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Engineering Methods in *Microsoft Excel*

Part 1:

by

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Abstract

This course is the first of a series on engineering methods in *Microsoft Excel* tailored to practicing engineers. The course series presents *Microsoft Excel* tools that can be used for a wide range of engineering analyses and data management. This course presents topics in linear algebra. Each topic includes a review of the fundamental principles followed by a real-life example encountered by a practicing engineer. The real-life examples are then formulated and implemented in *Microsoft Excel* and worked using the various *Excel* tools, spreadsheet techniques, and built-in functions. Examples from various engineering fields are used to demonstrate the concepts and methods learned throughout this course. Upon completion of this course, practitioners will be able to apply the methods learned to a variety of engineering problems, and also to identify situations in their fields of specialty where the innovative application of these tools and methods will be advantageous to their output and to their work product.



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1. INTRODUCTION

1.1 Overview

Engineers are constantly challenged with solving a wide range of complex analytical and computational problems in their fields of specialty. These problems involve analysis methodologies and the management of data. The application of computers enables repetitive, time-consuming and often tedious calculations to be conducted rapidly, efficiently, and less prone to errors. The application of computer tools also enables the results and outputs of such engineering analyses to be readily transferred and incorporated into reports and other engineering documents. An even greater advantage, in terms of productivity and efficiency, is realized when these calculations and outputs are replicated across numerous projects. Competence in computer skills predisposes engineers to pursue and develop more creative and innovative solutions to problems than otherwise.

Microsoft Excel is an electronic spreadsheet program developed by the *Microsoft* company, and is part of the *Microsoft Office* suite of products. A spreadsheet is a grid that organizes data and calculations into columns and rows. The intersection of a column and a row is called a cell. An electronic spreadsheet enables users to store, organize, manipulate, and analyze data in the cells of the spreadsheet. As of this publication, the software is into the 2016 release.

Microsoft Excel is widely and increasingly being used as a tool to assist engineers in conducting and replicating intricate calculations and analysis, designing complex systems, and managing large data sets. This course presents a selection of engineering analysis topics and how to formulate, implement and solve them in *Microsoft Excel*.

This course presents fundamental principles and engineering applications of problems in linear algebra, and demonstrates the *Excel* tools, methods, and strategies that can be used to formulate and solve them.

Upon completion of this course, participants will have gained insight into applying *Excel* tools, methods, and strategies in analyzing, formulating and implementing a variety of engineering related problems, as well as managing large sets of complex data. Participants will also be able to identify professional situations where the application of innovative *Excel* techniques will be of great benefit and advantage, and will enable practitioners to significantly improve their productivity, efficiency, and the quality of their work product.



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2. LINEAR ALGEBRA

2.1 Introduction

In this chapter, we shall conduct a review of the fundamentals of linear algebra, and see how the methods can be implemented in *Excel* on a spreadsheet.

2.2 Arrays

An array is an orderly list or arrangement of items or **elements**. The elements of the array are arranged in columns and rows. The number of rows and the number of columns are called the **dimensions** of the array. By convention, the elements in the array are listed by separating them by commas, or a space. Consider the following array called **A** that contains the following numerical elements,

$$\mathbf{A} = [2 \quad 0 \quad 1 \quad -1 \quad 3]$$

This array has 1 row and 5 columns. The array is therefore described as a 1x5 (“one-by-five”) array. Each element in the array is identified by its unique “address”, (or “position”), or **index** in the array. By convention, the following designation is used to identify an element in an array,

$$\mathbf{ArrayName}(\mathit{Row\#}, \mathit{Column\#})$$

where the first row is Row 1, and the first column is Column 1, and, so on. *Row#* and *Column#* are the indices of the element.

Therefore, for our array **A**, the element in the Row 1, Column 3, position, is the value 1. Using the notation,

$$\mathbf{A}(1, 3) = 1$$



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Consider another array, **B**, which has the elements,

$$\mathbf{B} = \begin{bmatrix} -1 \\ 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}$$

This array has five rows and one column. It is therefore a 5x1 array. We can see for example, that

$$\mathbf{B}(4, 1) = 0$$

An array that has one row and more than one column, or one column and more than one rows, is called a **vector**. The arrays A and B we just studied are vectors. A is a row vector, whereas B is a column vector. A multidimensional array – an array with more than one row and more than column, is called a **matrix** (plural is matrices).

2.3 Matrices

Consider the following array,

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -1 & 3 \end{bmatrix}$$

M is a 2x3 matrix.

Let *i* denote the number of rows of a matrix

Let *j* denote the number of columns of a matrix.



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If $i = j$, then the matrix is a square matrix. For example, the matrix

$$\mathbf{P} = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}$$

is a 2x2 square matrix. The matrix

$$\mathbf{Q} = \begin{bmatrix} 2 & 3 & 0 & -1 \\ 2 & 1 & 0 & -2 \\ 0 & 1 & 2 & 3 \\ 1 & -1 & 2 & -1 \end{bmatrix}$$

is a 4x4 square matrix.

For a i -by- i square matrix \mathbf{X} , the elements $\mathbf{X}(1,1)$, $\mathbf{X}(2,2)$, ..., $\mathbf{X}(i,i)$ form what is called the **main diagonal** of the matrix, whereas the elements $\mathbf{X}(1,i)$, $\mathbf{X}(2, i - 1)$, ..., $\mathbf{X}(i, 1)$ form the **minor diagonal** of the matrix. For example,

$\mathbf{X} = \begin{bmatrix} 2 & 3 & 0 & -1 \\ 2 & 1 & 0 & -2 \\ 0 & 1 & 2 & -3 \\ 1 & 1 & 2 & -1 \end{bmatrix}$

Minor diagonal

Main diagonal

2.4 Matrix Addition and Subtraction

Matrix addition (and subtraction) can be conducted on two matrices provided they have the same dimensions. For example a 2x3 matrix can be added to (or subtracted from) a 2x3 matrix, whereas the addition (or subtraction) of a 2x3 matrix and a 4x2 matrix is not possible. Matrix



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addition (and subtraction) is conducted by adding (or subtracting) the corresponding elements of the matrices.

Example: Consider the matrices

$$\mathbf{Y} = \begin{bmatrix} 1 & 4 & 5 \\ 1 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

and

$$\mathbf{Z} = \begin{bmatrix} -1 & 3 & 0 \\ -2 & 2 & 0 \\ 3 & 5 & 1 \end{bmatrix}$$

So,

$$\mathbf{Y} + \mathbf{Z} = \begin{bmatrix} 0 & 7 & 5 \\ -1 & 4 & 6 \\ 3 & 6 & 4 \end{bmatrix}$$

and

$$\mathbf{Y} - \mathbf{Z} = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 0 & 6 \\ -3 & -4 & 2 \end{bmatrix}$$

2.5 Scalar Multiplication

Scalar multiplication involves multiplying the matrix by a scalar value. This results in each element of the matrix being multiplied by the scalar value.



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Example: Consider the matrix

$$\mathbf{Y} = \begin{bmatrix} 1 & 4 & 5 \\ 1 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

Multiplying \mathbf{Y} by a scalar value of 5, yields,

$$5 * \mathbf{Y} = 5 * \begin{bmatrix} 1 & 4 & 5 \\ 1 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 20 & 25 \\ 5 & 10 & 30 \\ 0 & 5 & 15 \end{bmatrix}$$

2.6 Transpose of Matrix

A matrix is transposed by making the rows of the matrix, the columns of the matrix (or vice versa). So, for example, the first row of the original matrix becomes the first column of the transposed matrix and so on.

Another way of viewing the transpose of a matrix is that the indices of each element will be switch or swapped, or reversed in the transposed matrix. To generalize for an element of a matrix \mathbf{X} ,

$$\mathbf{X}(i, j) \xrightarrow{\text{Transpose}} \mathbf{X}^T(j, i)$$

where \mathbf{X}^T is the transpose matrix of \mathbf{X} .



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Example: The transpose of the matrix

$$\mathbf{Y} = \begin{bmatrix} 1 & 4 & 5 \\ 1 & 2 & 6 \\ 0 & 1 & 3 \end{bmatrix}$$

is

$$\mathbf{Y}^T = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 2 & 1 \\ 5 & 6 & 3 \end{bmatrix}$$

2.7 Matrix Multiplication

An $M \times N$ matrix can be multiplied by a $P \times Q$ matrix only if the “inner dimensions” are equal, that is, if $N = P$ (number of columns in first matrix equals number of rows in second matrix). The resulting matrix will be of dimensions $M \times Q$. If the inner dimensions are not equal, the product of the two matrices does not exist. To generalize,

$$X_{M \times N} * Y_{P \times Q} = Z_{M \times Q}$$

if $N = P$, otherwise the product of the input matrices does not exist, where \mathbf{X} , \mathbf{Y} are the input matrices, and \mathbf{Z} is the output matrix of the matrix multiplication process.

The elements of the output matrix are computed as follows. For any row i , an element

$$Z(i, j) = \sum_{i=1}^N X(i, j) * Y(j, i)$$

and we repeat for row $i = 1$ to M



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This process can be described graphically as follows.

$$\begin{bmatrix} X(1,1) & X(1,2) & X(1,3) \\ X(2,1) & X(2,2) & X(2,3) \\ X(3,1) & X(3,2) & X(3,3) \end{bmatrix} * \begin{bmatrix} Y(1,1) & Y(1,2) & Y(1,3) \\ Y(2,1) & Y(2,2) & Y(2,3) \\ Y(3,1) & Y(3,2) & Y(3,3) \end{bmatrix} = \begin{bmatrix} Z(1,1) & Z(1,2) & Z(1,3) \\ Z(2,1) & Z(2,2) & Z(2,3) \\ Z(3,1) & Z(3,2) & Z(3,3) \end{bmatrix}$$

$$\begin{bmatrix} X(1,1) & X(1,2) & X(1,3) \\ X(2,1) & X(2,2) & X(2,3) \\ X(3,1) & X(3,2) & X(3,3) \end{bmatrix} * \begin{bmatrix} Y(1,1) & Y(1,2) & Y(1,3) \\ Y(2,1) & Y(2,2) & Y(2,3) \\ Y(3,1) & Y(3,2) & Y(3,3) \end{bmatrix} = \begin{bmatrix} Z(1,1) & Z(1,2) & Z(1,3) \\ Z(2,1) & Z(2,2) & Z(2,3) \\ Z(3,1) & Z(3,2) & Z(3,3) \end{bmatrix}$$

$$\begin{bmatrix} X(1,1) & X(1,2) & X(1,3) \\ X(2,1) & X(2,2) & X(2,3) \\ X(3,1) & X(3,2) & X(3,3) \end{bmatrix} * \begin{bmatrix} Y(1,1) & Y(1,2) & Y(1,3) \\ Y(2,1) & Y(2,2) & Y(2,3) \\ Y(3,1) & Y(3,2) & Y(3,3) \end{bmatrix} = \begin{bmatrix} Z(1,1) & Z(1,2) & Z(1,3) \\ Z(2,1) & Z(2,2) & Z(2,3) \\ Z(3,1) & Z(3,2) & Z(3,3) \end{bmatrix}$$

$$\begin{bmatrix} X(1,1) & X(1,2) & X(1,3) \\ X(2,1) & X(2,2) & X(2,3) \\ X(3,1) & X(3,2) & X(3,3) \end{bmatrix} * \begin{bmatrix} Y(1,1) & Y(1,2) & Y(1,3) \\ Y(2,1) & Y(2,2) & Y(2,3) \\ Y(3,1) & Y(3,2) & Y(3,3) \end{bmatrix} = \begin{bmatrix} Z(1,1) & Z(1,2) & Z(1,3) \\ Z(2,1) & Z(2,2) & Z(2,3) \\ Z(3,1) & Z(3,2) & Z(3,3) \end{bmatrix}$$

and so on, and so forth.

The corresponding color coded elements (yellow, green, blue) are multiplied and all summed up to produce the element of the output matrix (in grey).

Example: Multiply the matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 2 & -1 \end{bmatrix}$$

by



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$$\mathbf{G} = \begin{bmatrix} 2 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$$

We have a 2x3 multiplying a 3x2. The inner dimensions are equal, so this matrix multiplication can proceed, and will result in a 2x2 matrix.

$$\mathbf{H} * \mathbf{G} = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 2 & -1 \end{bmatrix} * \begin{bmatrix} 2 & -2 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\mathbf{H} * \mathbf{G} = \begin{bmatrix} 1 * 2 + 1 * 1 + 3 * 1 & 1 * (-2) + 1 * 0 + 3 * 2 \\ 4 * 2 + 2 * 1 + (-1) * 1 & 4 * (-2) + 2 * 0 + (-1) * 2 \end{bmatrix}$$

$$\mathbf{H} * \mathbf{G} = \begin{bmatrix} 6 & 4 \\ 9 & -10 \end{bmatrix}$$

Another multiplication operation for matrices is the Element-by-Element Multiplication, also known as the Dot Product. As the name suggests, the element-by-element multiplication involves multiplying the corresponding elements in the two input matrices. Therefore, the element-by-element multiplication can only work if the two input matrices have identical vertical and horizontal dimensions. In this course we shall use “.*” (dot asterisk) to denote element-by-element multiplication.

Example: Given

$$\mathbf{P} = \begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$$



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Therefore, the dot product,

$$\mathbf{P} \cdot \mathbf{Q} = \begin{bmatrix} 3 & -2 \\ 1 & -6 \end{bmatrix}$$

2.8 Determinant of a Matrix

The determinant of a square matrix is a real number associated with the matrix. The determinant is computed from the elements of the matrix. Later on, we shall see that the determinant is an input in finding the inverse of a square matrix.

In this section we shall use the algorithm for computing the determinant of a 3x3 matrix to illustrate the fundamental principles as well as introduce new concepts.

Consider the following 3x3 matrix,

$$\mathbf{F} = \begin{bmatrix} 5 & -1 & 3 \\ 4 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

We can pick a row, or a column to base the process on. For convenience we shall pick the first row.

Step 1: For the first element “5”, let us picture eliminating the row and the column that this element belongs to.

$$\begin{bmatrix} 5 & -1 & 3 \\ 4 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$



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We will be left with a 2x2 matrix

$$\begin{bmatrix} -2 & 2 \\ 0 & 1 \end{bmatrix}$$

Step 2: Now compute the product of the elements along the main diagonal, subtract the product of the elements on the minor diagonal

$$[(-2)*1 - 2*0] = -2$$

So “-2” is called the **minor** of the element “5”.

Step 3: Multiply the minor by $(-1)^{(i+j)}$, where i and j are the row and column indices of the element whose minor we just computed (in this example, the indices of the element “5”). The result of this calculation is called the **cofactor** of the original element.

Step 4: Repeat the steps 1 through 3 above to compute the cofactors of the other elements on the first row of the original matrix.

Step 5: We now compute the summation of the product of each element on the first row by its cofactor. This summation is the determinant of the square matrix.

Example. The determinant for our matrix F,

$$\det(\mathbf{F}) \text{ or } |\mathbf{F}| = 5 * (-1)^{1+1} * \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} + (-1) * (-1)^{1+2} * \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} + 3 * (-1)^{1+3} * \begin{vmatrix} 4 & -2 \\ 1 & 0 \end{vmatrix}$$

$$= 5 * (1) * [(-2)(1) - 2(0)] + (-1) * (-1) * [4(1) - 2(1)] + 3 * (1) * [4(0) - (-2)(1)]$$

$$|\mathbf{F}| = -10 + 2 + 6 = -2$$



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So to generalize, for a 3x3 matrix with the elements

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Using the first row as the basis for the computation procedure, the determinant,

$$|\mathbf{F}| = f_{11} * (-1)^{i+j} * \begin{vmatrix} f_{22} & f_{23} \\ f_{32} & f_{33} \end{vmatrix} + f_{12} * (-1)^{i+j} * \begin{vmatrix} f_{21} & f_{23} \\ f_{31} & f_{33} \end{vmatrix} + f_{13} * (-1)^{i+j} * \begin{vmatrix} f_{21} & f_{22} \\ f_{31} & f_{32} \end{vmatrix}$$

It is pertinent to note that the notation used for the determinant is similar to that of the absolute value or the magnitude of a vector. The determinant of a matrix is neither of these, it is a real value associated with the square matrix and is calculated from the elements of the matrix. The determinant can be a positive or negative value. The determinant can also be zero in which case it has consequences on the inverse of the matrix which we shall study later in this course. Using the above algorithm for a 2x2 matrix it will be observed that the determinant of a 2x2 matrix reduces to the product of the elements on the main diagonal minus the product of the elements on the minor diagonal. For higher dimensions of square matrices, 4x4, 5x5, etc., etc.), the calculation of the determinant follows the same procedure but becomes more involving and considerably more tedious.

2.9 Inverse of a Matrix

Consider an NxN (square) matrix **A**. If **A** has a non-zero determinant, then there exists another NxN matrix, denoted **A**⁻¹, such that

$$\mathbf{A} * \mathbf{A}^{-1} = \mathbf{I}$$



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where **I** is an NxN **identity matrix**, otherwise \mathbf{A}^{-1} does not exist.

\mathbf{A}^{-1} is called the inverse matrix of the matrix **A**.

An identity matrix is one that has all elements on the main diagonal equal to one (1), and all other elements equal to zero (0). For example the 3x3 identity matrix is

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The identity matrix is analogous to the value one (1). If a square matrix is multiplied by its identity matrix, the result is that same matrix (analogous to multiplying a constant by unity). The algorithm to calculate the elements of the inverse matrix involves the following steps.

Step1: Replace each element of the original matrix with its cofactor.

Step 2: Transpose the matrix of cofactors. The resulting matrix is called the **adjoint matrix**.

Step 3: Divide each element of the adjoint matrix by the determinant of the original matrix. The resulting matrix is the inverse matrix of the original matrix.

Example: Let us compute the inverse of

$$\mathbf{K} = \begin{bmatrix} 1 & -1 & 3 \\ 4 & -2 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

The determinant of **K** is

$$\begin{aligned} |\mathbf{K}| &= 1 * (-1)^{1+1} * \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} + (-1) * (-1)^{1+2} * \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} + 3 * (-1)^{1+3} * \begin{vmatrix} 4 & -2 \\ 1 & 0 \end{vmatrix} \\ &= 1 * (1) * [(-2)(1) - 2(0)] + (-1) * (-1) * [4(1) - 2(1)] + 3 * (1) * [4(0) - (-2)(1)] \end{aligned}$$



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$$|\mathbf{K}| = -2 + 2 + 6 = 6$$

The elements of the matrix of cofactors are as follows

$$C_{11} = (-1)^{1+1} * \begin{vmatrix} -2 & 2 \\ 0 & 1 \end{vmatrix} = (1) * (-2 - 0) = -2$$

$$C_{12} = (-1)^{1+2} * \begin{vmatrix} 4 & 2 \\ 1 & 1 \end{vmatrix} = (-1) * (4 - 2) = -2$$

$$C_{13} = (-1)^{1+3} * \begin{vmatrix} 4 & -2 \\ 1 & 0 \end{vmatrix} = (1) * (0 + 2) = 2$$

$$C_{21} = (-1)^{2+1} * \begin{vmatrix} -1 & 3 \\ 0 & 1 \end{vmatrix} = (-1) * (-1 - 0) = 1$$

$$C_{22} = (-1)^{2+2} * \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = (1) * (1 - 3) = -2$$

$$C_{23} = (-1)^{2+3} * \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = (-1) * (0 + 1) = -1$$

$$C_{31} = (-1)^{3+1} * \begin{vmatrix} -1 & 3 \\ -2 & 2 \end{vmatrix} = (1) * (-2 + 6) = 4$$

$$C_{32} = (-1)^{3+2} * \begin{vmatrix} 1 & 3 \\ 4 & 2 \end{vmatrix} = (-1) * (2 - 12) = 10$$

$$C_{33} = (-1)^{3+3} * \begin{vmatrix} 1 & -1 \\ 4 & -2 \end{vmatrix} = (1) * (-2 + 4) = 2$$



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The matrix of cofactors is

$$\mathbf{C} = \begin{bmatrix} -2 & -2 & 2 \\ 1 & -2 & -1 \\ 4 & 10 & 2 \end{bmatrix}$$

We transpose the matrix of cofactors to obtain the adjoint matrix

$$\mathbf{K}_{adj} = \begin{bmatrix} -2 & 1 & 4 \\ -2 & -2 & 10 \\ 2 & -1 & 2 \end{bmatrix}$$

And then we divide each element of the adjoint matrix by the determinant of the original matrix to obtain the inverse matrix.

$$\mathbf{K}^{-1} = \frac{1}{6} * \begin{bmatrix} -2 & 1 & 4 \\ 0 & -2 & -10 \\ 2 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -0.333 & 0.166 & 0.666 \\ -0.333 & -0.333 & 1.666 \\ 0.333 & -0.166 & 0.333 \end{bmatrix}$$

2.10 Systems of Linear Equations

One application of matrices commonly used by engineers is in solving systems of linear equations.

Consider a system of linear equation with m equations and m unknown quantities (x_i) as follows.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2m}x_m &= b_2 \\ & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mm}x_m &= b_m \end{aligned}$$



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We may represent this system of linear equations in matrix notation as follows

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Or in a more simplified notation,

$$\mathbf{Ax} = \mathbf{b}$$

Where \mathbf{A} is the matrix of coefficients, \mathbf{x} is the vector of unknown quantities, and \mathbf{b} is the vector of the constants on the right hand sides of the equations. Multiplying both sides by \mathbf{A}^{-1} , we obtain

$$\mathbf{A} * \mathbf{A}^{-1}\mathbf{x} = \mathbf{A}^{-1} * \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1} * \mathbf{b}$$

In other words if we multiply the vector of constant terms on the right hand sides of the equations by the inverse of the matrix of coefficients, we obtain a vector whose elements are the values of the unknown quantities of the system of equations. That is

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix}^{-1} * \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Example: Solve the following system of linear equations.

$$x_1 + x_2 + x_3 = 6$$

$$2x_2 + 5x_3 = -4$$

$$2x_1 + 5x_2 - 3x_3 = 27$$



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In matrix form this will be,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 5 \\ 2 & 5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix}$$

or

$$[\mathbf{A}] \quad [\mathbf{x}] \quad [\mathbf{b}]$$

Inverting \mathbf{A} , we obtain

$$\mathbf{A}^{-1} = \begin{bmatrix} 1.24 & -0.32 & -0.12 \\ -0.40 & 0.20 & 0.20 \\ 0.16 & 0.12 & -0.08 \end{bmatrix}$$

and then

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \mathbf{A}^{-1} * \mathbf{b} = \begin{bmatrix} 1.24 & -0.32 & -0.12 \\ -0.40 & 0.20 & 0.20 \\ 0.16 & 0.12 & -0.08 \end{bmatrix} \begin{bmatrix} 6 \\ -4 \\ 27 \end{bmatrix} = \begin{bmatrix} 5.48 \\ 2.2 \\ -1.68 \end{bmatrix}$$

2.11 Matrices in Excel

In this section we shall implement all the matrix methods we have reviewed so far in *Excel*. In Excel matrices' elements can be entered directly into the cells of a spreadsheet, and then manipulated from there.

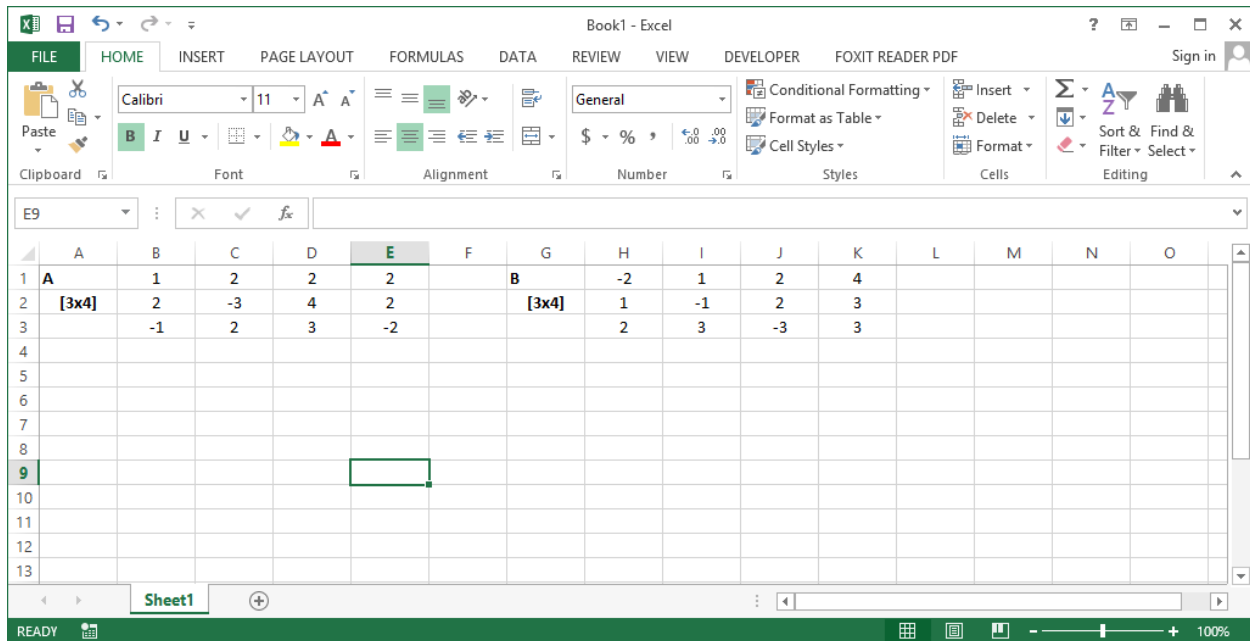
Open a new *Excel* file.

Go to the spreadsheet **Sheet1**



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Enter the elements of a matrix **A** and a matrix **B** as follows,



Addition and Subtraction

Let us add **A** and **B** to create a matrix called **C**.

Click on the cell location of the element **C(1,1)**.

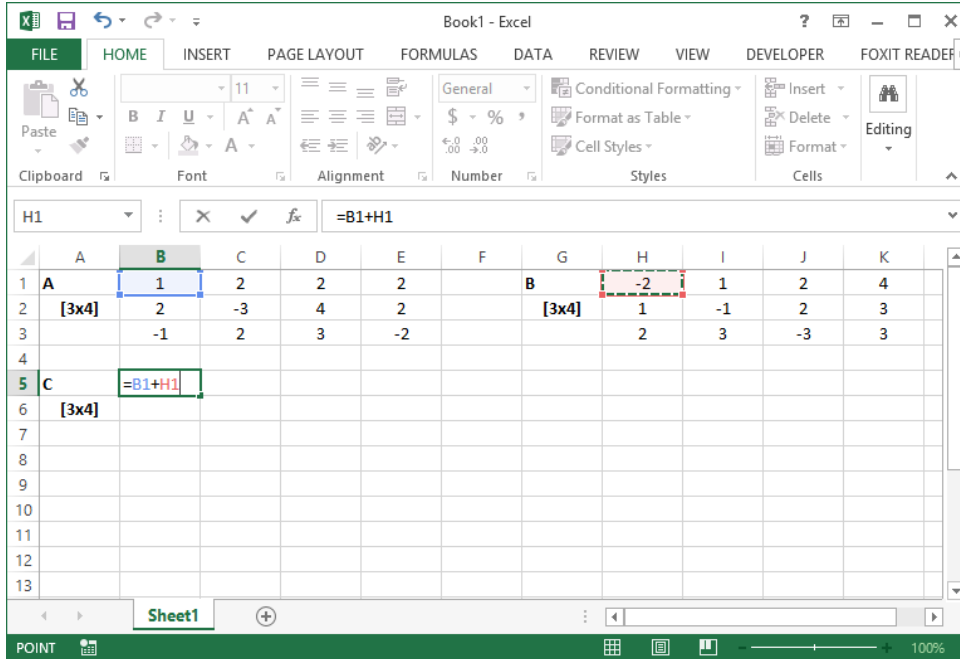
Type in the simple addition formula to add **A(1,1)** and its counterpart **B(1,1)**.

The formula is “= **B1** + **H1**”.



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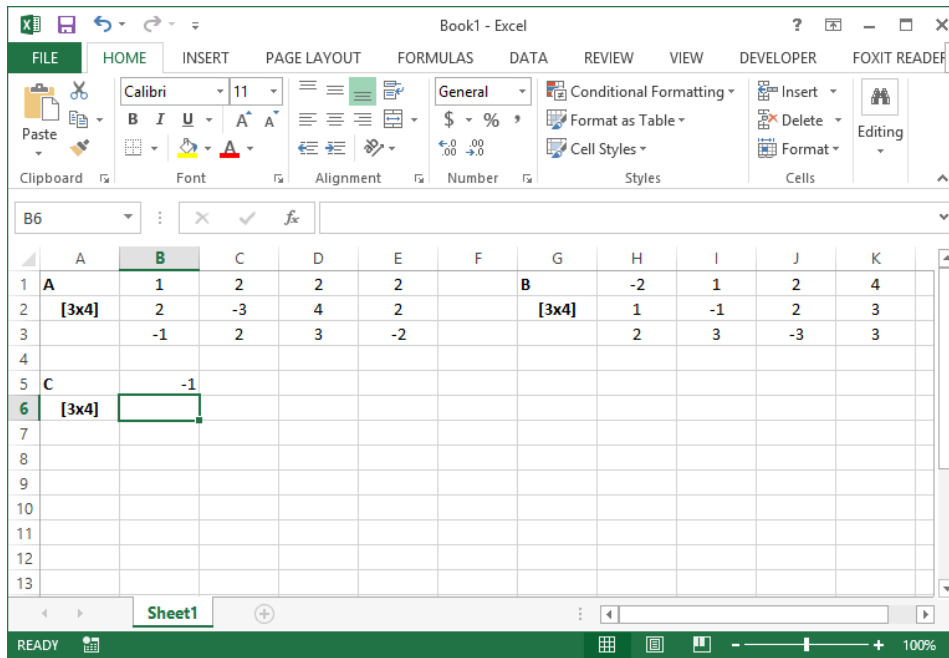
The formula may be typed in, or you may click on the relevant cells after the typing in the “=”.
The latter is commonly referred to as **Point-and-Click** formula entry.





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Hit **Enter** to see the calculated value of the element.



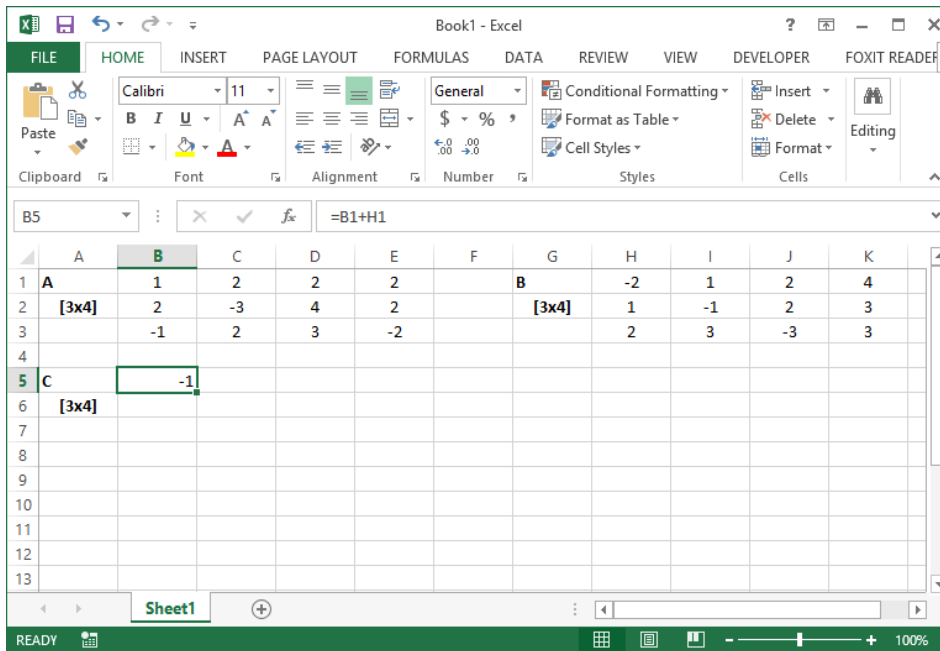
We shall now replicate this calculation for the other elements of the matrix C. We know that the addition of two 3x4 matrices will yield a 3x4 matrix. We shall copy the formula to fill two more rows and three more columns.



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Select the cell holding C(1,1).

Hover over the bottom right corner of the cell. Notice that the cursor changes to a “thin” plus (“+”) sign.

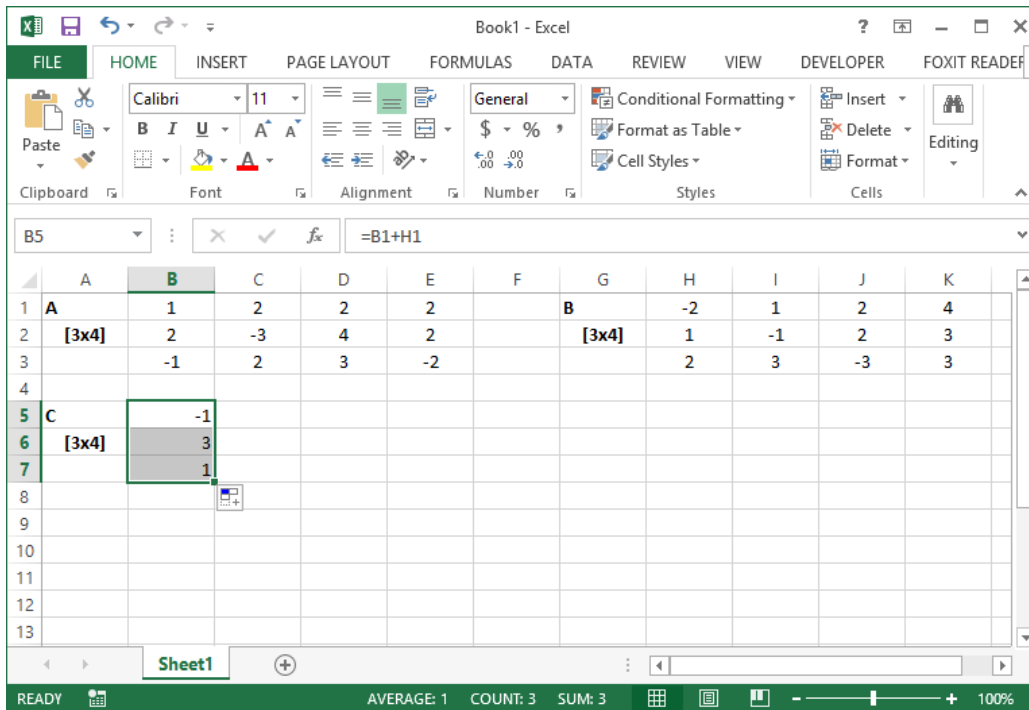


With the cursor now as a “thin” plus sign, click and hold down the cursor. With the cursor held down, drag the cursor two rows below the original cell. Release the cursor.



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The addition formula is copied and updated automatically for the relevant input cells (elements), thus calculating all of the elements for the first column. This type of copying data is called the **Fill Handle**.

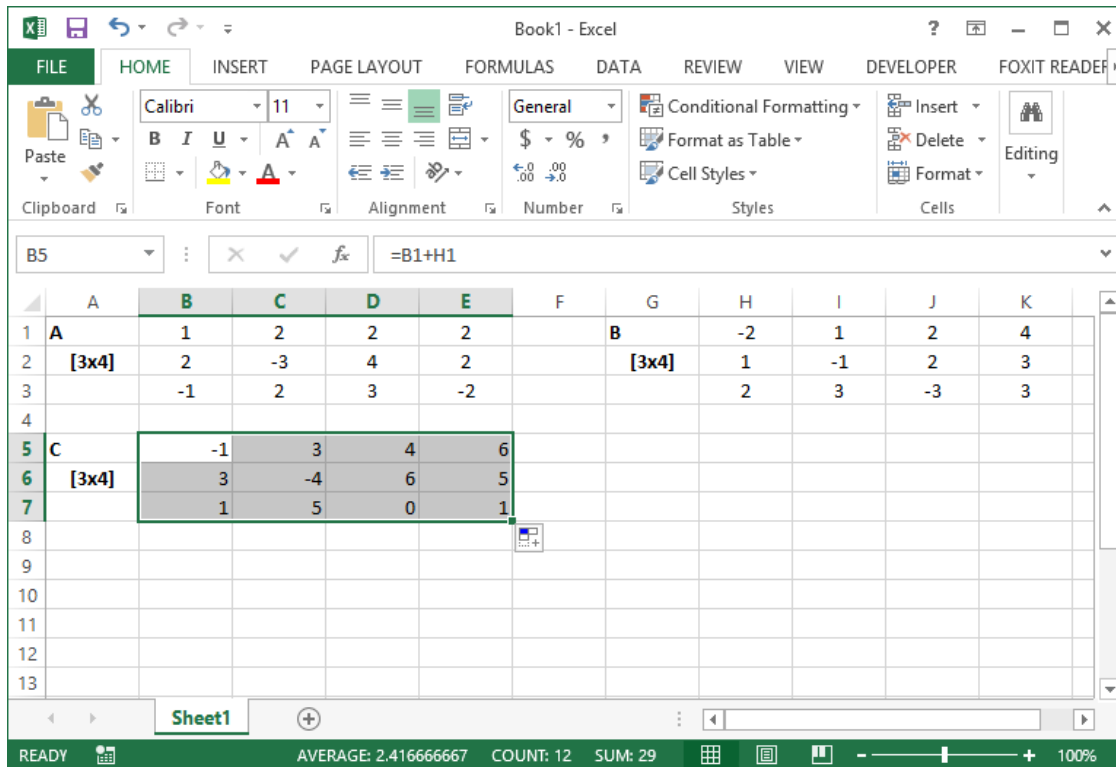


If not selected already, select the three elements of the first column of the matrix C.



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Hover over the bottom left corner of the selection until the cursor changes to the “thin” plus sign.
Click and hold down the cursor.
Drag the cursor over three additional columns to complete the four columns of the matrix.
Release the cursor.



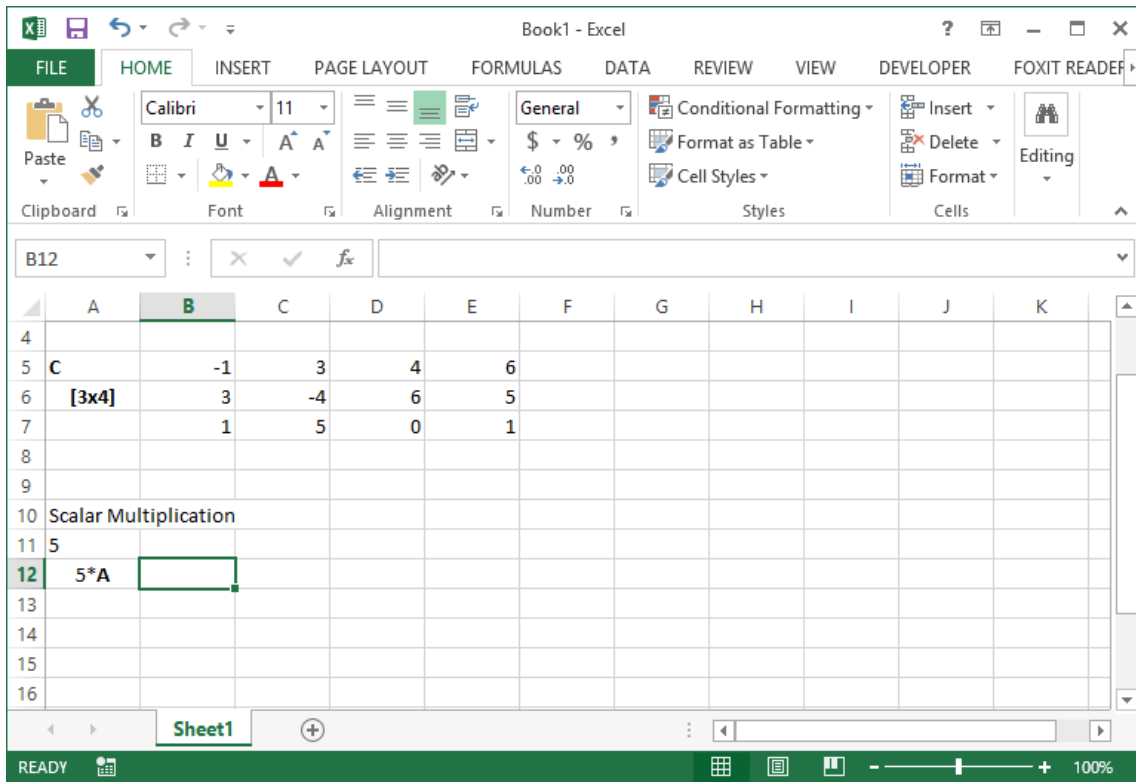


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Multiplication By Scalar

Let's multiply the matrix A on the spreadsheet by a scalar value of 5.

Enter the scalar value of the spreadsheet, and select the cell location of element (1,1) as follows.





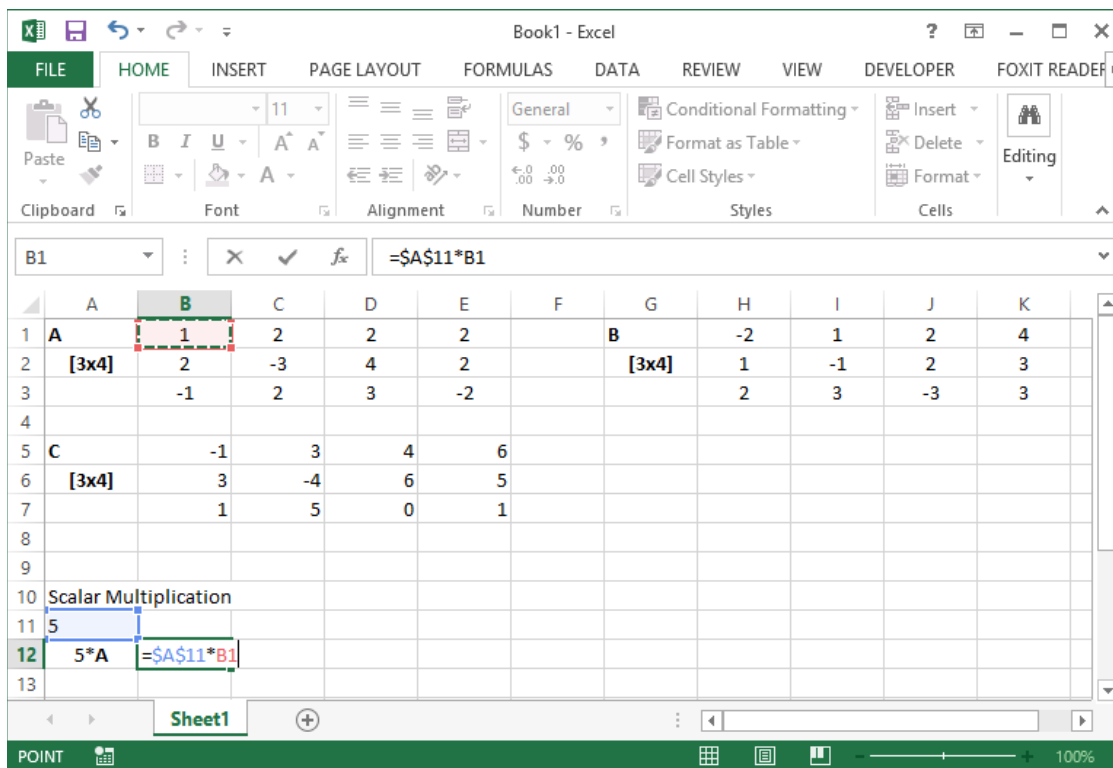
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In the cell that will hold element (1,1) type the equal sign (“=”).
By the point-and-click method, select the cell containing the scalar value.

Hit the **F4** button on your keyboard.

Type the multiply (“*”) sign.

Click on the element **A(1,1)**

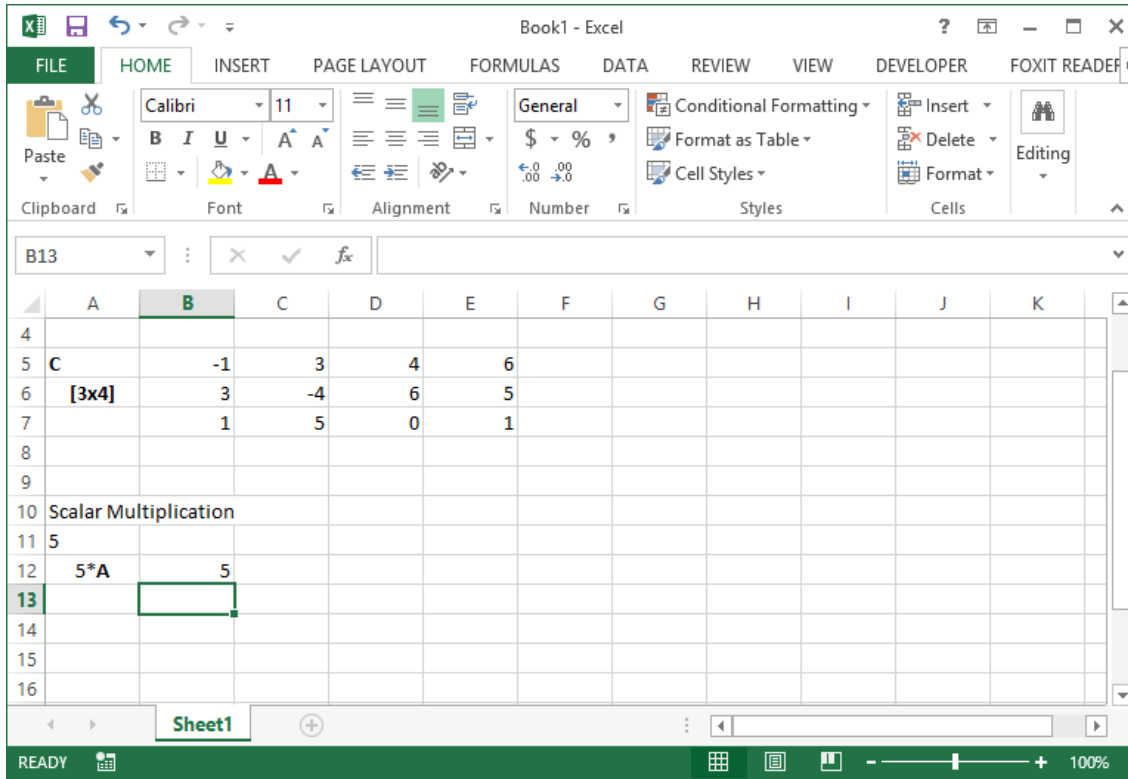


Hit **Enter** on your keyboard.



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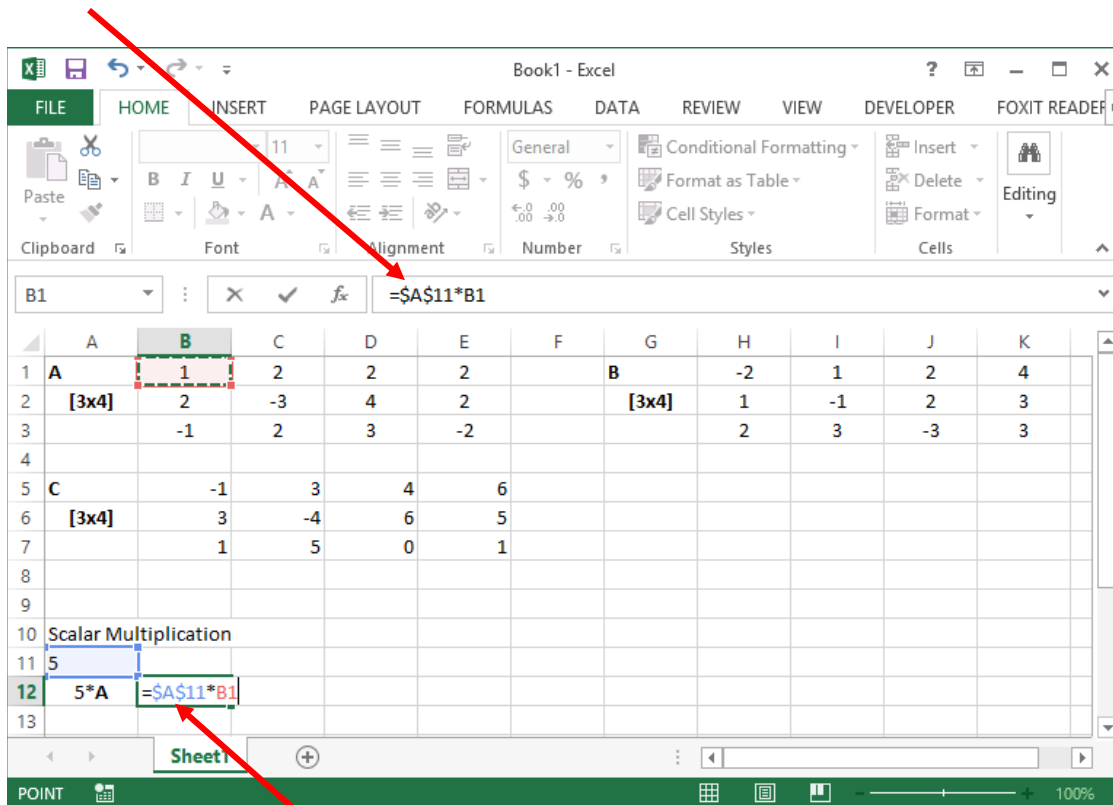
The element (1,1) of the new matrix is computed,





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In this procedure we hit **F4** after selecting the cell containing the scalar value. This makes that cell an **Absolute Reference**. When a formula containing an absolute reference is copied (by Fill Handle), the absolute reference does not update, hence locking in the cell so that the constant value it contains is applied to all cells that are covered by the fill handle operation. An absolute reference is identified by the dollar (\$) signs incorporated into the cell address upon hitting **F4**, or they may be typed in manually.





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Now let us complete the multiplication by scalar for the other elements of the new matrix.

Perform fill handle to complete the first column.

	A	B	C	D	E	F	G	H	I	J	K
4											
5	C	-1	3	4	6						
6	[3x4]	3	-4	6	5						
7		1	5	0	1						
8											
9											
10	Scalar Multiplication										
11	5										
12	5*A	5									
13		10									
14		-5									
15											
16											



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With the first column selected, fill handle to complete the remaining columns.

The screenshot shows the Microsoft Excel interface. The ribbon is set to 'HOME'. The active cell is B12, containing the formula $=\$A\$11*B1$. The spreadsheet contains the following data:

	A	B	C	D	E	F	G	H	I	J	K
4											
5	C	-1	3	4	6						
6	[3x4]	3	-4	6	5						
7		1	5	0	1						
8											
9											
10	Scalar Multiplication										
11	5										
12	5*A	5	10	10	10						
13		10	-15	20	10						
14		-5	10	15	-10						
15											
16											

The status bar at the bottom shows: READY, AVERAGE: 5.833333333, COUNT: 12, SUM: 70, and 100% zoom.



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Transpose of a Matrix

Let us transpose the matrix **B** that we created.

Select the cells of the matrix **B**.

Go to the HOME tab. Click on the **Copy** icon.

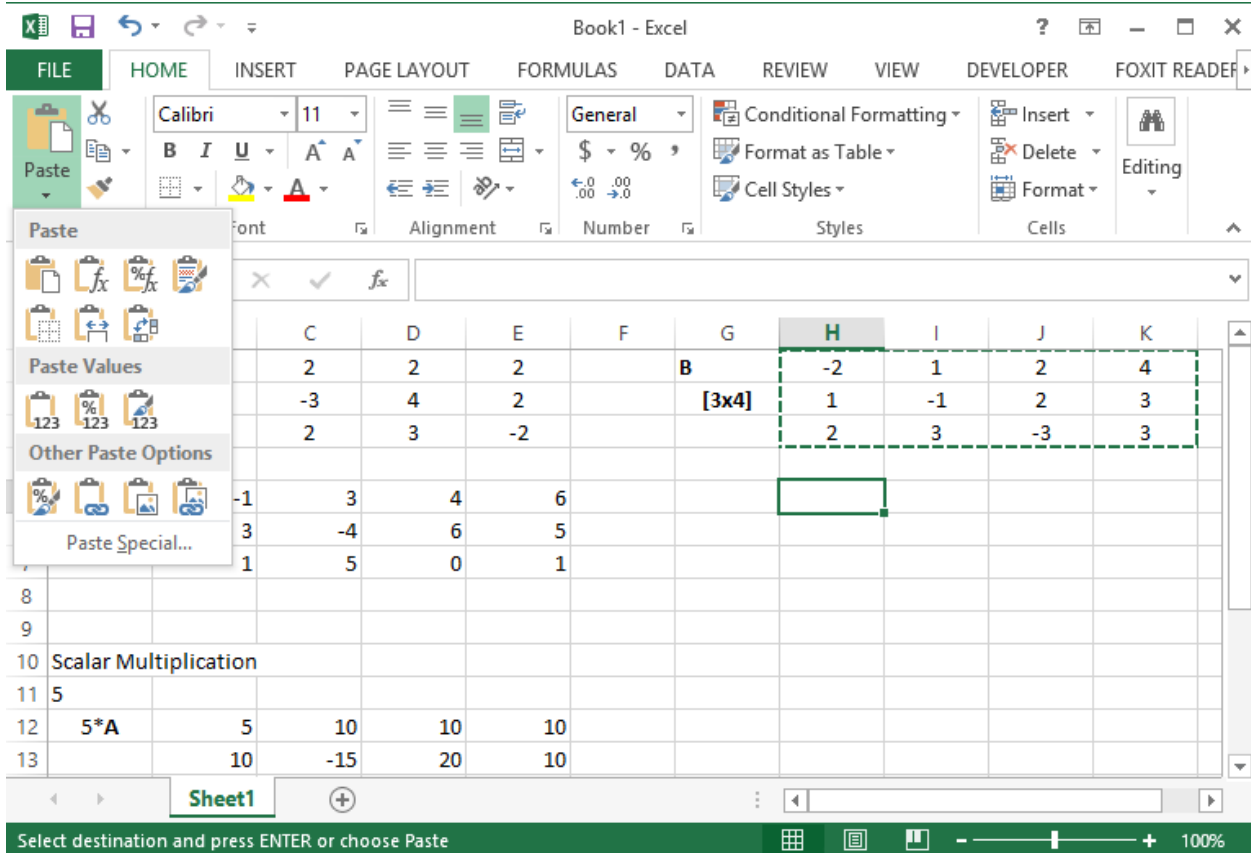
The screenshot shows the Microsoft Excel interface with the HOME tab selected. A red arrow points to the Copy icon in the Clipboard group. The spreadsheet contains a matrix B in cells H2:K4, which is currently selected. A tooltip for the Copy (Ctrl+C) icon is visible, stating: "Copy (Ctrl+C) Put a copy of the selection on the Clipboard so you can paste it somewhere else." The spreadsheet also shows other matrices A and C, and a section for Scalar Multiplication.

		E	F	G	H	I	J	K	
1	A								
2	[3x4]	2	-3	4	2				
3		-1	2	3	-2				
4									
5	C		-1	3	4	6			
6	[3x4]		3	-4	6	5			
7			1	5	0	1			
8									
9									
10	Scalar Multiplication								
11	5								
12	5*A	5	10	10	10				
13		10	-15	20	10				



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Click a cell where the element (1,1) of the output matrix will be positioned.
Click on the **Paste** icon dropdown button.

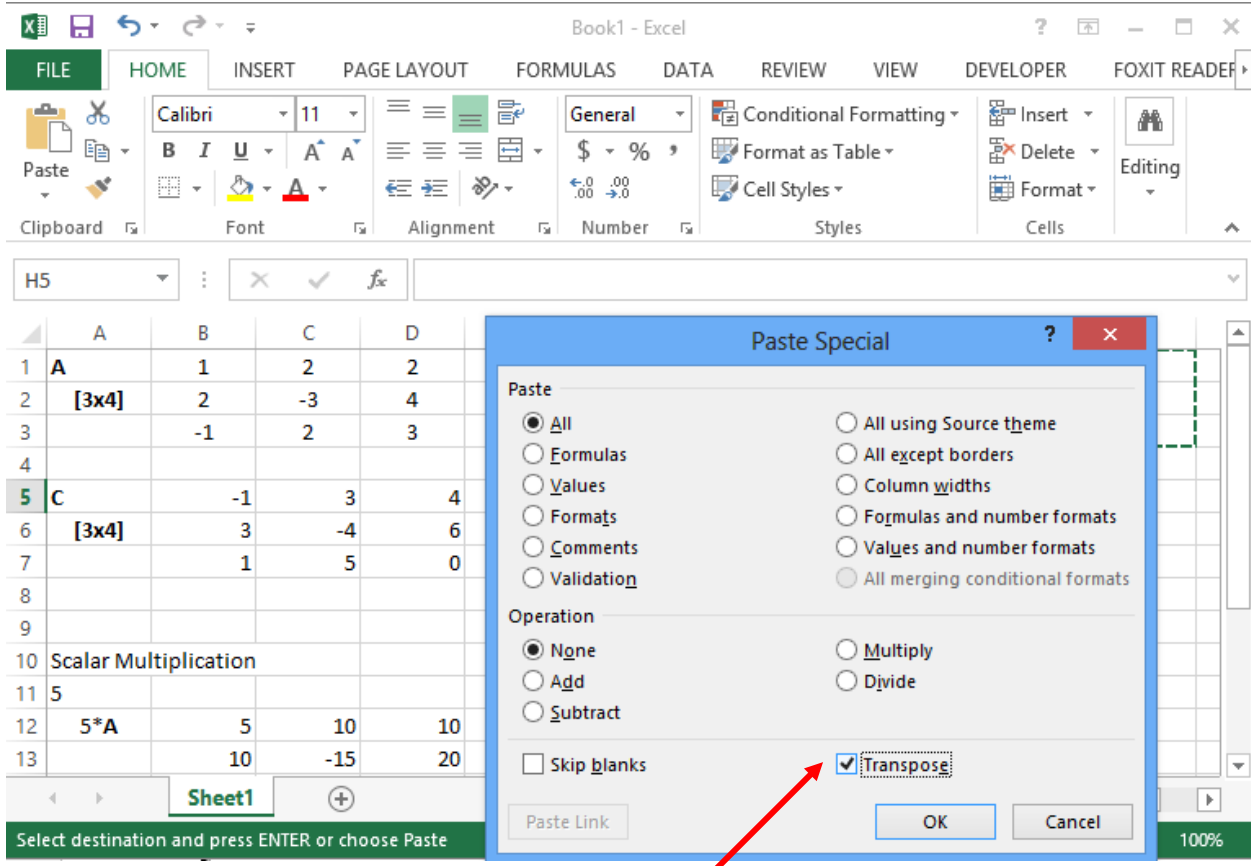


Select **Paste Special**.



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In the **Paste Special** window
Check the box for the **Transpose** option.



Click on the **OK** button.



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The matrix is copied and transposed as follows.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K
1	A	1	2	2	2		B	-2	1	2	4
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3
3		-1	2	3	-2			2	3	-3	3
4											
5	C	-1	3	4	6			-2	1	2	
6	[3x4]	3	-4	6	5			1	-1	3	
7		1	5	0	1			2	2	-3	
8								4	3	3	
9											
10	Scalar Multiplication										
11	5										
12	5*A	5	10	10	10						
13		10	-15	20	10						

The spreadsheet also shows the formula bar with the value -2 in cell H5. The status bar at the bottom indicates: AVERAGE: 1.25 COUNT: 12 SUM: 15.



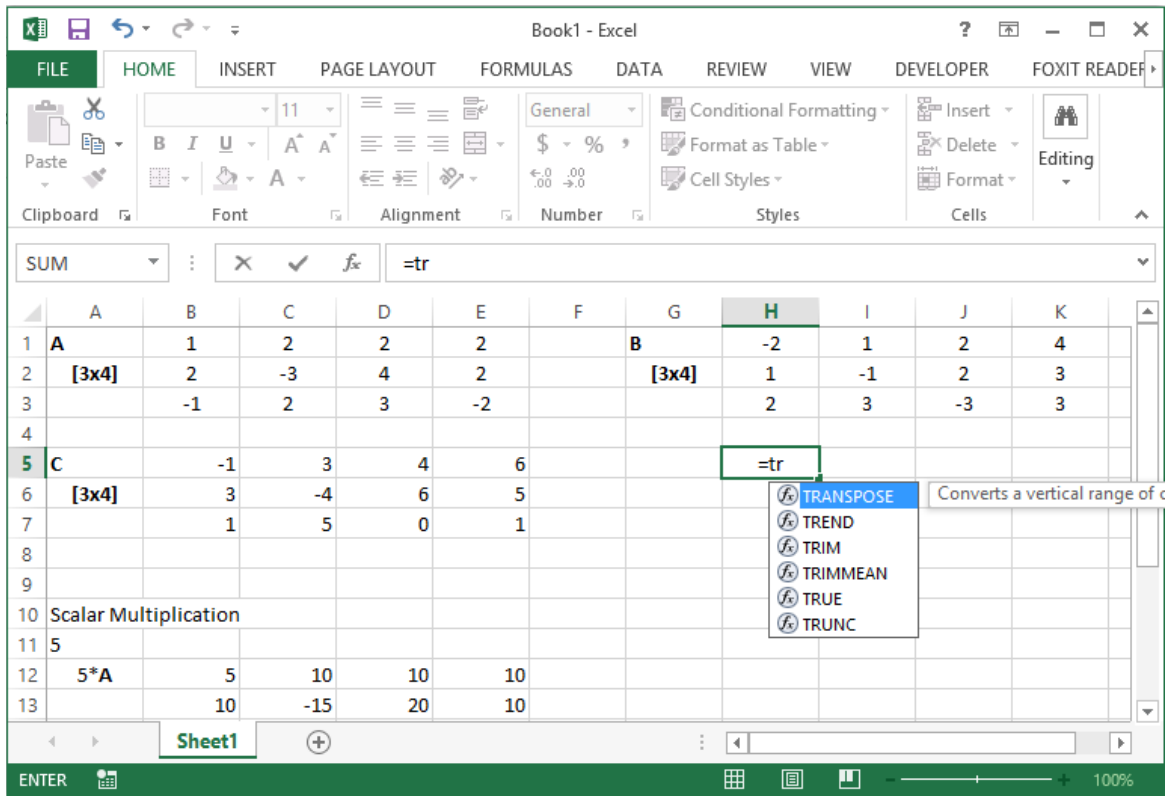
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Another method is by using the built-in TRANSPOSE function.

Let us start over. Delete the transposed matrix you just completed.

At the location of the element (1,1) of the transposed matrix type in the formula
“=Transpose”

Notice that as you start typing, a shortlist of possible built-in functions you are looking for appears, to assist you.



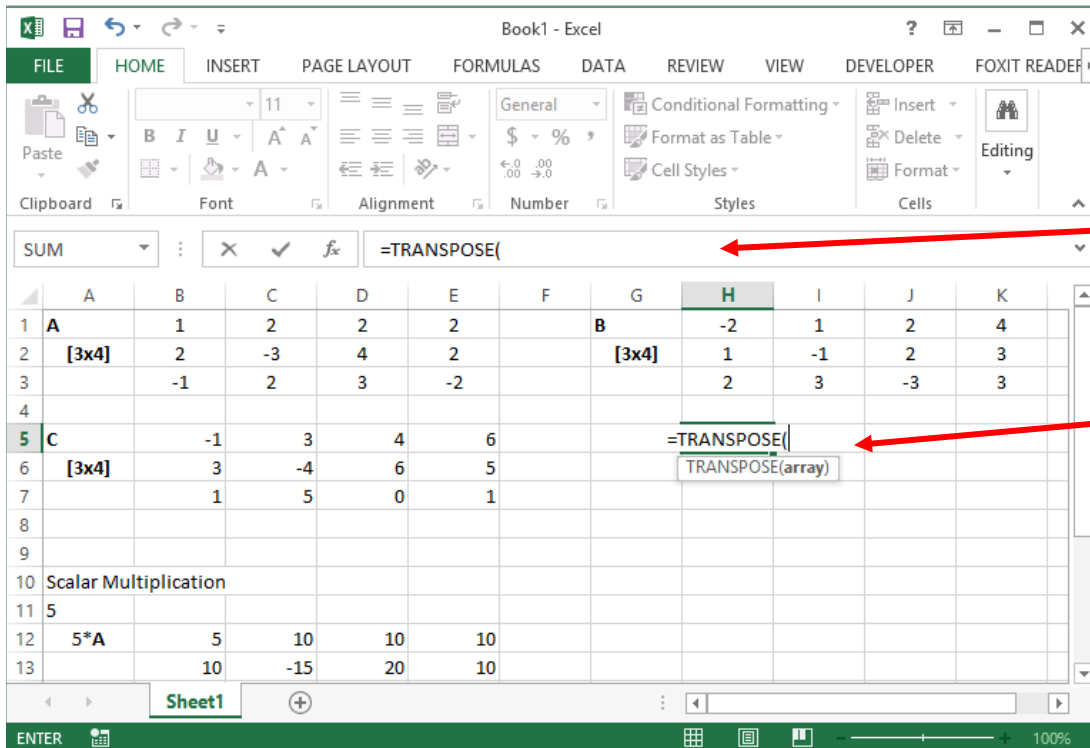


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Double click on the TRANSPOSE function to select it.
Notice the update that now appears in the Formula Bar.

In fact, the function could be have been entered directly in the Formula Bar after selecting the relevant cell on the spreadsheet.

Notice that the pick list of possible functions has reduced to the transpose function we are looking for. The tip now shows how the function is used. In our case we need to select the cells that hold the elements of the array whose transpose we are seeking to create.



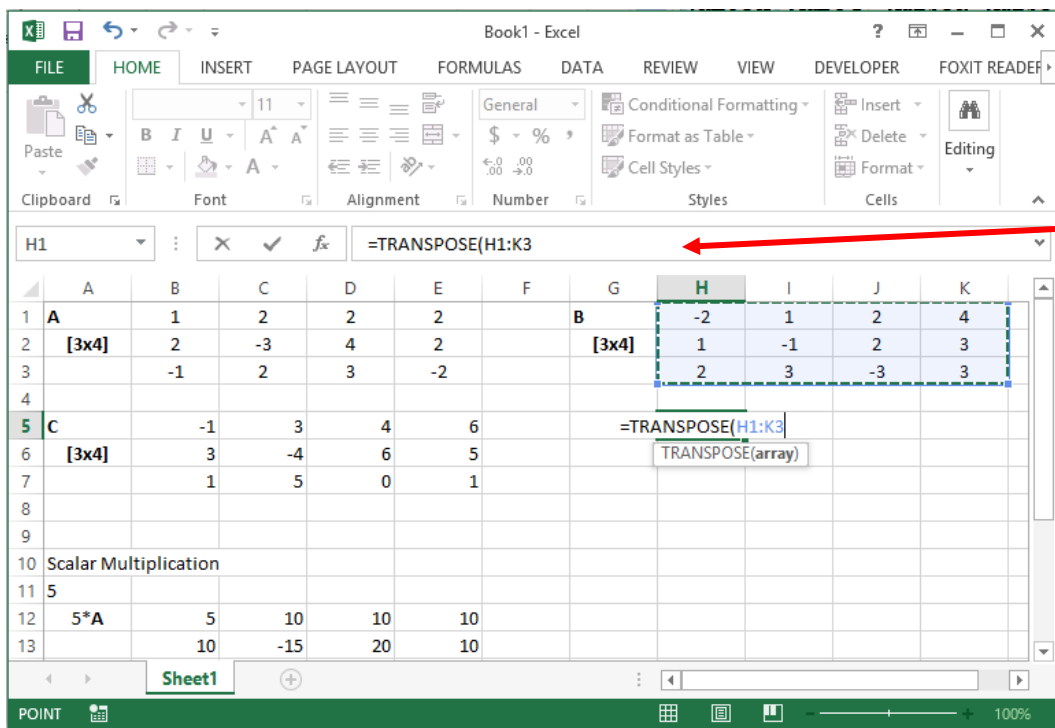


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Using the point-and click method, select the array we are transposing.

Notice how the matrix (array) is automatically specified in the formula bar and in the cell formula.

In fact instead of point-and click we could have alternately typed in the matrix cell references in the formula bar or in the cell formula.





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Hit **Enter** on your keyboard.

The screenshot shows an Excel spreadsheet with the following data:

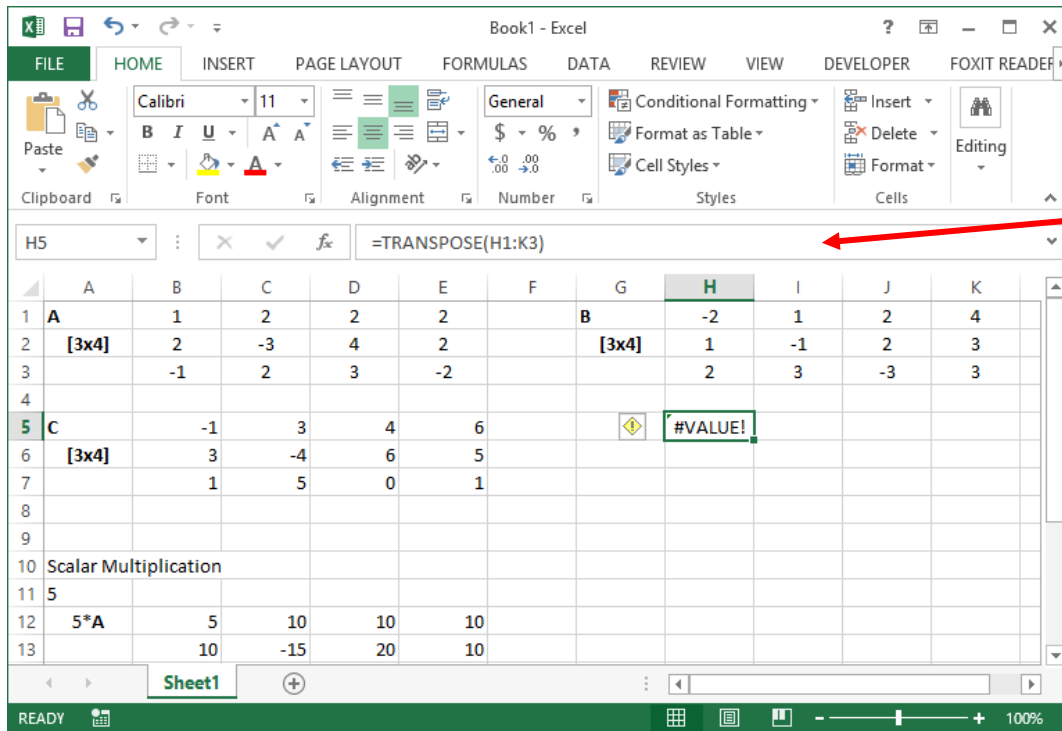
	A	B	C	D	E	F	G	H	I	J	K
1	A	1	2	2	2		B	-2	1	2	4
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3
3		-1	2	3	-2			2	3	-3	3
4											
5	C	-1	3	4	6			#VALUE!			
6	[3x4]	3	-4	6	5						
7		1	5	0	1						
8											
9											
10	Scalar Multiplication										
11	5										
12	5*A	5	10	10	10						
13		10	-15	20	10						



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Move the cursor back onto the cell of element (1,1) of the transpose matrix, which now contains “gibberish”.

Notice that the Formula Bar shows that we are indeed computing the transpose of the matrix **B**.



The matrix **B** is a 3x4 matrix so we know that the transpose of **B** will be a 4x3 matrix.



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Starting at the element (1,1) cell of the transpose matrix, select four (4) rows and three (3) columns.

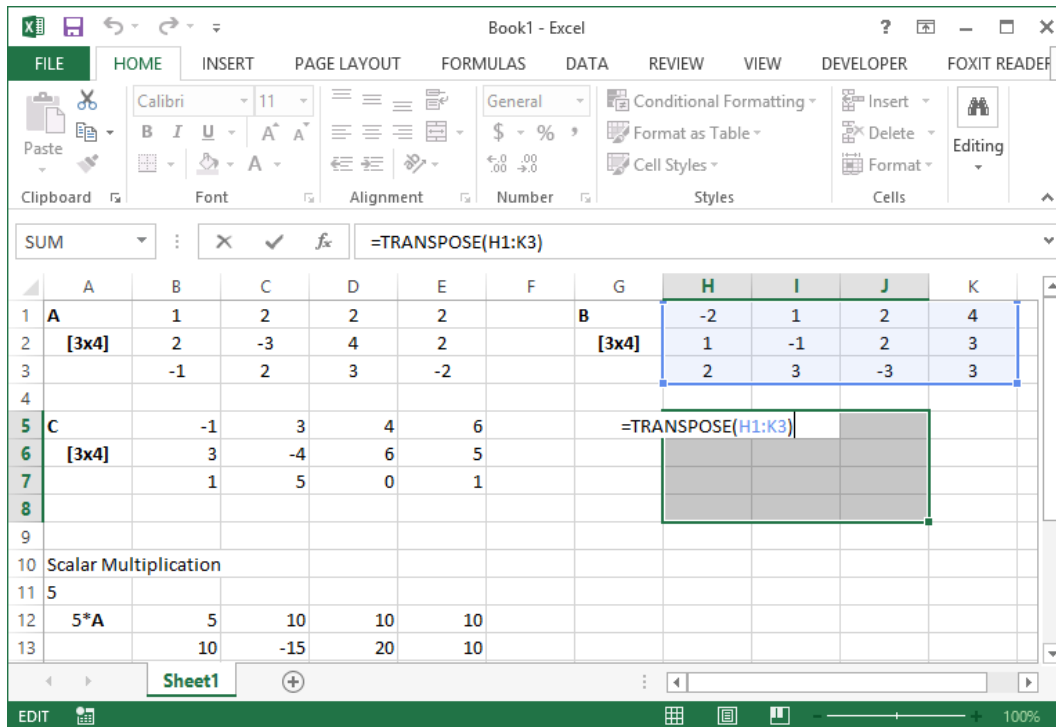
The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K
1	A	1	2	2	2		B	-2	1	2	4
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3
3		-1	2	3	-2			2	3	-3	3
4											
5	C		-1	3	4	6		#VALUE!			
6	[3x4]		3	-4	6	5					
7			1	5	0	1					
8											
9											
10	Scalar Multiplication										
11	5										
12	5*A	5	10	10	10						
13		10	-15	20	10						



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Hit **F2** on your keyboard.



Hold down the **Ctrl** button and the **Shift** button simultaneously.

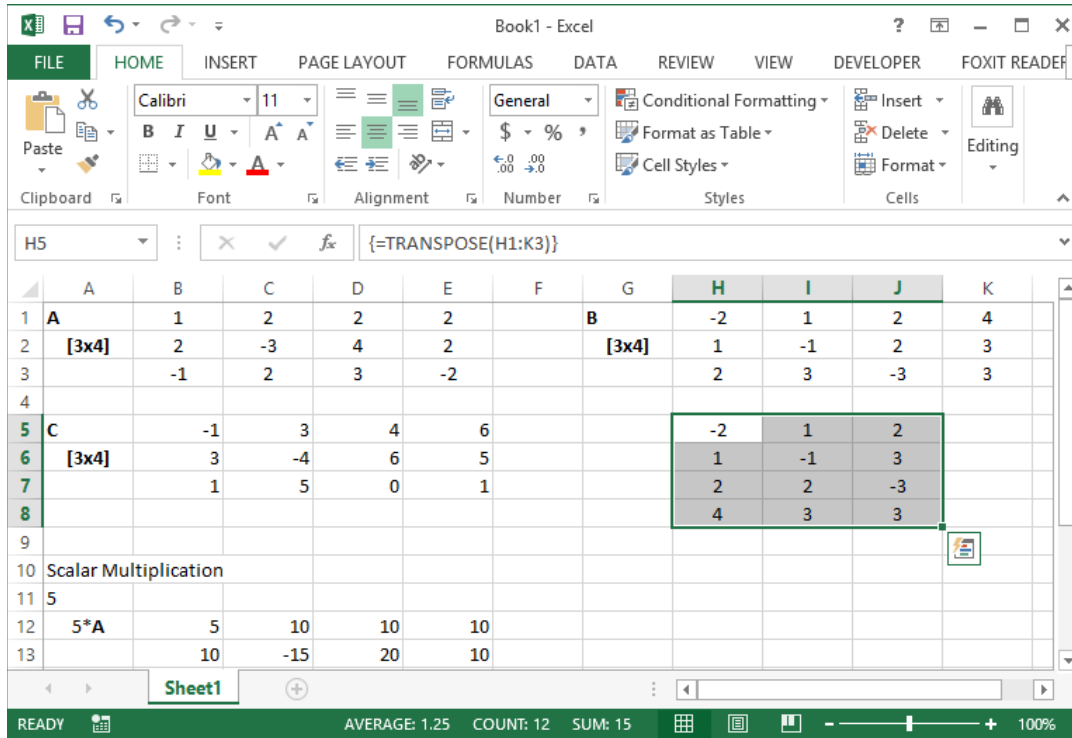
Hit the **Enter** button.

Release all buttons.



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Notice the transpose matrix has now been implemented.
Also notice how this new matrix is specified in the formula bar.



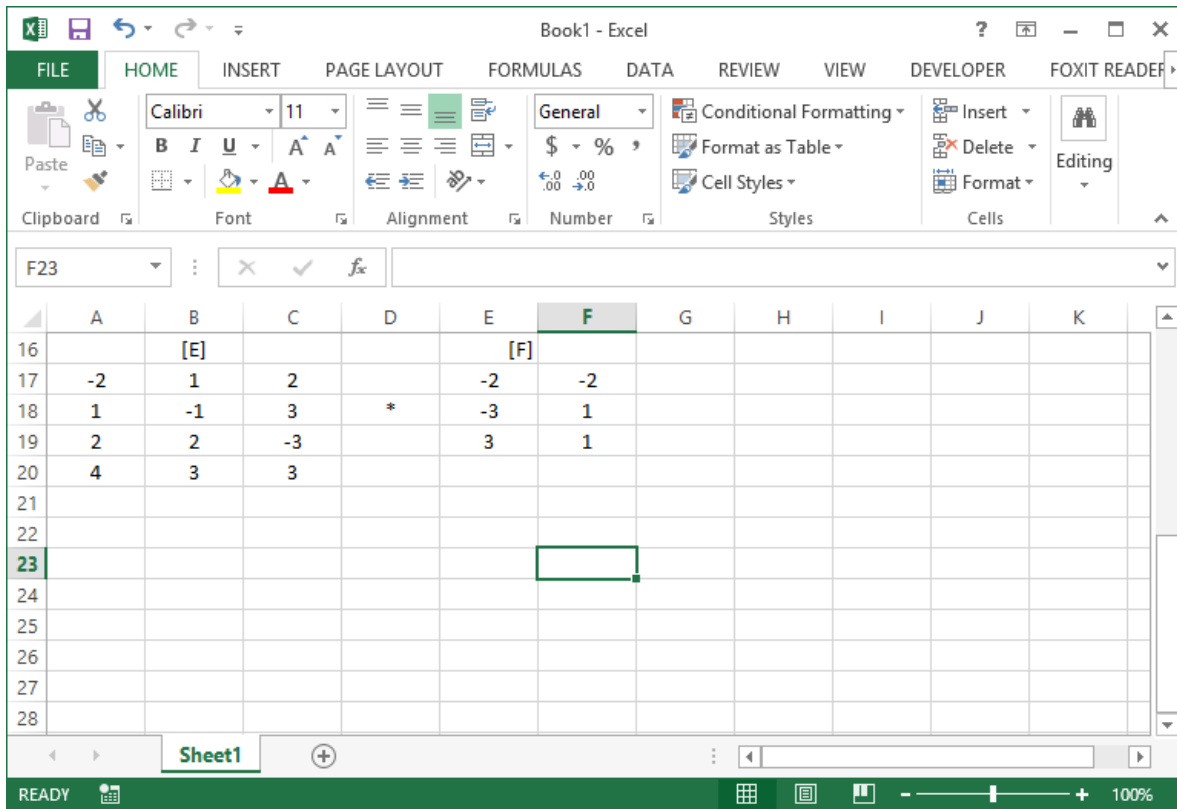
It is pertinent to note that the step of holding down the **Ctrl** button and the **Shift** button at the same time, and then hitting the **Enter** button is a common step in the *Excel* matrix functions and manipulations. For the remainder of this course we shall denote this step as **Ctrl-Shift+Enter**.



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Matrix Multiplication

Enter the following matrices **E** and **F** as shown below.

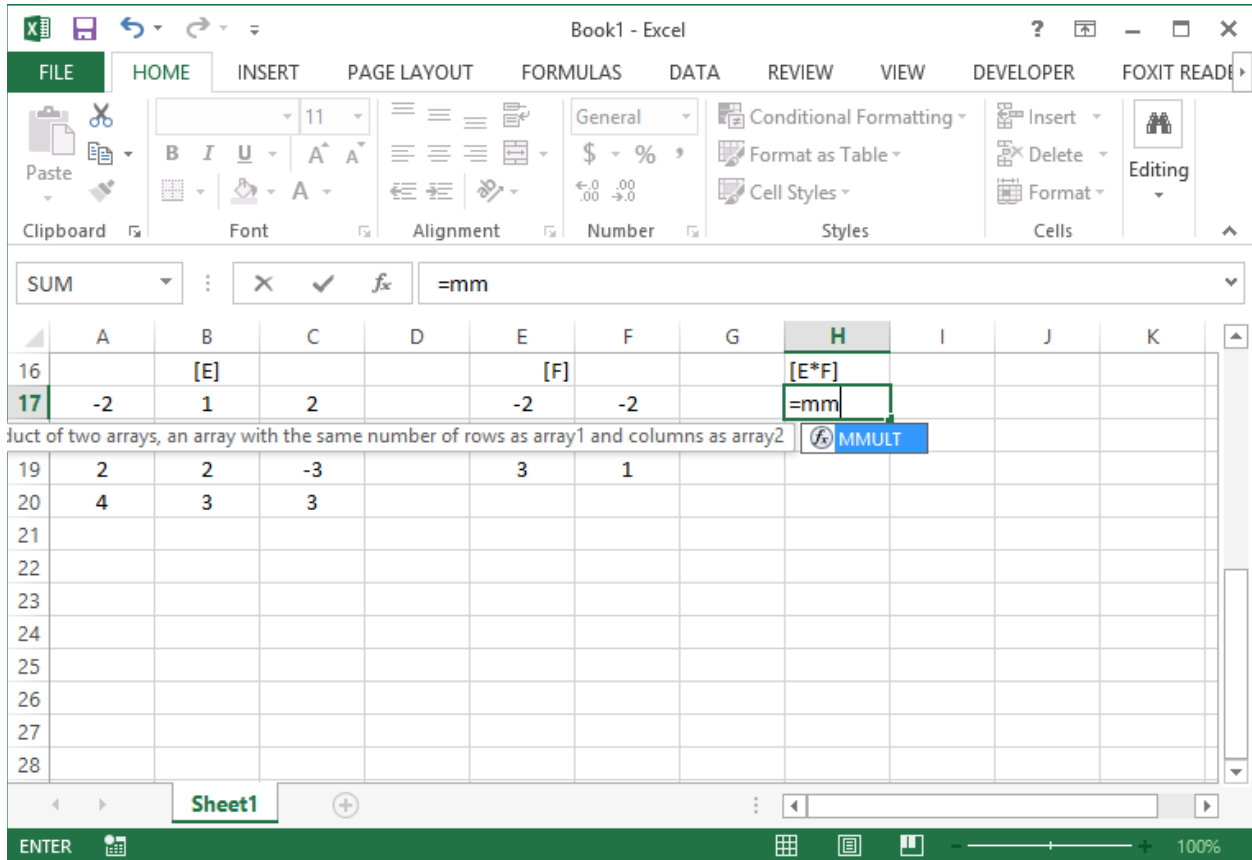


So we have a 4x3 matrix multiplying a 3x2 matrix. The inner dimensions are equal, therefore this matrix multiplication can proceed and will result in a 4x2 matrix output.



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Move the cursor to the cell location of the element (1,1) of the output matrix.
Type “=MMULT”. This is the *Excel* matrix multiplication built-in function.





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Double click on MMULT.

The screenshot shows the Microsoft Excel interface with the following data and formula:

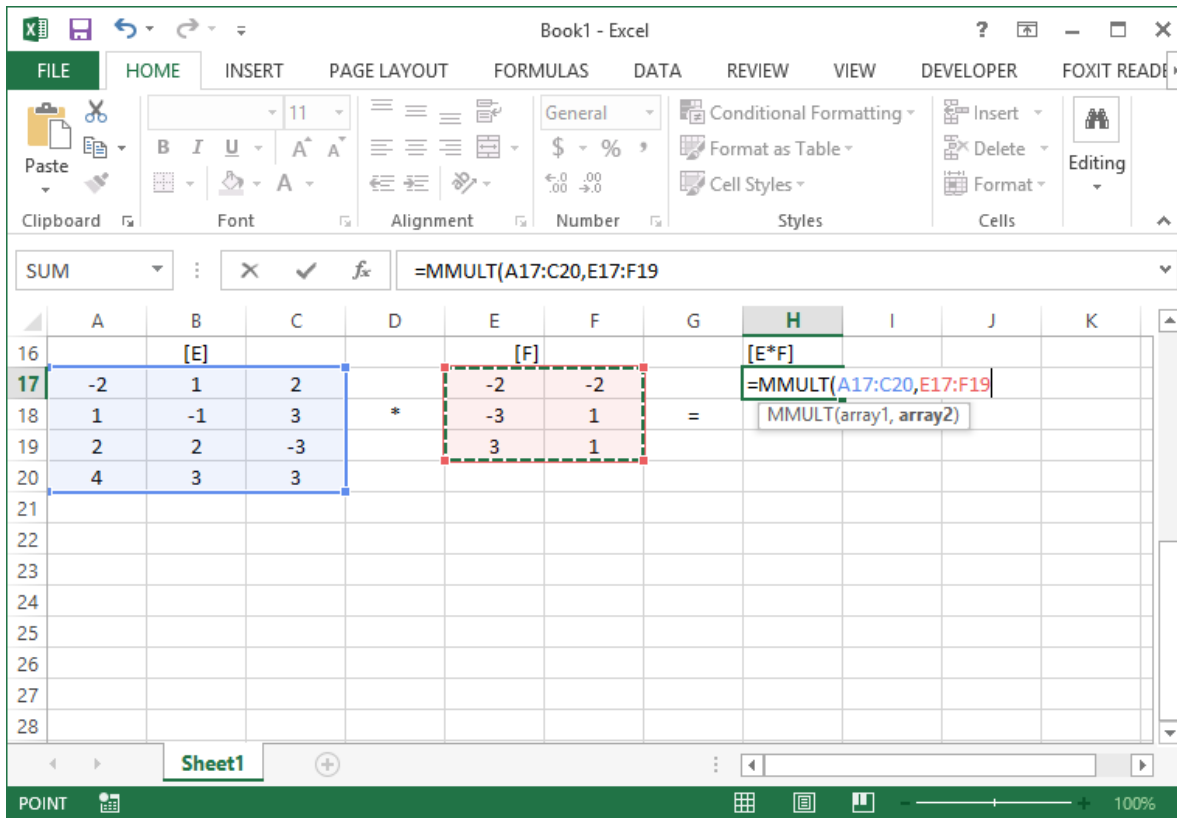
	A	B	C	D	E	F	G	H	I	J	K
16		[E]				[F]		[E*F]			
17	-2	1	2		-2	-2		=MMULT(
18	1	-1	3	*	-3	1	=	MMULT(array1, array2)			
19	2	2	-3		3	1					
20	4	3	3								
21											
22											
23											
24											
25											
26											
27											
28											

The formula bar shows: `=MMULT(`



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Use point-and click to select the elements of E.
Type in a comma (“,”).
Use point-and click to select the elements of F.

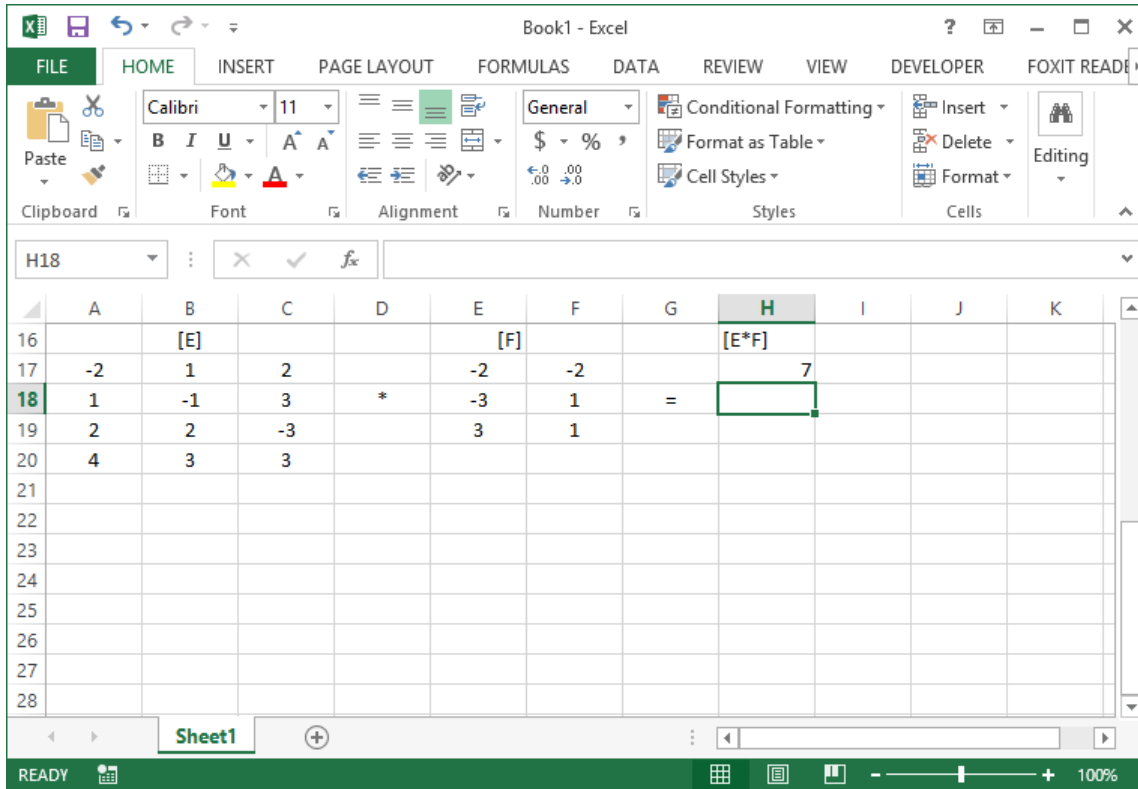


Hit Enter.



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The element (1,1) is computed. We must now fill in the other elements.



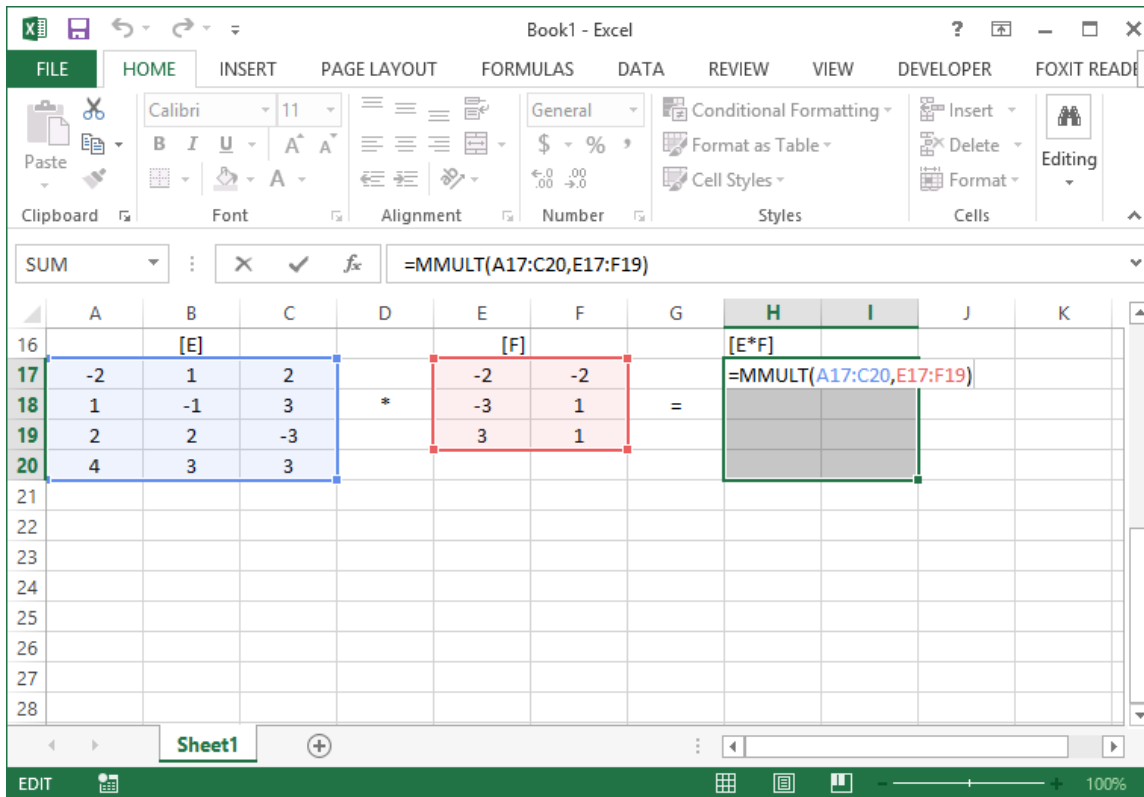
We know the output matrix will be a 4x2 matrix.



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Starting at cell location of the element (1,1) of the output matrix select four (4) rows and two (2) columns.

Hit **F2**.

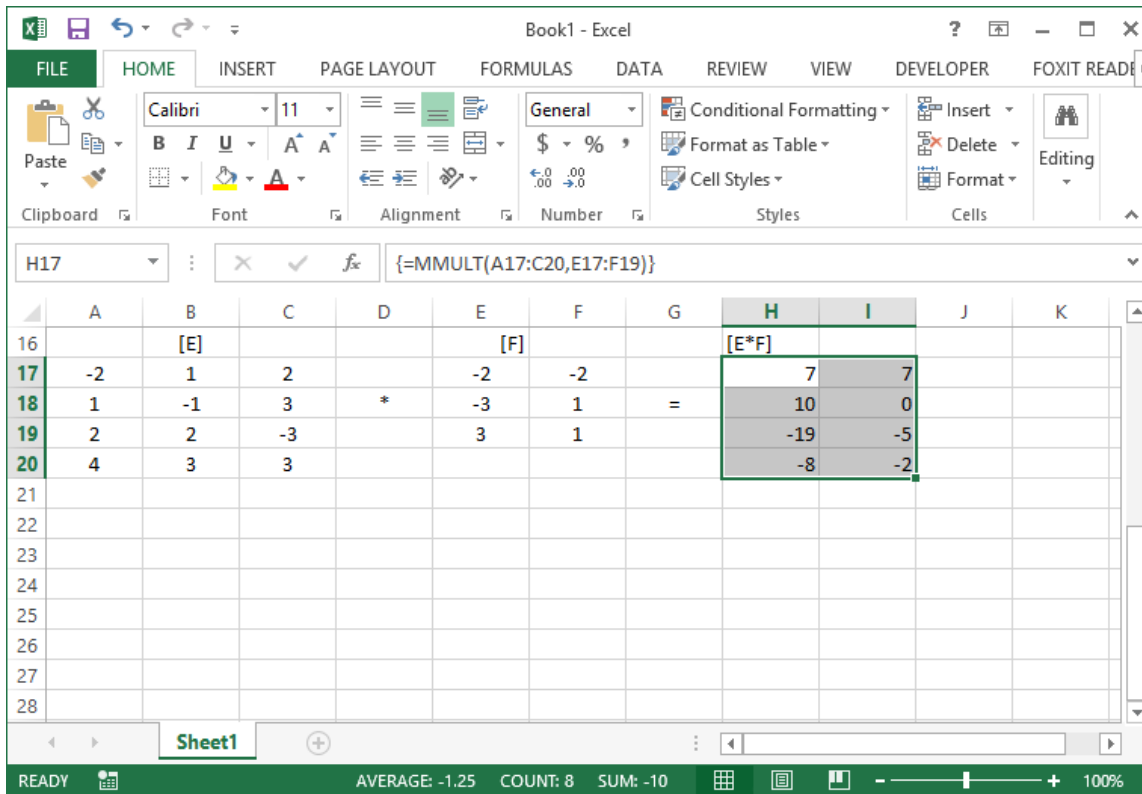




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Ctrl-Shift+Enter.

The output matrix of the multiplication of the two input matrices is as follows



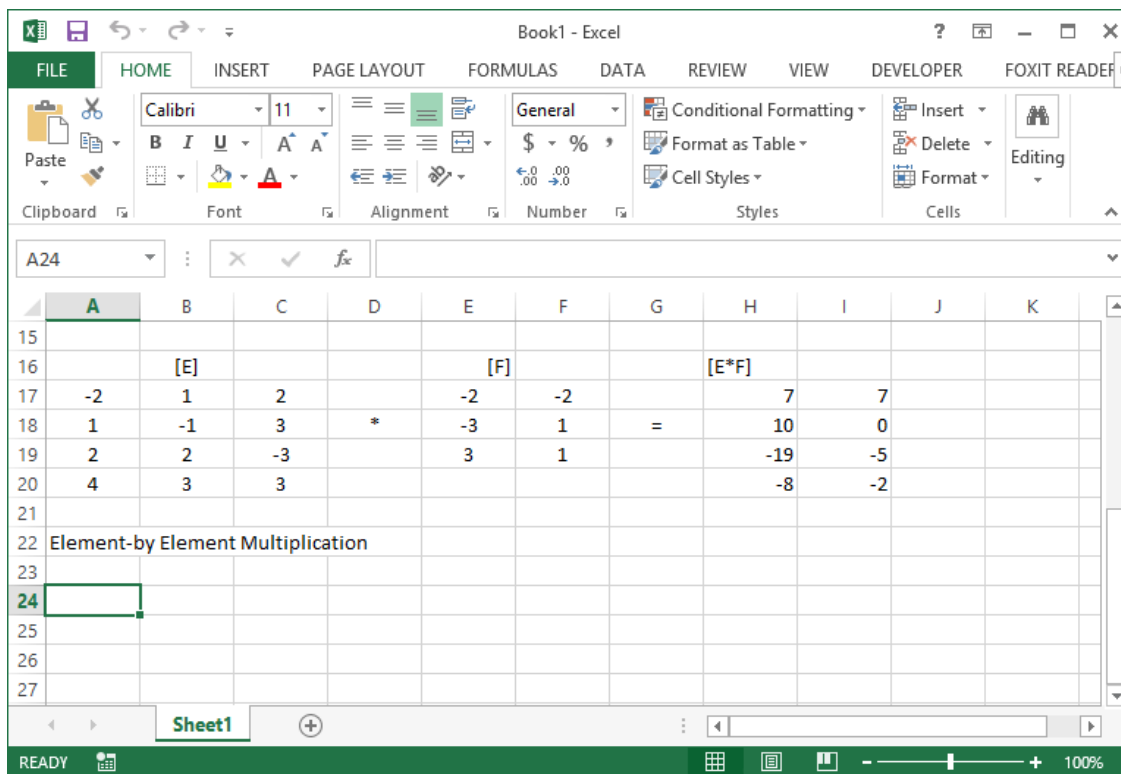


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Element-by-Element Multiplication

As the name suggests, element-by-element multiplication of two matrices involves multiplying together corresponding elements in the two matrices. This is analogous to the addition/subtraction of matrices. The two input matrices must have the exact same dimensions for this operation to be possible.

Let us conduct element-by-element multiplication on the matrices **A** and **B**.
Select the cell location for the cell(1,1) of the output matrix.
Type “=”





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Click on the cell of A(1,1).

Type “*”

Click on the cell of B(1,1)

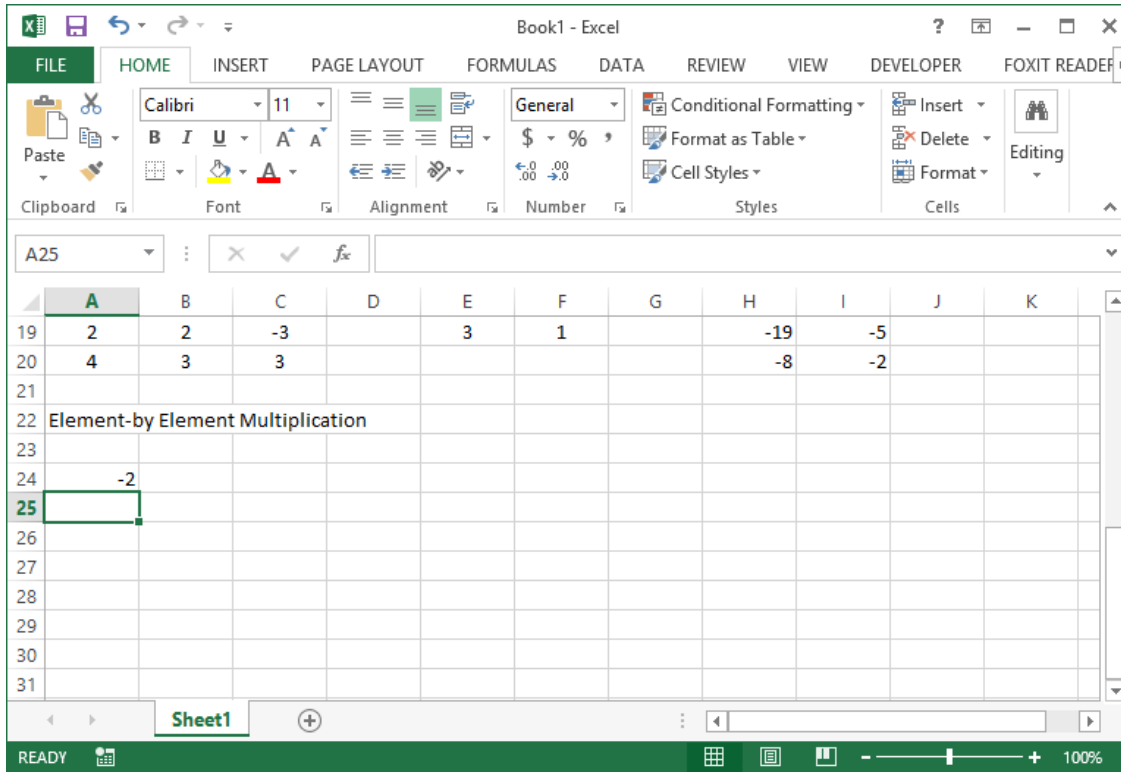
The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K
1	A	1	2	2	2		B	-2	1	2	4
2	[3x4]	2	-3	4	2		[3x4]	1	-1	2	3
3		-1	2	3	-2			2	3	-3	3
4											
5	C	-1	3	4	6			-2	1	2	
6	[3x4]	3	-4	6	5			1	-1	3	
7		1	5	0	1			2	2	-3	
8								4	3	3	
9											
10	Scalar Multiplication										
11	5										
12	5*A	5	10	10	10						
13		10	-15	20	10						



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Hit Enter.





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Use fill handle to complete the elements of the first row.

The screenshot shows the Microsoft Excel interface. The ribbon is set to 'HOME'. The formula bar displays the formula $=B1*H1$. The spreadsheet contains the following data:

	A	B	C	D	E	F	G	H	I	J	K
19	2	2	-3		3	1		-19	-5		
20	4	3	3					-8	-2		
21											
22	Element-by Element Multiplication										
23											
24	-2	2	4	8							
25											
26											
27											
28											
29											
30											
31											

The status bar at the bottom shows: READY, AVERAGE: 3, COUNT: 4, SUM: 12, and 100% zoom.



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With the first row selected, use fill handle to complete the remaining rows of the output matrix.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K
19	2	2	-3		3	1		-19	-5		
20	4	3	3					-8	-2		
21											
22	Element-by Element Multiplication										
23											
24	-2	2	4	8							
25	2	3	8	6							
26	-2	6	-9	-6							
27											
28											
29											
30											
31											

The formula bar shows the formula $=B1*H1$ and the active cell is A24. The status bar at the bottom shows: READY, AVERAGE: 1.66666667, COUNT: 12, SUM: 20, and 100% zoom.



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Determinant of a Matrix

Enter the elements of the matrix **J**.

The screenshot shows the Microsoft Excel interface with the following data:

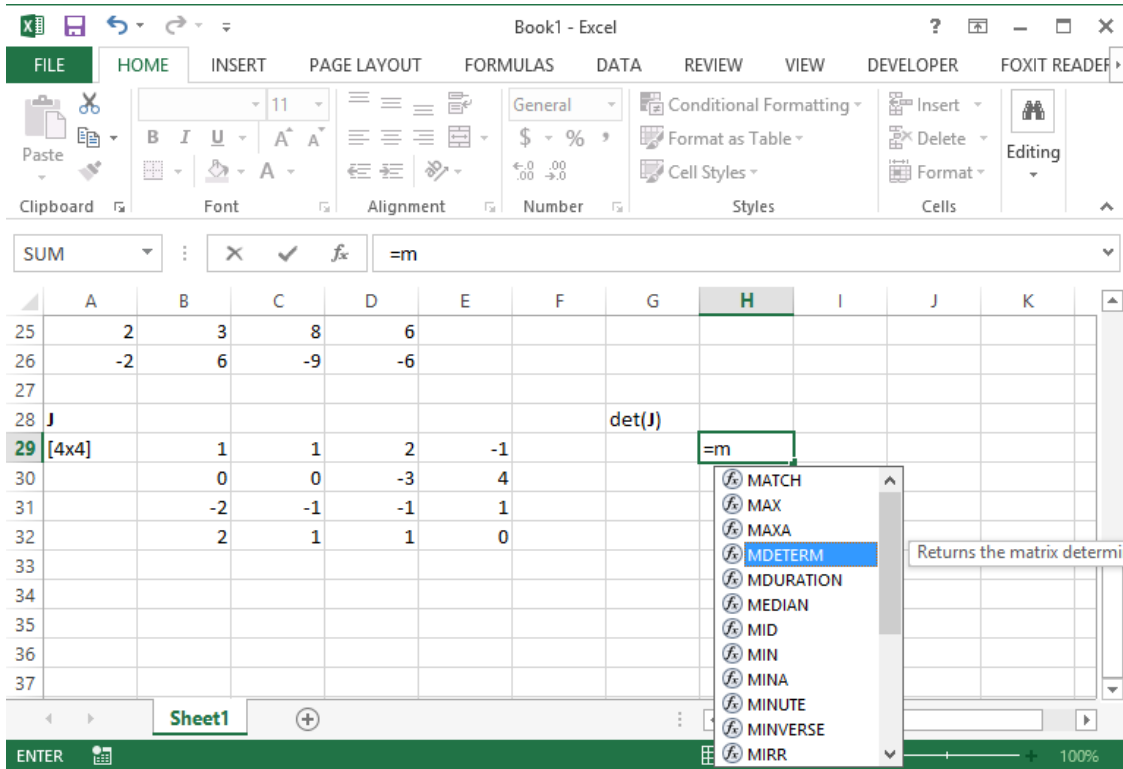
	A	B	C	D	E	F	G	H	I	J	K
25	2	3	8	6							
26	-2	6	-9	-6							
27											
28	J						det(J)				
29	[4x4]	1	1	2	-1						
30		0	0	-3	4						
31		-2	-1	-1	1						
32		2	1	1	0						
33											
34											
35											
36											
37											



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Select a cell that will hold the result.

Type “=MDETERM”



From the list of candidate functions that appears, double click on MDETERM.



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Select the elements of the matrix **J**.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K
25		2	3	8	6						
26		-2	6	-9	-6						
27											
28											
29	J	[4x4]	1	1	2	-1		det(J)			
30			0	0	-3	4					
31			-2	-1	-1	1					
32			2	1	1	0					
33											
34											
35											
36											
37											

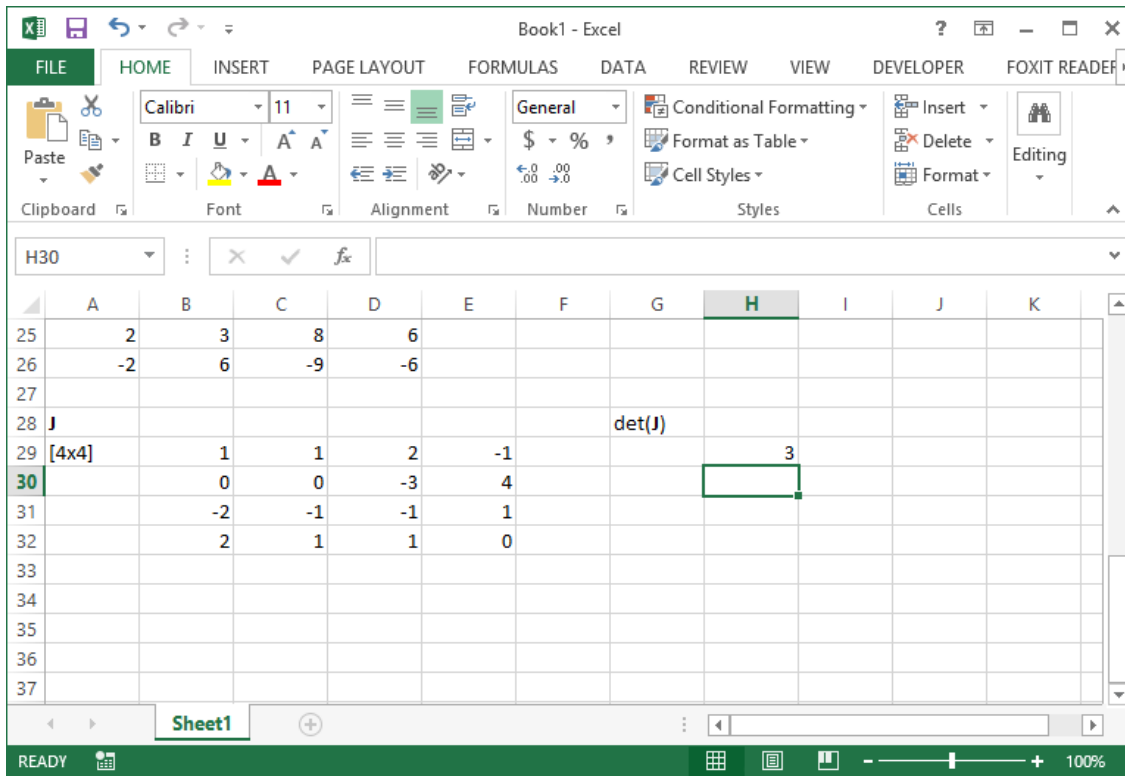
The formula bar shows the formula: `=MDETERM(B29:E32)`. A tooltip for the formula bar shows: `MDETERM(array)`.



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Type “)” to close parenthesis.
Hit **Enter**.

The determinant of the matrix is computed and displayed.





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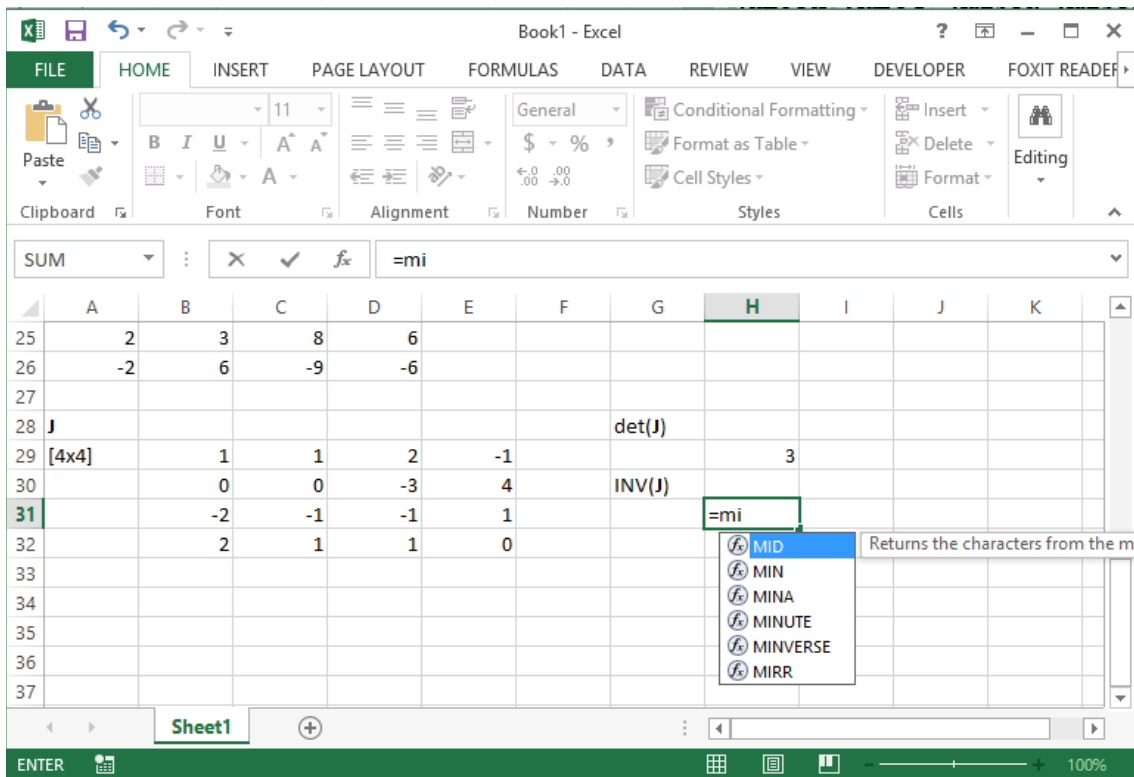
Inverse Matrix

Let us find the inverse of the matrix **J**.

Select the cell location of the element (1,1) of the inverse matrix.

Type “=”

Type the built-in function MINVERSE.





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From the list of candidate functions that appears as you start typing, double click on MINVERSE.

Select the elements of the matrix **J**.

The screenshot shows the Microsoft Excel interface with the following data and formulas:

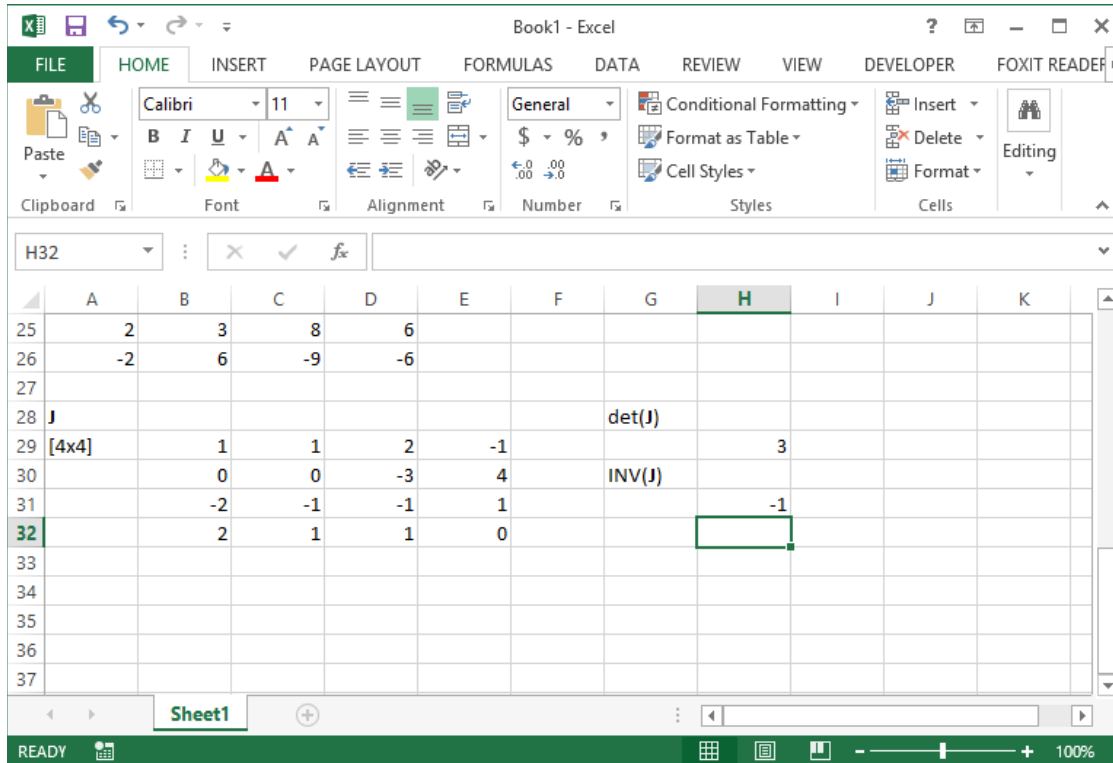
	A	B	C	D	E	F	G	H	I	J	K
25		2	3	8	6						
26		-2	6	-9	-6						
27											
28		J					det(J)				
29	[4x4]	1	1	2	-1			3			
30		0	0	-3	4		INV(J)				
31		-2	-1	-1	1			=MINVERSE(B29:E32)			
32		2	1	1	0			MINVERSE(array)			
33											
34											
35											
36											
37											

The formula bar shows: `=MINVERSE(B29:E32)`



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Type “)” to close parenthesis.
Hit **Enter**.



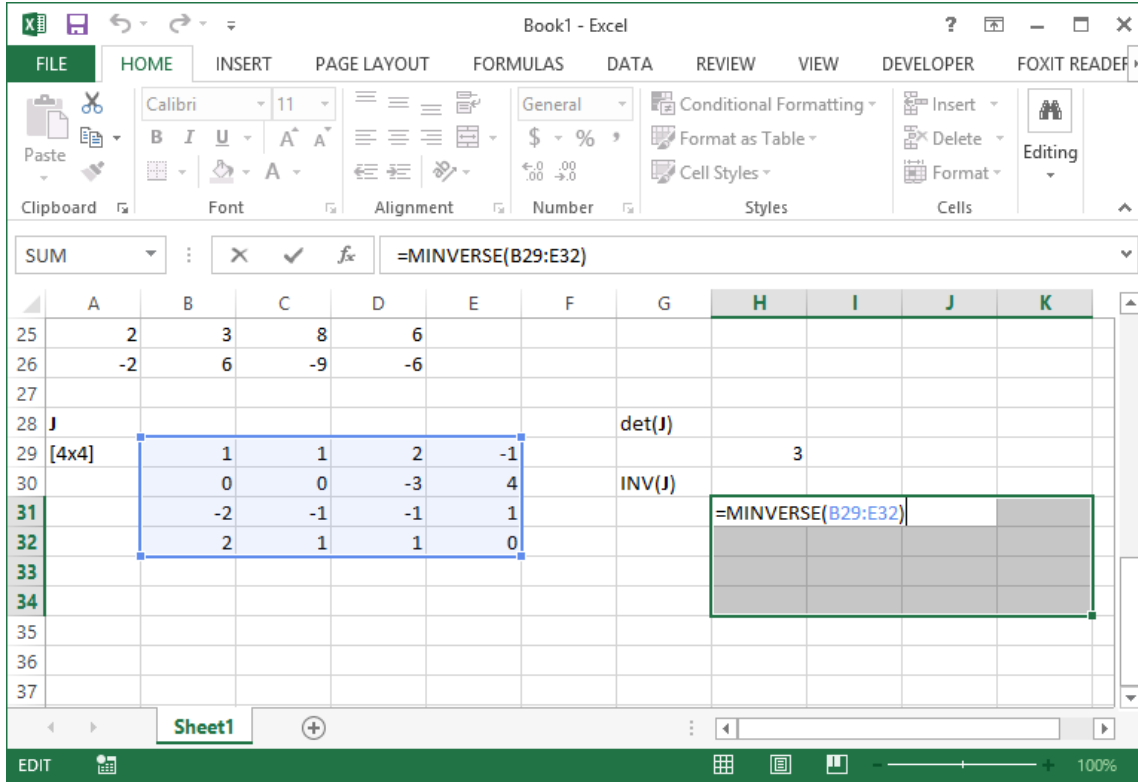
The element (1,1) of the inverse matrix is computed and displayed.

We know that the matrix **J** is a 4x4 square matrix, and that its inverse will also be a 4x4 square matrix.



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Starting at the element (1,1) of the inverse matrix, select four (4) rows and four (4) columns.
Hit **F2**.



Ctrl-Shift+Enter.



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The inverse matrix is displayed as follows

	A	B	C	D	E	F	G	H	I	J	K
25	2	3	8	6							
26	-2	6	-9	-6							
27											
28	J						det(J)				
29	[4x4]	1	1	2	-1		3				
30		0	0	-3	4		INV(J)				
31		-2	-1	-1	1		-1	-0.33333	0.333333	1.333333	1.333333
32		2	1	1	0		2	1	-2	-3	
33							0	-0.33333	1.333333	1.333333	
34							0	0	1	1	

Systems of Linear Equations

Consider the following problem.

A financial company is considering investing in three financial products. The financial institution has decided to invest \$500,000 in these three products, and wants a return on investment of \$38,000. The projected return on investment from Product 1, Product 2, and Product 3 are 6.5%, 7%, and 9% respectively. It has been decided that the Product 2 investment amount shall be twice the Product 3 investment amount.



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Let us do the following:

- Represent your investment plan as a system of linear equations.
- Represent this system of equations in matrix form.
- Solve the system of equations to determine how much the financial institution should invest in each in financial product.

Solution:

Let x_1 represent the amount invested in Product 1.

Let x_2 represent the amount invested in Product 2.

Let x_3 represent the amount invested in Product 3.

The system of linear equations is as follows:

$$x_1 + x_2 + x_3 = 500000 \quad \text{[total investment]}$$

$$0.065x_1 + 0.07x_2 + 0.09x_3 = 38000 \quad \text{[return on investment]}$$

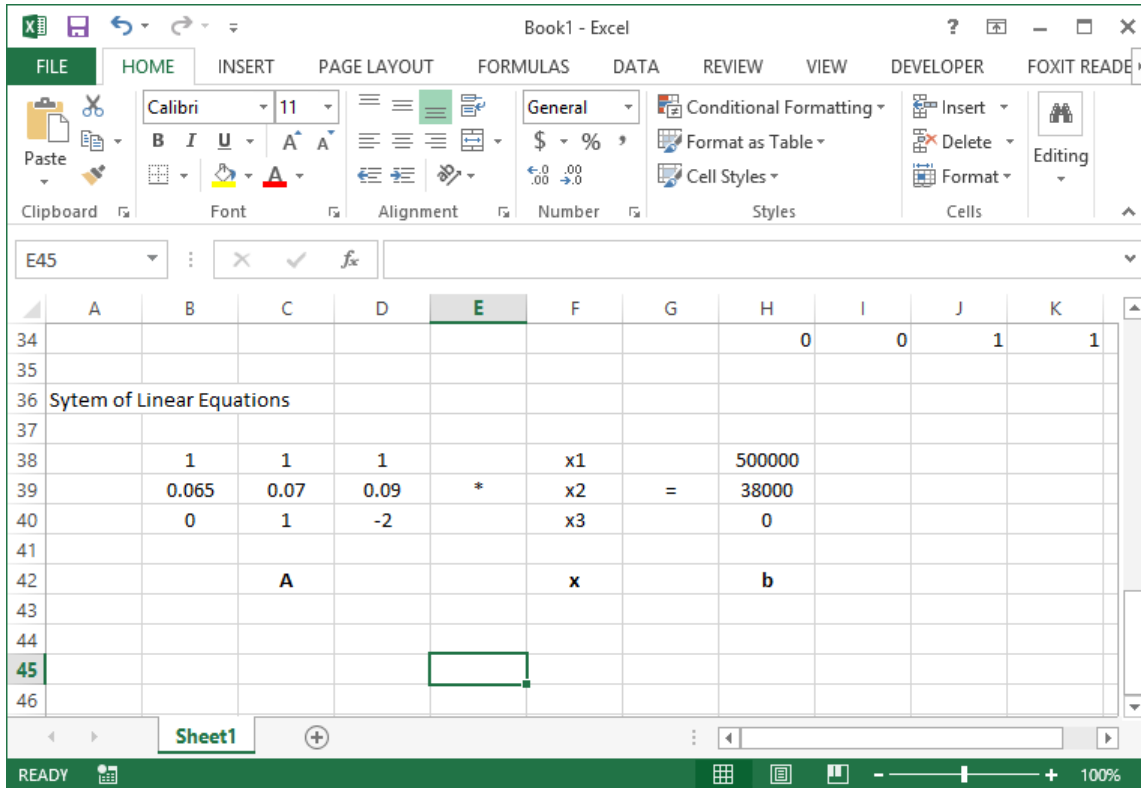
$$x_2 = 2x_3 \quad \text{or}$$

$$x_2 - 2x_3 = 0 \quad \text{[Product 2 versus Product 3]}$$



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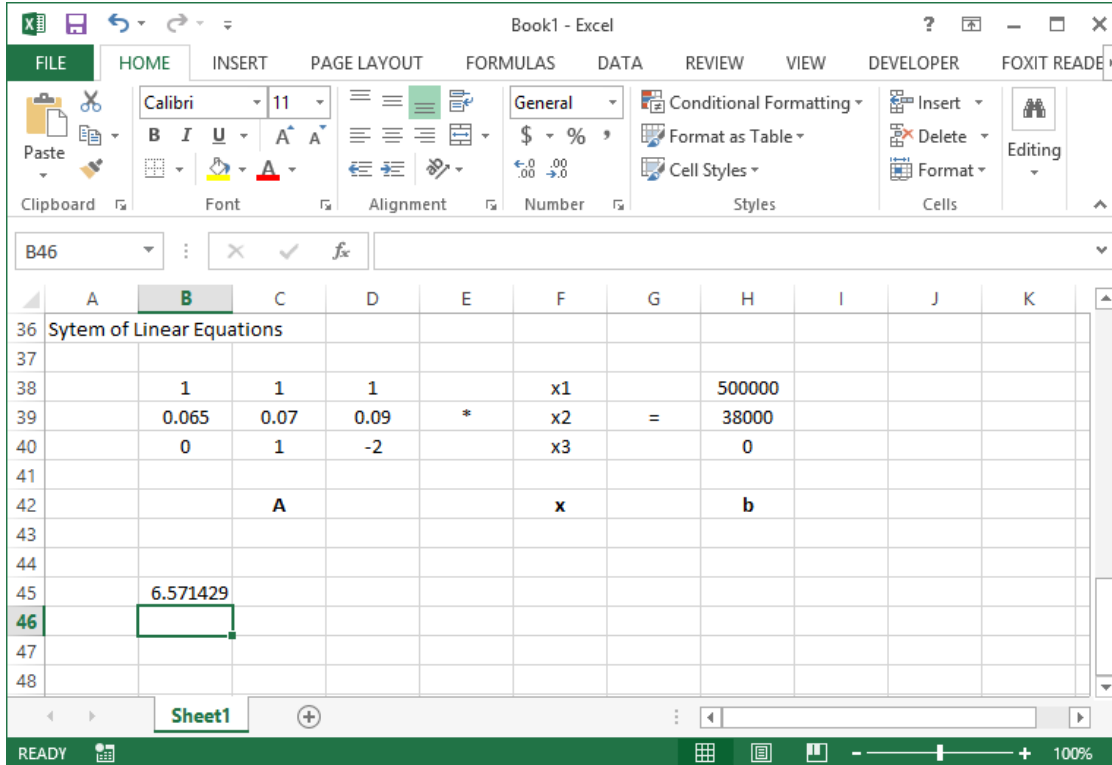
We may now go into *Excel* and create the matrix of coefficients, the vector of the unknown quantities, and the vector of right-hand-side constants, which will represent this system of equations.





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Type “)” to close parenthesis.
Hit **Enter**.



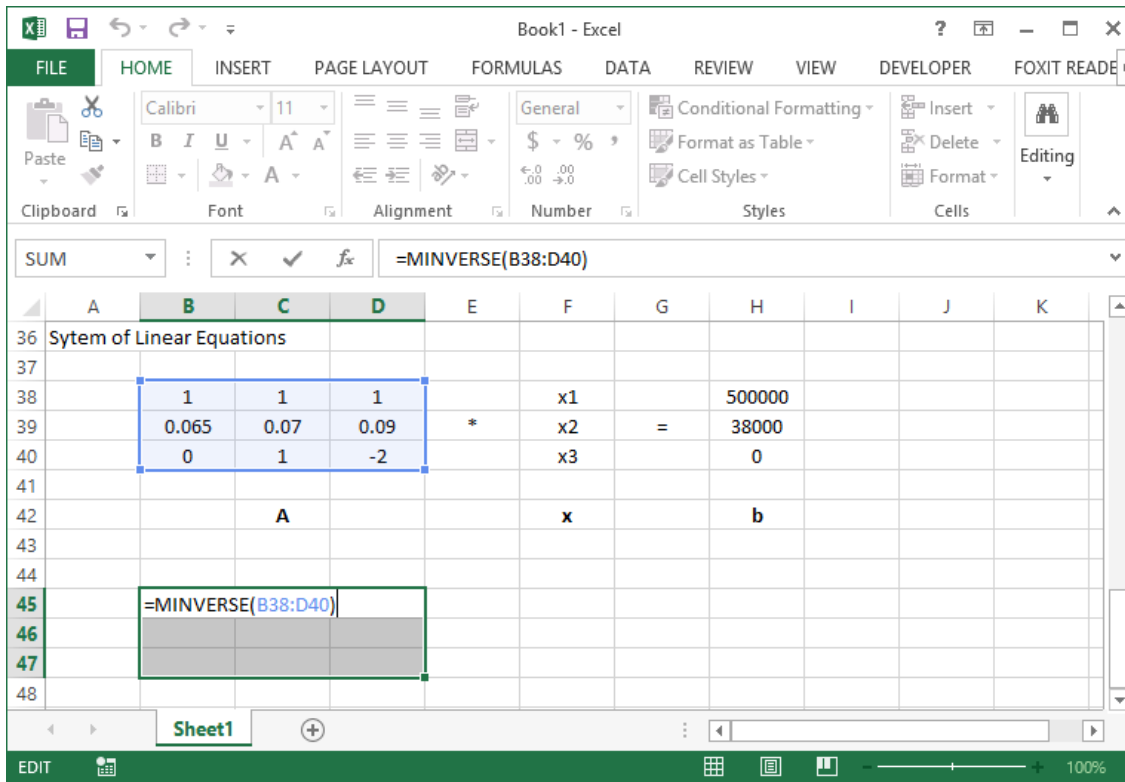
A is a 3x3 square matrix, and so its inverse will also be a 3x3 square matrix.



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Starting at the cell holding the element (1,1) of the inverse matrix, select three (3) rows and three (3) columns.

Hit **F2**.



Ctrl-Shift+Enter.



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The inverse matrix A^{-1} is computed and displayed as follows

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K
36	System of Linear Equations										
37											
38		1	1	1		x1	=	500000			
39		0.065	0.07	0.09	*	x2	=	38000			
40		0	1	-2		x3	=	0			
41											
42			A			x		b			
43											
44											
45		6.571429	-85.7143	-0.57143							
46		-3.71429	57.14286	0.714286							
47		-1.85714	28.57143	-0.14286							
48											

The formula bar shows the formula: `{=MINVERSE(B38:D40)}`



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We shall now multiply A^{-1} by the vector b to obtain the vector of unknown quantities that we are solving for.

The screenshot shows an Excel spreadsheet titled "Book1 - Excel" with the following data:

	A	B	C	D	E	F	G	H	I	J	K
36	System of Linear Equations										
37											
38		1	1	1		x1	=	500000			
39		0.065	0.07	0.09	*	x2	=	38000			
40		0	1	-2		x3	=	0			
41											
42			A			x		b			
43											
44											
45		6.571429	-85.7143	-0.57143							
46		-3.71429	57.14286	0.714286							
47		-1.85714	28.57143	-0.14286							
48											



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- Move the cursor to the location of the cell holding the element (1,1) of the output matrix **x**.
- Type “=MMULT”.
- Use point-and click to select the elements of A^{-1} .
- Type in a comma (“,”).
- Use point-and click to select the elements of **b**.

The screenshot shows an Excel spreadsheet with the following data:

	A	B	C	D	E	F	G	H	I	J	K
36	System of Linear Equations										
37											
38		1	1	1		x1		500000			
39		0.065	0.07	0.09	*	x2	=	38000			
40		0	1	-2		x3		0			
41											
42			A			x		b			
43											
44											
45		6.571429	-85.7143	-0.57143		=MMULT(B45:D47,H38:H40)					
46		-3.71429	57.14286	0.714286		MMULT(array1, array2)					
47		-1.85714	28.57143	-0.14286							
48											



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Type “)” to close the parenthesis.

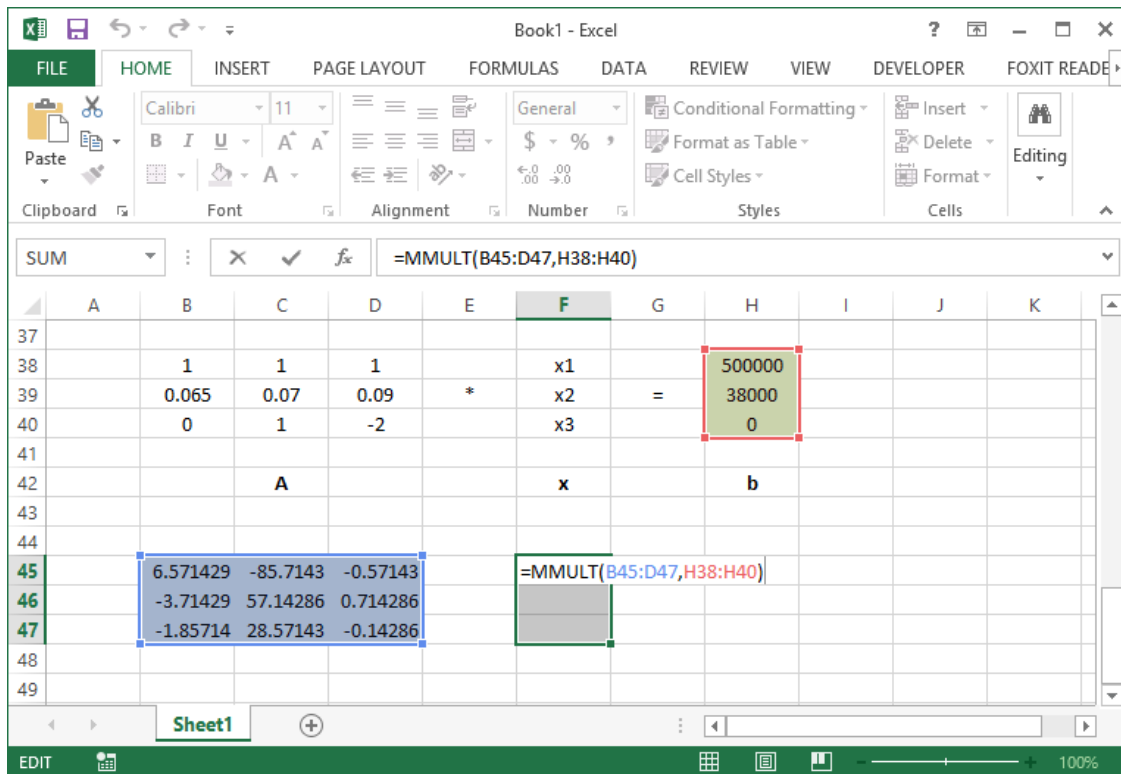
Hit **Enter**.

The element (1,1) is computed. We must now fill in the other elements.

We know the output of a 3x3 matrix multiplied by a 3x1 matrix will be a 3x1 matrix.

Starting at cell holding the element (1,1) of the output matrix select three (3) rows and one (1) column.

Hit **F2**.



Ctrl-Shift+Enter.



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The output matrix containing the values of the unknown quantities x_1 , x_2 , and x_3 is as follows,

	A	B	C	D	E	F	G	H	I	J	K
37											
38		1	1	1		x1		500000			
39		0.065	0.07	0.09	*	x2	=	38000			
40		0	1	-2		x3		0			
41											
42			A			x		b			
43											
44											
45		6.571429	-85.7143	-0.57143		28571.43					
46		-3.71429	57.14286	0.714286		314285.7					
47		-1.85714	28.57143	-0.14286		157142.9					
48											
49											

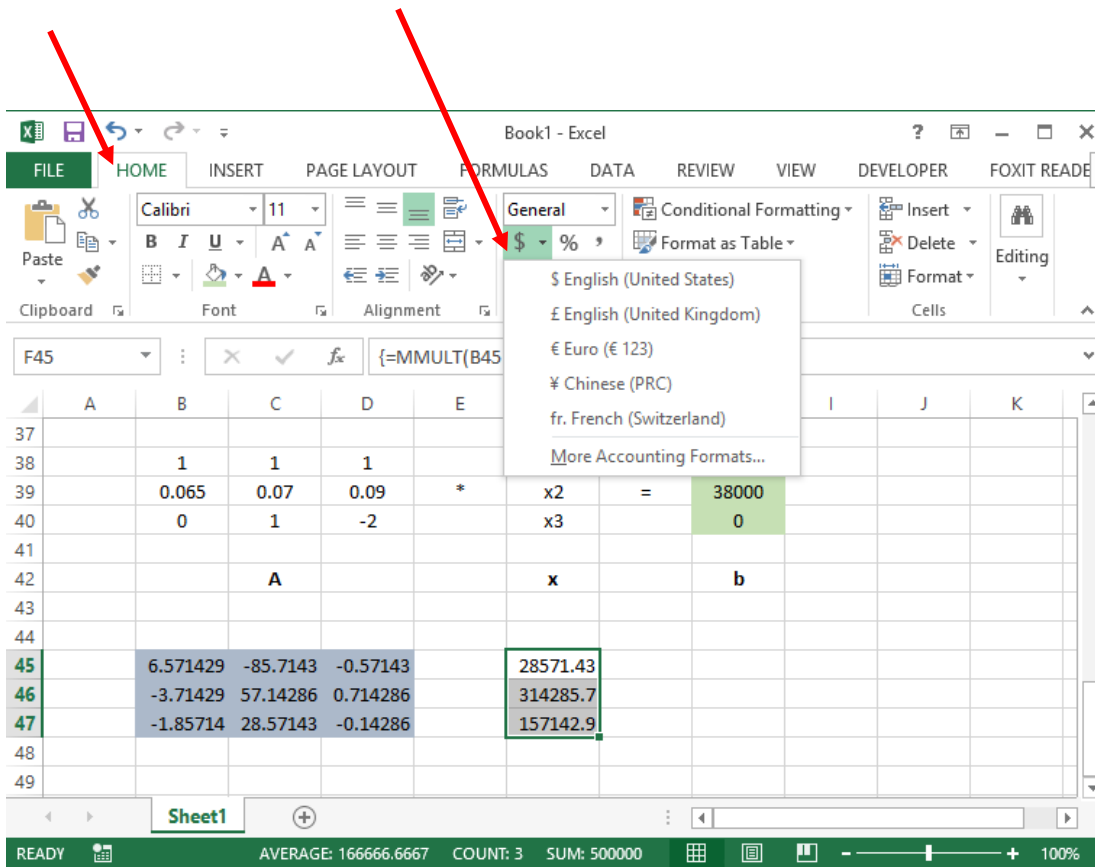


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Let us impart a number format that displays our results as monetary values.

Click on the HOME tab.

In the **Number** palette, click on the \$ drop down button



Select the **\$ English (United States)** option.



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We can now review the amounts of investment for each product that will enable the company to reach its goal while complying with the all the requirements of the investment plan.

The screenshot shows an Excel spreadsheet with the following data and formulas:

	A	B	C	D	E	F	G	H	I	J	K
37											
38		1	1	1		x1		500000			
39		0.065	0.07	0.09	*	x2	=	38000			
40		0	1	-2		x3		0			
41											
42			A			x		b			
43											
44											
45		6.571429	-85.7143	-0.57143		\$ 28,571.43					
46		-3.71429	57.14286	0.714286		\$314,285.71					
47		-1.85714	28.57143	-0.14286		\$157,142.86					
48											
49											

The formula bar shows: `{=MMULT(B45:D47,H38:H40)}`

The status bar at the bottom shows: AVERAGE: \$166,666.67 COUNT: 3 SUM: \$500,000.00

A red arrow points to the result in cell F47.



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3. CONCLUSION

This course is the first of a series on engineering methods in *Excel*. This course presented a broad overview of fundamental principles and engineering applications of linear algebra, and how to use *Excel* tools to analyze and solve these types of problems.

This course the fundamentals of arrays, and then we looked at how arrays can be organized and manipulated on an *Excel* spreadsheet. We looked at how to formulate and solve systems of linear equations using arrays and how these systems can be implemented and solved on a spreadsheet.

This course has enabled participants to identify professional situations where the innovative application of techniques learned in this course are relevant and will be of benefit to the productivity, efficiency, and quality of the work output of a practitioner.

Practitioners are strongly encouraged to look out for situations in their domains of expertise where engineering methods in *Excel* are applicable and will be of benefit to their work and their organization.

A successful application of engineering methods in *Excel* requires a careful and meticulous approach, and can only be mastered and retained by practice and repetition.

It has been my utmost pleasure presenting this topic to you. Thank you.



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Microsoft. (2017). *Microsoft Office help and training - Office Support*. Retrieved March 13, 2017, from Microsoft Office Home: <https://support.office.com/en-us/article/Define-and-solve-a-problem-by-using-Solver-9ed03c9f-7caf-4d99-bb6d-078f96d1652c>

Images were all drawn/ prepared by Kwabena. Ofosu

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