

A SunCam online continuing education course

# Engineering Methods in Microsoft Excel

## Part 1:

by

Kwabena Ofosu, Ph.D., P.E., PTOE



#### Abstract

This course is the first of a series on engineering methods in *Microsoft Excel* tailored to practicing engineers. The course series presents *Microsoft Excel* tools that can be used for a wide range of engineering analyses and data management. This course presents topics in linear algebra. Each topic includes a review of the fundamental principles followed by a real-life example encountered by a practicing engineer. The real-life examples are then formulated and implemented in *Microsoft Excel* and worked using the various *Excel* tools, spreadsheet techniques, and built-in functions. Examples from various engineering fields are used to demonstrate the concepts and methods learned throughout this course. Upon completion of this course, practitioners will be able to apply the methods learned to a variety of engineering problems, and also to identify situations in their fields of specialty where the innovative application of these tools and methods will be advantageous to their output and to their work product.



## TABLE OF CONTENTS

Abstract	2
1. INTRODUCTION	4
1.1 Overview	4
2. LINEAR ALGEBRA	5
2.1 Introduction	5
2.2 Arrays	5
2.3 Matrices	6
2.4 Matrix Addition and Subtraction	7
2.5 Scalar Multiplication	8
2.6 Transpose of Matrix	9
2.7 Matrix Multiplication	10
2.8 Determinant of a Matrix	13
2.9 Inverse of a Matrix	15
2.10 Systems of Linear Equations	18
2.11 Matrices in <i>Excel</i>	20
3. CONCLUSION	78
REFERENCES	79



#### A SunCam online continuing education course

## **1. INTRODUCTION**

## 1.1 Overview

Engineers are constantly challenged with solving a wide range of complex analytical and computational problems in their fields of specialty. These problems involve analysis methodologies and the management of data. The application of computers enables repetitive, time-consuming and often tedious calculations to be conducted rapidly, efficiently, and less prone to errors. The application of computer tools also enables the results and outputs of such engineering analyses to be readily transferred and incorporated into reports and other engineering documents. An even greater advantage, in terms of productivity and efficiency, is realized when these calculations and outputs are replicated across numerous projects. Competence in computer skills predisposes engineers to pursue and develop more creative and innovative solutions to problems than otherwise.

*Microsoft Excel* is an electronic spreadsheet program developed by the *Microsoft* company, and is part of the *Microsoft Office* suite of products. A spreadsheet is a grid that organizes data and calculations into columns and rows. The intersection of a column and a row is called a cell. An electronic spreadsheet enables users to store, organize, manipulate, and analyze data in the cells of the spreadsheet. As of this publication, the software is into the 2016 release.

*Microsoft Excel* is widely and increasingly being used as a tool to assist engineers in conducting and replicating intricate calculations and analysis, designing complex systems, and managing large data sets. This course presents a selection of engineering analysis topics and how to formulate, implement and solve them in *Microsoft Excel*.

This course presents fundamental principles and engineering applications of problems in linear algebra, and demonstrates the *Excel* tools, methods, and strategies that can be used to formulate and solve them.

Upon completion of this course, participants will have gained insight into applying *Excel* tools, methods, and strategies in analyzing, formulating and implementing a variety of engineering related problems, as well as managing large sets of complex data. Participants will also be able to identify professional situations where the application of innovative *Excel* techniques will be of great benefit and advantage, and will enable practitioners to significantly improve their productivity, efficiency, and the quality of their work product.



## 2. LINEAR ALGEBRA

#### **2.1 Introduction**

In this chapter, we shall conduct a review of the fundamentals of linear algebra, and see how the methods can be implemented in *Excel* on a spreadsheet.

#### 2.2 Arrays

An array is an orderly list or arrangement of items or **elements**. The elements of the array are arranged in columns and rows. The number of rows and the number of columns are called the **dimensions** of the array. By convention, the elements in the array are listed by separating them by commas, or a space. Consider the following array called **A** that contains the following numerical elements,

 $\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 & -1 & 3 \end{bmatrix}$ 

This array has 1 row and 5 columns. The array is therefore described as a 1x5 ("one-by-five") array. Each element in the array is identified by its unique "address", (or "position"), or **index** in the array. By convention, the following designation is used to identify an element in an array,

## ArrayName(Row#,Column#)

where the first row is Row 1, and the first column is Column 1, and. so on. *Row#* and *Colum#* are the indices of the element.

Therefore, for our array **A**, the element in the Row 1, Column 3, position, is the value 1. Using the notation,

A(1,3) = 1



Consider another array, **B**, which has the elements,

$$\mathbf{B} = \begin{bmatrix} -1\\2\\-2\\0\\1 \end{bmatrix}$$

This array has five rows and one column. It is therefore a 5x1 array. We can see for example, that

$$\mathbf{B}(4,1)=0$$

An array that has one row and more than one column, or one column and more than one rows, is called a **vector**. The arrays A and B we just studied are vectors. A is a row vector, whereas B is a column vector. A multidimensional array – an array with more than one row and more than column, is called a **matrix** (plural is matrices).

#### 2.3 Matrices

Consider the following array,

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -1 & 3 \end{bmatrix}$$

**M** is a 2x3 matrix.

Let i denote the number of rows of a matrix Let j denote the number of columns of a matrix.



If i = j, then the matrix is a square matrix. For example, the matrix

$$\mathbf{P} = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}$$

is a 2x2 square matrix. The matrix

$$\mathbf{Q} = \begin{bmatrix} 2 & 3 & 0 & -1 \\ 2 & 1 & 0 & -2 \\ 0 & 1 & 2 & 3 \\ 1 & -1 & 2 & -1 \end{bmatrix}$$

is a 4x4 square matrix.

For a i-by-i square matrix **X**, the elements **X** (1,1), **X** (2,2), ..., **X**( $_{i,i}$ ) form what is called the **main diagonal** of the matrix, whereas the elements **X**(1,*i*), **X**(2, *i* – 1), ..., **X**(*i*, 1) form the **minor diagonal** of the matrix. For example,



#### 2.4 Matrix Addition and Subtraction

Matrix addition (and subtraction) can be conducted on two matrices provided they have the same dimensions. For example a 2x3 matrix can be added to (or subtracted from) a 2x3 matrix, whereas the addition (or subtraction) of a 2x3 matrix and a 4x2 matrix is not possible. Matrix



addition (and subtraction) is conducted by adding (or subtracting) the corresponding elements of the matrices.

Example: Consider the matrices

$$\mathbf{Y} = \begin{bmatrix} 1 & 4 & 5 \\ 1 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

and

$$\mathbf{Z} = \begin{bmatrix} -1 & 3 & 0 \\ -2 & 2 & 0 \\ 3 & 5 & 1 \end{bmatrix}$$

So,

$$\mathbf{Y} + \mathbf{Z} = \begin{bmatrix} 0 & 7 & 5 \\ -1 & 4 & 6 \\ 3 & 6 & 4 \end{bmatrix}$$

and

$$\mathbf{Y} - \mathbf{Z} = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 0 & 6 \\ -3 & -4 & 2 \end{bmatrix}$$

#### **2.5 Scalar Multiplication**

Scalar multiplication involves multiplying the matrix by a scalar value. This results in each element of the matrix being multiplied by the scalar value.



Example: Consider the matrix

$$\mathbf{Y} = \begin{bmatrix} 1 & 4 & 5 \\ 1 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

Multiplying **Y** by a scalar value of 5, yields,

$$\mathbf{5} * \mathbf{Y} = 5 * \begin{bmatrix} 1 & 4 & 5 \\ 1 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 20 & 25 \\ 5 & 10 & 30 \\ 0 & 5 & 15 \end{bmatrix}$$

#### 2.6 Transpose of Matrix

A matrix is transposed by making the rows of the matrix, the columns of the matrix (or vice versa). So, for example, the first row of the original matrix becomes the first column of the transposed matrix and so on.

Another way of viewing the transpose of a matrix is that the indices of each element will be switch or swapped, or reversed in the transposed matrix. To generalize for an element of a matrix **X**,

$$X(i,j) \xrightarrow{\text{Transpose}} X^{T}(j,i)$$

where  $\mathbf{X}^{T}$  is the transpose matrix of  $\mathbf{X}$ .



Example: The transpose of the matrix

$$\mathbf{Y} = \begin{bmatrix} 1 & 4 & 5 \\ 1 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

is

$$\mathbf{Y}^{\mathbf{T}} = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 2 & 1 \\ 5 & 6 & 3 \end{bmatrix}$$

#### 2.7 Matrix Multiplication

An MxN matrix can be multiplied by a PxQ matrix only if the "inner dimensions" are equal, that is, if N = P (number of columns in first matrix equals number of rows in second matrix). The resulting matrix will be of dimensions MxQ. If the inner dimensions are not equal, the product of the two matrices does not exist. To generalize,

$$X_{MxN} * Y_{PxQ} = Z_{MxQ}$$

if N = P, otherwise the product of the input matrices does not exist, where **X**, **Y** are the input matrices, and **Z** is the output matrix of the matrix multiplication process.

The elements of the output matrix are computed as follows. For any row i, an element

$$\mathbf{Z}(\mathbf{i},\mathbf{j}) = \sum_{i=1}^{N} \mathbf{X}(\mathbf{i},\mathbf{j}) * \mathbf{Y}(\mathbf{j},\mathbf{i})$$

and we repeat for row i = 1 to M



This process can be described graphically as follows.

X(1,1) X(2,1) X(3,1)	<mark>X(1,2)</mark> X(2,2) X(3,2)	$ \begin{bmatrix} X(1,3) \\ X(2,3) \\ X(3,3) \end{bmatrix} * \begin{bmatrix} Y(1,1) \\ Y(2,1) \\ Y(3,1) \end{bmatrix} $	Y(1,2) Y(2,2) Y(3,2)	$ \begin{cases} Y(1,3) \\ Y(2,3) \\ Y(3,3) \end{bmatrix} = \begin{bmatrix} Z(1,1) \\ Z(2,1) \\ Z(3,1) \end{cases} $	Z(1,2) Z(2,2) Z(3,2)	Z(1,3) Z(2,3) Z(3,3)
[ <mark>X(1,1)</mark> X(2,1) X(3,1)	<mark>X(1,2)</mark> X(2,2) X(3,2)	$ \begin{bmatrix} X(1,3) \\ X(2,3) \\ X(3,3) \end{bmatrix} * \begin{bmatrix} Y(1,1) \\ Y(2,1) \\ Y(3,1) \end{bmatrix} $	Y(1,2) Y(2,2) Y(3,2)	$ \begin{array}{c} Y(1,3) \\ Y(2,3) \\ Y(3,3) \end{array} \end{bmatrix} = \begin{bmatrix} Z(1,1) \\ Z(2,1) \\ Z(3,1) \end{bmatrix} $	Z(1,2) Z(2,2) Z(3,2)	Z(1,3) Z(2,3) Z(3,3)
[ <mark>X(1,1)</mark> X(2,1) X(3,1)	<b>X(1,2)</b> X(2,2) X(3,2)	$ \begin{bmatrix} X(1,3) \\ X(2,3) \\ X(3,3) \end{bmatrix} * \begin{bmatrix} Y(1,1) \\ Y(2,1) \\ Y(3,1) \end{bmatrix} $	Y(1,2) Y(2,2) Y(3,2)	$ \begin{bmatrix} Y(1,3) \\ Y(2,3) \\ Y(3,3) \end{bmatrix} = \begin{bmatrix} Z(1,1) \\ Z(2,1) \\ Z(3,1) \end{bmatrix} $	Z(1,2) Z(2,2) Z(3,2)	Z(1,3) Z(2,3) Z(3,3) ]
$ \begin{bmatrix} X(1,1) \\ X(2,1) \\ X(3,1) \end{bmatrix} $	X(1,2) X(2,2) X(3,2)	$ \begin{array}{c} X(1,3) \\ X(2,3) \\ X(3,3) \end{array} * \begin{bmatrix} Y(1,1) \\ Y(2,1) \\ Y(3,1) \\ \end{array} $	Y(1,2) Y(2,2) Y(3,2)	$ \begin{cases} Y(1,3) \\ Y(2,3) \\ Y(3,3) \end{cases} = \begin{bmatrix} Z(1,1) \\ Z(2,1) \\ Z(3,1) \end{cases} $	Z(1,2) Z(2,2) Z(3,2)	Z(1,3) Z(2,3) Z(3,3)

and so on, and so forth.

The corresponding color coded elements (yellow, green, blue) are multiplied and all summed up to produce the element of the output matrix (in grey).

Example: Multiply the matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 2 & -1 \end{bmatrix}$$

by



$$\mathbf{G} = \begin{bmatrix} 2 & -2\\ 1 & 0\\ 1 & 2 \end{bmatrix}$$

We have a 2x3 multiplying a 3x2. The inner dimensions are equal, so this matrix multiplication can proceed, and will result in a 2x2 matrix.

$$\mathbf{H} * \mathbf{G} = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 2 & -1 \end{bmatrix} * \begin{bmatrix} 2 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$$
$$\mathbf{H} * \mathbf{G} = \begin{bmatrix} 1 * 2 + 1 * 1 + 3 * 1 & 1 * (-2) + 1 * 0 + 3 * 2 \\ 4 * 2 + 2 * 1 + (-1) * 1 & 4 * (-2) + 2 * 0 + (-1) * 2 \end{bmatrix}$$
$$\mathbf{H} * \mathbf{G} = \begin{bmatrix} 6 & 4 \\ 9 & -10 \end{bmatrix}$$

Another multiplication operation for matrices is the Element-by-Element Multiplication, also known as the Dot Product. As the name suggests, the element-by-element multiplication involves multiplying the corresponding elements in the two input matrices. Therefore, the element-by-element multiplication can only work if the two input matrices have identical vertical and horizontal dimensions. In this course we shall use ".\*" (dot asterisk) to denote element-by-element multiplication.

Example: Given

$$\mathbf{P} = \begin{bmatrix} 3 & 2\\ 1 & -2 \end{bmatrix}$$
$$\mathbf{Q} = \begin{bmatrix} 1 & -1\\ 1 & 3 \end{bmatrix}$$



Therefore, the dot product,

$$\mathbf{P} \cdot \mathbf{q} = \begin{bmatrix} 3 & -2 \\ 1 & -6 \end{bmatrix}$$

#### 2.8 Determinant of a Matrix

The determinant of a square matrix is a real number associated with the matrix. The determinant is computed from the elements of the matrix. Later on, we shall see that the determinant is an input in finding the inverse of a square matrix.

In this section we shall use the algorithm for computing the determinant of a 3x3 matrix to illustrate the fundamental principles as well as introduce new concepts.

Consider the following 3x3 matrix,

$$\mathbf{F} = \begin{bmatrix} 5 & -1 & 3 \\ 4 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

We can pick a row, or a column to base the process on. For convenience we shall pick the first row.

Step 1: For the first element "5", let us picture eliminating the row and the column that this element belongs to.

$$\begin{bmatrix} 5 & 1 & 3 \\ 4 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$



We will be left with a 2x2 matrix

$$\begin{bmatrix} -2 & 2 \\ 0 & 1 \end{bmatrix}$$

Step 2: Now compute the product of the elements along the main diagonal, subtract the product of the elements on the minor diagonal

[(-2)\*1 - 2\*0] = -2

So "-2" is called the **minor** of the element "5".

Step 3: Multiply the minor by  $(-1)^{(i+j)}$ , where *i* and *j* are the row and column indices of the element whose minor we just computed (in this example, the indices of the element "5"). The result of this calculation is called the **cofactor** of the original element.

Step 4: Repeat the steps 1 through 3 above to compute the cofactors of the other elements on the first row of the original matrix.

Step 5: We now compute the summation of the product of each element on the first row by its cofactor. This summation is the determinant of the square matrix.

Example. The determinant for our matrix F,

$$det(\mathbf{F}) \text{ or } |\mathbf{F}| = 5 * (-1)^{1+1} * \begin{vmatrix} -2 \\ 0 \end{vmatrix}^{2} \begin{vmatrix} 2 \\ 1 \end{vmatrix} + (-1) * (-1)^{1+2} * \begin{vmatrix} 4 \\ 1 \end{vmatrix}^{2} \begin{vmatrix} 4 \\ 1 \end{vmatrix}^{2} + 3 * (-1)^{1+3} * \begin{vmatrix} 4 \\ 1 \end{vmatrix}^{2} \begin{vmatrix} 4 \\ 0 \end{vmatrix}$$
$$= 5 * (1) * [(-2)(1) - 2(0)] + (-1) * (-1) * [4(1) - 2(1)] + 3 * (1) * [4(0) - (-2)(1)]$$

 $|\mathbf{F}| = -10 + 2 + 6 = -2$ 



So to generalize, for a 3x3 matrix with the elements

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Using the first row as the basis for the computation procedure, the determinant,

$$|\mathbf{F}| = f_{11} * (-1)^{i+j} * \begin{vmatrix} f_{22} & f_{23} \\ f_{32} & f_{33} \end{vmatrix} + f_{12} * (-1)^{i+j} * \begin{vmatrix} f_{21} & f_{23} \\ f_{31} & f_{33} \end{vmatrix} + f_{13} * (-1)^{i+j} * \begin{vmatrix} f_{21} & f_{22} \\ f_{31} & f_{32} \end{vmatrix}$$

It is pertinent to note that the notation used for the determinant is similar to that of the absolute value or the magnitude of a vector. The determinant of a matrix is neither of these, it is a real value associated with the square matrix and is calculated from the elements of the matrix. The determinant can be a positive or negative value. The determinant can also be zero in which case it has consequences on the inverse of the matrix which we shall study later in this course. Using the above algorithm for a 2x2 matrix it will be observed that the determinant of a 2x2 matrix reduces to the product of the elements on the main diagonal minus the product of the elements on the minor diagonal. For higher dimensions of square matrices, 4x4, 5x5, etc., etc.), the calculation of the determinant follows the same procedure but becomes more involving and considerably more tedious.

#### 2.9 Inverse of a Matrix

Consider an NxN (square) matrix **A**. If **A** has a non-zero determinant, then there exists another NxN matrix, denoted  $A^{-1}$ , such that

$$\mathbf{A} * \mathbf{A}^{-1} = \mathbf{I}$$



where  $\mathbf{I}$  is an NxN **identity matrix**, otherwise  $\mathbf{A}^{-1}$  does not exist.

 $A^{-1}$  is called the inverse matrix of the matrix A.

An identity matrix is one that has all elements on the main diagonal equal to one (1), and all other elements equal to zero (0). For example the 3x3 identity matrix is

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The identity matrix is analogous to the value one (1). If a square matrix is multiplied by its identity matrix, the result is that same matrix (analogous to multiplying a constant by unity). The algorithm to calculate the elements of the inverse matrix involves the following steps.

Step1: Replace each element of the original matrix with its cofactor.

Step 2: Transpose the matrix of cofactors. The resulting matrix is called the **adjoint matrix**.

Step 3: Divide each element of the adjoint matrix by the determinant of the original matrix. The resulting matrix is the inverse matrix of the original matrix.

Example: Let us compute the inverse of

$$\mathbf{K} = \begin{bmatrix} 1 & -1 & 3 \\ 4 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

The determinant of K is

$$|\mathbf{K}| = 1 * (-1)^{1+1} * \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} + (-1) * (-1)^{1+2} * \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} + 3 * (-1)^{1+3} * \begin{vmatrix} 4 & -2 \\ 1 & 0 \end{vmatrix}$$
$$= 1 * (1) * [(-2)(1) - 2(0)] + (-1) * (-1) * [4(1) - 2(1)] + 3 * (1) * [4(0) - (-2)(1)]$$



 $|\mathbf{K}| = -2 + 2 + 6 = 6$ 

The elements of the matrix of cofactors are as follows

$$\begin{aligned} \mathbf{C}_{11} &= (-1)^{1+1} * \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} = (1) * (-2 - 0) = -2 \\ \mathbf{C}_{12} &= (-1)^{1+2} * \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} = (-1) * (4 - 2) = -2 \\ \mathbf{C}_{13} &= (-1)^{1+3} * \begin{vmatrix} 4 & -2 \\ 1 & 0 \end{vmatrix} = (1) * (0 + 2) = 2 \\ \mathbf{C}_{21} &= (-1)^{2+1} * \begin{vmatrix} -1 & 3 \\ 0 & 1 \end{vmatrix} = (-1) * (-1 - 0) = 1 \\ \mathbf{C}_{22} &= (-1)^{2+2} * \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = (1) * (1 - 3) = -2 \\ \mathbf{C}_{23} &= (-1)^{2+3} * \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = (-1) * (0 + 1) = -1 \\ \mathbf{C}_{31} &= (-1)^{3+1} * \begin{vmatrix} -1 \\ -2 & 2 \end{vmatrix} = (1) * (-2 + 6) = 4 \\ \mathbf{C}_{32} &= (-1)^{3+2} * \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} = (-1) * (2 - 12) = 10 \\ \mathbf{C}_{33} &= (-1)^{3+3} * \begin{vmatrix} 1 & -1 \\ 4 & -2 \end{vmatrix} = (1) * (-2 + 4) = 2 \end{aligned}$$



The matrix of cofactors is

$$\mathbf{C} = \begin{bmatrix} -2 & -2 & 2\\ 1 & -2 & -1\\ 4 & 10 & 2 \end{bmatrix}$$

We transpose the matrix of cofactors to obtain the adjoint matrix

$$\mathbf{K}_{adj} = \begin{bmatrix} -2 & 1 & 4\\ -2 & -2 & 10\\ 2 & -1 & 2 \end{bmatrix}$$

And then we divide each element of the adjoint matrix by the determinant of the original matrix to obtain the inverse matrix.

$$\mathbf{K}^{-1} = \frac{1}{6} * \begin{bmatrix} -2 & 1 & 4 \\ 0 & -2 & -10 \\ 2 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -0.333 & 0.166 & 0.666 \\ -0.333 & -0.333 & 1.666 \\ 0.333 & -0.166 & 0.333 \end{bmatrix}$$

#### 2.10 Systems of Linear Equations

One application of matrices commonly used by engineers is in solving systems of linear equations.

Consider a system of linear equation with m equations and m unknown quantities ( $x_i$ ) as follows.

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1m}x_m = b_1$$
  
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2m}x_m = b_2$$
  
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
  
$$a_{m1}x_m + a_{m2}x_2 + \dots + a_{mm}x_m = b_m$$



We may represent this system of linear equations in matrix notation as follows

$[a_{11}]$	$a_{12}$		$a_{1m}$	$\begin{bmatrix} x_1 \end{bmatrix}$		$\begin{bmatrix} b_1 \end{bmatrix}$	
a <sub>21</sub>	$a_{22}$		$a_{2m}$	<i>x</i> <sub>2</sub>	_	<i>b</i> <sub>2</sub>	
:	:	:	:	:	-	:	
$a_{m1}$	$a_{m2}$		$a_{mm}$	$Lx_m$		$\lfloor b_m \rfloor$	

Or in a more simplified notation,

$$Ax = b$$

Where **A** is the matrix of coefficients, **x** is the vector of unknown quantities, and **b** is the vector of the constants on the right hand sides of the equations. Multiplying both sides by  $A^{-1}$ , we obtain

$$\mathbf{A} * \mathbf{A}^{-1}\mathbf{x} = \mathbf{A}^{-1} * \mathbf{b}$$
$$\mathbf{x} = \mathbf{A}^{-1} * \mathbf{b}$$

In other words if we multiply the vector of constant terms on the right hand sides of the equations by the inverse of the matrix of coefficients, we obtain a vector whose elements are the values of the unknown quantities of the system of equations. That is

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix}^{-1} * \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Example: Solve the following system of linear equations.

$$x_1 + x_2 + x_3 = 6$$
$$2x_2 + 5x_3 = -4$$
$$2x_1 + 5x_2 - 3x_3 = 27$$



In matrix form this will be,

 $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix}$  $\begin{bmatrix} \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{X} \end{bmatrix} \begin{bmatrix} \mathbf{X} \end{bmatrix} \begin{bmatrix} \mathbf{X} \end{bmatrix} \begin{bmatrix} \mathbf{A} \end{bmatrix}$ 

or

Inverting A, we obtain

$$\mathbf{A^{-1}} = \begin{bmatrix} 1.24 & -0.32 & -0.12 \\ -0.40 & 0.20 & 0.20 \\ 0.16 & 0.12 & -0.08 \end{bmatrix}$$

and then

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{A}^{-1} * \mathbf{b} = \begin{bmatrix} 1.24 & -0.32 & -0.12 \\ -0.40 & 0.20 & 0.20 \\ 0.16 & 0.12 & -0.08 \end{bmatrix} \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix} = \begin{bmatrix} 5.48 \\ 2.2 \\ -1.68 \end{bmatrix}$$

#### 2.11 Matrices in Excel

In this section we shall implement all the matrix methods we have reviewed so far in *Excel*. In Excel matrices' elements can be entered directly into the cells of a spreadsheet, and then manipulated from there.

Open a new *Excel* file. Go to the spreadsheet **Sheet1** 



Enter the elements of a matrix **A** and a matrix **B** as follows,

x∎	5	- @						Book1 - E	xcel					? 🗹	ı — 🗆	×
F	ILE HO	DME INS	SERT PA	AGE LAYOUT	FORM	ULAS D	ATA	REVIEW	VIEW D	EVELOPER	FOXIT RE	ADER PDF			Sign ir	n 🍳
Pas	te ✓	Calibri B I <u>U</u>	• 11 • 🗄 •	• A •		<b>_ ∛</b> ≁ ≡ €≣ #≣	II •	General \$ - %	▼ 9 (.00 0.00 0.00 0.0	Condi Forma	tional Format t as Table * yles *	tting - E	r Insert → Delete → Format →	∑ · A Z · Z · Sort 8 · Filter	' Ind & * Select *	
Clip	board 🗔		Font	5	A	lignment	Es.	Num	ber 🖓		Styles		Cells	Editi	ng	^
E9		▼ ÷ ⊃	X 🗸	$f_{x}$												~
	А	В	С	D	E	F	G	н	1	J	К	L	М	Ν	0	
1	Α	1	2	2	2		В	-2	1	2	4					
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3					
3		-1	2	3	-2			2	3	-3	3					_
4																-
6																
7																
8																
9																
10																
11																_
12																_
13																_
	<	Sheet1	+							-						
REA	DY 🔠												] 🛄 –		<b>— +</b> 10	0%

#### Addition and Subtraction

Let us add **A** and **B** to create a matrix called **C**.

Click on the cell location of the element C(1,1). Type in the simple addition formula to add A(1,1) and its counterpart B(1,1). The formula is "= B1 + H1".



The formula may be typed in, or you may click on the relevant cells after the typing in the "=". The latter is commonly referred to as **Point-and-Click** formula entry.

x	l 🔒 🔊	- @				Book1 - E	cel			? 🛧	- □	×
F	ILE H	OME IN:	SERT P.	AGE LAYOUT	FORM	IULAS	DATA F	EVIEW	VIEW D	DEVELOPER	FOXIT RE	ADEF►
Pa	ste	B I <u>U</u> 	• 11 • • A A • A •		₽ □ - ?	General \$ - % €.0 .00 .00 →.0	<ul> <li>✓ E Co</li> <li>✓ For</li> <li>✓ Cel</li> </ul>	nditional For mat as Table Il Styles -	matting -	Hinsert ▼ E× Delete ▼ Hind Format ▼	Editing	
Cli	oboard 🗔	Fon	t	G Alignmer	nt 🖬	Number	G)	Styles		Cells		~
H	L	* ÷ )	× 🗸	<i>f</i> ∞ =B1+	H1							~
	А	В	С	D	Е	F	G	н	1	J	К	
1	Α	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4	C	-014111										
6	[3x4]	-01401										
7												
8												
9												
10												
11												
12												
	4 F	Sheet1	(+)				:	4				
РО	NT 🔚							Ⅲ □			-+ 10	00%



Hit **Enter** to see the calculated value of the element.

X∄	E 5	- 0				Book1 - Ex	cel			? 📧	- □	×
FIL	E H	OME INS	SERT P/	AGE LAYOUT	FORM	IULAS	DATA	REVIEW	VIEW D	EVELOPER	FOXIT RE	ADEF 🕨
Past	te 💉	Calibri B I U - &	• 11 • • A A A • A •		■ 🖶 - ≫ -	General \$ - % ←.0 .00 .00 →.0	) ∰ ▼ 1 ₩ 2 ) ₩ 2	Conditional For Format as Table Cell Styles = Styles	rmatting * = *	Delete •	Editing	~
nc				s light				Sqies		- Centr		
BO		· · ·	~ ~	Jx								
	А	В	С	D	E	F	G	Н	1	J	K	
14	4	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4	_											
5	[2v4]	-1										
7	[584]											
8												
9							_					
10												
11												
12												
13												
4	) E	Sheet1	+									►
READ	рү 🔠							▦	<b>.</b> .			00%

We shall now replicate this calculation for the other elements of the matrix C. We know that the addition of two 3x4 matrices will yield a 3x4 matrix. We shall copy the formula to fill two more rows and three more columns.



Select the cell holding C(1,1).

Hover over the bottom right corner of the cell. Notice that the cursor changes to a "thin" plus ("+") sign.

XI .	5	- @				Book1 - Ex	cel			? 🖈	- □	×
FILE	н	OME IN:	SERT P	AGE LAYOUT	FORM	IULAS	DATA R	EVIEW	VIEW D	DEVELOPER	FOXIT RE	ADEF •
Paste	X ≌ - ∛	Calibri B I U - A	• 11 • • A A • A •		■ ₽ ■ = - ≫⁄ -	General \$ ▼ % €.000	<ul> <li>✓ E Con</li> <li>&gt; ✓ For</li> <li>✓ Cel</li> </ul>	nditional For mat as Table I Styles <del>+</del>	matting • • •	E Insert ▼ Delete ▼ Format▼	Attack Editing	
Clipboa	ard 🗔	Fon	it	🖫 Alignme	ent 🗔	Number	Ga .	Styles		Cells		^
B5		▼ ÷ ⊃	× 🗸	<i>f</i> <sub>x</sub> =B1	+H1							~
	А	В	с	D	E	F	G	Н	1	J	К	
1 <b>A</b>		1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4												
5 C	[2v/]	-1										
7	[344]											
8												
9												
10												
11												
12												
13												
	+	Sheet1	•	)			:	•				Þ
READY	1								- ·			00%

With the cursor now as a "thin" plus sign, click and hold down the cursor. With the cursor held down, drag the cursor two rows below the original cell. Release the cursor.



The addition formula is copied and updated automatically for the relevant input cells (elements), thus calculating all of the elements for the first column. This type of copying data is called the **Fill Handle**.

🛛 🔒 🕤	- @				Book1 - Ex	cel			? 📧		×
FILE +	IOME IN	SERT P	AGE LAYOUT	FORM	MULAS	DATA F	REVIEW	VIEW D	DEVELOPER	FOXIT RE	ADEF
Paste	Calibri B I U 	• 11 • • A <sup>*</sup> A <sup>*</sup>		■ 🗗 + ≫ +	General \$ - % €.0 .00 .00 →.0		nditional For rmat as Table II Styles *	matting •	Ensert  Delete  Format	H Editing	•
Cipboard is			Angrine	11. 31	Number	Call I	Styles		Cells		
B5	▼ : )	×	<i>f</i> <sub>x</sub> =B1+	H1							~
A	В	С	D	Е	F	G	Н	1	J	К	
1 <b>A</b>	1	2	2	2		В	-2	1	2	4	
2 <b>[3x4]</b>	2	-3	4	2		[3x4]	1	-1	2	3	
3	-1	2	3	-2			2	3	-3	3	
4	1										$\left  - \right  $
6 [3x4]	3										
7	1										
8		<b>-</b>									
9											
10											
12											$\left  \cdot \right $
13											
	Sheet1	(+)			!	:	•	1	<u> </u>		
READY 🔚			AVE	RAGE: 1	COUNT: 3	SUM: 3		<b>P</b> -		<b>-+</b> 1	00%

If not selected already, select the three elements of the first column of the matrix C.



Hover over the bottom left corner of the selection until the cursor changes to the "thin" plus sign. Click and hold down the cursor.

Drag the cursor over three additional columns to complete the four columns of the matrix. Release the cursor.

x	l 🔒 🍤	- @- ;	;			Book1 - E	cel			? 🛧		×
F	ILE H	OME IN	ISERT PA	AGE LAYOUT	FORM	IULAS	DATA F	EVIEW	VIEW D	DEVELOPER	FOXIT RE	ADEF
Pa	ste	Calibri B I U 	• 11 • • A <sup>*</sup> A <sup>*</sup>		■ = = = = = ≫/ -	General \$ - % €.0 .00 .00 →.0	→ 🛃 Co > 🐺 For 🐺 Ce	nditional Fo mat as Table I Styles *	rmatting + e +	Ensert  Delete  Format	Editing	
				si Alignme		Number	12	Styles		Cells		
ВЗ	)	•	~	Jx =B1	+H1							*
	Α	В	С	D	E	F	G	Н	1	J	К	<b>A</b>
1	Α	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4												
5	С	-1	3	4	6							
6	[3x4]	3	-4	6	5							
7		1	. 5	0	1							
8												-
9												
10												+
11												+
12												+
15												
	<	Sheet1	(+)					•				
RE4	NDY 🛅		AVE	RAGE: 2.4166	66667 CC	DUNT: 12	SUM: 29		- <b>D</b>		— <b>+</b> 1	00%



#### Multiplication By Scalar

Let's multiply the matrix A on the spreadsheet by a scalar value of 5. Enter the scalar value of the spreadsheet, and select the cell location of element (1,1) as follows.

x	5	- @	:			Book1 - E	cel			? 📧	- □	×
F	ILE HO	DME IN	SERT PA	AGE LAYOUT	FORM	IULAS	DATA	REVIEW	VIEW [	DEVELOPER	FOXIT RE	ADEF►
Pa	ste	Calibri B I U 	• 11 • • A A A		■ ₽ ■ □ • ≫••	General \$ - % €.0000	<ul> <li>▼</li> <li>₽</li> <li>₽</li> <li>₽</li> <li>₽</li> </ul>	Conditional Format as Ta Cell Styles •	Formatting • Ible •	E Insert ▼ E Delete ▼ E Format ▼	Editing	
Cli	pboard 🗔	For	it r	Alignm	ent 🗔	Number	Es .	Style	5	Cells		^
B	2	• : :	X V	$f_{x}$								~
	А	В	С	D	E	F	G	н	I	J	К	
4												
5	С	-1	3	4	6							
6	[3x4]	3	-4	6	5							
7		1	5	0	1							
8												
10	Scalar Mu	Itiplication										
11	5											
12	5* <b>A</b>											
13												
14												
15												
16												
		Sheet1	+					-				×
RE/	ADY 🛗								3 🗉 -		-+ 10	00%



In the cell that will hold element (1,1) type the equal sign ("="). By the point-and-click method, select the cell containing the scalar value.

Hit the **F4** button on your keyboard. Type the multiply ("\*") sign. Click on the element **A**(1,1)

x		<b>5</b> - ∂ -	Ŧ			Book1 - Exce	2			? 📧		i x
F	ILE	HOME	NSERT P.	AGE LAYOUT	FORM	ULAS D	ATA F	REVIEW	VIEW [	DEVELOPER	FOXIT RE	ADEF
Pa	ste	• B I L	- 11 - J - A^ A` ≥ - A -		= = = = - %/-	General \$ - % * €.0 .00	For For Ce	nditional Fo rmat as Tab II Styles -	rmatting -	E Insert ▼ E Delete ▼	H Editing	
Cli	pboard	Fa Fo	ont	Alignme	ent 🖬	Number	G.	Styles		Cells		~
B	1	• :	× 🗸	<i>f<sub>sc</sub></i> =\$A	\$11*B1							~
	Α	В	С	D	E	F	G	н	1	J	К	
1	Α	1	2	2	2		в	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4	c		1 2	4	6							
6	[3x4]	-	3 -4	6	5							
7	[SA1]		1 5	0	1							
8												
9												
10	Scalar I	Multiplicatio	n									_
11	5											
12	5*A	=\$A\$11*	81									+
13		Sheet	:1 (+)					4				
РО	INT 🔡	]							•		-+ 1	00%

Hit Enter on your keyboard.



## The element (1,1) of the new matrix is computed,

X	5	- @				Book1 - E	kcel				? 🗗	- □	×
F	ILE H	OME IN	SERT PA	AGE LAYOUT	FORM	IULAS	DATA	RE	VIEW	/IEW D	EVELOPER	FOXIT RE	ADEF
Pa	∎ X □ ⊡ - ste V	Calibri B I U 	• 11 • • A A A • A •		■ 🗗 + ● 🖽 + ॐ⁄ +	General \$ - % €.0000	• 9	🖶 Con 🐺 Forn 🐺 Cell	ditional For nat as Table Styles <del>*</del>	matting *	Insert ▼ Delete ▼ Format ▼	Editing	
Clij	board 🗔	For	nt r	Alignme	ent 🖫	Number	E.		Styles		Cells		~
<b>B</b> 1	.3	• : :	× 🗸	f <sub>x</sub>									*
	А	В	С	D	Е	F		G	н	1	J	К	
4													
5	С	-1	3	4	6		_						
6	[3x4]	3	-4	6	5								
7		1	5	0	1		_						
8							_						
10	Scalar Mu	Itiplication											
11	5												
12	5* <b>A</b>	5											
13													
14													
15							_						
16													
	4 - F	Sheet1	+						•				
REA	DY 🔠									<b>—</b> •			00%



In this procedure we hit **F4** after selecting the cell containing the scalar value. This makes that cell an **Absolute Reference**. When a formula containing an absolute reference is copied (by Fill Handle), the absolute reference does not update, hence locking in the cell so that the constant value it contains is applied to all cells that are covered by the fill handle operation. An absolute reference is identified by the dollar (\$) signs incorporated into the cell address upon hitting **F4**, or they may be typed in manually.

x		<b>5</b> - d	€~ ÷				Book1 - E	kcel			? 🖈	_	□ ×	
F	ILE	HOME	INSE	RT PA	AGE LAYOUT	FORM	ULAS	DATA F	EVIEW	VIEW D	DEVELOPER	FOXIT	READEF	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $								rmatting - e -	E Insert → Cells	Editing				
B1	B1 $\checkmark$ : $\times \checkmark f_{x}$ =\$A\$11*B1 $\checkmark$													
	А		в	С	D	E	F	G	н	1	J	к		
1	Α		1	2	2	2		В	-2	1	2	4		
2	[3x4	4]	2	-3	4	2		[3x4]	1	-1	2	3		
3			-1	2	3	-2			2	3	-3	3		
4	-													
5	C		-1	3	4	6								
0	[3X4	1	3	-4	0	3								
8			1	5	U	1								
9														
10	Scalar	Multipli	cation											
11	5	I												
12	5* <b>A</b>	=\$A	\$11 <b>*</b> B1											
13			N.											
	• •	S	heeti	÷				:	•				Þ	
PO	NT 🚦										· · · ·	+	100%	



Now let us complete the multiplication by scalar for the other elements of the new matrix.

Perform fill handle to complete the first column.

x	5	- @- <del>.</del>	;			Book1 - E	cel			? 🛧		×	
F	ILE H	OME IN	SERT PA	AGE LAYOUT	FORM	IULAS	DATA	REVIEW	VIEW D	DEVELOPER	FOXIT RE	ADEF	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $									rmatting • e •	Herensert → Construction → Enternation → Enternation →	H Editing		
Cli	pboard 🕞	For	nt r	a Alignm	ent 🕞	Number	Tai l	Styles		Cells		^	
B	B12 $\checkmark$ : $\times \checkmark f_{x}$ =\$A\$11*B1 $\checkmark$												
	А	В	с	D	E	F	G	н	1	J	К		
4													
5	С	-1	3	4	6								
6	[3x4]	3	-4	6	5								
7		1	5	0	1								
8 9													
10	Scalar Mu	Itiplication											
11	5												
12	5* <b>A</b>	5											
13		10											
14		-5											
15			<u></u> +									+	
10	4	Shoct1											
		Sheet	÷				:						
RÉ#	ADY 🛅		AV	/ERAGE: 3.333	333333 C	OUNT: 3	SUM: 10				-+ 1	00%	



With the first column selected, fill handle to complete the remaining columns.

x	5	- @	:			Book1 - Ex	cel			? 🗹	- □	×		
F	ILE H	OME IN	SERT PA	GE LAYOUT	FORM	ULAS	DATA	REVIEW	VIEW D	DEVELOPER	FOXIT RE	ADEF 🕨		
Pa	ste	Calibri B I U	• 11 • • A <sup>*</sup> A <sup>*</sup>		■ ■ ■ ■ ■ ■ ■ ■ ■ ■	General \$ - % €.000	→ 🔮 Ca > 🐺 Fa	onditional Fo ormat as Tabl ell Styles •	rmatting ▼ e ▼	Ensert ▼ Delete ▼ EFormat ▼	Editing			
Ciij	board S	For	IT D	Alignme	ent G	Number	Dai I	Styles		Cells				
B1	B12 $\checkmark$ : $\times$ $\checkmark$ $f_x$ =\$A\$11*B1 $\checkmark$													
	А	В	С	D	E	F	G	н	I.	J	К			
4														
5	С	-1	3	4	6									
6	[3x4]	3	-4	6	5							_		
7		1	5	0	1							_		
9														
10	Scalar Mu	ltiplication												
11	5													
12	5* <b>A</b>	5	10	10	10									
13		10	-15	20	10									
14		-5	10	15	-10							_		
15														
10	4	Sheet1	(+)				:	•						
REA	DY 🔚		AVE	RAGE: 5.8333	33333 CO	UNT: 12	SUM: 70		<b>II</b>		+ 1(	00%		



## Transpose of a Matrix

Let us transpose the matrix **B** that we created.

Select the cells of the matric **B**. Go to the HOME tab. Click on the **Copy** icon.

x		5-∂-				Book1 - E	xcel			? 📧	i — C	1 × 1
F	ILE	HOME	INSERT F	PAGE LAYOUT	FORM	IULAS	DATA F	REVIEW	VIEW [	DEVELOPER	FOXIT RE	ADEF
Pa: Clip	ste	Calibri B I T T F	- 11 <u>U</u> - A ≠ 2 - A + Font	v = = v = v = v = v = v = v = v = v = v = v =	<ul> <li>■</li> <li>●</li> <li>●</li> <li>●</li> <li>&gt;</li> <li>&gt;</li></ul>	General \$ - % €.0000 Number	✓	nditional F rmat as Tak II Styles * Styles	ormatting + ble +	Ensert -	Editing	~
H1	Pu Cli	<b>py (Ctrl+C)</b> t a copy of th pboard so yo	ne selection or ou can paste it	n the	E	F	G	Н	1	J	К	▼ ▲
1	A	somewhere else.			2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4 5	с		-1	3 4	6							
6	[3x4]		3 -4	4 6	5							
7			1 !	5 0	1							
8												
9												
10	Scalar N	Multiplicati	on									
11	5											
12	5* <b>A</b>		5 10	0 10	10							
13		:	10 -1	5 20	10							-
	$\leftarrow$	Shee	t1 (+	)			:	•				►
Sele	ect destin	ation and pre	ess EN AV	/ERAGE: 1.25	CC	DUNT: 12	SUM: 15		- ""		+ 1	00%



Click a cell where the element (1,1) of the output matrix will be positioned. Click on the **Paste** icon dropdown button.

🕅 🔒 🐬 👌	Ŧ			Book1 - Ex	cel			? 📧	- □	×
FILE HOME	INSERT P	AGE LAYOUT	FORM	ULAS	DATA R	EVIEW V	IEW D	EVELOPER	FOXIT RE/	ADEF ►
Paste	- 11 - <u>□</u> - A A → A -		■ <b>F</b> r [ ■ <b>□</b> • ≫r•	General \$ - % 	<ul> <li>✓ Part Con</li> <li>&gt; Part For</li> <li>↓ Cel</li> </ul>	nditional Fori mat as Table I Styles <del>-</del>	matting ▼ ▼	E Insert ▼ Delete ▼ Format▼	Editing	
Paste	Font	Alignme	nt 🖬	Number	Faile -	Styles		Cells		~
💼 🕏 🕅 🕅	× ✓	$f_{x}$								~
C 🛱 🛱	с	D	E	F	G	н	I.	J	K	
Paste Values	2	2	2		В	-2	1	2	4	
Ê 🕯 🏅	-3	4	2		[3x4]	1	-1	2	3	
123 123 123	2	3	-2			2	3	-3	3	
Other Paste Options										
🐉 👝 🛋 😹	-1 3	4	6							
Paste Special	3 -4	6	5							
=r	1 5	0	1							
8										
9										
10 Scalar Multiplicat	ion									
11 5										
12 <b>5*A</b>	5 10	10	10							
13	10 -15	20	10							-
← → She	et1 🕂				:	4				Þ
Select destination and p	ess ENTER or ch	ioose Paste				⊞ 🗉	<b>—</b> —		— <b>+</b> 10	0%

Select Paste Special.



## In the **Paste Special** window

Check the box for the **Transpose** option.

x	6.5	- @			Book1 - E	xcel		?	– 🗆 X				
F	ILE H	OME IN	SERT PA	AGE LAYOUT	FORMULAS	DATA REVIEW	VIEW D	EVELOPER	FOXIT READEF •				
Pa	ste pboard	Calibri B I U - Son Fon	• 11 • • A A • A • t □		General ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓		Formatting • able •	Ensert • Delete • Format • Cells	Editing				
H	5	▼] ÷ [ 2	×	Jx									
_	A	В	С	D		Paste Spe	ecial	? 🔁					
1	Α	1	2	2	Paste								
2	[3x4]	2	-3	4		C	All using Sou	rca thama					
3		-1	2	3			<ul> <li>All using source (<u>m</u>eme</li> <li>All except borders</li> <li>Column widths</li> </ul>						
4					O Values								
5	С	-1	3	4	Eormats		Column <u>w</u> ittins     Eormulas and number formate						
6	[3x4]	3	-4	6	Comments		Values and number formats						
7		1	5	0	Validation		) All merging c	onditional form	ats				
8					On continu		, an incrying c	onarionariona					
9					Operation	~	· · · · ·						
10	Scalar Mu	ltiplication			None		<u>Multiply</u>						
11	5				O A <u>d</u> d	C	Divide						
12	5* <b>A</b>	5	10	10	<u>Subtract</u>								
13		10	-15	20	Skip <u>b</u> lanks	-	Transpos <u>e</u>						
		Sheet1	(+)		De ste Link		01	Canaal					
Sel	ect destinati	on and press	ENTER or ch	oose Paste	Paste LINK		OK	Cancel	100%				

Click on the **OK** button.



The matrix is copied and transposed as follows.

x	5	- @				Book1 - E	cel			? 🗹	i — 🗆	×	
F	ILE H	IOME INS	SERT PA	AGE LAYOUT	FORM	ULAS	DATA F	REVIEW	/IEW D	EVELOPER	FOXIT RE	ADEF►	
Pa	ste 💉	Calibri B I U - S Fon	• 11 • • A A • A • t		■ 🗗 ד ■ 🖬 ד ≫ ד ent 🕠	General     • <t< td=""><td>Ensert •</td><td>Editing</td><td>^</td></t<>				Ensert •	Editing	^	
HS	H5 $\checkmark$ : $\times \checkmark f_x$ -2 $\checkmark$												
	А	В	С	D	E	F	G	н	1	J	K		
1	Α	1	2	2	2		В	-2	1	2	4		
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3		
3		-1	2	3	-2			2	3	-3	3		
4													
5	С	-1	3	4	6			-2	1	2			
6	[3x4]	3	-4	6	5			1	-1	3			
7		1	5	0	1			2	2	-3			
8								4	3	3	-		
9	Caslanda										🔁 (Ctrl) 🔻		
10	scalar Mu	intiplication											
12	J 5*A	E	10	10	10								
12	эд	10	_15	20	10								
13	4	Sheet1	-13	20	10		:						
Sel	ect destinati	ion and press	ENTER or c	AVERAGE	E: 1.25 CO	UNT: 12	SUM: 15		- ·			00%	


Another method is by using the built-in TRANSPOSE function.

Let us start over. Delete the transposed matrix you just completed.

At the location of the element (1,1) of the transposed matrix type in the formula "=Transpose"

Notice that as you start typing, a shortlist of possible built-in functions you are looking for appears, to assist you.

x		o- ∂	:			Book1 - Ex	cel			? 🗹		×
F	ILE	HOME IN	SERT PA	AGE LAYOUT	FORM	IULAS	DATA R	REVIEW	VIEW D	EVELOPER	FOXIT RE	ADEF +
Pa	ste pboard	B I U		= = = = = = €	= ₽ = ₽ • • • • • • • • •	General \$ - % €.0 -00 Number	<ul> <li>✓ E Co</li> <li>? For</li> <li>☑ Cel</li> <li>☑</li> </ul>	nditional For mat as Table Il Styles ~ Styles	matting *	Ensert ▼ E× Delete ▼ EFormat ▼ Cells	Editing	*
SI	JM		× 🗸	f <sub>∞</sub> =tr								~
	А	В	С	D	Е	F	G	н	1	J	К	
1	Α	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4	c	-1	3	4	6			=tr				
6	[3x4]	3	-4	6	5			_cr €_T	RANSPOSE	Converts a	vertical ra	nge of o
7		1	5	0	1			۲. T	REND			- III
8								€ T	RIM			
9								(Fr) T	RIMMEAN RUF			
10	Scalar M	lultiplication						۰ ا	RUNC			
11	5											
12	5* <b>A</b>	5	10	10	10							
13		10	-15	20	10							
	4 F	Sheet1	+									
EN	TER 🔠										- + 10	00%



Double click on the TRANSPOSE function to select it. Notice the update that now appears in the Formula Bar.

In fact, the function could be have been entered directly in the Formula Bar after selecting the relevant cell on the spreadsheet.

Notice that the pick list of possible functions has reduced to the transpose function we are looking for. The tip now shows how the function is used. In our case we need to select the cells that hold the elements of the array whose transpose we are seeking to create.

x	l 🗄 5	- @				Book1 - Ex	cel			? 🗹	- 🗆	×		
F	ILE H	OME INS	SERT PA	AGE LAYOUT	FORM	ULAS	DATA R	EVIEW	VIEW E	DEVELOPER	FOXIT RE/	ADEF +		
Pa	te →	B I <u>U</u> 	• 11 • • A A • A •			General \$ <b>~ %</b> €.0 .00	<ul> <li>✓ E Cor</li> <li>۶ For</li> <li>♥ Cell</li> </ul>	nditional For mat as Table Styles -	matting -	E Insert → E Delete → E Format →	Editing			
Cli	pboard 🗔	Fon	t G	Alignme	ent 🖬	Number	G.	Styles		Cells		^		
SUM $\checkmark$ : $\times \checkmark f_x$ =TRANSPOSE(														
1	Α	1	2	2	2		В	-2	1	2	4			
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3			
3		-1	2	3	-2			2	3	-3	3			
4														
5	C [2v4]	-1	3	4	6		=	TRANSPOS	E()					
7	[5X4]	3	-4	0	1		-	TRANSPOS	c(array)					
8		-	5	Ū	-									
9														
10	Scalar Mu	Itiplication												
11	5													
12	5* <b>A</b>	5	10	10	10									
13		10	-15	20	10							-		
	+ F	Sheet1	+				:	4				Þ		
EN	TER 🔚							⊞ 🗉	<b>—</b> —		- + 10	0%		



Using the point-and click method, select the array we are transposing.

Notice how the matrix (array) is automatically specified in the formula bar and in the cell formula.

In fact instead of point-and click we could have alternately typed in the matrix cell references in the formula bar or in the cell formula.

x∎	5	- ⊘ - <del>-</del>				Book1 - E	xcel			? 🗹	- □	×	
FIL	.E ⊦	IOME INS	SERT PA	AGE LAYOUT	FORM	ULAS	DATA R	EVIEW	/IEW D	EVELOPER	FOXIT RE/	ADEF +	
Past	te V	B I <u>U</u> 	• 11 • • A A • A •		=	General \$ ~ % €.000	<ul> <li>Control</li> <li>For</li> <li>Cel</li> </ul>	nditional For mat as Table I Styles •	matting -	E Insert ▼ E Delete ▼ E Format ▼	Editing		
Clipt	board 🕞	Fon	it r	Alignme	ent 🖬	Number	Gi I	Styles		Cells		^	
H1 $\checkmark$ : $\times \checkmark f_x$ =TRANSPOSE(H1:K3 $\checkmark$													
1	4	1	2	2	2		В	-2	1	2	4		
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3		
3		-1	2	3	-2			2	3	-3	3		
4	~		2		6		то		11.10			_	
5 IV	[3v/l]	-1	-/	4	5		=1K		F(array)				
7	[344]	1	5	0	1		-	110110100	c(array)				
8													
9													
10 5	Scalar Mu	ultiplication											
11 5	5												
12	5* <b>A</b>	5	10	10	10								
13		10	-15	20	10								
4	•	Sheet1	$\oplus$				:	4				▶	
POIN	п 🔚								- 11			0%	



# Hit **Enter** on your keyboard.

x		5 ¢ ;	÷			Book1 - E	xcel			? 📧	- C	: ×
F	ILE	HOME IN	ISERT PA	AGE LAYOUT	FORM	ULAS	DATA	REVIEW V	IEW I	DEVELOPER	FOXIT R	EADEF 🕨
Pa	ste	Calibri	• 11 • • A A		■ 🗗 ד ■ 🖽 ד ॐ⁄ ד	General \$ ▼ % €_0 .00	▼ ■ C > ■ F C	onditional Forr ormat as Table ell Styles •	matting • •	E Insert ▼ E Delete ▼ E Format ▼	Editing	
Cli	pboard	For For	nt r	a Alignme	ent 🗔	Number	Est.	Styles		Cells		^
H	6	* : :	X 🗸	$f_{x}$								¥
	Α	В	С	D	Е	F	G	Н	1	J	К	
1	Α	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4	C	-1	3	4	6			#VALUE!				
6	[3x4]	3	-4	6	5			WALVE.				
7		1	5	0	1							
8												
9												
10	Scalar N	Aultiplication	ı									_
11	5											
12	5* <b>A</b>	5	10	10	10							
13		10	-15	20	10							
	4 F.	Sheet1	+					•				Þ
RE/	ADY 🔠								- 🛄			00%



Move the cursor back onto the cell of element (1,1) of the transpose matrix, which now contains "gibberish".

Notice that the Formula Bar shows that we are indeed computing the transpose of the matrix **B**.

x	5	- @				Book1 - E	kcel			? 🛧		×
F	FILE HO	OME INS	SERT PA	AGE LAYOUT	FORM	IULAS	DATA R	REVIEW V	IEW [	DEVELOPER	FOXIT RE	ADEF
Pa	ste	Calibri B I U 	• 11 • • A A A • A •		■ Ē + ⊗? -	General \$ - % €.0 .00 .00 →.0	Co     E     Co     E     Co	nditional Form mat as Table Il Styles *	natting ▼ ▼	E Insert *	H Editing	
	ppoard 12	FON		Alignm	ent iai	Number		Styles		Cells		
H	5	▼ E ⊃	< 🗸	<i>f</i> <sub>∞</sub> =TR	ANSPOSE	(H1:K3)						
	Α	В	С	D	E	F	G	Н	1	J	К	
1	Α	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4								[				
5	C [2v4]	-1	3	4	0			#VALUE!				
7	[384]	1	-4	0	1							
8		-			-							
9												
10	Scalar Mu	Itiplication										
11	5											
12	5* <b>A</b>	5	10	10	10							
13		10	-15	20	10							
	$\bullet  \to $	Sheet1	+				:	4				Þ
RE/	ADY 🔠								<b>I</b> -		<b>+</b> 1	00%

The matrix **B** is a 3x4 matrix so we know that the transpose of **B** will be a 4x3 matrix.



Starting at the element (1,1) cell of the transpose matrix, select four (4) rows and three (3) columns.

X	5	- 0				Book1 - E	xcel			? 🛧		×
F	ILE H	OME IN	SERT PA	AGE LAYOUT	FORM	ULAS	DATA F	REVIEW V	IEW E	DEVELOPER	FOXIT RE	ADEF
Pa	ste pboard	Calibri B I U - D Fon			■ 🛱 + ≫ - ent 5	General \$ - % €.0000 Number	✓ Cell ✓ Cell	nditional Forr rmat as Table II Styles <del>-</del> Styles	matting * *	E Insert → Delete → Format → Cells	Editing	~
H	5	* ÷ )	X 🗸	f <sub>x</sub> =TR	ANSPOSE(	H1:K3)						~
	А	В	С	D	Е	F	G	н	1	J	К	-
1	Α	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4								-				
2	C [2v/l]	-1	-4	4	5		V	#VALUE!				
7	[3,4]	1	-4	0	1							
8		-			-							
9												
10	Scalar Mu	ltiplication										
11	5											
12	5* <b>A</b>	5	10	10	10							
13		10	-15	20	10							
	$\leftarrow \rightarrow$	Sheet1	$( \cdot )$				:	4				Þ
RE/	ADY 🔠							⊞ 🗉	<b>—</b>		— <b>+</b> 1	00%



# Hit **F2** on your keyboard.

x	l 🖬 🔊	- @- ;	÷			Book1 - Exc	el			? 🛧	- [	×
F	ILE H	IOME IN	ISERT P	AGE LAYOUT	FORM	ULAS D	DATA F	REVIEW	VIEW [	DEVELOPER	FOXIT R	EADEF •
Pa	te ste	Calibri B I U	• 11 • • A A • A •		: : : : : : : : : : : : : :	General \$ <b>~ %</b> ! €.0 .00	<ul> <li>✓ E Co</li> <li>✓ For</li> <li>✓ Ce</li> </ul>	nditional Fo rmat as Tab Il Styles •	ormatting ~ ole ~	<ul> <li>Insert ▼</li> <li>Delete ▼</li> <li>Format ▼</li> </ul>	A Editing	
Cli	pboard 🕞	For	nt r	Alignme	nt 🖬	Number	Est.	Styles		Cells		^
SU	JM	•	X 🗸	f <sub>∞</sub> =TRA	ANSPOSE(	H1:K3)						٣
	А	В	С	D	E	F	G	н	1	J	К	
1	Α	1	2	2	2		В	-2	1	2	4	Ī.
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4	C	-1	3	4	6		=TR		(H1·K3)			
6	[3x4]	3	-4	6	5		-110					
7		1	5	0	1							
8												
9												
10	Scalar Mu	ultiplicatior	1									
11	5	-										
12	5*A	10	10	10	10							+
15	4	Shoot1	-13	20	10		:					
501	- •-	Sheet		_			:					
EDI	i iii										+ 1	

Hold down the **Ctrl** button and the **Shift** button simultaneously. Hit the **Enter** button.

Release all buttons.



Notice the transpose matrix has now been implemented. Also notice how this new matrix is specified in the formula bar.

X	5	- @				Book1 - E	xcel			? 📧	- C	×
F	FILE H	OME IN	SERT PA	AGE LAYOUT	FORM	ULAS	DATA R	EVIEW	VIEW D	EVELOPER	FOXIT R	EADEF •
Pa	ste	Calibri B I U - A	• 11 • • A A A • <u>A</u> •		■ 🗗 - ■ 🖽 - ≫ - ent 5	General \$ ▼ % €.0 .00 Number	E Con     E Con     E Con     E Con     E Con     E Con	nditional For mat as Table I Styles <del>*</del> Styles	matting • • •	Delete Format Cells	Editing	~
H	5	* : )	×	<i>f</i> <sub>x</sub> {=TR	ANSPOSE(	H1:K3)}						~
	А	В	С	D	Е	F	G	Н	1	J	к	
1	Α	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4												
5	C	-1	3	4	6			-2	1	2		
6	[3x4]	3	-4	6	5			1	-1	3		
0		1	2	U	1			2	2	-3		
•								4	5	3	<i></i>	
10	Scalar Mu	ltiplication									<u></u>	+
11	5											
12	5* <b>A</b>	5	10	10	10							
13		10	-15	20	10							•
	<	Sheet1	+	I				•		- i i		Þ
RE/	ADY 🔠			AVERAGE	E: 1.25 CO	UNT: 12	SUM: 15		•	-	+ 1	00%

It is pertinent to note that the step of holding down the **Ctrl** button and the **Shift** button at the same time, and then hitting the **Enter** button is a common step in the *Excel* matrix functions and manipulations. For the remainder of this course we shall denote this step as **Ctrl-Shift+Enter**.



## Matrix Multiplication

Enter the following matrices  $\mathbf{E}$  and  $\mathbf{F}$  as shown below.

XI 🔒	<del>ن</del> ج	<i>∂</i>				Book1 - E	xcel			? 🖻		×
FILE	HOM	IE IN	SERT	PAGE LAYOUT	FORM	IULAS	DATA	REVIEW	VIEW D	DEVELOPER	FOXIT RE	ADEF >
Paste	G Ca B - B	alibri I <u>U</u> - <mark></mark>	• 11 • A <sup>^</sup> A • A •		■	General \$ - % €.0 -00	<ul> <li>▼ ■ Co</li> <li>&gt; ■ Fo</li> <li>↓ Co</li> </ul>	onditional Fo ormat as Tab ell Styles *	ormatting <del>*</del> le <del>*</del>	Insert ▼ Insert ▼ Delete ▼ Format ▼	Editing	
Clipboard	Es .	For	it	🖬 Alignme	ent 🗔	Number	E.	Styles		Cells		~
F23	Ŧ	: :	×	$f_{x}$								~
A		В	С	D	Е	F	G	н	1	J	К	
16		[E]			[F]							
17 -2	2	1	2		-2	-2						
18 1		-1	3	*	-3	1						
19 2		2	-3		3	1						
20 4		3	3									
21												
22							_					
23							_					
24												+
25												+
20												+
28												ΗĻ
		Sheet1	(1	9			:	4				•
READY	<b>a</b>							▦	<b>—</b>	-	<b>+</b> 1	00%

So we have a 4x3 matrix multiplying a 3x2 matrix. The inner dimensions are equal, therefore this matrix multiplication can proceed and will result in a 4x2 matrix output.



# Move the cursor to the cell location of the element (1,1) of the output matrix. Type "=MMULT". This is the *Excel* matrix multiplication built-in function.

x	6	- ¢- ;	;			Book1 - Exc	el			? 🗹	- 🗆	×
F	ILE H	IOME IN	ISERT F	PAGE LAYOUT	FORM	IULAS	DATA F	REVIEW	VIEW	DEVELOPER	FOXIT RE	ADE •
Pa	ste	B I U	• 11 • A A		= = = = = →	General \$ <b>- %</b> €.00 .00 .00	<ul> <li>✓ ■ Co</li> <li>&gt; ■ Fo</li> <li>↓ Ce</li> </ul>	nditional Fo rmat as Tabl II Styles -	rmatting ∗ e *	E Insert ▼ E Delete ▼ III Format ▼	Editing	
Cli	pboard 🗔	For	nt	G Alignme	ent 🕤	Number	G.	Styles		Cells		^
su	JM	<b>*</b>	× ✓	<i>f</i> ∞ =mn	n							~
	А	В	С	D	Е	F	G	Н	1	J	к	
16		[E]			[F]			[E*F]				
17	-2	1	2		-2	-2		=mm				
luct	of two array	/s, an array w	ith the sam	e number of ro	ws as array	/1 and colur	mns as array	2 🛞 MMU	JLT			
19	2	2	-3		3	1						_
20	4	3	3									-
21												
23												
24		_					_					
25												
26												
27												
28												
		Sheet1	+				:	•				F
ENT	TER 🔚							⊞ 🗉	<b>II</b>		-+ 100	0%



# Double click on MMULT.

X∎	5	- @- <del>.</del>	;		I	Book1 - Exce	2			? 🗈	- 🗆	×
FI	LE H	OME IN	SERT P	AGE LAYOUT	FORM	ULAS D	ATA	REVIEW	VIEW	DEVELOPER	FOXIT RE	ADE 🕨
Pas	► <b>*</b>	B I <u>U</u> 	• 11 • • A •		= E   = Ξ - ≫	General \$ ▼ % €.0 .00	→ ■ C → → → → → → → → → → → → → → → → → → →	onditional For ormat as Table ell Styles -	matting •	智 Insert 、 它 Delete 、 III Format、	Editing	
Clip	board 🗔	For	nt	G Alignme	ent 🖬	Number	<b>G</b>	Styles		Cells		~
SU	М	• : :	× ✓	∫x =MN	/ULT(							~
	А	В	С	D	E	F	G	н	L I	J	К	
16		[E]			[F]			[E*F]				
17	-2	1	2		-2	-2		=MMULT(				
18	1	-1	3	*	-3	1	=	MMULT	(array1, ar	ray2)		
19	2	2	-3		3	1						
20	4	3	3									
21												
22												-
23												
24												
26												
27												
28												ΠĻ
	l F	Sheet1	+	)			:	•				Image: A state of the state
ENTE	R 🔠								<b>.</b>		-+ 100	)%



Use point-and click to select the elements of E. Type in a comma (",").

Use point-and click to select the elements of F.

x∎	5	- ð- ;	;		I	Book1 - Exce	2l			? 🛧	- □	×
FII	LE H	OME IN	SERT P	AGE LAYOUT	FORM	IULAS E	DATA F	REVIEW	VIEW	DEVELOPER	FOXIT RE	ADE
Past	∎ 🔏 ⊡ 📑 ∓ te 💉	B I <u>U</u> 	• 11 • 11 • 11 • 11 • 11 • 11 • 11 • 1		= = = = > > >	General \$ ▼ % €.0 .00	<ul> <li>✓</li> <li>✓</li></ul>	nditional For mat as Table Il Styles ~	rmatting - e -	E Insert ▼ E Delete ▼ III Format ▼	Editing	
Clip	board 🗔	For	nt	a Alignme	ent 🖬	Number	Gi .	Styles		Cells		^
SUI	М	• : :	× ✓	<i>f</i> ∞ =MN	/ULT(A17	:C20,E17:F:	19					~
	А	В	С	D	Е	F	G	Н	1	J	К	
16		[E]			[F]			[E*F]		_		
17	-2	1	2		-2	-2		=MMULT	A17:C20,	E17:F19		
18	1	-1	3	*	-3	1	=	MMULT	(array1, arr	ray2)		
19	2	2	-3	i	3	1						_
20	4	3	3									
21												
22												$-\square$
23												+
25												
26												
27												
28												Ţ
		Sheet1	÷				:	4				Þ
POIN	п 🔚										-+ 100	0%

Hit Enter.



The element (1,1) is computed. We must now fill in the other elements.

X≣	<b>5</b>	- @ - ;	Ŧ			Book1 - Ex	cel				? 📧	- 🗆	×
FIL	E H	OME IN	ISERT F	AGE LAYOUT	FORM	IULAS	DATA	RI	EVIEW	VIEW [	DEVELOPER	FOXIT R	EADE >
Paste	. X ⊡ - • ✓	Calibri B I U	• 11 • A A		■	General \$ ▼ % €.00 →.0	• 9	🖶 Cor 🐺 Forr 🐺 Cell	ditional Fo nat as Tabl Styles <del>+</del>	rmatting * e *	Insert ▼ Insert ▼ Delete ▼ Format ▼	Editing	
Clipb	oard 🗔	For	nt	🖬 Alignme	ent 🖫	Number	E.		Styles		Cells		~
H18		•	X 🗸	f <sub>x</sub>									~
	А	В	С	D	Е	F		G	н	1.1	J	к	
16		[E]			[F]				[E*F]				
17	-2	1	2		-2	-2			-	7			
18	1	-1	3	*	-3	1	_	=					
19	2	2	-3		3	1							
20	4	3	3										
21							_						
22							_						
23							_						
24							_						
25							_						
20													+
28										_			뉘닏
	•	Sheet1	(÷	)				:	4				⊥Ľ
RFAD	y 🖭			·			_	E		m		-+ _10	0%

We know the output matrix will be a 4x2 matrix.



Starting at cell location of the element (1,1) of the output matrix select four (4) rows and two (2) columns.

Hit **F2**.

X≣	5	· @• ;	Ŧ			Book1 - Exce	el			? 🗹	- 🗆	×
FI	LE H	OME IN	ISERT P	AGE LAYOUT	FORM	IULAS [	DATA F	REVIEW	VIEW	DEVELOPER	FOXIT RE	ADE
Pas	te	Calibri B I U	• 11 • • A A		= = = → → →	General \$ - % €.00 .00	<ul> <li>✓ ■ Co</li> <li>&gt; ■ For</li> <li>↓ Ce</li> </ul>	nditional Fo rmat as Table II Styles •	rmatting - e -	E Insert ▼ E Delete ▼ E Format ▼	H Editing	
Clip	board 🖓	For	nt	G Alignm	ent 🕞	Number	E.	Styles		Cells		~
SU	М	• : :	× ✓	f≈ =MI	MULT(A17	:C20,E17:F	19)					٣
	А	В	С	D	E	F	G	Н	- I	J	к	
16		[E]			[F]			[E*F]				
17	-2	1	2	Ī	-2	-2		=MMULT	(A17:C20,	E17:F19)		
18	1	-1	3	*	-3	1	=					
19	2	2	-3		3	1						
20	4	3	3							_		
21												
22												
23												
24												
25												+
20												+
21												ΗL
20	•	Sheet1	(+	)				4				• •
EDIT	· 🔠							▦	<b>.</b> - ·		- + 10	0%



#### Ctrl-Shift+Enter.

# The output matrix of the multiplication of the two input matrices is as follows

x∎	5	- 0	Ŧ			Book1 - Exc	:el			? 🛧	- □	×
FIL	E H	OME IN	ISERT P	PAGE LAYOUT	FORM	1ULAS	DATA F	REVIEW \	/IEW D	EVELOPER	FOXIT RE	ADE
Past	. X ⊡ - *	Calibri B I U	• 11 • A A		■ = = = *	General \$ ▼ % €.0 .00 .00 →.0	<ul> <li>▼ ■ Co</li> <li>&gt; ■ Fo</li> <li>↓ Ce</li> </ul>	nditional For rmat as Table Il Styles <del>+</del>	matting ▼ ▼	Ensert ▼ E Delete ▼ E Format ▼	H Editing	
Clipb	oard 🗔	Fo	nt	🖬 Alignm	ient 🗔	Number	E.	Styles		Cells		^
H17	7	• :	× ✓	<i>f<sub>x</sub></i> {=M	MULT(A17	:C20,E17:F	19)}					~
	А	В	С	D	E	F	G	н	1	J	к	
16		[E]			[F]			[E*F]				
17	-2	1	2		-2	-2		7		7		
18	1	-1	3	*	-3	1	=	10	(	)		
19	2	2	-3		3	1		-19	-5	5		_
20	4	3	3					-8	-2	2		_
21												_
22												
23												
24												
25												+
20												+
28												$+ \square$
4	×.	Sheet	1 (+	)			:	4				•
READ	Y 🔠			AVERAGE:	-1.25 CO	UNT: 8 SL	JM: -10	III ■	•		— <b>+</b> 10	0%



## **Element-by-Element Multiplication**

As the name suggests, element-by-element multiplication of two matrices involves multiplying together corresponding elements in the two matrices. This is analogous to the addition/ subtraction of matrices. The two input matrices must have the exact same dimensions for this operation to be possible.

Let us conduct element-by-element multiplication on the matrices **A** and **B**. Select the cell location for the cell(1,1) of the output matrix. Type "="

X	5	· @• ;	Ŧ			Book1 - E	xcel				? 🖪	i — I	: ×
F	ILE H	OME IN	ISERT F	PAGE LAYOUT	FORM	IULAS	DATA	RE	VIEW V	IEW D	EVELOPER	FOXIT R	EADEF
Pa	∎ X ⊡ ⊡ - ste	Calibri B I U	• 11 • A <sup>*</sup> A • <u>A</u> •		■ 🗗 + ■ 🖽 + ≫⁄ +	General \$ - % €.00 -000	• F	🚽 Cond V Form V Cell S	ditional Forn nat as Table Styles <del>-</del>	matting *	Ensert -	Editing	
Clip	pboard 🗔	For	nt	G Alignme	ent 🖫	Number	E.		Styles		Cells		^
A2	24	•	× ✓	f <sub>x</sub>									~
	Α	В	С	D	Е	F	G	5	н	1	J	К	
15													
16		[E]			[F]				[E*F]				
17	-2	1	2		-2	-2			7	7	'		_
18	1	-1	3	*	-3	1	=	-	10	0			
19	2	2	-3		3	1			-19	-5			
20	4	3	3						-8	-2			
21	Element-	by Elemen	t Multiplic	ation			_						
23													
24													
25													
26													
27													
	4 - F	Sheet1	•	)				:	4				Þ
REA	DY 🔠							[	# 8	•		<b>+</b> 1	00%



Click on the cell of **A**(1,1). Type "\*" Click on the cell of **B**(1,1)

x	5	- @				Book1 - E	xcel			? 📧	- □	×
F	ILE H	OME IN:	SERT PA	AGE LAYOUT	FORM	ULAS	DATA F	REVIEW	VIEW D	DEVELOPER	FOXIT RE	ADEF +
Pa	te ste	B I <u>U</u> 	• 11 • • A • • A •		=	General \$ - % €.0 ->.0	<ul> <li>Co</li> <li>For</li> <li>Ce</li> </ul>	nditional Fo rmat as Tabl II Styles ~	rmatting ~ e ~	E Insert → E Delete → E Format →	Editing	
Cli	pboard 🗔	Fon	it G	Alignme	ent 🖬	Number	G I	Styles		Cells		~
H	1	▼ ÷ ⊃	x 🗸	<i>f<sub>x</sub></i> =B1	*H1							~
	Α	В	С	D	Е	F	G	Н	I	J	К	
1	Α	1	2	2	2		В	-2	1	2	4	
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3	
3		-1	2	3	-2			2	3	-3	3	
4												
5	C	-1	3	4	6			-2	1	2		
6	[3x4]	3	-4	6	5			1	-1	3		
7		1	5	0	1			2	2	-3		
8								4	3	3		
9												
10	Scalar Mu	Itiplication										
11	5											
12	5* <b>A</b>	5	10	10	10							
13		10	-15	20	10							-
	4 F	Sheet1	$( \cdot )$				:	4				Þ
PO	INT 🔠							⊞ 🗉			- + 1	00%



# Hit Enter.

X≣	5	- @ - <del>.</del>				Book1 - Ex	cel				? 🗹	— C	: ×
F	ILE HO	DME IN	SERT F	PAGE LAYOUT	FORM	IULAS	DAT	A RI	EVIEW \	/IEW D	EVELOPER	FOXIT R	ADEF 🕨
Pas	te →	Calibri B I U 	• 11 • A <sup>*</sup> A • <u>A</u> •		₽	General \$ - % €.00 -0.0	•	🖶 Cor 🐺 Forr 🐺 Cell	nditional For mat as Table Styles •	matting •	Insert •	Editing	
Clip	poard 1	For	11	Alignme	nt Ial	Number	Tai I		Styles		Cells		
A2	5	• ÷ )	X 🗸	$f_x$									~
	Α	В	С	D	Е	F		G	н	I.	J	К	
19	2	2	-3		3	1			-19	-5	5		
20	4	3	3						-8	-2	2		
21													
22	Element-k	y Element	t Multiplio	cation									
23							_						_
24	-2												
25													
26							_						$+ \parallel \parallel$
21							_						+
20							_						
30													+
31													
	<	Sheet1	÷	)				:	4				
REA	.DY 🔡		_							<b>.</b>	-	-+ 1	00%



💵 🖯 🚽 🚽 🕫 ?  $\square$  × Book1 - Excel **\*** -HOME INSERT PAGE LAYOUT FORMULAS DATA REVIEW DEVELOPER FOXIT READEF VIEW = = = = Ж Calibri - 11 -General -🖶 Conditional Formatting 👻 🚰 Insert 🔻 **#**1 Ē -BIU - A A = = = = • \$ - % , 🐺 Format as Table 🕶 환 Delete 🔹 Editing Paste s' Ш - <u>А</u>-€ 🚈 🗞 -00. 0.→ 0.← 00. Cell Styles -🗮 Format -Alignment Clipboard 🗔 Font G. G. Number 🗔 Styles Cells ^  $f_{x}$ =B1\*H1 Y A24  $\times$ ~ С D Е F ٠ Α В G Н L J Κ 3 -5 19 2 2 -3 1 -19 3 3 -2 20 4 -8 21 22 Element-by Element Multiplication 23 24 -2 2 4 8 25 ₽₽ 26 27 28 29 30 31 Sheet1 Ð Þ 1 ⊞ 100% READY AVERAGE: 3 COUNT: 4 SUM: 12 

Use fill handle to complete the elements of the first row.



With the first row selected, use fill handle to complete the remaining rows of the output matrix.

x	l 🔒 🍤	- ð	:			Book1 - E	kcel			? 🛧	- 0	i x
F	ILE HO	DME IN	SERT PA	AGE LAYOUT	FOR	MULAS	DATA	REVIEW V	/IEW D	EVELOPER	FOXIT R	EADEF►
Pa	te ste	Calibri B I U	• 11 • • A A • A •	= = : : ::::::::::::::::::::::::::::::	■ ि ≡ ⊡ • ≫••	General \$ ▼ % €.0 .00 .00 →.0	→ 12 C	onditional Fori ormat as Table ell Styles <del>-</del>	matting ▼ ▼	Insert ▼ ■ Delete ▼ ■ Format ▼	Editing	
Clip	pboard 🗔	For	nt r	Alignm	ient G	Number	Est.	Styles		Cells		~
A	24	• : :	X 🗸	<i>f</i> <sub>x</sub> =B1	l*H1							~
	Α	В	С	D	E	F	G	Н	I.	J	К	
19	2	2	-3		3	1		-19	-5	;		
20	4	3	3					-8	-2	2		
21												
22	Element-b	by Element	t Multiplica	ation								
23	2	2	1	0								
25	-2	2	- 4	6								
26	-2	6	-9	-6								
27					<b>.</b>							
28												
29												
30												
31												
	4 - F	Sheet1	÷					•				Þ
REA	ADY 🔠		AVE	RAGE: 1.6666	566667 (	COUNT: 12	SUM: 20		<b>—</b>		+ 1	00%



# Determinant of a Matrix

Enter the elements of the matrix **J**.

🕅 🗄 🗲 👌	Ŧ		Book	- Excel			? 🛧	- □	×
FILE HOME IN	ISERT PAG	E LAYOUT	FORMULAS	DATA	REVIEW	VIEW [	DEVELOPER	FOXIT RE	ADEF +
Calibri Paste ▼ ◆ B I U	• 11 • • A <sup>•</sup> A <sup>•</sup>		Gener	al • % • 🗐	불 Conditional I Format as Ta Cell Styles ▼	Formatting • ble •	Insert ▼ Delete ▼ Format ▼	Editing	
Clipboard 🗔 Fo	nt 🕞	Alignment	ra Num	ber 🕞	Style	S	Cells		^
H29 🔻 :	$\times \checkmark f_s$	κ							~
A B	С	D	E	F (	G H	1	J	К	
25 <b>2</b> 3	8 8	6							
26 <b>-2</b> 6	j -9	-6							
27									_
28 J	1		1	det(J	) 				
30 (		-3	-1						
31 -2	2 -1	-1	1						
32 2	2 1	1	0						
33									
34									
35									
30									
Si Chast	1				: 4				
Sneet					: •			_	P



Select a cell that will hold the result. Type "=MDETERM"

x		<del>6</del> -	@- ;	Ŧ				Book1 - E	xcel					?	Ť	_	n x
F	ILE	НО	ME IN	ISERT	PAG	SE LAYOUT	FORM	ULAS	DATA	REV	IEW \	/IEW	DEVE	LOPER		FOXIT	READEF
Pa	ste	•	B I ∐ 	- 11 - A	Ă			General \$ ▼ % €.0 .00 .00 →.0	· E	Condi Forma Cell Si	itional Fori at as Table tyles *	matting -		Delete	≂ . at .	Editing	9
Cli	oboard	Б <u>а</u>	For	nt	E.	Alignme	nt 🖬	Number	G.		Styles			Cells			~
SL	M		• :	× ✓	f.	<del>ا</del> س =m											~
	А		В	С		D	E	F	G		н	1		J		к	
25		2	3		8	6											
26		-2	6	; -	.9	-6											
27																	
28	J								det(J)								
29	[4x4]		1		1	2	-1			=	m						
30			0	)	0	-3	4				€ MATC	н	^				
31			-2	-	-1	-1	1				€ MAX						
32			2	2	1	1	0				E MAXA	5DM		Retur	ns th	e matrix	determin
33											C MDUE			Recui		e macin	
34											€ MEDI	AN					
35											🕭 MID						
36											🕭 MIN						
37											€ MINA						
	4 - F		Sheet1	(	Ð					:	🕞 MINU 🕞 MINVI	TE ERSE					Þ
ENT	ER 🖁									I	🕖 MIRR		$\mathbf{v}$			-+	100%

From the list of candidate functions that appears, double click on MDETERM.



# Select the elements of the matrix **J**.

x∎		<del>5</del> -	¢	Ŧ			Book1 - Exc	el			? 🛧		×
F	ILE	но	ME IN	ISERT P/	AGE LAYOUT	FORM	ULAS E	DATA R	EVIEW	VIEW D	EVELOPER	FOXIT RE	ADEF
Pas	te	•	в I Ц	• 11 • • A A • A •		= = = = →	General \$ ▼ % €.0 .00	Cor     For     Cel	nditional Fo mat as Tab I Styles -	ormatting * le *	E Insert ▼ E Delete ▼ H Format ▼	Editing	
Clip	board	Fa	Fo	nt r	Alignm	ent 🕤	Number	Est.	Styles		Cells		~
B2	9		• :	X 🗸	<i>f</i> <sub>x</sub> =MI	DETERM(B	29:E32						~
	А		В	С	D	E	F	G	н	1	J	К	
25		2	3	8	6								
26		-2	6	-9	-6								
27		_											
28	J	_						det(J)			1		
29	[4x4]	_	1	. 1	2	-1			=MDETE	RM(B29:E32	2		
30		-	0	0	-3	4			MDETE	RM(array)			
31		-	-2	-1	-1	1							
32		-	2	1	1	0							
34													
35													
36													
37													-
	• •		Sheet1	+				:	•				Þ
POI	NT 🔡									• •		- + 1	00%



Type ")" to close parenthesis. Hit **Enter**.

The determinant of the matrix is computed and displayed.

x	. 5	- @				Book1 - E	kcel			? 📧	_ [	×
F	ILE HO	DME IN	SERT PA	AGE LAYOUT	FORM	ULAS	DATA	REVIEW	VIEW D	DEVELOPER	FOXIT RE	EADEF •
Pa	ste	Calibri B I U 	• 11 • • A A A		: ₽ ( : ⊡ - %••	General \$ ₹ % €.0 .00	<ul> <li>▼ ■ C</li> <li>&gt; ■ F</li> <li>□ ■ C</li> </ul>	onditional For ormat as Table ell Styles •	matting •	Insert ▼ Insert ▼ Delete ▼ Format ▼	H Editing	
Cli	pboard 🗔	Fon	it r	Alignme	nt G	Number	G.	Styles		Cells		~
H	30	▼ ÷ ⊃	X V	$f_{x}$								۷
	Α	В	С	D	E	F	G	Н	1	J	к	
25	2	3	8	6								
26	-2	6	-9	-6								
27												
28	l						det(J)					
29	[4x4]	1	1	2	-1			3				_
30		0	0	-3	4				-			
31		-2	-1	-1	1							
22		2	1	1	U							
34												$+ \parallel \parallel$
35												
36												
37												ŢĻ
	4 - F	Sheet1	+					•	:			•
RE/	ADY 🔠								- E		<b>+</b> 1	00%



Inverse Matrix

Let us find the inverse of the matrix **J**.

Select the cell location of the element (1,1) of the inverse matrix.

Type "="

Type the built-in function MINVERSE.

x	5	- @- ;	;			Book1 - E	kcel			? 🛧	- □	×
F	ILE HO	DME IN	SERT P	AGE LAYOUT	FORM	ULAS	DATA	REVIEW	VIEW	DEVELOPER	FOXIT RE	ADEF •
Pa	■ X □ ste ✓	B I ∐ 	• 11 • • A • • A •		= = = = *	General \$ - % €.0 ->.0	<ul> <li>✓</li> <li>✓</li></ul>	onditional Fo ormat as Tabl ell Styles -	rmatting ⊤ e ⊤	<ul> <li>Insert ▼</li> <li>Delete ▼</li> <li>Format ▼</li> </ul>	Editing	
Clij	board 🗔	For	nt r	Alignmo	ent 🔤	Number	Gi I	Styles		Cells		~
SU	M	• :	× ✓	<i>f</i> <sub>∞</sub> =mi								~
	А	В	С	D	Е	F	G	н	1	J	К	
25	2	3	8	6								
26	-2	6	-9	-6								
27												
28	J						det(J)		-			
29	[4x4]	1	1	2	-1				3			
30		0	1	-3	4		(J) VNI	-mi				
32		-2	-1	-1	0			-IIII	F	Returns the chara	acters fron	n the m
33								🛞 MIN				
34								€ MIN/	A			
35								E MINU				
36								- 🖉 MIRF				
37												
	• • • • • • • • • • • • • • • • • • •	Sheet1	+				:	4				Þ
ENT	ER 🔠							▦	•			00%



From the list of candidate functions that appears as you start typing, double click on MINVERSE.

Select the elements of the matrix **J**.

XI 🔒 🧉	<b>)</b> -	Ŧ			Book1 - Ex	cel			? 🗹	- 0	×
FILE	HOME IN	ISERT P	AGE LAYOUT	FORM	ULAS	DATA I	REVIEW V	IEW D	DEVELOPER	FOXIT RE	ADEF
Paste	B I U	• 11 • • A A • A •		= ■	General \$ <b>- %</b> €.00 →.0	<ul> <li>✓ ■ Co</li> <li>&gt; ■ Fo</li> <li>↓ Co</li> </ul>	onditional Forr rmat as Table Il Styles -	natting - -	<ul> <li>Insert ▼</li> <li>Delete ▼</li> <li>Format ▼</li> </ul>	Editing	
Clipboard	Foi Foi	nt	Alignm	ent 🖫	Number	Faile -	Styles		Cells		~
B29	• :	X 🗸	<i>f</i> <sub>x</sub> = <b>M</b>	INVERSE(B	29:E32						*
A	В	С	D	E	F	G	Н	I.	J	К	
25	2 3	8	6								
26	-2 6	i -9	-6								
27											
28 J			2			det(J)					
29 [4x4]		. 1	2	-1			3				
31	-2	-1	-1			1147(3)	=MINVERS	F(B29:E3	2		
32	2	1	1	0			MINVERS	E(array)			
33											
34											
35											
36											
37											
	Sheet	+				:	•				Þ
POINT 🔡								- <b>-</b>		- + 10	00%



Type ")" to close parenthesis. Hit **Enter**.

x∎	5	- @- ;	;			Book1 - Ex	cel			? 🛧		×
FI	LE HO	DME IN	SERT P	AGE LAYOUT	FORM	IULAS	DATA	REVIEW	VIEW [	DEVELOPER	FOXIT RE	ADEF +
Pas	∎ X □ ⊡ - te V	Calibri B I U	• 11 • • A A		■ 🗗 - ■ 🖽 - ≫⁄	General \$ - % €.00 -000	<ul> <li>→</li> <li>&gt;</li> <li>&gt;</li> <li>&gt;</li> <li>&gt;</li> <li>&gt;</li> <li>&gt;</li> <li></li> <li></li></ul>	onditional For ormat as Table ell Styles *	matting *	E Insert ▼ E Delete ▼ E Format ▼	Editing	
Clip	board 🗔	For	nt r	Alignme	ent 🖬	Number	Est.	Styles		Cells		~
H3	2	<b>▼</b> : :	X 🗸	$f_{\mathcal{K}}$								~
	А	В	С	D	Е	F	G	н	- I	J	к	
25	2	3	8	6								
26	-2	6	-9	-6								
27												
28	J						det(J)					
29	[4x4]	1	1	2	-1			3	; 			
30		-2	-1	-5	4		INV(J)	-1				
32		2	1	1	0							
33									<b>.</b>			
34												
35												
36												
37												
	• •	Sheet1	$\oplus$				:	4				Þ
REA	DY 🔠								- III -		<b>+</b> 1	00%

The element (1,1) of the inverse matrix is computed and displayed.

We know that the matrix  $\mathbf{J}$  is a 4x4 square matrix, and that its inverse will also be a 4x4 square matrix.



Starting at the element (1,1) of the inverse matrix, select four (4) rows and four (4) columns. Hit **F2**.

x		5	· @ ·	Ŧ			Book1 - Exe	cel			? 🛧	- □	×
F	ILE	НС	DME IN	ISERT P	AGE LAYOUT	FORM	ULAS	DATA F	REVIEW	VIEW D	EVELOPER	FOXIT RE	ADEF 🕨
Pa	ste	Ŧ	Calibri B I U	• 11 • • A • • A •		= È   =	General \$ <b>▼ %</b> €.0 .00	<ul> <li>✓ E Co</li> <li>✓ For</li> <li>✓ Ce</li> </ul>	nditional For rmat as Table Il Styles -	matting -	Insert ▼ Image: The service of the	Atta Editing	
Clij	oboard	E	Fo	nt	Alignm	ent 🗔	Number	5	Styles		Cells		~
SU	M		•	×	<i>f</i> <sub>x</sub> =MI	NVERSE(B	29:E32)						~
	А		В	С	D	Е	F	G	Н	1	J	K	
25		2	3	8 8	6								
26		-2	6	i -9	-6								
27								dot(I)					
28	J [4x4]		1	1	2	-1		der(J)		1			
30	[-7-]	_		) 0	-3	4		INV(J)		, 			
31		_	-2	2 -1	-1	1			=MINVER	SE(B29:E32	2)		
32			2	2 1	1	0							
33													
34													
35													
30													
57	4 F		Sheet					:					
EDI	т									m	·	-+ _1	00%

Ctrl-Shift+Enter.



x	5	- @- ;	F			Book1 - E	xcel			? 🖪	] _ 🗆	×
F	ILE HO	OME IN	ISERT PA	AGE LAYOUT	FORM	ULAS	DATA	REVIEW V	IEW DE	VELOPER	FOXIT REA	ADEF +
Pa Cli	ste	Calibri B I U T - Sort	• 11 • • A <sup>^</sup> A <sup>^</sup>	] = = = 	■ 🗗 - [ ■ 🖽 - ≫ ent 5	General \$ ▼ % €.0 .00 Number	→ 🚰 Co > 🐺 Fo 🐺 Ce	onditional Forr ormat as Table ell Styles + Styles	natting ▼ ▼	Ensert  Cells	Editing	~
H	31	• : :	X V	<i>f</i> <sub>x</sub> {=MI	NVERSE(B	29:E32)}						~
	А	В	С	D	E	F	G	н	1	J	К	
25	2	3	8	6								
26	-2	6	-9	-6								
27							dot(I)					- 1
20	J [4x4]	1	1	2	-1		uer(J)	3				-
30		0	0	-3	4		INV(J)					
31		-2	-1	-1	1			-1	-0.33333	0.333333	1.333333	
32		2	1	1	0			2	1	-2	-3	
33								0	-0.33333	1.333333	1.333333	_
34								0	0	1	1	
35											2	
37												
	4 F	Sheet1	+	II				4				
REA	ADY 🔡	A	VERAGE: 0.1	66666667 (	COUNT: 16	SUM: 2.66	56666667		• -		<b> +</b> 10	0%

The inverse matrix is displayed as follows

#### Systems of Linear Equations

Consider the following problem.

A financial company is considering investing in three financial products. The financial institution has decided to invest \$500,000 in these three products, and wants a return on investment of \$38,000. The projected return on investment from Product 1, Product 2, and Product 3 are 6.5%, 7%, and 9% respectively. It has been decided that the Product 2 investment amount shall be twice the Product 3 investment amount.



Let us do the following:

a. Represent your investment plan as a system of linear equations.

b. Represent this system of equations in matrix form.

c. Solve the system of equations to determine how much the financial institution should invest in each in financial product.

Solution:

Let  $x_1$  represent the amount invested in Product 1. Let  $x_2$  represent the amount invested in Product 2. Let  $x_3$  represent the amount invested in Product 3.

The system of linear equations is as follows:

$x_1 + x_2 + x_3 = 500000$	[total investment]
$0.065x_1 + 0.07x_2 + 0.09x_3 = 38000$	[return on investment]
$x_2 = 2x_3$	or

 $x_2 - 2x_3 = 0$  [Product 2 versus Product 3]



We may now go into *Excel* and create the matrix of coefficients, the vector of the unknown quantities, and the vector of right-hand-side constants, which will represent this system of equations.

x∎	B 5	- @				Book1 - Ex	cel			? 🛧	- 🗆	×
FILE	H	OME IN:	SERT	PAGE LAYOUT	FORM	/IULAS	DATA	REVIEW	VIEW D	EVELOPER	FOXIT RE	ADE •
Paste	× ] <sub>[] ↓</sub>	Calibri B I U 	• 11 • A <sup>*</sup> A • A •	- = = = A = = = €≣ ₽Ξ	■	General \$ ▼ % €.0 .00	• E	Conditional For Format as Table Cell Styles •	matting +	Ensert •	Editing	
Clipbo	oard 🗔	Fon	t	🖫 Alignm	ent 🖫	Number	E.	Styles		Cells		~
E45		▼ ÷ ⊃	×	$f_{x}$								~
	А	В	С	D	E	F	G	н	1	J	к	
34								0	(	0 1	1	
35												
36 Sy	ytem of	Linear Equa	ations									_
37		1	1	1				500000				_
38		1	1	1 0.09	*	×1	_	38000				_
40		0.005	1	-2		x3	-	0				
41		_										
42			Α			x		b				
43												
44												
45						. <b> </b>						
46												
	•	Sheet1	(+	)				•				F
READY	1 🔚								•		- + 100	0%



Let us find the inverse of the matrix **A**.

Select the cell location of the element (1,1) of the inverse matrix.

Type "="

Type the built-in function MINVERSE.

Select the elements of the matrix **A**.

XI 🗖	<b>5 -</b> @ - :	Ŧ			Book1 - Exc	el			? 🛧	- □	×
FILE	HOME IN	ISERT P	AGE LAYOUT	FORM	IULAS	DATA	REVIEW	VIEW	DEVELOPER	FOXIT RE	EADE •
Paste	- B I U	- 11 - A^ A - A -		8° ₽	General \$ <b>~ %</b> €.0 .00	<ul> <li>✓</li> <li>✓</li></ul>	onditional Fo rmat as Tabl Il Styles -	rmatting → e →	E Insert ▼ E Delete ▼ III Format ▼	Editing	
Clipboard	Fail Fo	nt	G Alignme	nt 🕤	Number	G.	Styles		Cells		^
B38	- · ·	× ✓	f <sub>x</sub> =MIN	VERSE(E	338:D40						~
A	В	С	D	Е	F	G	н	I.	J	K	
37											
38	1	1	1		x1		500000				
39	0.065	0.07	0.09	*	x2	=	38000				
40	0	1	-2		x3		0				
41											
42		Α			x		b				
43											
44											
45	=MINVER	SE(B38:D4	0								
46	MINVER	SE(array)									
47											
48											
49											-
	Sheet	1 (+	)			:	4				F
POINT 📲							III III	<b>I</b> I]		- + 10	00%



Type ")" to close parenthesis. Hit **Enter**.

x II	<b>5</b> - @-	Ŧ			Book1 - Ex	cel				? 🛧		1 ×
FILE	HOME IN	NSERT P	PAGE LAYOUT	FORM	IULAS	DATA	RE	VIEW	VIEW [	DEVELOPER	FOXIT R	EADE
Paste	K         Calibri           Image: a filled and a	- 11 · ! - A A ≥ - <u>A</u> -		₽ ₽ ₽ ₽ ₽ ₽ ₽	General \$ ▼ % €.00 →.00	•	🗄 Con Forn Cell	ditional For nat as Table Styles *	matting • • •	E Insert → Delete → Format →	Editing	
Clipboar	rd 🖬 Fo	nt	🖫 Alignmei	nt 🖬	Number	E I		Styles		Cells		~
B46		$\times$ $\checkmark$	$f_{\mathcal{K}}$									~
	A B	С	D	Е	F		G	н	1	J	К	
36 Syte	em of Linear Equ	ations										
37												
38	1	1	1		x1			500000				
39	0.065	0.07	0.09	*	x2	-	=	38000				
40	0	1	-2		x3	_		0				
41		Α			×	_		b				
43								-				
44												
45	6.571429	Э										
46												
47												
48												
	> Sheet	1 +	)				: [	4				Þ
READY							B	∄ 🗉	•		<b>+</b> 1	00%

A is a 3x3 square matrix, and so its inverse will also be a 3x3 square matrix.



Starting at the cell holding the element (1,1) of the inverse matrix, select three (3) rows and three (3) columns.

Hit **F2**.

x	5	· @ · -	:			Book1 - Exc	:el			? 🕋	- □	×
F	ILE H	OME IN	SERT F	PAGE LAYOUT	FORM	IULAS	DATA	REVIEW	VIEW	DEVELOPER	FOXIT RE	ADE •
Pa	ste	Calibri B I U	• 11 • A A		= = = = → →	General \$ <b>- %</b> €.00 .000	▼ ■ C 2 ■ C	onditional Fo ormat as Tabl ell Styles -	rmatting - e -	Hosert ▼ E Delete ▼ H Format ▼	H Editing	
Clip	board 🕞	For	nt	G Alignme	ent 🔤	Number	G.	Styles		Cells		^
SL	M	* : I	× ✓	f <sub>≭</sub> =MI	NVERSE(E	338:D40)						*
	А	В	С	D	E	F	G	н	I.	J	К	
36	Sytem of	Linear Equ	ations									
37												
38		1	1	1		×1		500000				
39		0.065	0.07	0.09	*	x2	=	38000				
40		0	1	-2		x3		0				
41			•									
42			А			X		a				
45												
45		=MINVERS	SE(B38:D4	0)								
46				1								
47												
48												
	< >	Sheet1	(+	)				4				•
EDI	T 🔚							TH I	<b>P</b> –		-+ 10	0%

Ctrl-Shift+Enter.



The inverse matrix  $\mathbf{A}^{-1}$  is computed and displayed as follows

x∎	E 5	- @				Book1 - Ex	cel			? 📧	- □	×
F	ILE HO	DME INS	SERT PA	GE LAYOUT	FOR	MULAS	DATA	REVIEW	VIEW	DEVELOPER	FOXIT RE	EADE 🕨
Pa	kan	Calibri B I U 	• 11 • • A A •		■	General \$ - % €.0 .00 .00 →.0	<ul> <li>▼</li> <li>▼</li> <li>▼</li> <li>▼</li> <li>▼</li> </ul>	Conditional Fo Format as Tab Cell Styles <del>*</del>	rmatting • e •	<ul> <li>Insert ▼</li> <li>Delete ▼</li> <li>Format ▼</li> </ul>	H Editing	
Clip	board 🗔	Fon	it 15	Alignm	ent 🖫	Number	Es l	Styles		Cells		^
B4	5	▼ ÷ ⊃	× 🗸	<i>f</i> <sub>x</sub> {=M	INVERSE(	B38:D40)}						۷
	А	В	С	D	Е	F	G	н	I.	J	К	
36	Sytem of I	Linear Equa	ations									
37												
38		1	1	1		x1		500000				
39		0.065	0.07	0.09	*	x2	=	38000				
40		0	1	-2		x3		0				
41												
42			Α			X		b				
43												
44		6 571420	05 7140	0 57142								+
45		-2 71/29	-85.7145	-0.57143								
47		-1 85714	28 57143	-0 14286								+
48		1.00714	20107140	3.142.00	⁄囯							뉘닏
	<	Sheet1	+					: •				
REA	DY 🔠		AV	ERAGE: 0.11	111111	COUNT: 9	SUM: 1		•		-+ 10	00%



We shall now multiply  $A^{-1}$  by the vector **b** to obtain the vector of unknown quantities that we are solving for.

x∎	H 5	- ð				Book1 - Fx	cel				? क	1	×
EII F			SERT D		FORM		 DΔΤΔ	RF	VIEW	VIEW		FOXIT R	
						//ULAS	DAIA			VIL VV	m		LADU
	<u> </u>	Calibri	• 11 •		= =	General		(≩ Con	ditional Fo	rmatting *	Ensert •		
Paste		в <u>г</u> <u></u>	· A A		≣ 🛱 ▾	\$ - %	,	🛃 Forr	nat as Tabl	e▼	🖹 Delete	Editing	
+	- <b>S</b> *	🗄 🖌 🖄	- <u>A</u> -	€≣÷≣	*? -	00. 0.→ 00. →.0	E	📝 Cell	Styles 🔹		Format	* *	
Clipb	oard 🗔	Fon	t r	Alignm	ent 🗔	Number	Es .		Styles		Cells		~
F45		<b>-</b> : 3	x J	fx									~
	А	В	C	D	E	F		G	Н	1	J	K	
36 <b>S</b>	ytem of I	Linear Equa	tions										
37							_			_			_
38		1	1	1		x1			500000				
39		0.065	0.07	0.09	*	x2		=	38000				
40		0	1	-2		х3	_		0	_			
41			Δ				_						
42			A			x			U				
43													+
45		6.571429	-85.7143	-0.57143									
46		-3.71429	57.14286	0.714286			-						
47		-1.85714	28.57143	-0.14286									
48													-
	•	Sheet1	+					:	4				Þ
READ	Y 🔚							F	THE INTERNATION OF THE PROPERTY OF THE PROPERT	<b>m</b> -		<b>+</b> 10	00%


Move the cursor to the location of the cell holding the element (1,1) of the output matrix **x**. Type "=MMULT".

Use point-and click to select the elements of  $A^{-1}$ .

Type in a comma (",").

Use point-and click to select the elements of **b**.

x	l 🖯 5	- @				Book1 - Exc	el			? 🛧	- □	×
F	ILE H	OME INS	SERT PA	AGE LAYOUT	FORM	IULAS I	DATA I	REVIEW	VIEW	DEVELOPER	FOXIT RE	ADE •
Pa	ste	B I ∐ 	• 11 • • A A • A •	$ \begin{array}{c} \blacksquare \blacksquare$					E Insert ▼ E Delete ▼ E Format ▼	H Editing		
Clij	pboard 🕞	Fon	t G	Alignme	ent 🕤	Number	E.	Styles		Cells		^
H	38	- i )	× v	f≈ =MM	NULT(B45	:D47,H38:H	140					~
	А	В	С	D	E	F	G	н	I	J	К	
36	Sytem of	Linear Equa	tions									
37												
38		1	1	1		x1		500000	-			
39		0.065	0.07	0.09	*	x2	=	38000				
40		0	1	-2		X3		0	4			
41			Δ			x		b				
43								~				
44												
45		6.571429	-85.7143	-0.57143		=MMULT(	B45:D47,H	138:H40				
46		-3.71429	57.14286	0.714286		MMULT	(array1, <b>arr</b> a	ay2)				
47		-1.85714	28.57143	-0.14286								
48												
	4 - F	Sheet1	$( \mathbf{+} )$				:	•				Þ
POI	NT 🔚							⊞			- + 10	0%



Type ")" to close the parenthesis. Hit **Enter**.

The element (1,1) is computed. We must now fill in the other elements. We know the output of a 3x3 matrix multiplied by a 3x1 matrix will be a 3x1 matrix.

Starting at cell holding the element (1,1) of the output matrix select three (3) rows and one (1) column.

Hit **F2**.

x∎		o- @- ≠				Book1 - Excel					- 🗆	×
F	LE	HOME IN:	SERT PA	GE LAYOUT	FORM	NULAS	DATA	REVIEW	VIEW [	DEVELOPER	FOXIT RE	ADE 🕨
Pas	te	Calibri B I U	• 11 • • A A • A •		= = = = %	General \$ ▼ % €.00 →.0	· E	Conditional Fo Format as Tab Cell Styles •	rmatting * le *	≌ Insert ▼ ≌× Delete ▼	Editing	
Clip	board	Fon Fon	it 5	Alignme	ent 🔤	Number	E.	Styles		Cells		^
SU	SUM ▼ : X ✓ f <sub>x</sub> =MMULT(B45:D47,H38:H40) ×											
	А	В	С	D	Е	F	G	н	1	J	К	
37												
38		1	1	1		x1		500000	Ī			
39		0.065	0.07	0.09	*	x2	=	38000				_
40		0	1	-2		x3		0	-			
41			Α			x		b				
43												
44												
45		6.571429	-85.7143	-0.57143		=MMULT	(B45:D4	7,H38:H40)				
46		-3.71429	57.14286	0.714286								
47		-1.85714	28.57143	-0.14280								
49												
	< >	Sheet1	(+)					: 4				LI♥ ▶
EDI	r 🔚								<b>U</b> -		- + 10	0%

Ctrl-Shift+Enter.



The output matrix containing the values of the unknown quantities  $x_1$ ,  $x_2$ , and  $x_3$  is as follows,

X≣	<b>5</b>	- @				Book1 - Exc	el			? 🛧	- □	×
FI	LE H	OME IN:	SERT PA	AGE LAYOUT	FORM	IULAS [	ATA	REVIEW	/IEW	DEVELOPER	FOXIT RE	ADE
Pas	u み □ te ❤	Calibri B I U T - 2	• 11 • • A A • A		■ = = = ≫ -	General \$ ▼ % €.0 .00 .00 →.0	▼ ■ C > ■ C > ■ C	onditional For ormat as Table ell Styles •	matting •	E Insert ▼ Delete ▼ Format ▼	Editing	
Clip	board 🕞	Fon	it G	Alignm	ent 🖫	Number	Gi i	Styles		Cells		^
F45	5	▼ ÷ ⊃	X 🗸	$f_{\mathcal{K}} = M$	MULT(B45	:D47,H38:H	140)}					~
	А	В	С	D	Е	F	G	н	I.	J	к	
37												
38		1	1	1		×1		500000				
39		0.065	0.07	0.09	*	x2	=	38000				-
40		0	1	-2		x3		0				
41 42			Α			x		b				
43												
44		6 571400	05 7140	0 57142		20571.42						
45		-3 71429	-85.7145 57 14286	0 714286		314285 7						
47		-1.85714	28.57143	-0.14286		157142.9						
48							•					
49												<b>—</b>
	(	Sheet1	+				:	4				•
REAL	DY 🔠		AVERAG	E: 166666.666	57 COUN	T: 3 SUM: 5	00000	⊞	• •	-	— <b>+</b> 10	0%



Let us impart a number format that displays our results as monetary values.

Click on the HOME tab. In the **Number** palette, click on the \$ drop down button

١				١								
x∎	5	- @				Book1 - Exce	1			? 🛧	- 0	×
FILE	н		SERT PA	AGE LAYOUT		IULAS D	ATA R	EVIEW \	/IEW	DEVELOPER	FOXIT RE	ADE .
Paste	× ⊫ - *	Calibri B I U	• 11 • • A A • <u>A</u> •		■ 🛱 + •	General \$ - % 3 \$ Engli	▼ Para Cor ▼ For sh (United S	nditional For mat as Table States)	matting -	Insert ▼ Delete ▼ Format ▼	Editing	
Clipb	oard 🖫	Fon	t G	Alignm	ent 🖬	£ Engli	sh (United K	(ingdom)		Cells		
F45		* E 🗆	× 🗸	<i>f</i> <sub>x</sub> {=M	MULT(B45	€ Euro	(€ 123)					~
37	Α	В	С	D	E	¥ Chinese (PRC) fr. French (Switzerland)				J	К	
38		1	1	1		More A	Accounting	Formats				+-
39		0.065	0.07	0.09	*	x2	=	38000				
40		0	1	-2		x3		0				
41												
42			Α			x		b				
43												_
44												
45		6.571429	-85.7143	-0.57143		28571.43						
46		-3./1429	57.14286	0.714286		314285.7						+
47		-1.85714	28.57143	-0.14286		15/142.9						+
48												$+ \square$
-+2	Þ	Sheet1	$( \div )$				:	4				
READ	( 🚡		AVERAG	E: 166666.66	67 COUNT	: 3 SUM: 5	00000		<b>—</b> -		— <b>+</b> 10	0%

Select the **\$ English (United States)** option.



We can now review the amounts of investment for each product that will enable the company to reach its goal while complying with the all the requirements of the investment plan.

X≣	5	- @				Book1 - Excel				? 🛧	- □	×	
FILE	E H	OME IN	SERT PA	AGE LAYOUT	FORM	ULAS DAT	A REVIEV	N VIEW	DEVEL	.OPER	FOXIT REA	DE •	
Paste		Calibri B I U T - D	• 11 • • A A A		■	Accounting ▼ \$ ▼ % *	Condition Format a Cell Style	onal Formatt as Table = es =	ing ▼ 🖀	Insert ▼ Delete ▼ Format ▼	Editing		
Clipb	oard 🗔	For	it r	Alignm	ent 🗔	Number 🕞		Styles		Cells		^	
F45	F45         ▼         ⋮         ×         ✓         fx         {=MMULT(B45:D47,H38:H40)}         ×												
	А	В	С	D	Е	F	G	Н	I	J	К		
37 38		1	1	1		x1		500000				-	
39		0.065	0.07	0.09	*	x2	=	38000					
40		0	1	-2		x3		0					
41													
42			Α			X		b				-	
43												-	
44		6 571/120	95 71/12	0 571/2		¢ 20 571 42						-	
45		-3 71429	57 14286	0 714286		\$ 20,371.43						-1	
47		-1.85714	28.57143	-0.14286		\$157,142.86						-	
48												11	
49												-	
	Þ	Sheet1	+				: 💽					•	
READ	Y 🛅	A	VERAGE: \$1	66,666.67	COUNT: 3	SUM: \$500,000.	00 🌐			-	-+ 1009	%	



# **3. CONCLUSION**

This course is the first of a series on engineering methods in *Excel*. This course presented a broad overview of fundamental principles and engineering applications of linear algebra, and how to use *Excel* tools to analyze and solve these types of problems.

This course the fundamentals of arrays, and then we looked at how arrays can be organized and manipulated on an *Excel* spreadsheet. We looked at how to formulate and solve systems of linear equations using arrays and how these systems can be implemented and solved on a spreadsheet.

This course has enabled participants to identify professional situations where the innovative application of techniques learned in this course are relevant and will be of benefit to the productivity, efficiency, and quality of the work output of a practitioner.

Practitioners are strongly encouraged to look out for situations in their domains of expertise where engineering methods in *Excel* are applicable and will be of benefit to their work and their organization.

A successful application of engineering methods in *Excel* requires a careful and meticulous approach, and can only be mastered and retained by practice and repetition.

It has been my utmost pleasure presenting this topic to you. Thank you.



## REFERENCES

Larson, R., & Hostetler, R. (2007). Precalculus. Houghton Mifflin Company.

Microsoft. (2017). *Microsoft Office help and training - Office Suport*. Retrieved March 13, 2017, from Microsoft Office Home: https://support.office.com/en-us/article/Define-and-solve-a-problem-by-using-Solver-9ed03c9f-7caf-4d99-bb6d-078f96d1652c

Images were all drawn/ prepared by Kwabena. Ofosu

0