A SunCam online continuing education course

Orifice and Venturi Pipe Flow Meters

For Liquid and Gas Flow

by

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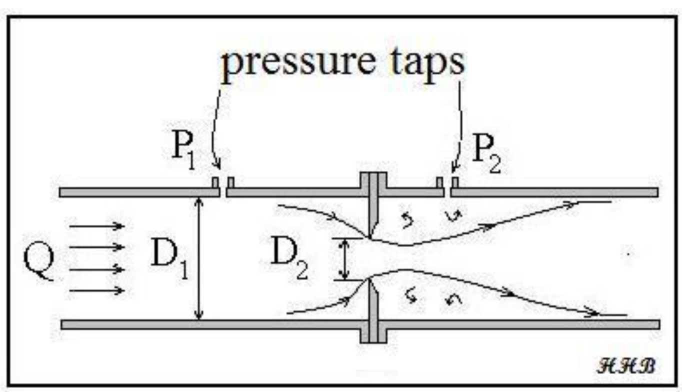
1. Introduction

The flow rate of a fluid flowing in a pipe under pressure is measured for a variety of applications, such as monitoring of pipe flow rate and control of industrial processes. Differential pressure flow meters, consisting of orifice, flow nozzle, and venturi meters, are widely used for pipe flow measurement and are the topic of this course. All three of these meters use a constriction in the path of the pipe flow and measure the difference in pressure between the undisturbed flow and the flow through the constriction. That pressure difference can then be used to calculate the flow rate. This course will provide general background information about differential pressure flow meters and the format of the equation used for calculating liquid flow rate through any of them. There will also be presentation and discussion of equations used for calculation of gas flow through a differential pressure meter and the parameters in those equations. There will be descriptions of each of these meters and their particular equations, along with example calculations. Use of the ideal gas law to calculate the density of a gas at known temperature and pressure and use of an ISO 5167 equation to calculate the value of an orifice coefficient are additional topics related to orifice and venturi meter calculations that are included in this course. A spreadsheet to assist with orifice/venturi/flow nozzle meter calculations and ISO 5167 calculation of an orifice coefficient is also provided. Note that there is provision for user input to select either U.S. units of S.I. units in each of the spreadsheet worksheets.



Orifice Meter Parameters

2. Learning Objectives

At the conclusion of this course, the student will

- Be able to calculate the liquid flow rate from measured pressure difference, fluid properties, and meter parameters, using the provided equations for venturi, orifice, and flow nozzle meters.

- Be able to calculate the gas flow rate from measured pressure difference, fluid properties, and meter parameters, using the provided equations for venturi, orifice, and flow nozzle meters.

- Be able to estimate the density of a specified gas at specified temperature and pressure using the Ideal Gas Equation.

- Be able to determine which type of ISO standard pressure tap locations are being used for a given orifice meter.

- Be able to calculate the orifice coefficient, C_o, for specified orifice and pipe diameters, pressure tap locations and fluid properties using ISO 5167 equations.

- Be able to calculate the Reynolds number for specified pipe flow conditions.

3. Topics Covered in this Course

I. Differential Pressure Flow Meter Background

II. Gas Flow Calculations for Differential Pressure Flow Meters

III. The Ideal Gas Law for Calculating the Density of a Gas

IV. The Venturi Meter

V. The Orifice Meter

VI. ISO 5167 for Determination of an Orifice Coefficient

VII. The Flow Nozzle Meter

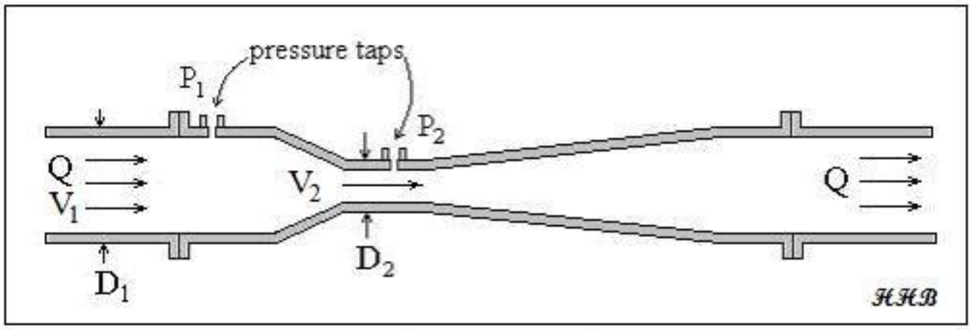
VIII. Summary

IX. References

4. Differential Pressure Flow Meter Background

Orifice meters, venturi meters, and flow nozzle meters are three commonly used examples of differential pressure flow meters. These three meters function by placing a constricted area in the flow path of the fluid flowing through the pipe, thus causing an increase in the fluid velocity as it goes through the constricted area. As indicated by the Bernoulli Equation ($\mathbf{p + 1/2\rho V^2 + \rho gh = constant}$), if the velocity (V) increases, with the density (ρ) remaining constant, then either pressure (p) or elevation, (h) must decrease. For a flow meter in a horizontal pipe, the elevation will not change, so the increase in velocity must be accompanied by a decrease in pressure. This is the principle used in the differential pressure flow meter.

A general equation will now be derived for calculating the flow rate from the measured difference between the pressure of the approach fluid and the pressure of the fluid in the constricted area of flow. The parameters shown in the venturi diagram below will be used to represent those parameters in general for a differential pressure flow meter.



Venturi Meter Parameters

The Bernoulli equation written between cross-section 1 in the approach fluid flow and cross-section 2 in the constricted area of flow is shown below:

$$P_1 + \frac{\rho V_1^2}{2} + \rho g h_1 = P_2 + \frac{\rho V_2^2}{2} + \rho g h_2 \qquad \text{Eqn (1)}$$

Note that the Bernoulli equation is for “ideal flow”, neglecting frictional effects and non-ideal flow factors. Also note that the density, ρ, has been assumed to remain the same for the approach fluid and the fluid flowing through the constricted area in Equation (1). If the pipe and meter are horizontal, then $h_1 = h_2$ and the ρgh terms “drop out” of the equation, giving:

$$P_1 + \frac{\rho V_1^2}{2} = P_2 + \frac{\rho V_2^2}{2} \qquad \text{Eqn (2)}$$

The volumetric flow rate through the pipe (and meter) can be introduced by substituting the expressions: $V_1 = Q/A_1$ and $V_2 = Q/A_2$ into the equation. Then solving for Q gives:

$$Q_{ideal} = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \qquad \text{Eqn (3)}$$

Note that Equation (3) remains exactly the same for either U.S. units or S.I. units. A consistent set of each of those units is shown in the parameter list below:

- **Q_{ideal}** is the ideal flow rate through the meter (neglecting viscosity and other friction effects), in cfs for U.S. units (or m^3/s for S.I. units)

- **A_2** is the constricted cross-sectional area perpendicular to flow, in ft^2 for U.S. units (or m^2 for S.I. units)

- **P_1** is the approach pressure in the pipe, in lb/ft^2 for U.S. units (or N/m^2 for S.I. units)

- **P_2** is the pressure in the meter, in the constricted area, in lb/ft^2 for U.S. units (or N/m^2 for S.I. units)

- β is the diameter ratio = D_2/D_1 = (diam. at A_2)/(pipe diam.), dimensionless

- ρ is the fluid density in $slugs/ft^3$ for U.S. units (or kg/m^3 for S.I. units)

The volumetric flow rate calculated from this equation is called Q_{ideal}, because the equation was derived from the Bernoulli equation, which is for ideal flow and doesn't include the effects of frictional losses. The method of taking into account friction losses and other non-ideal factors for differential pressure flow meters is to put a discharge coefficient, C, into the equation for Q, giving:

$$Q = C A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \qquad \text{Eqn (4)}$$

Where:

- **Q** is the flow rate through the meter (and through the pipe), in cfs for U.S. units (or m^3/s for S.I. units)

- **C** is the discharge coefficient, which is dimensionless

All of the other parameters are the same as defined above. The discharge coefficient, C, will be less than one, because the actual pipe/meter flow rate will be less than the ideal flow rate due to fluid friction losses.

Note that Equation (4) works well for flow of liquids through differential flow meters, because the difference in pressure between the upstream pipe and the constricted region typically has negligible effect on the density. Thus the density can be taken as constant. For gas flow through a differential flow meter, however, the difference in the upstream pressure, P_1, and the constricted pressure, P_2, typically has a significant effect on the density. Thus a variation of Equation (4) is typically used for gas flow calculations as discussed in the next section.

5. Gas Flow Calculations for Differential Pressure Meters

In order to account for the effect of changing pressure on the density of a gas as it flows through a differential pressure flow meter, equation (5), shown below, is typically used for gas flow calculations. This equation is equation (4) with the compressibility factor form of the ideal gas law (discussed further in the next section of the course) used as an expression for the gas density and incorporation of the compressibility factor, Y, for the gas as discussed below.

$$Q = C\,A_2\,Y\sqrt{\frac{2ZRT_1(P_1 - P_2)}{(MW)P_1(1 - \beta^4)}} \qquad \text{Eqn (5)}$$

In this equation Q, C, A_2, P_1, P_2, and β are as defined above for Equations (3) and (4). Note, however, that P_1 in the denominator must be the absolute pressure in psia (or in Pa for S.I. units).

- Z is the compressibility factor of the gas at P1 and T1. (Z is discussed further in the next section of the course.)
- R is the Ideal Gas Law Constant (345.23 psia-ft^3/slugmole-^{o}R for the U.S. units used above or 8.3145 kN-m/kgmole-K for S.I. units).
- MW is the molecular weight of the gas.
- T_1 is the upstream absolute temperature of the gas in the pipe in ^{o}R for U.S. units (or K for S.I. units).
- Y is the Expansion Factor of the gas, which is dimensionless. (Equations for Y are given below.)

The expansion factor, Y, is needed for gas flow through a differential pressure flow meter in order to account for the decrease in gas density due to the decreased pressure in the constricted portion of the flow meter. ISO 5167 – 2:2003 (reference #3 at the end of the course) gives the following equation for the expansion factor, Y, for flow through orifice meters meeting ISO 5167 – 2:2003 requirements.

$$\mathbf{Y = 1 - (0.351 + 0.265\beta^4 + 0.93\beta^8)[1 - (P_2/P_1)^{1/k}]} \qquad \text{Eqn (6)}$$

This equation is for $\mathbf{P_2/P_1 \geq 0.75}$. The parameters, $\boldsymbol{\beta}$, $\mathbf{P_1}$, and $\mathbf{P_2}$ are the diameter ratio, inlet pressure and pressure at the constriction, as defined above, and **k** is the specific heat ratio ($\mathbf{C_p/C_v}$) for the gas flowing through the meter.

ISO 5167 – 4:2003 (reference #4 at the end of the course) gives the following equation for the expansion factor, Y, for flow through venturi meters meeting ISO 5167 – 4:2003 requirements. This equation for Y is also typically used for flow nozzle meter calculations with gas flows.

$$Y = \sqrt{\left(\frac{k\,\tau^{2/k}}{k-1}\right)\left(\frac{1-\beta^4}{1-\beta^4\,\tau^{2/k}}\right)\left(\frac{1-\tau^{(k-1)/k}}{1-\tau}\right)} \qquad \text{Eqn (7)}$$

In Equation (7), **k** and **β** are the specific heat ratio and diameter ratio respectively, as defined above, and **τ** is the pressure ratio, $\mathbf{P_2/P_1}$.

The next section provides discussion of the ideal gas law for calculating the density of a gas and use of the compressibility factor, Z, to extend the ideal gas law to calculations for nonideal gases.

6. The Ideal Gas Law for Calculating the Density of a Gas

The density of the flowing fluid, ρ, is needed for any of the differential pressure meter calculations. If the fluid is a liquid, then the density depends primarily on temperature. A suitable value for the density of a common liquid at the operating temperature in the pipe can usually be obtained from a textbook, a handbook, or an internet source. For a gas, however, the density depends upon both temperature and pressure. A convenient way to get the density of a gas at a specified temperature and pressure is through the use of the ideal gas law.

A commonly used form of the ideal gas law is: **PV = n RT**, an equation giving the relationship among the temperature, **T**, pressure, **P**, and volume, **V**, of **n** moles of a gas that can be treated as an ideal gas. The ideal gas law constant, **R**, also appears in this equation. Note that in this equation the quantity of gas is expressed in terms of the number of moles, rather than as the mass of the gas. Fortunately, this can be taken care of by using the definition of molecular weight as the mass of any compound in one mole of that compound. In other words: **MW = m/n or n = m/MW**. That is the number of moles in a mass **m** of a gas is equal to the mass divided by the molecular weight. Substituting **n = m/MW** into the Ideal Gas Law equation gives: **PV = (m/MW)RT**. Solving for **m/V** (which is equal to the density) gives:

$$m/V = \rho = (MW)P/RT \qquad \text{Eqn (8)}$$

Where:

- ρ is the density of the gas in slugs/ft3 for U.S. units (or in kg/m^3 for S.I. units)
- **P** is the absolute pressure of the gas in psia for U.S. units (or in N/m^2 absolute for S.I. units)
- **T** is the absolute temperature of the gas in ^oR for U.S. units (or K for S.I. units)
- **MW** is the molecular weight of the gas in slugs/slugmole (or kg/kgmole for the S.I. units used here)
- **R** is the ideal gas law constant for the combination of units used for the other parameters. In this case, R = **345.23 psia-ft^3/slugmole-^{o}R** for the U.S. units shown above (or **8.3145 kN-m/kgmole-K** for the S.I. units given above)

Example #1: Use the ideal gas law to calculate the density of air at 65^{o}F and 25 psig. Assume that local atmospheric pressure is 14.7 psi. Also, note that the molecular weight of air is 29.

Solution: The temperature and pressure both need to be converted to absolute units. For the temperature, the conversion is ^{o}R = ^{o}F + 459.67. Thus 65^{o}F = 65 + 459.67 ^{o}R = 525^{o}R. For pressure, recall that a gauge measures the difference between absolute pressure and ambient atmospheric pressure, so absolute pressure = gauge pressure + atmospheric pressure. For this example, P = 25 + 14.7 psia = 39.7 psia.

Substitution into Equation (8) above for gas density gives:

$$\rho = \mathbf{MW\ P/RT} = (29)(39.7)/(345.23)(525) = 0.00636$$

with units of: (slugs/slugmole)(psia)/[(psia-ft^3/slugmole-^{o}R)(^{o}R) = slugs/ft^3

Thus the answer is: **ρ = 0.00636 slugs/ft^3**

Example #2: For a calculation with S.I. units, find the density of methane at 35^{o}C and 300 kPa gauge pressure. Assume that atmospheric pressure is 1 atm (= 101.325 kPa). Note that the molecular weight of methane is 16.0.

Solution: The conversion to absolute temperature is K = ^{o}C + 273.15, so the absolute temperature is 35 + 273.15 K = 308 K. Adding the given gauge pressure and atmospheric pressure gives: P = 300 + 101.325 kPa absolute = 401.325 kPa abs (kPa = kN/m^2).

Substitution into the equation (5) for gas density gives:

$$\rho = \mathbf{MW\ P/RT} = (16)(401.325)/(8.3145)(308) = 2.51$$

with units of: (kg/kgmole)(kPa abs)/[(kN-m/kgmole-K)(K) = kg/m^3

Thus the answer is: **ρ = 2.51 kg/m^3**

What conditions are needed to use the Ideal Gas Law? This is an important question, because any gas flowing through an orifice, venturi, or flow nozzle meter, is actually a real gas, not an ideal gas. Fortunately, many real gases follow ideal gas behavior almost exactly over a wide range of practical temperatures and pressures. The ideal gas law works best for gases with relatively simple molecules, at **temperatures that are well above the critical temperature** of the gas and at **pressures that are well below the critical pressure** of the gas.

As an aid for gas density calculations, the table below gives molecular weight, critical temperature and critical pressure for several common gases. The critical temperature and critical pressure aren't used in the calculation. They are just provided as a reference, to check that a given set of gas conditions are indeed at a temperature well above the critical temperature and pressure well below the critical pressure of that gas. Similar information can be found for most other gases of interest through an internet search.

Gas	Mol. Wt.	Crit. Temp, ^oC	Crit. Press, atm
Air	29.0	-140.5	37.25
Carbon Dioxide	44.0	31	72.9
Carbon Monoxide	28.0	-140.3	34.53
Nitrogen	28.0	-147	33.54
Oxygen	32.0	-118.6	50.14
Methane	16.0	-82.4	45.8
Propane	44.1	96.9	42.1

For the gases shown in the table, it can be seen that many practical sets of operating conditions will have temperature well above the critical temperature and pressure well below the critical pressure, so the ideal gas law will be suitable for calculating gas density.

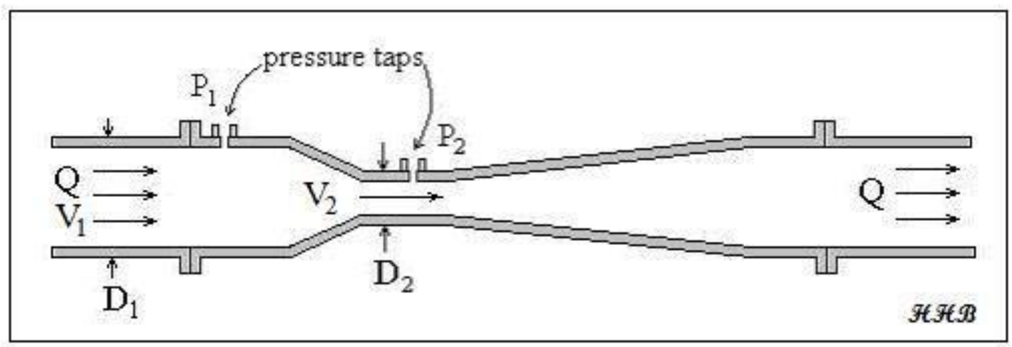
For gas flow in which the temperature is too low and/or the pressure is too high to assume ideal gas behavior, the compressibility factor, **Z**, can be used for the calculations in the compressibility factor form of the ideal gas law [$\mathbf{pV = nZRT}$ or $\boldsymbol{\rho} = \mathbf{(MW)P/ZRT}$].

The compressibility factor for a gas is a function of its reduced temperature ($\mathbf{T_R = T/T_c}$) and its reduced pressure ($\mathbf{P_R = P/P_c}$). Graphs, equations and other correlations are available for compressibility factor, **Z**, as a function of $\mathbf{T_R}$ and $\mathbf{P_R}$, for gases in general and for some particular gases.

The next several sections will provide descriptions and discussion of venturi meters, orifice meters, and flow nozzle meters.

7. The Venturi Meter

The diagram of a venturi meter below shows the general shape and flow pattern for this type of differential pressure flow meter.



Venturi Flow Meter

The converging cone, through which fluid enters a venturi meter, typically has a cone angle of 15^o to 20^o. This cone on the inlet side of the meter converges to the throat diameter, which is where the area of flow is at its minimum, the velocity is at its maximum, and the pressure is at its minimum. The diverging exit section of the venturi meter uses a cone angle of 5^o to 7^o, to bring the meter diameter back to the full pipe diameter. As shown in the diagram, D_2 is the diameter of the venturi throat and P_2 is the pressure in the throat. D_1 and P_1 are the diameter and the pressure for the pipe before the flow enters the converging section of the meter.

The design of a venturi meter, with its smooth contraction on the inlet side and gradual expansion back to the pipe diameter, leads to very little frictional loss through the meter. The discharge coefficient for a venturi meter is often called the venturi coefficient, C_v. the equation for flow rate through a venturi meter, thus becomes:

$$Q = C_v A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$ Eqn (9)

Due to the small frictional loss in a venturi meter, the venturi coefficient is fairly close to one, typically in the range from 0.95 to nearly one. From reference #3 at the end of this course (ISO 5167-1:2003) the venturi coefficient for cast iron or machined venturi meters is given as 0.995. C_v is given as 0.985 for welded sheet metal meters that meet ISO specifications. All of these C_v values are for Reynolds number between 2 x 10^5 and 10^6. Venturi meter manufacturers will often provide information about the venturi coefficient for their meters.

Example #3: Calculate the flow rate of water at 45°F, if the pressure difference, $P_1 - P_2$, is measured as 10 inches of Hg for a venturi meter with a 1 inch diameter throat in a 3 inch diameter pipe. The manufacturer has specified C_v = 0.98 for this meter under these flow conditions.

Solution: The density of water at 45°F is 1.94 slugs/ft^3. The other parameters needed (together with the given value for C_v) are β, A_2, and $P_1 - P_2$.

$\beta = D_2/D_1 = 1 \text{ in}/3 \text{ in} = 0.3333$

$A_2 = \pi D_2^2/4 = \pi(1/12)^2/4 \text{ ft}^2 = 0.005454 \text{ ft}^2$

$P_1 - P_2 = (10 \text{ in Hg})(70.73 \text{ lb/ft}^2/\text{ in Hg}) = 707.3 \text{ lb/ft}^2$

Substituting all values into Eqn (9) gives:

$$Q = (0.98)(0.005454)\sqrt{\frac{2(707.3)}{(1.94)(1 - 0.3333^4)}} = \underline{\mathbf{0.145\ cfs}}$$

This type of calculation can be conveniently done with the spreadsheet file that was provided with this course. The screenshot on the next page shows the solution to this example with the first worksheet of that spreadsheet workbook. The given input values for D_1, D_2, $P_1 - P_2$, ρ, and C are entered in the blue boxes in the left column in the spreadsheet. Then the formulas in the yellow boxes in the right column calculate the indicated parameters, finishing with the flow rate through the meter. Note that this spreadsheet is set up to enter the measured pressure difference in psi, so I converted the given pressure difference in inches of mercury to 4.91 psi.

Example #4: Calculate the flow rate of water at 15^{o}C, if the pressure difference, $P_1 - P_2$, is measured as 8.5 kPa for a venturi meter with a 25 mm diameter throat in a 75 mm diameter pipe. The manufacturer has specified $C_v = 0.98$ for this meter under these flow conditions.

Solution: The density of water at 15^{o}C is 1000 kgs/m^3. The other parameters needed (together with the given value for C_v) are β, A_2, and $P_1 - P_2$.

$$\beta = D_2/D_1 = 25\text{ mm}/75\text{ mm} = 0.3333$$

$$A_2 = \pi D_2^2/4 = \pi(0.025)^2/4\ \text{m}^2 = 0.000491\ \text{m}^2$$

$$P_1 - P_2 = (8.5\text{ kPa})(1000\text{ Pa/kPa}) = 8500\text{ Pa}$$

Substituting all values into Eqn (9) gives a calculated flow rate of: **Q = 0.00200 m^3/s**

This calculation can also be conveniently done with the course spreadsheet. The second screenshot below shows the solution to Example #4 using the first worksheet of the course spreadsheet, with “S.I. Units” selected in the dropdown list near the top of the worksheet.

Calculation of Flow Rate for Orifice, Venturi or Flow Meters (known value for C - U.S. or S.I. units)

Instructions: Enter input data for your calculation into the blue boxes. The spreadsheet will carry out the calculations in the yellow boxes.

Click on the blue cell below and on the arrow to the right of it. Then use the drop down list to select the units you want to use:

U.S. Units

Inputs			**Calculations**		
Pipe diam, D_1 =	3	in	Pipe diam, D_1 =	0.25	ft
Constricted diam, D_2 =	1	in	Constricted diam, D_2 =	0.083333	ft
			Constricted area, A_2 =	0.005454	sq ft
Measured pressure diff., $P_1 - P_2$ (psi) =	4.91	psi			
			Diam. Ratio, β (D_2/D_1) =	0.333	
Fluid density, ρ =	1.94	slugs/cu ft			
Meter Coefficient, C =	0.98		Measured pressure diff., $P_1 - P_2$ (psf) =	707	lb/sq ft
			Pipe Flow Rate, Q =	**0.145**	**cfs**

Spreadsheet Solution to Example #3

Calculation of Flow Rate for Orifice, Venturi or Flow Meters (known value for C - U.S. or S.I. units)

Instructions: Enter input data for your calculation into the blue boxes. The spreadsheet will carry out the calculations in the yellow boxes.

Click on the blue cell below and on the arrow to the right of it. Then use the drop down list to select the units you want to use:

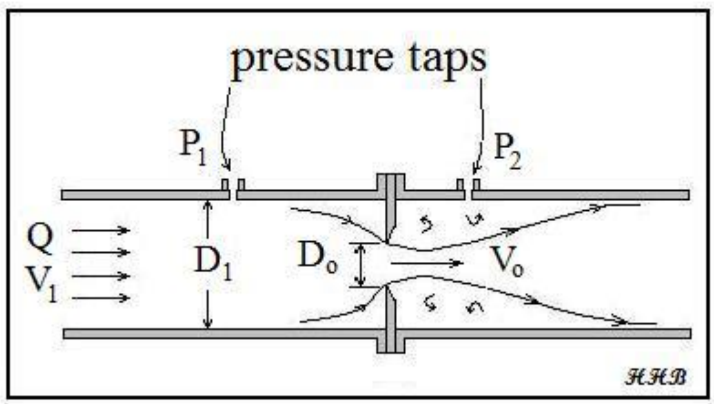
S.I. Units

Inputs			**Calculations**		
Pipe diam, D_1 =	75	mm	Pipe diam, D_1 =	0.075	m
Constricted diam, D_2 =	25	mm	Constricted diam, D_2 =	0.025	m
			Constricted area, A_2 =	0.000491	sq m
Measured pressure diff., $P_1 - P_2$ (psi) =	8.5	kPa			
			Diam. Ratio, β (D_2/D_1) =	0.333	
Fluid density, ρ =	1000	kg/cu m			
Meter Coefficient, C =	0.98		Measured pressure diff., $P_1 - P_2$ (psf) =	8500	Pa
			Pipe Flow Rate, Q =	**0.00200**	**cu m/s**

Spreadsheet Solution to Example #4

8. The Orifice Meter

The diagram of an orifice meter below shows the general shape and flow pattern for this type of differential pressure flow meter. As the diagram shows, this is quite a simple device, consisting of a circular plate with a hole in the middle, usually held in place between pipe flanges. The orifice meter is the simplest of the three differential pressure meters, but due to its abrupt decrease in flow area and abrupt transition back to full pipe diameter, it has the greatest frictional pressure loss of the three for a given fluid, flow rate, pipe diameter, and constricted diameter.



Orifice Flow Meter

Also, as shown in the diagram, the constricted diameter, D_2, is the diameter of the orifice, often represented by D_o, making $A_2 = A_o$. The discharge coefficient for an orifice meter is often called an orifice coefficient. All of this results in the following as the equation for the flow rate through an orifice meter.

$$Q = C_o A_o \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$

Eqn (10)

The next section will cover determination of orifice coefficients using the ISO 5167 equation. But first an example calculation with specified orifice coefficient:

Example #5: Calculate the flow rate of water at 45ºF, if the pressure difference, $P_1 - P_2$, is measured as 10 inches of Hg for an orifice meter with a 1 inch orifice diameter in a 3 inch diameter pipe. The orifice coefficient for this meter has been determined to be $C_o = 0.62$ under these flow conditions.

Solution: As calculated in Example #3, $A_o = 0.005454$ ft^2, $\beta = 0.3333$, and $\rho = 1.94$ slugs/ft^3, and $P_1 - P_2 = 707.3$ lb/ft^2. Substituting these values along with $C_o = 0.62$ into Eqn (10) gives:

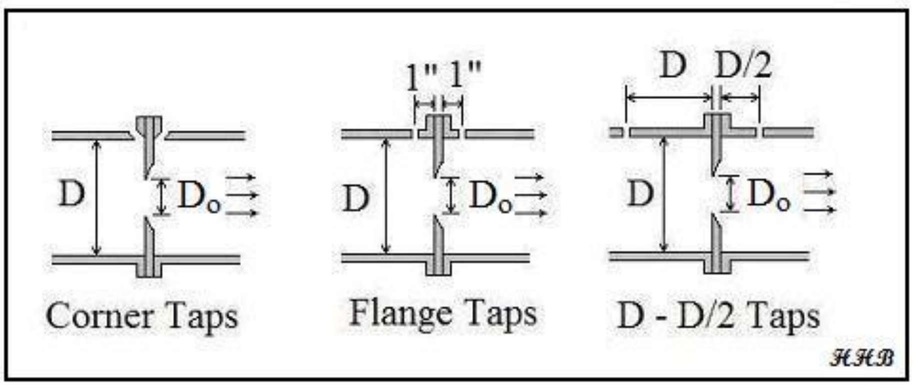
$$Q = (0.62)(0.005454)\sqrt{\frac{2\,(707.3)}{(1.94)(1 - 0.3333^4)}} = \underline{\mathbf{0.09188\ cfs}}$$

Note that for water flow through a venturi meter with the same pipe diameter, constricted diameter and pressure drop, as calculated in Example #3, the flow rate was **0.145 cfs**. The greater frictional loss through an orifice meter in comparison with a venturi meter, as evidenced by the lower value for the discharge coefficient, resulted in a lower flow rate through the orifice meter than through a venturi meter with the same values for meter constriction diameter, pipe diameter, fluid density, and pressure difference.

9. Use of ISO 5167 for Determination of an Orifice Coefficient

Prior to 1991, when the ISO 5167 standard for determination of orifice coefficients came out, the downstream pressure tap was preferentially located at the vena-contracta, the minimum jet area. The vena contracta is located downstream of the orifice plate as shown in the orifice meter diagrams above, however, the distance of the vena contracta from the orifice plate changes with changing orifice hole diameter. As a result, if different orifice plates were to be used in a given meter in order to change the range of flow measurement, it would be necessary to change the downstream pressure tap for each orifice hole size, in order to keep it at the vena contracta location.

In 1991, the ISO 5167 international standard specified three standard types of pressure tap configurations for orifice meters with an equation for calculation of orifice coefficients for any of those three standard configurations. This approach allows changing orifice hole size in a given meter while keeping the pressure tap locations constant. The three standard pressure tap configurations (corner taps, D – D/2 taps, and flange taps), are shown in the diagram below.



ISO 5167 Standard Pressure Tap Configurations

As shown in the diagram, the distance of the pressure taps from the orifice plate is specified as a fixed distance or as a function of the pipe diameter, rather than as a function of the orifice diameter as with the vena contracta pressure tap.

The ISO 5167 standard includes an equation for calculating the orifice coefficient, C_o, as a function of the Reynolds number in the pipe, the diameter ratio, β, the pipe diameter, D_1, and the distances of the pressure taps from the orifice plate, L_1 and L_2. The equation for C_o, given below, can be used for an orifice meter with any of the three standard pressure tap configurations shown above, but it is not suitable for use with any other arbitrary values for L_1 and L_2. Note that Equation (11) for C_o, shown on the next page, is given in reference #3 for this course, ISO 5167-2:2003. An earlier, slightly different version of the equation for C_o is given in the U.S. Dept. of the Interior Bureau of Reclamation, *Water Measurement Manual*, reference #2 at the end of this course.

$$C_o = 0.5961 + 0.0261\,\beta^2 - 0.216\,\beta^8 + 0.000521\,(\beta\,10^6/Re)^{0.7}$$

$$+ (0.0188 + 0.0063\,A)\beta^{3.5}(10^6/Re)^{0.3} + (0.043 + 0.080\,e^{-10L1/D1} - 0.123e^{-7L1/D1})$$

$$(1 – 0.11A)[\beta^4/(1 - \beta^4)] - 0.031(M'_2 – 0.8\,M'^{1.1}_2)\,\beta^{1.3} \qquad \text{Eqn (11)}$$

Where: $A = (19{,}000\beta/Re)^{0.8}$ $\qquad$ $M'_2 = 2\,(L_2/D_1)/(1 – \beta)$

If $D_1 < 2.8$ in, then add the following term to C_o: $\quad 0.011(0.75 – \beta)(2.8 – D_1)$

The parameters in equation (11) are as follows:

- C_o is the orifice coefficient, which is dimensionless [defined by equation (7)]
- L_1 is the distance of the upstream pressure tap from the face of the plate in inches (or mm for S.I. units)
- L_2 is the distance of the downstream pressure tap from the face of the plate in inches (or mm for S.I. units)
- D_1 is the pipe diameter in inches (or mm for S.I. units)
- β is the ratio of orifice diameter to pipe diameter (D_o/D), which is dimensionless
- Re is the Reynolds number in the pipe = D_1V/ν = $D_1V\rho/\mu$, which is dimensionless with D in ft (or m for S.I. units)
- V is the average velocity of the fluid in the pipe in ft/sec [$V = Q/(\pi D_1^2/4)$, with D_1 in ft (or m for S.I. units)]

- ν is the kinematic viscosity of the flowing fluid in ft^2/sec (or m2/s for S.I. units)
- ρ is the density of the flowing fluid in $slugs/ft^3$ (or kg/m^3 for S.I. units)
- μ is the dynamic viscosity of the flowing fluid in $lb\text{-}sec/ft^2$ (or Pa-s for S.I. units)

Note that L_1 and L_2 are shown in the diagram above to be as follows: $L_1 = L_2 = 1$ inch (or 25.4 mm) for flange taps; $L_1 = L_2 = 0$ for corner taps; and $L_1 = D_1$ & $L_2 = D_1/2$ for D – D/2 taps.

The ISO 5167 standard includes several requirements as follows in order to use Eqn (11):

- For all three pressure tap configurations:
  - $d \geq 0.5$ in (or $d \geq 12.5$ mm)
  - $2 \text{ in} \leq D_1 \leq 40 \text{ in}$ (or $50 \text{ mm} \leq D_1 \leq 1000 \text{ mm}$)
  - $0.1 \leq \beta \leq 0.75$
- For corner taps or (D – D/2) taps:
  - $Re > 5000$ for $0.1 \leq \beta \leq 0.56$
  - $Re > 16{,}000\ \beta^2$ for $\beta > 0.56$
- for flange taps:
  - $Re > 5{,}000$
  - $Re > 170\ \beta^2 (25.4\ D_1)$ with D_1 in inches

    (or $Re > 170\ \beta^2 D_1$ with D_1 in mm)

Calculation of the Reynolds Number – The fluid properties needed to calculate C_o (ν or ρ & μ) are typically available in textbooks or handbooks or from websites. The table below gives values of viscosity and density of water in U.S. units over a range of temperatures from 32°F to 70°F, for use in the examples and quiz problems for this course.

Density and Viscosity of Water

Temperature, °F	Density, slugs/ft³	Dynamic Viscosity, lb-s/ft²	Kinematic Viscosity, ft²/sec
32	1.94	3.732×10^{-5}	1.924×10^{-5}
40	1.94	3.228×10^{-5}	1.664×10^{-5}
50	1.94	2.730×10^{-5}	1.407×10^{-5}
60	1.938	2.334×10^{-5}	1.204×10^{-5}
70	1.936	2.037×10^{-5}	1.052×10^{-5}

Example #6: Calculate the Reynolds number for a flow rate of 0.50 cfs of water at 60°F through a 6 inch diameter pipe.

Solution: The velocity in the pipe can be calculated from the equation, $V = Q/A = Q/(\pi D^2/4) = (0.50)/(\pi[(6/12)^2]/4) = 2.546$ ft/sec. From the table above, for water at

60°F, kinematic viscosity, $\nu = 1.204 \times 10^{-5}$. From the problem statement, D = 6 inches = 0.5 ft. Substituting all of these values into the expression for Reynolds number gives:

$$\textbf{Re} = DV/\nu = (0.5)(2.546)/(1.204 \times 10^{-5}) = \underline{\textbf{105,751}}$$

Calculation of the Orifice Coefficient, C_o – An iterative procedure is needed to calculate C_o using equation (11), because the pipe velocity, V (needed to calculate Reynolds number), isn't known until a value for C_o is determined, and Re is needed to calculate C_o. A typical approach (with known values for D, D_o, $P_1 – P_2$, L_1 & L_2, and fluid temperature and properties) is to i) assume a value for Re, ii) proceed to calculate C_o with the ISO 5167 equation, iii) calculate Q and V from the orifice equation, and iv) use the calculated value of V to calculate Re and compare with the assumed value of Re. If calculated Re is different from the assumed value, then replace the assumed Re value with the calculated Re and repeat the calculations. Repeat as many times as necessary until the two Re values are the same. The calculation of C_o isn't very sensitive to the value of Re, so this procedure converges to a solution pretty rapidly. The next example illustrates this type of calculation.

Example #7: Calculate the orifice coefficient and flow rate, for flow of water at 50^{o}F through a 5" diameter orifice in a 12" diameter pipeline, with the pressure difference measured as 1.20 psi. The orifice meter has flange taps.

Solution: From the table above, the density of water at 50^{o}F is 1.94 slugs/ft^3 and its viscosity is 0.0000273 lb-sec/ft^2. For flange taps, $L_1 = L_2 = 1$". Since values are known for β, D_1, L_1, and L_2, the value of C_o can be calculated using Equation (11) if a value for the Reynolds number, Re, is assumed. With the calculated value for C_o, the flow rate, Q, can then be calculated using Equation (10). Then the velocity, V, can be calculated and a new value for Re can be calculated. If the calculated value of Re is not the same as the assumed value, then the process should be repeated using the calculated value of Re. This overall process is repeated until the calculated value of Re is the same as the most recent assumed value. The resulting values for C_o and Q are the final values for those parameters.

The table below shows values of the assumed value for Re, the calculated value of C_o, the calculated value of Q, and the calculated value of Re for four iterations starting with an assumed value of 10,000 for Re. The first iteration resulted in calculation of C_o = 0.603 and a calculated Re of 100,843. The subsequent iterations leading to a final solution of C_o = 0.603 and Q = 1.115 cfs are summarized below:

Iteration #	Assumed Re	Calculated C_o	Calculated Q	Calculated Re
1	10,000	0.611	1.130 cfs	102,231
2	102,231	0.603	1.114 cfs	100,839
3	100,839	0.603	1.115 cfs	100,843
4	100,843	0.603	1.115 cfs	100,843

The fourth iteration gave the same value for assumed and calculated Re along with the final result:

C_o = 0.603 and Q = 1.115 cfs

The calculations for Example #7 can be done with the second worksheet of the spreadsheet provided with the course. That worksheet includes a description of how to use Excel's Goal Seek tool to carry out the iterative calculation in an automated manner. The screenshot on the next page shows the solution to Example #6 using the course spreadsheet. The screenshot shows the values given for D_1, D_o, $P_1 – P_2$, ρ and μ in the blue cells on the left. The spreadsheet makes the calculations in the yellow cells on the right side of the worksheet. There is a dropdown menu for selecting whether the orifice meter has flange taps, corner taps or D – D/2 taps and there is a blue cell for entering an initial estimate for the Reynolds number. Then the instructions can be followed to carry out the iterative solution using Excel's Goal Seek tool.

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Liquid Flow/Orifice Calculations - Flow Rate - U.S. Units

For Large Bore Pipes (2 in $\leq D_1 \leq$ 40 in.) and $P_2/P_1 \geq 0.75$

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

Inputs			**Calculations**		
Pipe Diam, D_1 (in) =	12	in	Pipe Diam, D_1 (ft) =	1.00	ft
Orifice Diam., D_o (in) =	5	in	Orifice Diam., D_o (ft) =	0.417	ft
Measured pressure			Orifice Area, A_o =	0.136354	ft^2
diff., $P_1 - P_2$ =	1.20	lb/in^2	Pipe Area, A_1 =	0.7854	ft^2
Fluid Density, ρ =	1.94	$slugs/ft^3$	Diam. Ratio, β =	0.417	$(= D_o/D_1)$
Fluid Viscosity, μ =	0.0000273	$lb\text{-}sec/ft^2$	A =	0.1306	
			M'_2 =	0.286	
Click on the blue cell below and the arrow					
to the right of it. Then use the drop down			Orifice Coeff., C_o =	0.603	
list to select the pressure tap			(see eqn for C_o below)***		
configuration*:			Press. Diff., $P_1 - P_2$ =	172.8	lb/ft^2
	Flange Taps				
			Pipe Flow Rate, Q =	1.115	cfs
			Pipe Velocity, V =	1.42	ft/sec
Assumed value of			Upstream Press.		
Reynolds No., Re =	100,843	(in pipe)	Tap Loc., L_1^* =	1.0	in
(Enter an initial value to start the calculation.)			Downstr. Press.		
			Tap Loc., L_2^* =	1.0	in
Diff. between assumed & calculated					
Reynolds Number, ΔRe =	0.000		Reynolds Number, Re =	100,843	(in pipe)
			(calculated value)		
	Pipe Flow Rate, Q =		**1.115**	**cfs**	

NOTE: Use Excel's "Goal Seek" to find the flow rate by an iterative calculation as follows: Place the cursor o cell C34 and click on "goal seek" (in the "Tools" menu of older versions and under "Data - What If Analysis" in newer versions of Excel). Make entries to "Set cell: "C34" To value: "0" By changing cell: "C30", and click on "OK". The calculated value of Q will appear in cell F37, and cell C34 should show zero if the process worked properly. Note that the blue cell, C30, needs an initial estimate for Re to start the process.

Example #8: Use the ISO 5167 equation [Eqn (11)] to calculate the orifice coefficient, C_o , for flow of water at 50°F through orifice diameters of 0.6, 1.5, and 2.1 inches, each in a pipeline of 3 inch diameter with a measured pressure difference of 2.5 psi. Repeat this calculation for each of the three standard pressure tap configurations: i) flange taps, ii) corner taps, and iii) D – D/2 taps.

Solution: These calculations were done using the second worksheet of the course spreadsheet in the same manner as shown in the screenshot above for Example #7. The results are summarized in the following tables:

Flange Taps ($L_1 = L_2 = 1$")

D, in	D_o, in	β	Re	C_o
3	0.6	0.2	8,244	**0.602**
3	1.5	0.5	53,713	**0.608**
3	2.1	0.7	118,087	**0.614**

Corner Taps ($L_1 = L_2 = 0$)

D, in	D_o, in	β	Re	C_o
3	0.6	0.2	8,253	**0.602**
3	1.5	0.5	53,780	**0.609**
3	2.1	0.7	117,184	**0.609**

D – D/2 Taps ($L_1 = 3$" $L_2 = 1.5$")

D, in	D_o, in	β	Re	C_o
3	0.6	0.2	8,242	**0.602**
3	1.5	0.5	53,709	**0.608**
3	2.1	0.7	118,510	**0.616**

This example was included to show typical C_o values for an orifice meter. As the results show, the value of C_o stays within a fairly narrow range (0.602 to 0.616) for $0.2 \leq \beta \leq 0.7$, for all three of the pressure tap configurations.

10. Orifice Meter Gas Flow Calculations

Calculations for gas flow through an orifice meter are similar to those illustrated in the last section for liquid flow, but Equation (5) must be used for Q along with Equation (6) for the Expansion Factor, Y. Also note that values are needed for the upstream pressure of the gas and for the compressibility factor and specific heat ratio of the gas.

Orifice meter gas flow calculations will be illustrated in the next example by considering air flow through an orifice meter with the same pipe diameter, orifice diameter and measured pressure drop as for water flow in Example #6.

Example #9: Calculate the orifice coefficient and flow rate, for flow of air at 50°F through a 5" diameter orifice in a 12" diameter pipeline, with the pressure difference measured as 1.20 psi. The orifice meter has flange taps. The pressure in the pipe upstream of the meter is 20 psia.

Solution: The viscosity of air at 50°F is 4×10^{-7} lb-sec/ft^2, the specific heat ratio for air is 1.4. The gas temperature is much greater than the critical temperature of air and the gas pressure is much less than the critical pressure of air, so the value of the compressibility factor, Z, can be taken as 1.

Carrying out the iterative calculation by hand would be a rather tedious process, because it would involve calculating P_2/P_1, from the given values of P_1 and $P_1 – P_2$, calculating the expansion factor, Y, with Equation (6), and then use of the rather long Equation (11) for C_o, and use of Equation (5) for calculating Q, several times in the iterative calculation. With a spreadsheet set up for the calculation, however, like worksheet 3 in the course spreadsheet workbook, the calculation is quite straightforward. The screenshot below shows the solution to Example #9, using worksheet 3 from the course spreadsheet.

Gas Flow/Orifice Calculations - Flow Rate - U.S. or S.I. Units

For Large Bore Pipes (2 in < D1 < 40 in.) and P2/P1 > 0.75

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

Inputs			**Calculations**		
Pipe Diam, D_1 =	12	in	Pipe Diam, D_1 =	1.000	ft
Orifice Diam., D_o =	5	in	Orifice Diam., D_o =	0.417	ft
Measured pressure			Orifice Area, A_o =	0.1364	sq ft
diff., $P_1 - P_2$ (psi) =	1.2	psi	Pipe Area, A_1 =	0.7854	sq ft
Abs. Press. in Pipe, P_1 =	20	psia	Diam. Ratio, β =	0.417	(= D_o/D_1)
Temerature in Pipe, T_1 =	50	deg F	**A** =	0.0580	
Fluid Viscos., μ =	0.0000004	lb-sec/sq ft	M'_2 =	0.286	
Gas Mol. Wt., **MW** =	29	lb/lbmole	Orifice Coeff., C_o =	0.601	
Sp. Ht. Ratio			(see eqn for C_o below)***		
of gas (C_p/C_v), **k** =	1.4		Abs. Temp. in Pipe, T_1 =	509.67	°R
Compress. Factor			Press. Diff., $P_1 - P_2$ =	172.8	lb/sq ft
of gas, **Z** =	1		Pressure Ratio, P_2/P_1 =	0.9400	
Ideal Gas Law					
Constant, **R** =	345.23		Expansion Factor, **Y** =	0.984	
	(psia-cu ft/slugmole-deg R)				
			Fluid density, ρ =	0.00330	slugs/cu ft
Click on the blue cell below and the arrow to the right of it. Then use the drop down list to select the pressure tap configuration*:			Pipe Flow Rate, **Q** =	26.5250	cfs
			Pipe Velocity, **V** =	33.8	ft/sec
	Flange Taps		Upstream Press.		
			Tap Loc., L_1* =	1.0	in
Assumed value of			Downstr. Press.		
Reynolds No., **Re** =	278,314	(in pipe)	Tap Loc., L_2* =	1.0	in
(Enter an initial value to start the calculation.)					
Diff. between assumed & calculated			Reynolds Number, **Re** =	278,314	(in pipe)
Reynolds Number, Δ**Re** =	0.000		(calculated value)		
	Pipe Flow Rate, **Q** =		**26.52**	**cfs**	

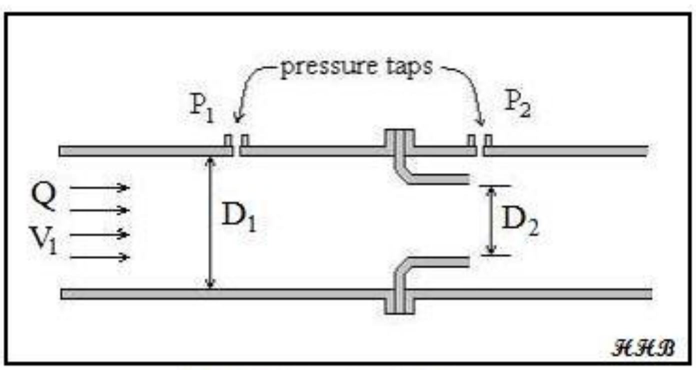
NOTE: Use Excel's "Goal Seek" to find the flow rate by an iterative calculation as follows: Place the cursor o cell C46 and click on "goal seek" (in the "Tools" menu of older versions and under "Data - What If Analysis" in newer versions of Excel). Make entries to "Set cell: "C46" To value: "0" By changing cell: "C43", and click on "OK". The calculated value of Q will appear in cell F48, and cell C46 should show zero if the process worked properly. Note that the blue cell, C43, needs an initial estimate for Re to start the process.

Note that values provided for pipe diameter, orifice diameter, measured pressure drop, absolute pressure in the pipe, and temperature in the pipe were entered in the blue cells on the left side of the worksheet. Also values were entered for the viscosity of air at 50°F, the molecular weight of air (29), the specific heat ratio of air (1.4), a value of 1 for the compressibility factor, and 345.23 psia-ft^3/slugmole-°R for the ideal gas law constant. The dropdown list was used to specify flange taps for the pressure tap configuration, and an initial estimate for the Reynolds number was entered in the blue cell near the bottom on the left side of the worksheet.

Then Excel's Goal Seek process was carried out, as described at the bottom of the screenshot, to carry out the iterative calculation of C_o, Re, and the air flow rate through the meter (and pipe), Q. The final solution for the flow rate of air is 26.5 cfs.

11. The Flow Nozzle Meter

The diagram of a flow nozzle meter below shows its general configuration. It consists of a fairly short nozzle, usually held in the pipe between pipe flanges. A flow nozzle meter is thus less expensive and simpler than a venturi meter, but not quite as simple or inexpensive as an orifice meter. The frictional loss and the discharge coefficients for flow nozzle meters are between typical values for venturi meters and orifice meters, but closer to those for venturi meters. A typical range for flow nozzle discharge coefficients is between 0.94 and 0.99.



Flow Nozzle Meter

F

If we refer to the area of the nozzle opening as A_n and refer to the discharge coefficient for a flow nozzle meter as C_n, then the equation for calculating the flow rate through a flow nozzle meter becomes:

$$Q = C_n A_n \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}} \qquad \text{Eqn (12)}$$

The calculation of liquid flow rate through a flow nozzle meter with known nozzle coefficient is essentially the same as that illustrated earlier for a venturi meter, and such calculations can be done with the first worksheet of the course spreadsheet.

For calculation of a gas flow rate through a flow nozzle meter, Equation (5) should be used to calculate the flow rate, Q, with Equation (7) used to calculate the Expansion Factor, Y. Those two equations are reproduced below. This type of calculation is illustrated with Example #9, below the two equations.

$$Q = C\,A_2\,Y \sqrt{\frac{2ZRT_1(P_1 - P_2)}{(MW)P_1(1 - \beta^4)}} \qquad \text{Eqn (5)}$$

$$\boxed{Y = \sqrt{\left(\frac{k\,\tau^{2/k}}{k - 1}\right)\left(\frac{1 - \beta^4}{1 - \beta^4\,\tau^{2/k}}\right)\left(\frac{1 - \tau^{(k-1)/k}}{1 - \tau}\right)}} \qquad \text{Eqn (7)}$$

Example #10: Calculate the flow rate of air at 50°F through a flow nozzle meter with a 5" nozzle diameter in a 12" diameter pipeline, with the pressure difference measured as 1.20 psi. The nozzle coefficient of the meter is known to be 0.984. The pressure in the pipe upstream of the meter is 20 psia.

Solution: Note that this is the same pipe diameter, constricted diameter, measured pressure difference, and upstream air temperature and pressure as that used in Example #9 for air flow through an orifice meter. This does not require an iterative calculation, because the value of the nozzle coefficient is specified.

As in Example #9, the specific heat ratio for air (**k**) is **1.4**. The gas temperature is much greater than the critical temperature of air and the gas pressure is much less than the critical pressure of air, so the value of the compressibility factor (**Z**) can be taken as **1**.

In order to calculate the Expansion Factor, Y, with Equation (7), P_2/P_1 can be calculated from: $P_2/P_1 = [P_1 - (P_1 - P_2)]/P_1 = (20 - 1.2)/20 = 0.94$. Using $\tau = 0.94$, $k = 1.4$, and $\beta = 5/12$ in Equation (7), gives **Y = 0.966**.

The parameter values to go into Equation (5) are thus as follows:

- Nozzle Coefficient, **C** = 0.984 (given)
- Nozzle Area, $\mathbf{A_2} = \pi(5/12)^2/4 = 0.1364$ ft^2
- Expansion Factor, **Y** = 0.966 (calculated above)
- Compressibility Factor, **Z** = 1
- Ideal Gas Law constant, **R** = 345.23 psia-ft^3/slugmole-^{o}R
- Upstream pipe absolute temperature, $\mathbf{T_1}$ = 50 + 460 = 510 ^{o}R
- Measured Pressure Drop, $\mathbf{P_1 - P_2}$ = (1.2)(144) = 172.8 lb/ft^2
- Molecular Weight of air, **MW** = 29
- Upstream Pipe Pressure, $\mathbf{P_1}$ = 20 psia (given)
- Diameter Ratio (D_n/D_1), $\boldsymbol{\beta}$ = 5/12 = 0.417

Substituting all of these values into Equation (5) gives:

$$Q = (0.984)(0.1364)(0.966)\sqrt{\frac{(2)(1)(345.23)(510)(172.8)}{(29)(20)(1 - 0.417^4)}} = \mathbf{42.63\ cfs}$$

12. Summary

The orifice meter, venturi meter and flow nozzle meter all use a restriction placed in the flow area to increase the fluid velocity and thus decrease the fluid pressure in the restricted area. The pressure difference between that in the undisturbed flow and that in the restricted area can then be used to calculate the flow rate through the meter using the equations presented and discussed in this course. The ideal gas law can be used to calculate the density of a gas at specified temperature and pressure, for use in these calculations. The ISO 5167 procedure for calculating the discharge coefficient for an orifice meter was also presented and discussed. Calculation of liquid flow rate and gas flow rate through the meters was illustrated with several worked examples.

13. References

1. Munson, B. R., Young, D. F., & Okiishi, T. H., *Fundamentals of Fluid Mechanics,* 4th Ed., New York: John Wiley and Sons, Inc, 2002

2. U.S. Dept. of the Interior, Bureau of Reclamation, 2001 revised, 1997 third edition, *Water Measurement Manual*, available for on-line use or download at: http://www.usbr.gov/pmts/hydraulics_lab/pubs/wmm/index.htm

3. International Organization of Standards - ISO 5167-2:2003 Measurement of fluid flow by means of pressure differential devices inserted in circular cross-section conduits running full, Part 2: Orifice Plates. Reference number: ISO 5167-2:2003.

4. International Organization of Standards - ISO 5167-4:2003 Measurement of fluid flow by means of pressure differential devices inserted in circular cross-section conduits running full, Part 4: Venturi Tubes. Reference number: ISO 5167-4:2003.

5. Bengtson, H.H., “Excel Spreadsheets for Orifice and Venturi Flow Meters,” an online article at www.engineeringexcelspreadsheets.com.

6. Bengtson, H.H., "Spreadsheets for ISO 5167 Orifice Plate Flow Meter Calculations," an online informational article at www.engineeringexcelspreadsheets.com