(50)}\right)\right] = 0.302$$

Example: Consider a two-unit standby redundant system that has constant switch failure rate of λ_s. If the switch fails, the system fails. In this system both units have identical time to failure pdf's given by f(t)=λexp(-λt).

(a). Find the system reliability function

(b). If λ_s =0.01/hr and the subsystems both have a constant failure rate λ=0.02/hr, Find R(50) hr.

Solution

$$R_s(t) = P[(t_1 > t \cap t_{sw} > t \cup (t_1 \le t \cap t_2 > (t - t_1) \cap t_{sw} > t)]$$

$$= R_1(t)R_{sw}(t) + R_{sw}(t)\int_0^t f_1(t_1)R_2(t - t_1)dt_1 R_s(t) = e^{-\lambda_s t}\left[R(t) + \int_0^t f(t_1)R(t - t_1)dt_1\right]$$

$$R_s(t) = e^{-0.01t}\left[e^{-0.02t} + \int_0^t 0.02e^{-0.02t_1}e^{-0.02(t-t_1)}dt_1\right] = R_s(t) = e^{-0.03t}[1 + 0.02t], R(50) = e^{-0.03(50)}[1 + 0.02(50)] = 0.45$$

Many other types of switch failure may be encountered in practical situations. For example, a switch may fail to hold a subsystem on line or the switch may inadvertently sense a failure.

3.5 Shared Load Models

In this type of configuration, the parallel subsystems share the load equally and as a subsystem fails, the surviving subsystem must sustain the increased load. Thus as successive subsystems fail the failure rate of the surviving components increases.

Example: A shared load parallel configuration would be when cables are used to support a load or bolts are used to support a machine component. In each case the supporting cables or bolts equally share in the support of the system.

Define the following system parameters

f_h (t) = pdf for time to failure under half load, f_f(t) = pdf for time to failure under full load

In the enduing analysis, it would be assumed that when failure occurs, the survivor follows the pdf f(t) and that the pdf does not depend on the interval of elapsed time.

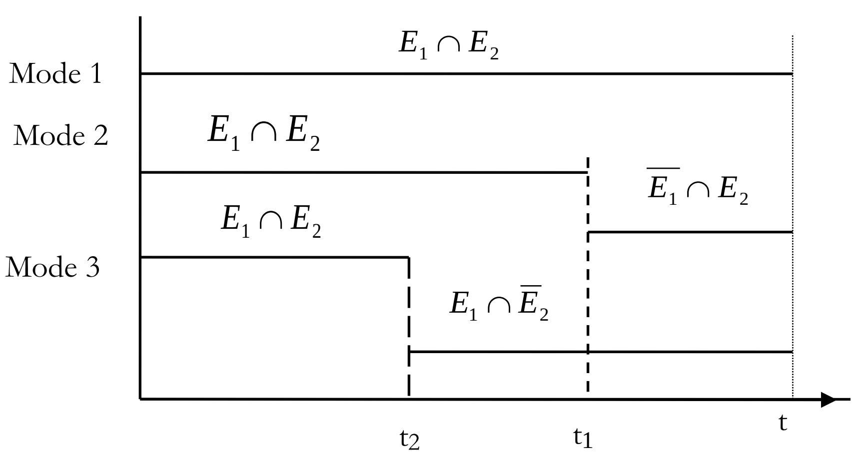


Fig 14: Success Modes for Shared Load Parallel System

The y-axis represents the success modes
Mode 1: both are working till time t
Mode 2: both work for a while, subsystem 1 fails and subsystem 2 works to completion.
Mode 3: both subsystems work for a while, subsystem 2 fails, and subsystem 1 works to completion.
Let us consider each of the modes separately:

1. For Mode 1: both components survive, hence:

   $P[t_1>t \cap t_2>t] = [R_h(t)]^2$, where $R_h(t)=\int_t^{\infty} f_h(\tau)d\tau$

2. For Mode 2, both units work for a while, then subsystem 1 fails at t_1, and subsystem 2 works to completion to time t:

   $P[t_1 \le t$, under half load$) \cap (t_2>t_1$, under half load$) \cap$ $(t_2>t-t_1$ under full load$)]$

   $$= \int_0^t f_h(t_1)R_h(t_1)R_f(t-t_1)dt_1, \sim where \sim R_f(t) = \int_0^{\infty} f(\tau)d\tau$$

3. For Mode 3. The third mode is identical to the second. Both work for a while, then subsystem 2 fails at t_2, and subsystem 1 works to completion to time t

   If we assume the components are identical then the system reliability is:

   $$R_s(t) = [R_h(t)]^2 + 2\int_0^t f_h(t_1)R_h(t_1)R_f(t-t_1)dt_1$$

   Let: λ_h= half load failure rate and λ_f = full load failure rate

   $$R_s(t) = e^{-2\lambda_h t} + 2\int_0^t \lambda_h e^{-\lambda_h \tau} e^{-\lambda_h \tau} e^{-\lambda_f (t-\tau)} d\tau$$

   $$R_s(t) = e^{-2\lambda_h t} + \frac{2\lambda_h}{(2\lambda_h - \lambda_f)}\left[e^{-\lambda_f t} - e^{-\lambda_h t}\right], t \ge 0$$

Example: Assume that the shared load parallel system has constant failure rate; and with λ_f =0.001/hr, and λ_h =0.05/hr, find the reliability of the system at t=300 hrs. Notice the failure rate at full load is much higher than that at half load. The reason is that, at full load, a component is at higher risk of failure than when it is working at half load.

$$R_s(t) = e^{-2\lambda_h t} + \frac{2\lambda_h}{(2\lambda_h - \lambda_f)}\left[e^{-\lambda_f t} - e^{-\lambda_h t}\right]$$

$$R_s(300) = e^{-0.05(300)} + \frac{2(0.05)}{(2(0.05) - 0.001)}\left[e^{-0.001(300)} - e^{-0.05(300)}\right] = 0.74831$$

Repairable Systems (Availability Analysis)

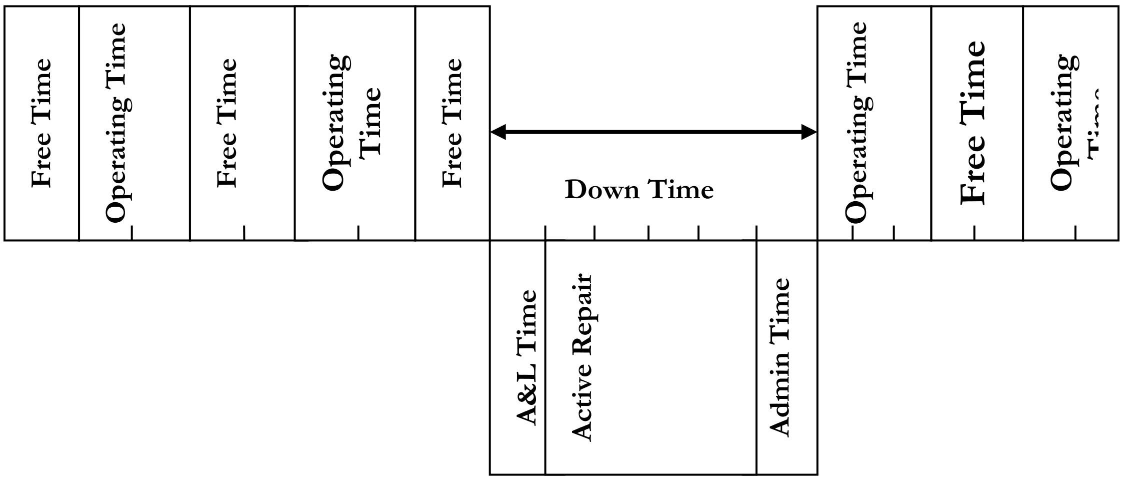
In many classes of systems where maintenance (preventive, predictive, and corrective) plays a central role, reliability is no longer the main focus. In the case of repairable systems (as a result of corrective maintenance), we are interested in:

- the probability of failure, • the number of failures, • the time required to make repairs

For such considerations, two new metrics (or parameters) of system effectiveness become the focus of attention, namely **i) Availability, ii) Maintainability**. Other related measures include:

- reparability
- operational readiness
- intrinsic availability, and
- serviceability

Figure 15: System Effectiveness Measures



4.1 Definition of Measures of System Effectiveness

4.1.1 Serviceability

This is the ease with which a system can be repaired. It is a characteristic of the system design and must be planned at the design phase. It is difficult to measure on a numeric scale.

4.1.2 Reparability

This is the probability that a system will be restored to a satisfactory condition in a specified interval of active repair time. This metric is very valuable to management since it helps to quantify workload for the repair crew. A major issue with reparability is the issue of secondary failure during repair or maintenance. Secondary failure is the failure of an item due to the failure of another item either due to repair, maintenance or sheer inducement and may also affect performance.

4.1.3 Operational Readiness (OR)

This is the probability that a system is operating or can operate satisfactorily when the system is used under stated conditions. This includes free (idle) time:

$$OR = \frac{Operating\ time + idle\ time}{Operating\ time + idle\ time + downtime}$$

4.1.4 Availability (A)

This is the probability that a system is available when needed **or** the probability that a system is available for use at a given time. It is simply the proportion of time the system is in an operating state, and it considers only operating time and down time. It excludes free or idle time:

$$A = \frac{Operating\ time}{Operating\ time + downtime}$$

4.1.5 Intrinsic Availability (AI)

This is defined as the probability that a system is operating in a satisfactory manner at any point in time. In this context, time is limited to operating and active repair time. Intrinsic availability is more restrictive than availability. Its numeric value is always more than that for availability:

$$A_I = \frac{Operating\ time}{Operating\ time + active\ repair\ time}$$

Summary of the Effectiveness Measures Based on Fig 15

The figure is not drawn to scale. the tick marks divide the time horizon into units of one as shown on the diagram

$$OR(Operational\ Readiness) = \frac{Operating\ time + idle\ time}{Operating\ time + idle\ time + downtime} = \frac{14}{20}$$

$$A(Availability) = \frac{Operating\ time}{Operating\ time + downtime} = \frac{9}{15}$$

$$AI(Intrinsic\ \ Avaliability) = \frac{Operating\ time}{Operating\ time + active\ repair\ time} = \frac{9}{13}$$

4.1.6 Maintainability

This is the probability that a system can be repaired in a given interval of downtime.

4.2 System Availability

For a repairable system, a fundamental parameter of interest is availability defined as:

A(t)= probability that a system is performing satisfactorily at time t and considers only operating time and downtime. This definition refers to point availability and is often not a true measure of the system performance. Often it is necessary to determine interval or mission availability defined as:

$$A^*(T) = \frac{1}{T}\int_0^T A(t)dt$$

This is the value of the point availability averaged over some interval of time T. This time interval may represent the design life of the system or the time to accomplish some mission.

The steady state or asymptotic availability is given by:

$$A^*(\infty) = \lim_{T\to\infty} it \ \frac{1}{T}\int_0^T A(t)dt$$

If the system or its components cannot be repaired, then the point availability at time t is simply the probability that it has not failed between time 0 and t, hence:

$$A(t) = R(t)$$

$$\textit{Substituting the value } A(t) = R(t); \Rightarrow: A^*(T) = \frac{1}{T}\int_0^T R(t)$$

$$\textit{As } T \Rightarrow \infty, \int_0^T R(t) \Rightarrow MTTF, \quad \textit{Hence } A^*(\infty) = \frac{MTTF}{\infty} = 0 \Rightarrow\Rightarrow \therefore A^*(T) = 0$$

This result is quite intuitive given our assumption. Since all systems eventually fail and there is no repair, then the availability averaged over an infinitely long time is zero.

The asymptotic availability is especially useful when both the failure and repair processes are driven by the exponential distribution. It is also useful for evaluating the overall availability, since for a reasonable time, period T, availability is insensitive to the details of repair and failure process.

Example: A non-repairable system has a known MTTF, and is characterized by a constant failure rate. The system mission availability must be 0.95. Find the maximum design life that can be tolerated in terms of the MTTF?

$$R(T) = e^{-\lambda T}$$

$$A^*(T) = \frac{1}{\lambda T}\int_0^T e^{-\lambda t} = \frac{1}{\lambda T}(1 - e^{-\lambda T})$$

Expanding the exponential assuming: (λT <<<1)

$$A^*(T) = \frac{1}{\lambda T}(1 - 1 + \lambda T - \frac{1}{2}(\lambda T)^2 + ...)$$

$$A(T) \approx 1 - \frac{1}{2}\lambda T \Rightarrow 0.95 = 1 - \frac{1}{2}\lambda T$$

$$\lambda T = 0.1, \textit{ but } MTTF = \frac{1}{\lambda}, \Rightarrow T = (0.1)(MTTF)$$

4.2.1 Computation of Availability

In order to calculate availability, one must take the repair rate into account; even though it may be large compared to the failure rate. In other words:

- Repair time of 5 hours is equal to a rate of (1/5)=0.2
- MTTF of 400 hours is equal to a rate of (1/400)=0.0025

4.2.2 Repair Function

Assuming that the repair rate is constant, this means that the probability density function for the repair function μ(t) is the exponential.

$$f(t) = \mu e^{-\mu t}, \text{ with the } MTTR(\text{mean time to repair}) = 1/\mu$$

Although the exponential may not reflect the details of the distribution very accurately, it provides a reasonable approximation for predicting availability values since these tend to depend more on the MTTR than on the details of the distribution. Therefore, even when the pdf of the repair time distribution is clustered about the MTTR, and resulting distribution does not seem to be the exponential, the constant repair rate (the exponential) model seems to adequately predict the asymptotic reliability.

So long as failures are revealed immediately, the time to repair is the primary factor that determines availability. If the system is not in continuous operation (as in standby), failures may occur but will remain unrevealed. The primary loss of availability will be failures in standby mode that are not detected until an attempt to use the system. A primary solution to this failure type is periodic testing. While periodic testing may help detect more failures, it may also lead to loss of availability, due to the downtime for testing. The longer it takes to detect failures, the less is the system availability

4.2.3 Availability Modeling

Consider a two state system (working or failed). The A(t) and Ã(t) are the probabilities that the system is operational or failed at any time t. The initial conditions are thus:

A(t)= 1, Ã(t) =0, and A(t) + Ã(t) = 1

Differential equations can be used to develop the equation for availability. Consider the change in A(t) between t and t+Δt. There are two contributions or possibilities:

- λΔt is the conditional probability of failure during Δt, given that the system was available at time t.
- μΔt is the conditional probability of repair during Δt given system failure.

Some assumptions:

- P[system failure during Δt] = λΔt
- P[repair during Δt|system failure] = μΔt
- A(t+ Δt) =A(t)[1-λΔt]+[1-A(t)] μΔt
- Either the system was available in time t and did not fail in the interval Δt ***OR*** it failed during Δt with prob. (1-A(t)) and was repaired with probability μΔt

Hence

$$A(t+\Delta t) = A(t)[1-\lambda\Delta t] + [1-A(t)]\mu\Delta t \Rightarrow\Rightarrow A(t) - \lambda A(t)\Delta t + \mu\Delta t - \mu A(t)\Delta t$$

$$Hence: \frac{A(t+\Delta t) - A(t)}{\Delta t} = -(\lambda+\mu)A(t) + \mu$$

$$As\ \Delta t \to 0,\ we\ have\ \frac{d}{dt}A(t) = -(\lambda+\mu)A(t) + \mu$$

$$The\ solution\ to\ this\ differential\ equation\ is\ given\ by: A(t) = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda}e^{-(\mu+\lambda)t}$$

$$A(t) = -(\lambda+\mu)A(t) + \mu$$

Integration factor (IF) approach:

$$\frac{d}{dt}A(t) + (\mu+\lambda)A(t) = \mu, \quad IF = p = e^{\int(\mu+\lambda)dt} = e^{(\mu+\lambda)t}$$

$$\frac{d}{dt}\left(A(t)e^{(\mu+\lambda)t}\right) = \mu\ e^{(\mu+\lambda)t} \Rightarrow A(t)e^{(\mu+\lambda)t} = \mu\int e^{(\mu+\lambda)t} + C$$

$$A(t)e^{(\mu+\lambda)t} = \frac{\mu}{(\mu+\lambda)}e^{(\mu+\lambda)t} + C,\ at\ t=0, A(0)=1$$

$$Hence\ \ C = 1 - \frac{\mu}{(\mu+\lambda)} = \frac{\lambda}{(\mu+\lambda)}$$

$$\therefore A(t) = \frac{\mu}{(\mu+\lambda)} + \frac{\lambda}{(\mu+\lambda)}e^{-(\mu+\lambda)t}$$

As t increases, the availability A(t) clearly approaches a constant value. The steady state availability is given by:

$$A^* = \lim_{t\to\infty} A(t) = \frac{\mu}{(\lambda+\mu)} \qquad = \frac{Mean\ \ repair\ \ rate}{Mean\ \ failure\ rate + Mean\ \ repair\ \ rate}$$

$$If\ \ we\ rewrite\ \frac{\mu}{(\lambda+\mu)}\ as\ \frac{\frac{1}{\lambda}}{\frac{1}{\lambda}+\frac{1}{\mu}} = \frac{1}{\lambda}\left[\frac{1}{\frac{1}{\lambda}+\frac{1}{\mu}}\right] = \frac{1}{\lambda}\left[\frac{1}{\frac{\lambda+\mu}{\lambda\mu}}\right] = \frac{\lambda\mu}{\lambda}\left[\frac{1}{\lambda+\mu}\right] = \frac{\mu}{\lambda+\mu}$$

$$hence\ A^*(\infty) = \frac{Mean\ \ time\ to\ \ failure}{Mean\ time\ to\ repair + Mean\ time\ to\ \ failure}$$

Example: In the following table (Table 4), the times (in days) over a 6-month period at which failure of a production line occurred (t_f) and times (t_r) at which the plant was brought back on line following repair are as shown. Question:

(a). Calculate the 6-month availability from the plant data
(b). Estimate the MTTF and the MTTR from the data
(c). Estimate interval (steady state) availability

a) During the 6 months (182.5 days) there were 10 failures

$$\tilde{A}(T) = \frac{1}{T}\sum_{i=1}^{10}(t_{ri} - t_{fi})$$

$$\frac{1}{182.5}[0.2 + 0.6 + 0.4 + 1.9 + 0.3 + 0.9 + .. + 3.3 + .1]$$

$$\tilde{A}(T) = 0.0630 \Rightarrow A(T) = 1 - 0.0630 = 0.937$$

Table 4: Failure and Repair Times of a production Line

S/N	t_f	t_r	S/N (Cont)	t_f	t_r
1	12.80	13.0	6	56.40	57.30
2	14.20	14.8	7	62.70	62.80
3	25.40	25.80	8	131.20	134.90
4	31.40	33.40	9	146.70	150.00
5	35.30	35.60	10	177.00	177.10

b). Let $t_{r0} = 0$, *we first estimate MTTFand then MTTR from the data*

$$MTTF = \frac{1}{N}\sum_{i=1}^{10}(t_{fi} - t_{ri-1})$$

$$= \frac{1}{10}[12.8 + 1.2 + 10.6 + 5.6 + 2.0 + 20.8 + 5.4 + 68.4 + 11.8 + 27.0]$$

$$MTTF = \frac{1}{10}(165.6) = 16.56\ days, \qquad MTTR = \frac{1}{N}\sum_{i=1}^{10}(t_{ri} - t_{fi})$$

$$= \frac{1}{10}[0.2 + 0.6 + 0.4 + 1.9 + 0.3 + 0.9 + 0.1 + 3.7 + 3.3 + 0.1] = 1.15\ days$$

c). $$A^*(\infty) = \frac{\mu}{\mu + \lambda} = \frac{\mu/\mu}{(\mu/\mu) + (\lambda/\mu)} = \frac{1}{1 + \frac{MTTR}{MTTF}} = \frac{1}{1 + \frac{1.15}{16.5}} = 0.936$$

Redesign of the Automobile Braking System Using Redundancy Concepts

To accomplish this task, we will examine four designs configurations, and compute their reliabilities with the goal of determining the optimal design. We will use the following notations and symbols:

M → Master Cylinder

W → Wheel Cylinder

L → Hydraulic Lines

Note: Safe breaking is achieved when either the front break works or the rear break works or both.

R(M)=0.995, R(W_i)=0.999, R(L_i)=0.999

5.1 Basic Brake Design (Design a)

This is the traditional brake design with two front and two rear cylinders connected to hydraulic lines and a master cylinder, design #a.

Figure 16a. Basic Brake Design

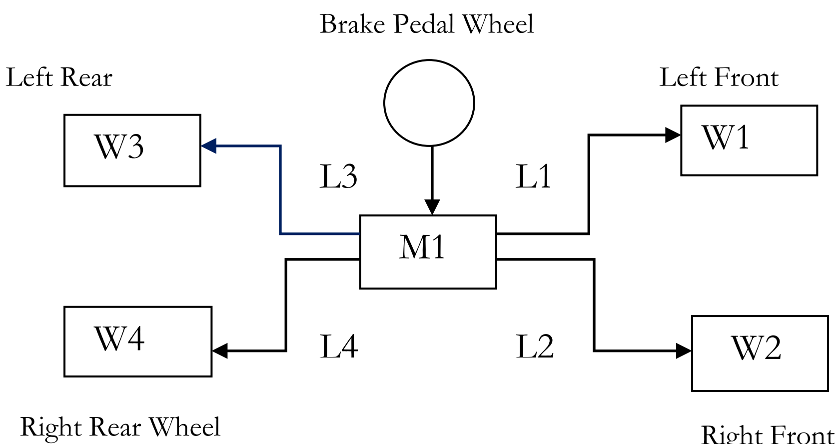
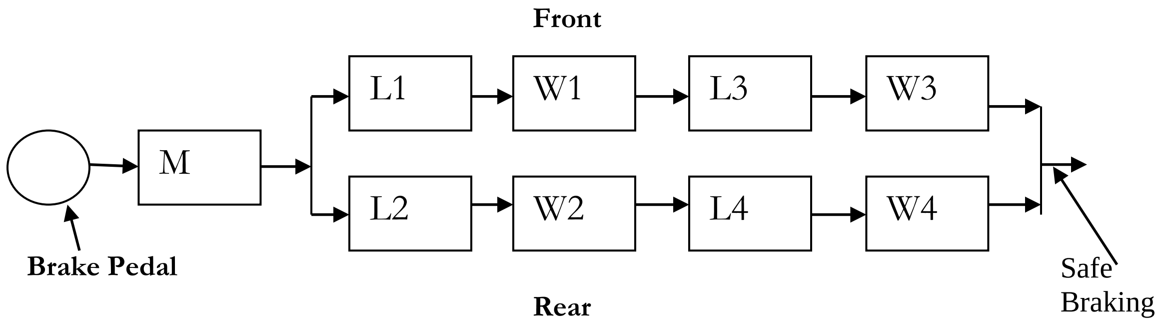


Figure 16b. Basic Brake Design



From Fig 16b, R(Basic Design a)

$R_a = R_{M1}\{1 - [(1 - R_{W1}R_{L1}R_{W2}R_{L2})(1 - R_{W3}R_{L3}R_{W4}R_{L4})]\} = 0.995[1 - (1 - 0.999^4)^2] = 0.99498$

5.2 Unit or System Redundancy (Design b)

Install a duplicate set of brake shoes and cylinder on each wheel, and feed these with separate hydraulic lines attached to a second master cylinder. This results in two separate systems, and doubles the cost, weight and volume of the system.

Figure 17a. Unit/System Redundancy

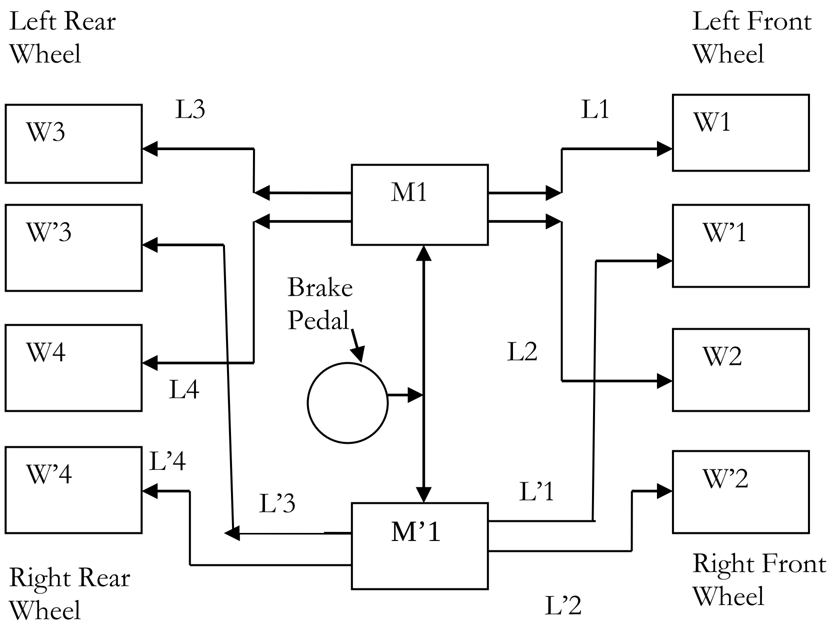
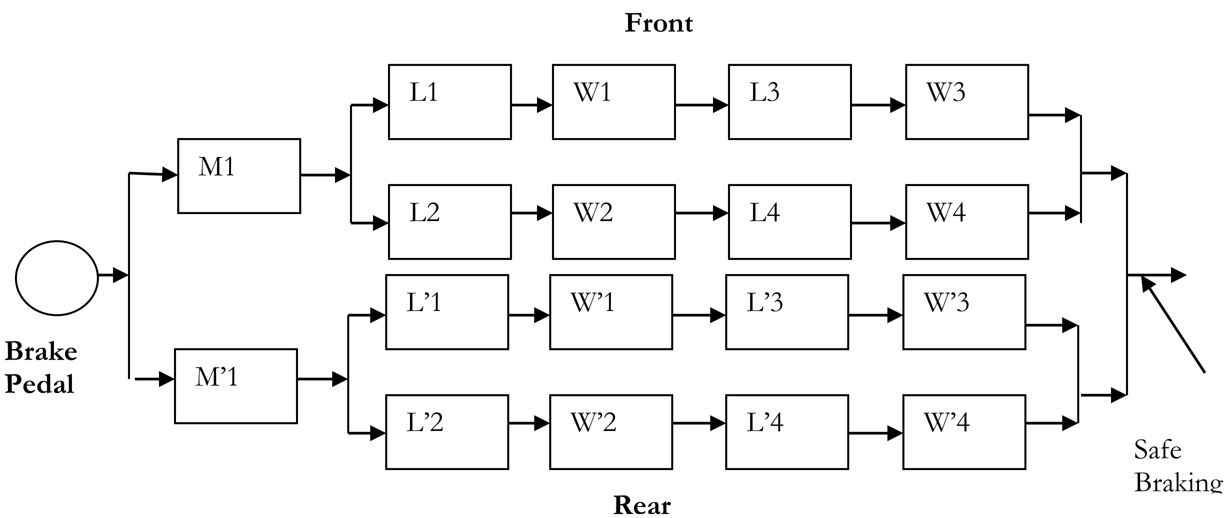


Figure 17b. Unit/System Redundancy



Reliability of Design b: R_a from Fig 16a &b =0.99498

$$R_b = 1 - [(1 - R_a)(1 - R_a)]$$

$$= 1 - (1 - 0.99498)2 = 0.9999748$$

5.3 Component Redundancy (Design c)

Parallel two master cylinders and run two parallel hydraulic lines to each wheel which connects to a parallel pair of wheel cylinders. In this case, each component is in parallel. Components are individually paralleled.

Figure 18a. Component Redundancy

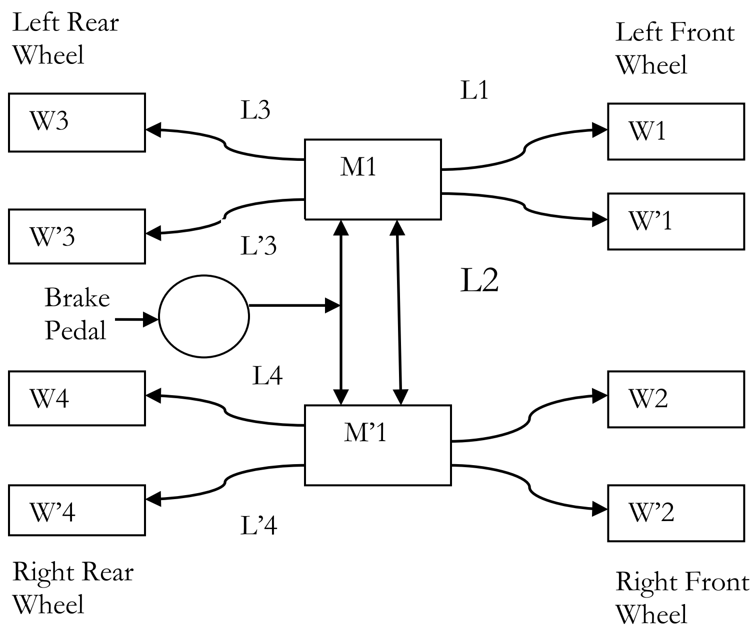
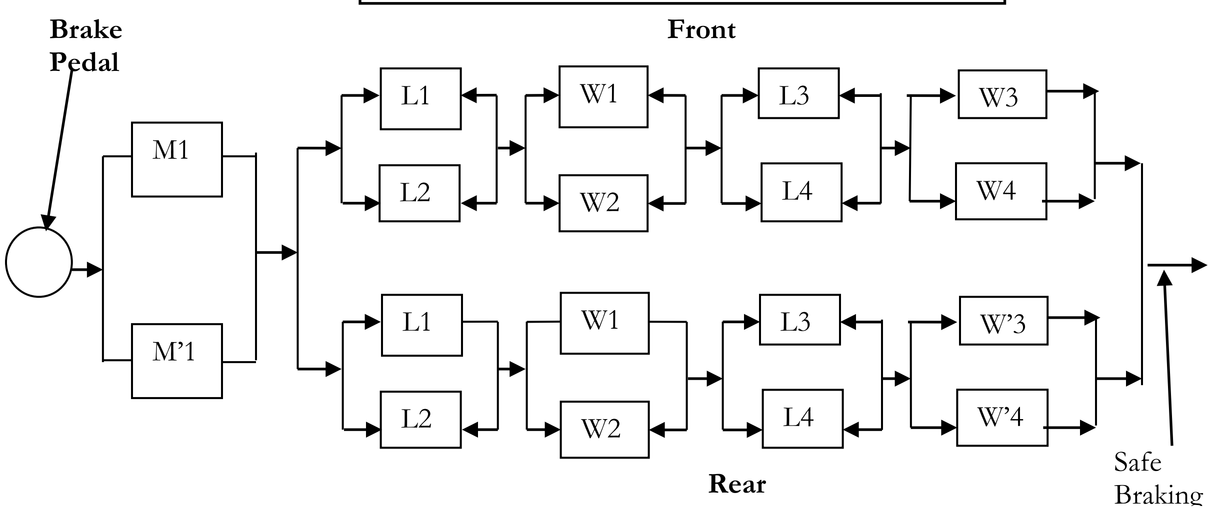


Figure 18b. Component Redundancy



Reliability for Design c

For the hydraulic lines:

$R_L = 1-[(1-R_{Li})(1-R_{Li})]$ for each set of parallel hydraulic lines

For the Wheel Cylinder:

$R_W = 1-[(1-R_{Wi})(1-R_{Wi})]$ for each set of parallel wheel cylinders

For each of these in series:

For the Top half: $$\left\{1-\prod_{i=1}^{2}\left(1-R_{L_i}\right)\right\}^2\left\{1-\prod_{i=1}^{2}\left(1-R_{W_i}\right)\right\}^2 = (0.999998)(0.999998) = 0.999996$$

For the Lower half: $$\left\{1-\prod_{i=1}^{2}\left(1-R_{L_i}\right)\right\}^2\left\{1-\prod_{i=1}^{2}\left(1-R_{L_i}\right)\right\}^2 = (0.999998)(0.999998) = 0.999996$$

Reliability of the wheel and hydraulic line subsystem:

$$R_{WL} = \{1-(1-0.999996)(1-0.999996)\} = 1$$

For the master cylinder subsystem:

$$R_M = 1-(1-0.9995)(1-0.9995) = 0.999975$$

$$R_C = R_{WL} * R_M = 0.999975(1) = 0.999975$$

5.4 Hybrid/Compromise Redundancy (Design d)

In a compromise system, a single brake pedal activates two separate master cylinders. One master cylinder feeds a set of hydraulic lines, which connects to the front wheel brakes, and the other master cylinder operates the rear wheel brake cylinder through its own set of lines.

Figure: 19a: Hybrid/Compromise

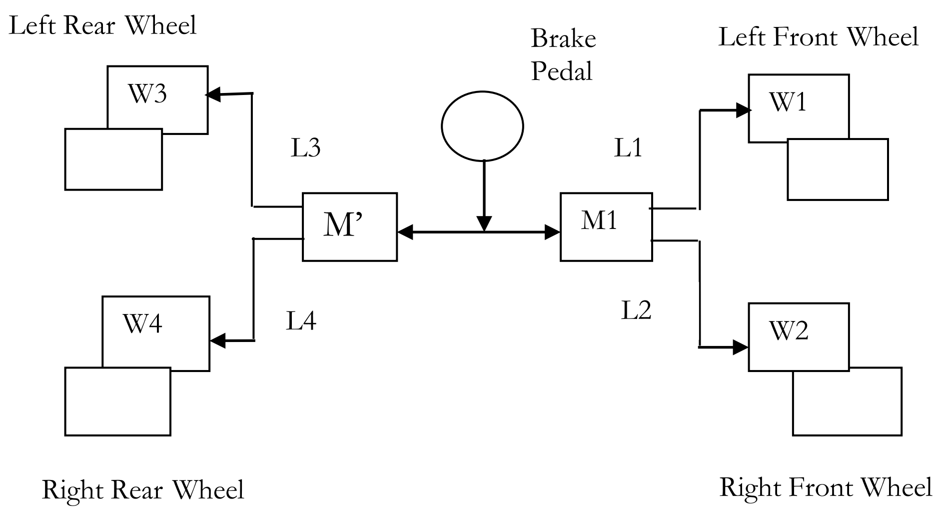
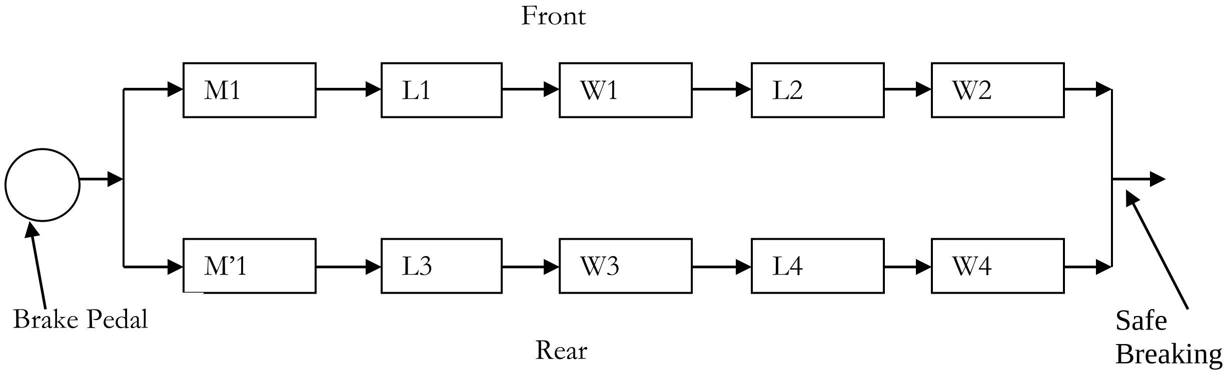


Figure: 19b: Hybrid/Compromise Redundancy



$R_F = R_{M1}R_{L1}R_{W1}R_{L2}R_{W2} = (0.95)(0.999^4)$ (R_F=Reliability Front, R_R=Reliability rear)

$R_F = R_R$, Hence: $R_d = 1- [(1-R_F)(1-R_R)] = 1- [1-(0.995)(0.999^4)]^2 = 0.9999195$

Maintenance

Relatively few systems can operate without a breakdown and so maintenance is needed to keep the system running to ensure minimal interruption to the production activity. The objective of maintenance is to increase the reliability (or more appropriately, the availability in case of a repairable system) of the system over the long haul; by reducing the aging and wear-out effects due to corrosion, fatigue and other internal and environmental problems that negatively affect the long-term operability of the system. Although the primary metric for judging effective maintenance is the resulting increase in reliability after maintenance, the criteria most often considered is system availability especially in the case of repairable or maintained system. Availability is defined in this case as the probability that the system will be operational when needed. The distinction is clear. Generally, reliability in a strict sense refers to unmaintained or irreparable systems; whereas availability refers to maintained or repairable systems. For satellites and one-shot space systems, we talk about reliability; whereas for cars or machinery, we talk about availability, again in the strictest sense. We also refer to the notion of idealized maintenance which is a very rare case where maintenance returns the system to as-good-as-new condition.

Considerable maintenance benefits can be realized when the maintenance intervals are chosen such that for a given system the positive effects of wear-out time (increasing failure rate) is greater than the negative effects of wear-in time (decreasing failure rate). This may apply more especially in a system with different components. In such a case, it would be better to perform maintenance only on those elements for which the wear-out effect dominate. For example, one may chose to replace worn spark plugs in a car rather than replace a fuel injector (which may be defective) with a new one. There are three basic types of maintenance, namely: Preventive, Predictive and Corrective and they are delineated as follows:

- Preventive (PM) – involves greasing, oiling, changing filters
- Predictive (PdM) – Inspections
- Corrective – Repairs

In general, maintenance does not return the system to as good as new condition. A system that has undergone any form of maintenance can be in one of several states after repair, namely:

a) **As good as new**
b) **New better than old**
c) **As bad as old**
d) **Worse than old**

6.1 Preventive Maintenance (PM)

Definition of Preventive Maintenance (PM): "Schedule of planned maintenance actions aimed at the prevention of breakdowns and failures." Preventive Maintenance is the planned maintenance of plant infrastructure and equipment with the goal of improving equipment life by preventing excess depreciation and impairment. This type of maintenance includes, but is not limited to: adjustments, cleaning, lubrication, repairs, and replacements for the express purpose for the extension of equipment life. However, by its nature, it can also lead to common-mode failures where related or connected component can be damaged due to the maintenance of its neighbor. Preventive Maintenance standards provide the fundamental principles and crucial guidelines for establishing a successful preventive maintenance program. Due to the varying needs of different plants, the type and amount of preventive maintenance required also varies greatly from plant to plant. Due to this, it is extremely difficult to establish a successful preventive maintenance program without proper guidelines and instructions regarding the specific plant or equipment. The primary goal of PM is to preserve and enhance equipment reliability. Therefore, any planned activity that increases the life of the plant or equipment/component and helps such an entity to run more efficiently is desirable. Examples of PM include tasks such as:

- Oil changes,
- Greasing,
- Changing filters,
- Belt tightening.

Preventive maintenance should be performed on equipment as recommended by the original Equipment Manufacturer (OEM). However, we must determine if time spent to perform PM is greater than the replacement cost. If the PM cost is higher than the replacement cost, then consideration should be given to replacement of the unit. Typically, equipment manufacturers outline preventive maintenance procedures and guidelines in the OEM manuals including:

- Oil and/or grease types, and quantities
- Time periods (weekly, monthly, quarterly)
- V-belt inspections & Torque settings
- General visual inspections

These guidelines should be used when creating a PM program

In a recent study, the US Department of Energy (DOE) reported that every 10 minutes an average furnace runs, it unleashes the equivalent energy of 3.5 sticks of dynamite in an effort to raise awareness of the importance of regular PM of a common household furnace. The lesson, or point to be taken away from this study is that not performing PM wastes energy and costs money.

In addition to the guidelines and procedures that manufacturers provide in their manuals, the American Standards Institute (ANSI) specify standards and recommendations for PM to help businesses determine the type and frequency of inspections and maintenance procedures, define the minimum requirements for servicing and maintaining plant equipment, serve as a comprehensive maintenance checklist, and supplement more specific instructions, manufacturer publications, and other standards. Through the application of these standards, industrial firms can improve automation and operate more efficiently, produce higher quality products, minimize energy consumption, reduce insurance inventories and business loss due to production delays, and increase overall safety levels. In addition, preventive maintenance measures can drastically reduce errors in day-to-day operations, as well as increase the overall preparedness of plants in the case of an emergency.

For PM, we must choose the maintenance interval for which the positive effect of wearout time is greater than the negative effect of wearin time. Typically, PM is performed on these components for which the wearout effect dominates. Even with wearout present, the constant failure rate model may suffice. Wearin like burn-in is a period of stabilization for the components.
As a general rule, only trained, qualified maintenance personnel should perform PM activities.
PM Training is important to:

- Ensure proper techniques and procedures are followed.
- Reduce Over greasing which is often worse than not greasing enough.
- Reduce improper tightening which increase shaft wear and shortens shaft life.
- Reduce common-mode failures due to poor maintenance practices.
- Ensure proper lubricants are used so as not to shorten equipment life

Benefits of PM

- Increases life of equipment
- Reduces failures and breakdowns
- Reduces costly down time
- Decreases cost of replacement

6.2 Predictive Maintenance (PdM)

Definition – Predictive Maintenance Techniques are techniques that help determine the condition of in-service equipment in order to predict when maintenance should be performed. The primary goal is to minimize disruption of normal system operations, while allowing for budgeted and scheduled repairs. It also involves data analytics and rigorous mathematical methods as well as:

- Vibration Analysis
- Infrared Thermography

- Oil Analysis
- Visual Inspections

Benefits of PdM

- Provides increased operational life
- Results in decrease of downtime
- Allows for scheduled downtime
- Allows for money to be budgeted for repairs
- Lowers need for extensive parts inventory
- DOE reports an estimated 8-12% cost savings by having an effective PdM program

According to the USDOE a good PdM program will lead to:

- Reduction in maintenance costs – 25-30%
- Elimination of breakdowns – 70-75%
- Reduction of downtime- 35-45%
- Increase in production – 20-25%

PdM is often performed by a contract and specialized technician who:

- Are qualified and trained on latest technology
- Possess the proper equipment
- Are able to provide trending and historical data in report form

Some of the techniques used to implement effective PdM programs and techniques include:

- Oil Analyses
- Thermography
- Vibration Analyses (VA)

1). **Oil Analyses is a long term program that may take years before its benefits are seen**.
Oil analyses include oil analysis and wear particles analysis

a) Oil analysis determines:
Condition of oil, Quality of the lubricant, and Suitability for continued use

b) Wear particle analysis determines:
i). Mechanical condition of machine components. ii). Identifies particle size, types, etc.

Oil Analysis results may:

- Detail the types of metal fragments in the sample
- Show a continued increase in particle size
- Recommend increasing sampling intervals
- Recommend shutting down machine

2). Thermography

This is used for electrical infrared inspections to detect hot spots, load imbalances and corrosion at a safe distance, and to detect failures due to excessive heat. Specific applications include:

- Indoor equipment such as MCC's (Motor Control Center) disconnect switches & transformers.
- Outdoor equipment such as substations, transformers and outdoor circuit breakers.

3). Vibration

Vibration tests are usually done on large equipment, such as blowers, pumps, etc. to:

- Determines if bearings or components are loose, moving or wearing.
- Allows for scheduled repair of equipment.
- Provide trending that enables shutdown of equipment BEFORE failure and major damage.

6.3 Corrective Maintenance (CM)

Definition – Repair of equipment/machinery in order to bring it to its original operating condition

Corrective maintenance is a form of maintenance that is implemented when the system breaks down or when there is a fault or problem in a system, with the goal of restoring the system to an operating condition. In some cases, it may not be possible to predict or prevent a failure, thus corrective maintenance becomes the only option. In other instances, a system can require repairs as a result of insufficient preventive maintenance, and in some other situations, it may be desirable to focus on corrective, rather than preventive, repairs as part of a maintenance strategy. The process of corrective maintenance begins with the failure, and a diagnosis of the failure to determine why it occurred. The diagnosis process may include a physical inspection of the system, the use of a diagnostic equipment to evaluate the system, interviews with users, and a number of other steps. It is important to determine what caused the problem in order to take appropriate action and to recognize that multiple failures of components or software may occur simultaneously.

6.4 Summary

In summary, proper orchestrated maintenance programs have significant payoffs including but not limited to: ● <u>Keeps equipment running longer</u>. ●<u>Allows for scheduled, budgeted repairs</u>

● .<u>Reduces unscheduled down time</u>, ● <u>Makes life less stressful</u>

Summary

Reliability Engineering is concerned with the design, implementation, and prediction of the life profile of a system or component using a disciplined analysis approach that has strong roots in statistics, mathematics and engineering. Given a system, subsystem or component, one of the major challenges of reliability analysis is to provide an understanding of the inherent failure mechanisms that undergird such a system and to develop the appropriate analytical scheme to determine the system's life profile. The problem becomes even more daunting given the phenomenon of aging and related transient phenomenon, as well as the practical realities of little or no data. Today, these challenges still persist especially as products get more and more miniaturized and as companies try to shorten the time to market to gain market share. This first of the two-course sequence has examined some of the basic issues related to reliability, such as:

- Understand the various viewpoints of reliability, especially the engineering design viewpoint.
- The use of nonparametric approach to estimate the reliability function.
- Understand the performance measures used to characterize reliability.

- Appropriate reliability based intervention strategies that lead to optimally maintained system.
- Availability, Maintainability and Performability measures.

The second sequence will focus on the all important area of dependency analysis, interference theory, data analysis and testing.

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