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Electrical Engineering Fundamentals for Non-Electrical Engineers

by

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Introduction

We all interact with electricity on a daily basis. Most of us start off the day waking up to an alarm clock, then we turn on the lights, take a hot shower, and make a cup of coffee, etc. Electricity is engrained in our daily lives, from the electricity in the wall sockets to power our alarm clocks and coffee pots, to the electricity wired in the ceiling for lights and a bathroom fan, to the heater elements in an electric water heater. It is easy to take electricity for granted, but we truly appreciate it when the power goes out for a few hours. That's when we realize that we have lost our TV, air conditioners, refrigerators, and the children go into panic mode with a loss of the internet and social media.

Electrical Engineering Fundamentals for Non-Electrical Engineers is a course designed to promote an understanding of the fundamentals of electricity. The course covers the differences between Alternating Current (AC) and Direct Current (DC) power sources by explaining the behavior of the voltage and current for both types of sources. The fundamental circuit building blocks including resistors, capacitors and inductors are covered including their behavior in series and parallel circuits as well as transient analysis. The course covers Ohm's law and Kirchoff's Laws and their application to performing circuit analysis. This course also includes a brief introduction to imaginary numbers and phasors as related to current, voltage, and impedance.

Definitions

- Alternating Current (AC) – Current flowing from a source that can be represented by a sine wave, which results in current flowing both directions.
- Amperes or Amps (A) – The amount of charge flowing past a given point per unit time.
 $1 \text{ Amp} = 1 \text{ Coulomb} / \text{second}$
- Coulomb (C) – SI unit of charge
- Cycles – Number of times a complete sine wave occurs.
- Direct Current (DC) – Current flowing from a source which does not change polarity, which results in current flowing in only one direction.
- Farad (F) – SI unit of capacitance. $1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$
- Henry (H) – SI unit of inductance. $1 \text{ Henry} = \frac{1 \text{ Volt} * 1 \text{ Second}}{1 \text{ Amp}}$
- Hertz (Hz) – SI unit of frequency. $1 \text{ Hertz} = \frac{1 \text{ cycle}}{1 \text{ second}}$
- Joules (J) – SI unit of energy. $3.60 \times 10^6 \text{ Joules} = 1 \text{ kWh}$



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- Ohms (Ω) – SI unit of resistance. $1 \text{ Ohm} = \frac{1 \text{ Volt}}{1 \text{ Amp}}$
- RMS – Root Mean Square value
- SI – International System of Units
- Volts (V) – A measure of the difference in electrical potential between two points.
 $1 \text{ Volt} = \frac{1 \text{ Joule}}{1 \text{ Coulomb}}$
- Watts (W) – SI unit of power. $1 \text{ Watt} = \frac{1 \text{ Joule}}{1 \text{ second}}$

Power Sources

Let's start with some of the fundamental building blocks. Every circuit needs a power source. There are two basic types of power sources, Alternating Current (AC) and Direct Current (DC). Alternating current, as the name implies, changes the polarity of the source at some frequency. Direct current maintains the same polarity; therefore, the positive and negative terminals are always at the same location. Figure 1 shows a standard symbol for an AC source. Figure 2 shows two standard symbols for a DC source.



Figure 1 Alternating Current (AC) Source

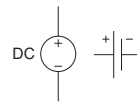


Figure 2 Direct Current (DC) Sources

Examples of an AC source could be a generator or a wall outlet in your house. Examples of a DC source could be a car battery or simple AA battery.

So what is the difference between AC and DC? It may best be illustrated in the examples below. Figure 3 shows a simple DC circuit. Figure 4 shows that the voltage of the 12 VDC source. Figure 5 shows the current through the 12Ω resistor.



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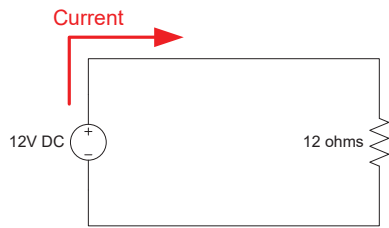


Figure 3 Simple DC Circuit

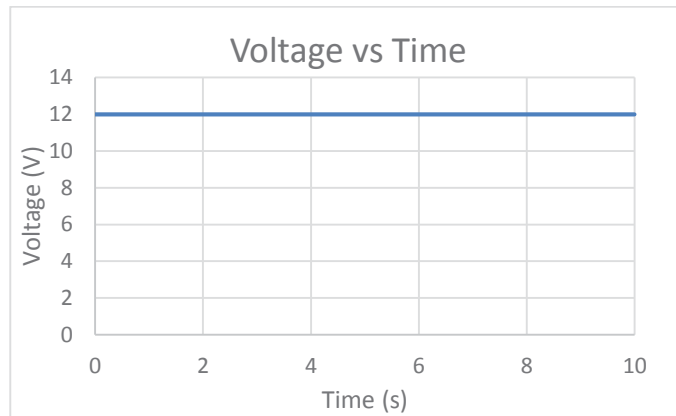


Figure 4 Voltage vs Time for an ideal DC source

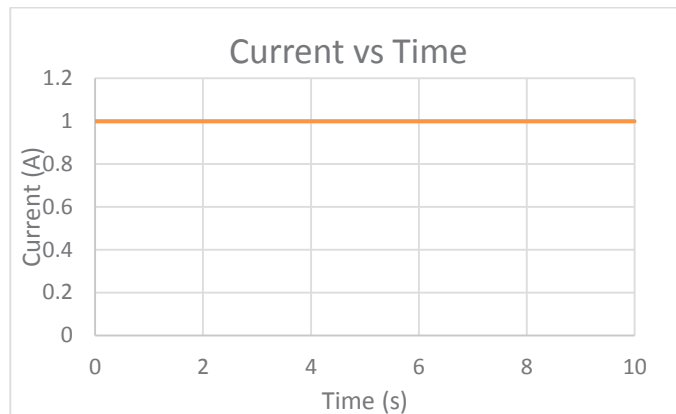


Figure 5 Current vs Time for an ideal DC source

In Figure 3 Simple DC Circuit, current leaves the positive terminal (+) of the 12 Volt DC source, flows through the 12 Ohm (Ω) resistor, and then returns to the negative terminal (-) of the 12V DC source. In an ideal voltage source, there is no change in voltage over time due to temperature, load, or internal battery resistance. Figure 4 Voltage vs Time for an ideal DC source shows that the voltage of 12 V source is constant with time. Figure 5 Current vs Time for an ideal DC source shows that the current through the resistor is constant with time.

Note: There are two conventions for current flow. (1) Conventional flow assumes that current flows from the positive terminal (+) to the negative terminal (-). This is also called hole flow. (2) Electron flow assumes current flows from the negative terminal (-) to the



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positive terminal (+). This is the path of the flowing electrons. For simplicity, conventional flow will be used in all examples.

Figure 5 was developed using a relationship called Ohm's Law. Ohm's law states that in a circuit containing only linear components, the expression that governs the relationship between voltage, current, and resistance is:

$$V = I * R$$

Where:

V is the voltage measured in volts (V)

I is the current measured in amps (A)

R is the resistance measured in ohms (Ω)

Note: An example of a non-linear component is a light bulb that varies the resistance with temperature.

In an Alternating Current (AC) circuit, we see a much different behavior. Figure 6 shows a simple AC circuit when the voltage of the source is greater than zero. Figure 7 shows the voltage vs. time for the AC source and the RMS value for voltage. Figure 8 shows the simple AC circuit when the voltage of the source is less than zero. Figure 9 shows the current vs. time for the current flowing through the 12Ω resistor.



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Figure 6 Simple AC Circuit when $V > 0$

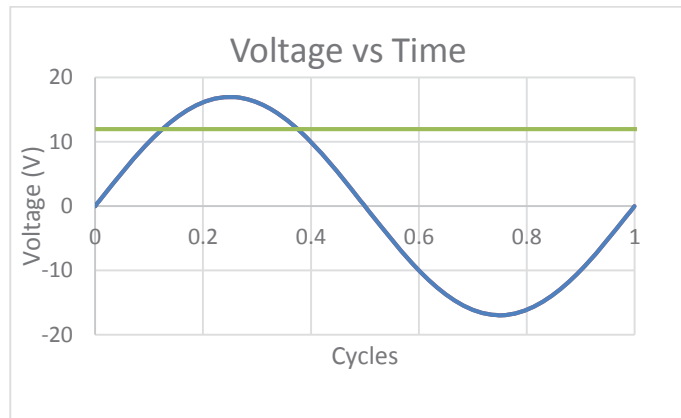


Figure 7 Voltage vs Time for an AC source

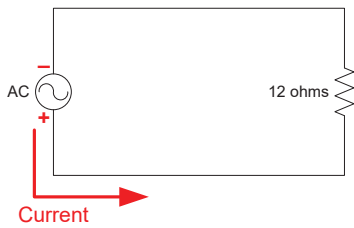


Figure 8 Simple AC Circuit when $V < 0$

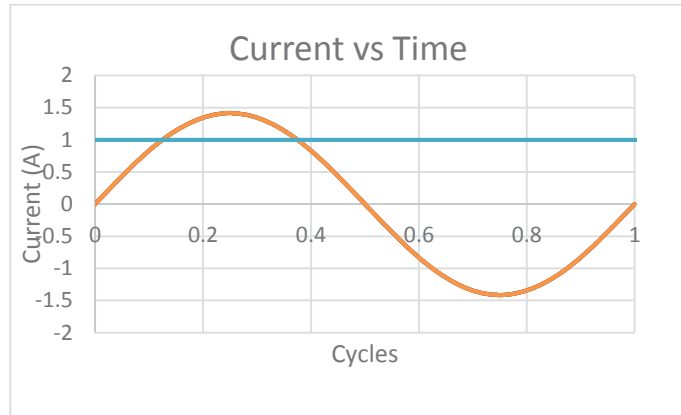


Figure 9 Current vs Time for an AC source

As seen in Figure 6, current leaves the positive terminal of the 12 Volt AC source, flows through the 12 Ohm (Ω) resistor, and then returns to the negative terminal of the 12V AC source. When the voltage source drops below 0 (from .5 cycles to 1 cycle), current flows the opposite direction as depicted in Figure 8.

The green line shown in Figure 7 and the light blue line in Figure 9 are the RMS values for the voltage and current respectively. AC voltages and currents can be expressed in a number of different ways.

V_{peak} – the value from a reference (normally zero) to the peak of the sine wave.

$V_{\text{peak to peak}}$ – The value from the positive peak to the negative peak of a sine wave.



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V_{RMS} – the root mean square value of the sine wave. This is the most commonly used term to express the magnitude.

Note: Three phase systems are a little more complicated and those variants are not covered.

Let's take a look at a household application. If you were able to hook up an oscilloscope, you would see that the voltage coming out of the wall outlets looks like a sine wave as described above in Figure 7. While the voltage values are typically expressed as RMS, you may also hear this referred to as line to neutral voltage (V_{LN}) or hot to neutral voltage (V_{HN}). Figure 10 shows a typical household wall outlet. The RMS voltage you will see from hot to neutral or hot to ground is approximately 120V. Voltage from neutral to ground should be 0V.

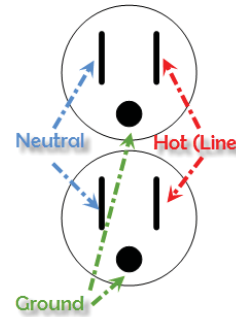


Figure 10 Household Wall Outlet



Figure 12 Hot to neutral voltage measurement

The following figures illustrate the RMS and peak values in a typical household. Figure 12 shows a Fluke Multimeter connected to a wall outlet. The black lead of the multimeter is connected to the neutral and the red lead is connected to the hot. Figure 11 shows the resulting reading. As we expected, the RMS voltage is approximately 120V. In this case it is 124.3V. To find the expected peak voltage, we take $120V_{RMS}$ times the square root of two.

$120 V * \sqrt{2} = 169.7$. As you can see, the positive peak value is 171.5V and the negative peak is -173.2V. The peak to peak voltage is $171.5 + 173.2 = 344.7V$. All of the voltage values are as expected.



Figure 11 Voltage values



Series vs. Parallel

A series circuit is a circuit where current has only one path to travel. A parallel circuit has more than one path for current to flow. The following figures show a simple series circuit and a simple parallel circuit.

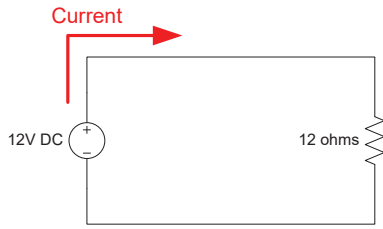


Figure 13 Simple Series Circuit

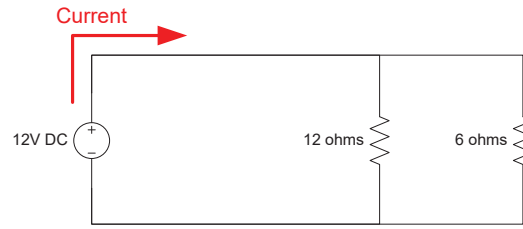
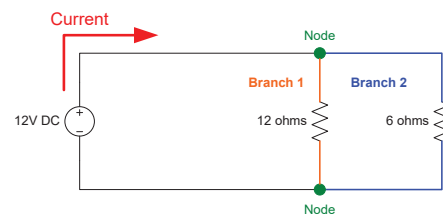


Figure 14 Simple Parallel Circuit

As you can see in Figure 13, current has only one path to flow, which makes this a series circuit. However, in Figure 14, current will flow through both the 12 ohm resistor and through the 6 ohm resistor. Multiple current paths makes Figure 14 a parallel circuit.

Current Behavior at a Node

In order to analyze parallel circuits, we first have to discuss what happens when current is given more than one path to choose from. Imagine connecting a garden hose and fire hose to the same water faucet. As you can imagine, more water will flow through the fire hose than the garden hose. The same concept holds true for current. When current reaches a node, where it is given two paths, more current will flow where there is less resistance. Figure 15 shows two nodes and two branches. A node is a position where current can flow in more than one direction. The resistance in Branch 1 is 12Ω and the resistance in Branch 2 is 6Ω . Branch 2 has $\frac{1}{2}$ of the resistance of Branch 1 and therefore will have 2 times the current flow. We can prove this using Ohm's law.





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Branch 1:

$$V = I * R$$

$$I = \frac{V}{R}$$

$$I = \frac{12V}{12\Omega}$$

$$I = 1A$$

Branch 2:

$$V = I * R$$

$$I = \frac{V}{R}$$

$$I = \frac{12V}{6\Omega}$$

$$I = 2A$$

Resistors

A resistor is a device that provides opposition to current flow. Resistance is measured in ohms (Ω). Resistors can vary in size from microelectronics to load banks requiring large enclosures. Figure 16 shows the symbol for a resistor.



Figure 16 Schematic Symbol for a Resistor

Resistors are used for a variety of applications: limiting current, voltage dividers, tuning circuits, controlling capacitor charge/discharge rates, and providing a known load for testing large equipment. In a series circuit, each resistor will have the same current through it, but will have voltage drops corresponding to their resistance. Figure 17 shows a circuit containing several resistors and the voltage drop across each resistor. Given that the total current is 2A, we can use Ohm's law to see how much voltage is dropped across each resistor.

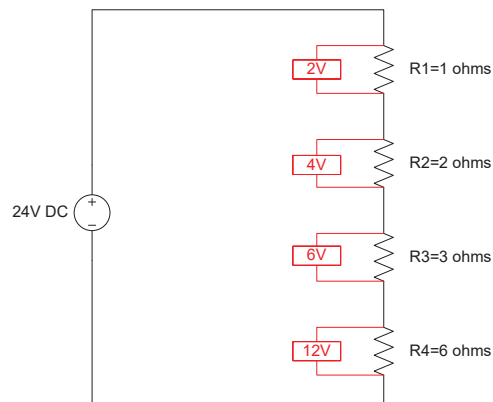


Figure 17 Voltage Drop Across Resistors

The values shown in red represent the voltage drop across each resistor. For R1:



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$$V = I * R$$
$$V = 2A * 1\Omega$$
$$V = 2V$$

The same method applies to R2, R3, R4. If you notice, the voltage drops add up to the source voltage. $V_{R1} + V_{R2} + V_{R3} + V_{R4} = 2V + 4V + 6V + 12V = 24V$. This is an important concept used in circuit analysis.

Combining Resistors in Series and Parallel Circuits

In order to make circuit analysis easier, we often combine circuit elements to make the circuit simpler. There are several techniques for reducing circuits to their simplest form. We will focus on finding the total resistance of a circuit containing only resistive elements.

Note: When other circuit elements are present, it is necessary to use more advanced techniques such as Thevenin equivalency.

To calculate the total resistance of a series circuit, simply add all of the resistors.

$$\text{Total Resistance } (R_T) = \sum_{x=1}^n R_x = R_1 + R_2 + \dots + R_n$$

Example in Figure 18:

$$R_T = R_1 + R_2 + R_3 + R_4 + R_5$$
$$R_T = 1\Omega + 2\Omega + 8\Omega + 4\Omega + 5\Omega$$
$$R_T = 20\Omega$$

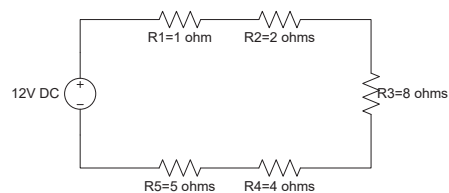


Figure 18 Series Circuit

Knowing the total resistance, we can draw an equivalent circuit as a 12V source and one 20Ω resistor.

To calculate the resistance of a parallel circuit, the resistors add as inverses.

$$\text{Total Resistance } (R_T) = \frac{1}{\sum_{x=1}^n \frac{1}{R_x}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$



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Example in Figure 19:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}$$
$$R_T = \frac{1}{\frac{1}{2\Omega} + \frac{1}{8\Omega} + \frac{1}{8\Omega} + \frac{1}{4\Omega}}$$
$$R_T = 1\Omega$$

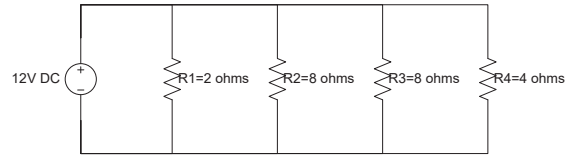


Figure 19 Parallel Circuit

Given the value of the total resistance, we can now redraw an equivalent circuit as a 12V source and one 1Ω resistor as shown in Figure 20. R_{eq} is used to denote that this is an equivalent resistance to the total circuit resistance.

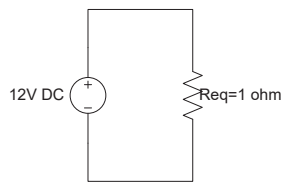


Figure 20 Equivalent Circuit

When circuits have both series and parallel, the same principles apply. First, we combine the series resistors in the parallel branches. Then, we combine the parallel branches. Lastly, we combine the equivalent parallel branch resistance with any remaining series resistors.

Figure 21 shows a circuit containing both series and parallel components.

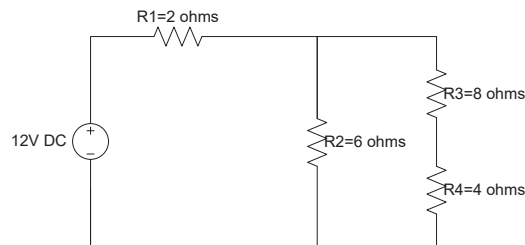
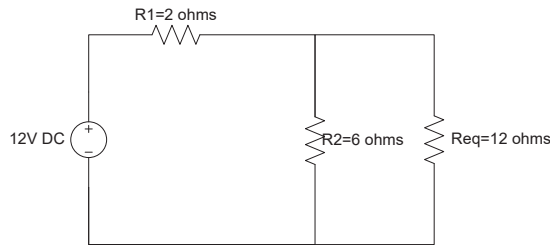


Figure 21 Series and Parallel Circuit

1. Start with the series resistors in the second parallel branch (R3 and R4). Resistors in series add by summing their values, so $R_{eq} = R3 + R4 = 8\Omega + 4\Omega = 12\Omega$.



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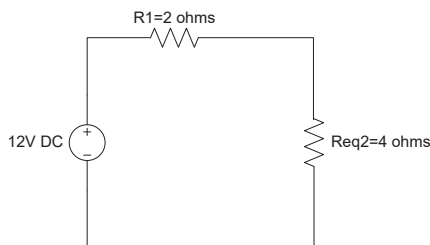
2. Next, we have to combine R_2 and R_{eq} , which add as inverses because they are in parallel.

$$R_{eq2} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_{eq}}}$$

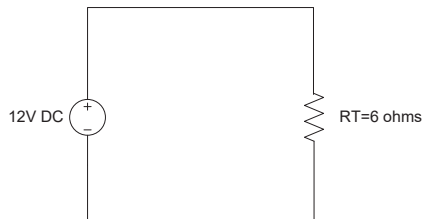
$$R_{eq2} = \frac{1}{\frac{1}{6\Omega} + \frac{1}{12\Omega}}$$

$$R_{eq2} = \frac{1}{\frac{3}{12\Omega}}$$

$$R_{eq2} = 4\Omega$$



3. Finally, we combine the last two remaining resistors in series. $R_T = R_1 + R_{eq2} = 2\Omega + 4\Omega = 6\Omega$.





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Capacitors

A capacitor is a device that provides opposition to a change in voltage. Capacitors store energy in the form of an electric field. There are many different types of capacitors, but the general principle is that there are two charged plates separated by a dielectric material. Capacitance is measured in Farads (F). The geometry of the capacitor, distance between plates, and dielectric material will all affect the capacitance. Figure 16 shows the symbol for a capacitor.

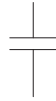


Figure 22 Schematic Symbol for a Capacitor

Capacitors are used for a variety of applications: storing energy, providing power factor correction, circuit isolation, starting circuits, and much more.

Capacitor Behavior While Charging

The following figure shows a voltage source, a switch, a resistor, and a capacitor.

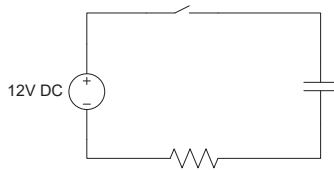


Figure 23 Simple Capacitive Circuit



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In Figure 23, the switch is initially open and the capacitor is initially fully discharged and $\tau = 2\text{s}$. At time = 0, the switch is closed. Figure 24 shows the voltage that would be seen at the capacitor. As you can see from the graph, the capacitor voltage starts at zero and gradually charges up to the match the source voltage.

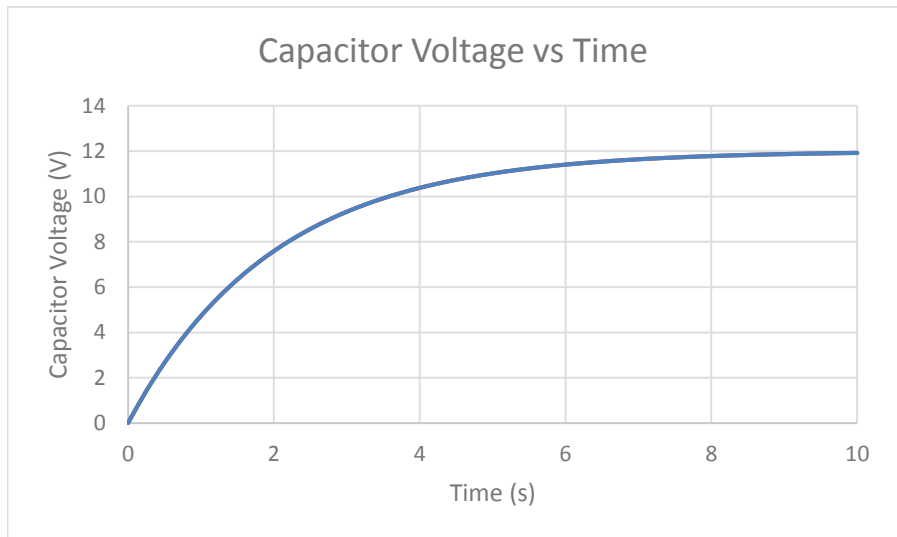


Figure 24 Voltage vs Time for a Capacitor Charging

Capacitor voltage over time can be determined using the following equation:

$$V_C = V_S \left(1 - e^{-\frac{t}{\tau}} \right)$$

Where:

- V_C is the capacitor voltage
- V_S is the source voltage
- t is the time after the switch closing
- τ is the time constant

After 5 time constants, or 10 seconds, the voltage is approximately the steady state value. For a capacitor, the time constant is:

$$\tau_C = RC$$

Where:

- τ is the time constant for the capacitor
- R is the resistance in the capacitor charge path
- C is the capacitance

Current through the capacitor also has an exponential behavior. Current decays exponentially to zero. Assume that the total current is 3A and $\tau = 2\text{s}$.



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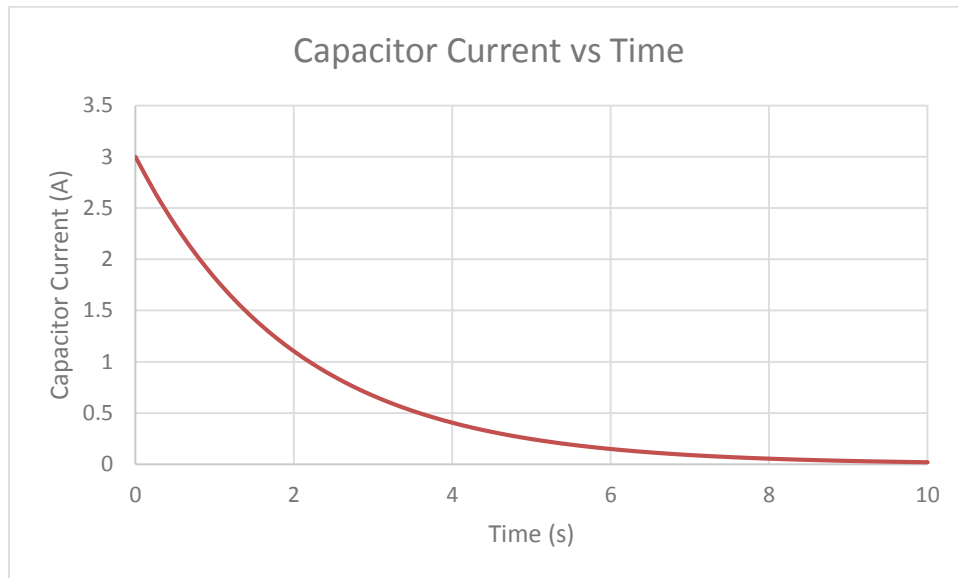


Figure 25 Current vs Time for a Capacitor Charging

Capacitor current over time can be determined using the following equation:

$$I_C = I_{Max} e^{\frac{-t}{\tau}}$$

Where:

I_C is the capacitor current

I_{Max} is the maximum current

t is the time after the switch closing

τ is the time constant

After 5 time constants, or 10 seconds, the current is approximately the steady state value of zero.

Capacitor Behavior While Discharging

The following figure shows a voltage source, a switch, a capacitor, and two resistors.

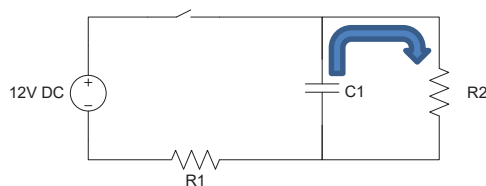


Figure 26 Simple Capacitive Circuit



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In Figure 26, the switch is initially closed and the capacitor has fully charged through resistor R1. Assume $\tau = 3\text{s}$. At time = 0, the switch is opened. The capacitor then begins to discharge through R2. Figure 27 shows the voltage that would be seen at the capacitor. As you can see from the graph, the capacitor voltage starts at the source voltage and gradually discharges to zero.

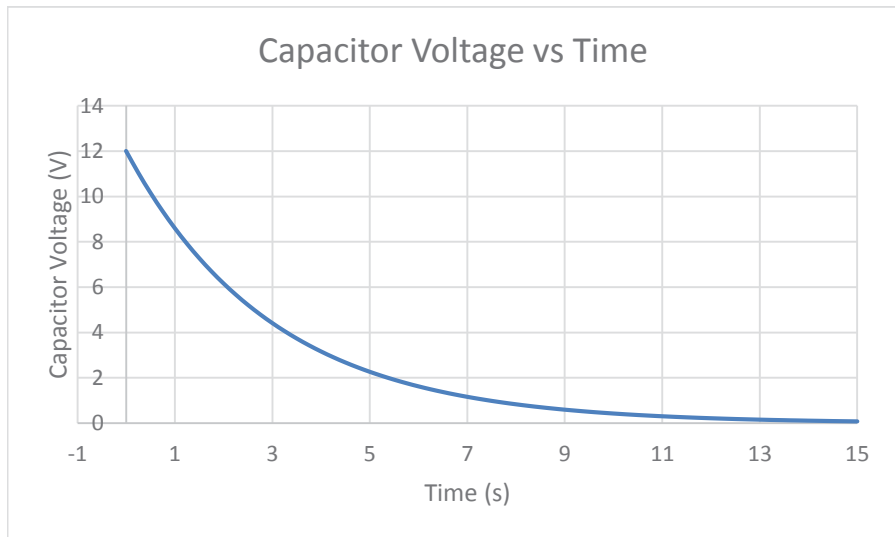


Figure 27 Voltage vs Time for a Capacitor Discharging

Capacitor voltage over time can be determined using the following equation:

$$V_C = V_S e^{-\frac{t}{\tau}}$$

Where:

- V_C is the capacitor voltage
- V_S is the source voltage
- t is the time after the switch closing
- τ is the discharge time constant

After 5 time constants, or 15 seconds, the voltage is approximately the steady state value. For a capacitor, the time constant is:

$$\tau_C = RC$$

Where:

- τ is the time constant for the capacitor
- R is the resistance in the capacitor discharge path
- C is the capacitance

Current through the capacitor has an exponential behavior. The capacitor is acting as the source for the discharge circuit. This results in current flowing the opposite, or negative, direction



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through the capacitor. As the source voltage decays, the current through the circuit also decays exponentially to zero. Assume that the total current is 4A and $\tau = 3s$.

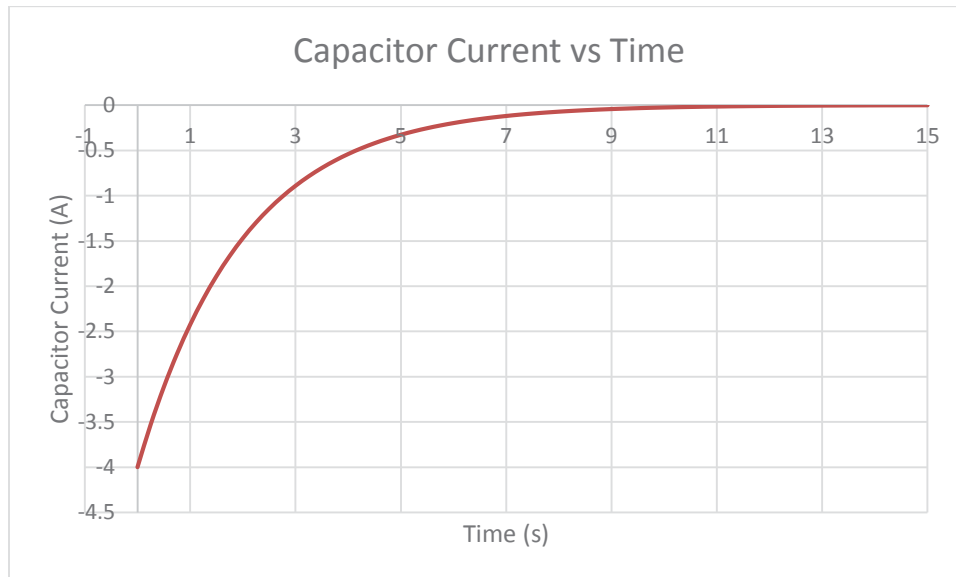


Figure 28 Capacitor Current vs Time for a Discharging Capacitor

Capacitor current over time can be determined using the following equation:

$$I_C = -I_{Max} e^{-\frac{t}{\tau}}$$

Where:

I_C is the capacitor current

I_{Max} is the maximum current

t is the time after the switch opening

τ is the time constant

After 5 time constants, or 15 seconds, the current is approximately the steady state value of zero.

Inductors

An inductor is a device that provides opposition to a change in current. Inductors store energy in the form of a magnetic field. An inductor in the simplest form is a coil of wire. Inductance is measured in Henries (H). The geometry of the inductor, thickness of wire, number of turns, and distance between coils will all affect the inductance. Figure 29 shows the symbol for an inductor.



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Figure 29 Schematic Symbol for an Inductor

Inductors are used for a variety of applications: filter circuits, sensors, transformers.

Inductor Behavior While Charging

The following figure shows a voltage source, a switch, a resistor, and an inductor.

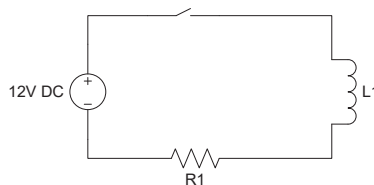


Figure 30 Simple Inductive Circuit

In Figure 30, the switch is initially open, the inductor magnetic field is fully discharged, and $\tau = 4s$. At time = 0, the switch is closed. Figure 31 shows the voltage that would be seen at the inductor. As you can see from the graph, the inductor voltage starts at the source voltage and gradually drops zero.

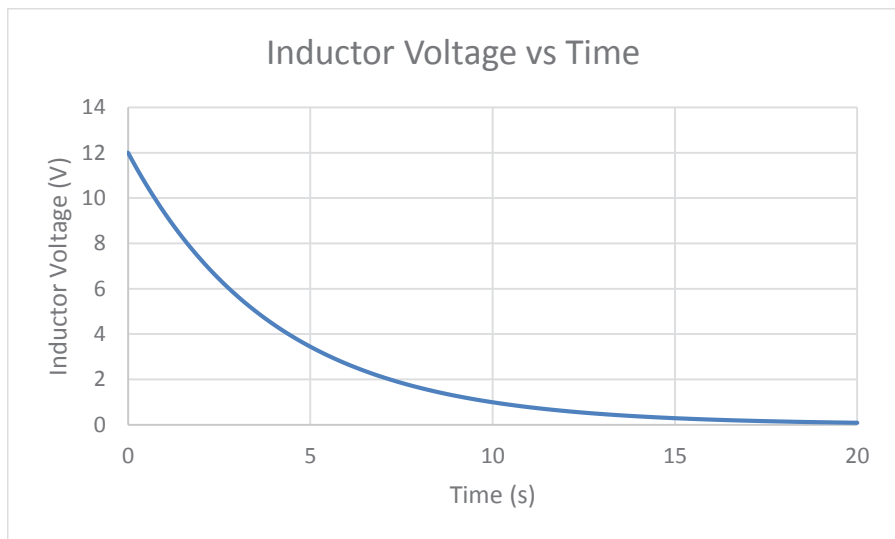


Figure 31 Voltage vs Time for a Capacitor Charging



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Inductor voltage over time can be determined using the following equation:

$$V_L = V_S e^{-\frac{t}{\tau}}$$

Where:

V_L is the inductor voltage

V_S is the source voltage

t is the time after the switch closing

τ is the inductor charging time constant

After 5 time constants, or 20 seconds, the voltage is approximately the steady state value of zero. For an inductor, the time constant is:

$$\tau_L = \frac{L}{R}$$

Where:

τ is the time constant for the inductor

L is the inductance

R is the resistance in the inductor charge path

Current through the inductor has an exponential behavior as well. When the switch is closed, the inductor opposes a change in current. As the magnetic field reaches equilibrium, the current increases to a maximum value. Assume that the total current is 6A and $\tau = 4s$.

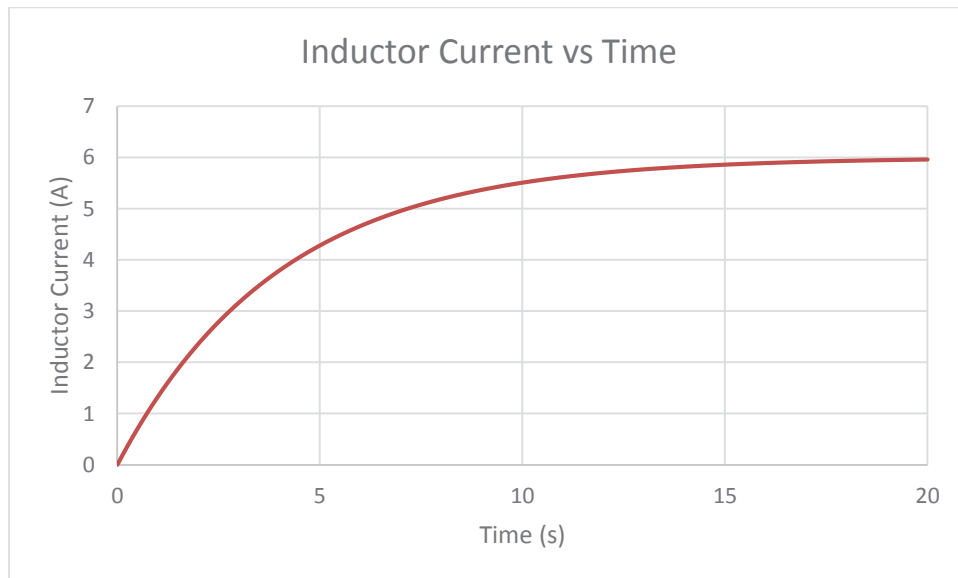


Figure 32 Current vs Time for an Inductor Charging

Inductor current over time can be determined using the following equation:



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$$I_L = I_{Max} \left(1 - e^{-\frac{t}{\tau}} \right)$$

Where:

I_L is the inductor current

I_{Max} is the maximum current

t is the time after the switch closing

τ is the inductor time constant

After 5 time constants, or 20 seconds, the current is approximately the steady state value of 6A.

Inductor Behavior While Discharging

The following figure shows a voltage source, a switch, an inductor, and two resistors.

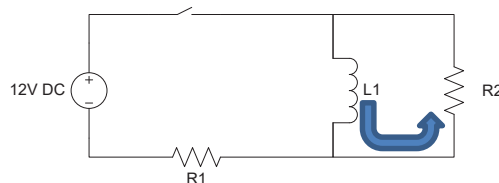


Figure 33 Simple Inductive Circuit

In Figure 26, the switch is initially closed and the inductor has fully charged through resistor R1. Assume $\tau = 4s$. At time = 0, the switch is opened. The inductor then begins to discharge through R2. Figure 27 shows the voltage that would be seen at the inductor. As you can see from the graph, the inductor voltage starts at the negative source voltage and gradually discharges to zero.

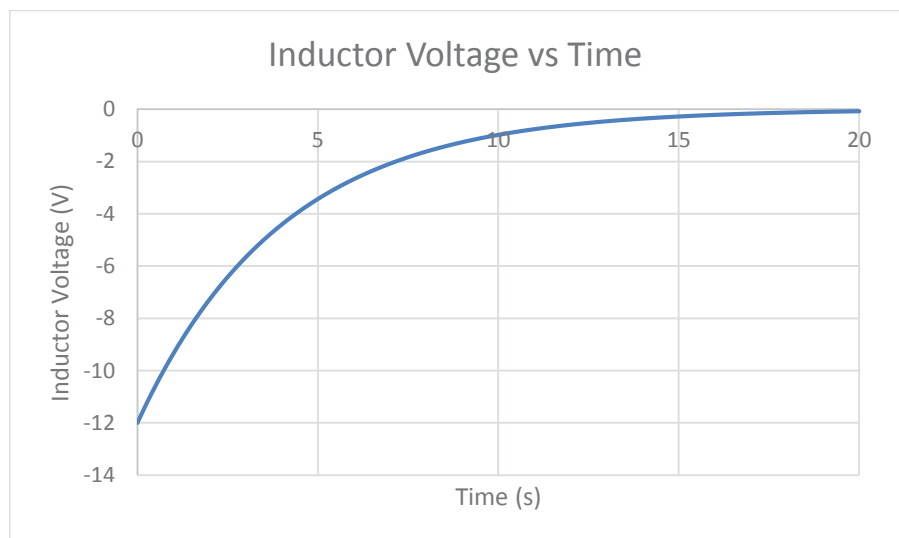


Figure 34 Voltage vs Time for a Capacitor Discharging



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Current will keep flowing in the same direction through the inductor, so the bottom terminal of inductor L1 becomes the positive voltage source. See the blue arrow in Figure 26. Note however that the direction of current through resistor R2 is in the opposite direction.

Inductor voltage over time can be determined using the following equation:

$$V_L = -V_S e^{\frac{-t}{\tau}}$$

Where:

V_C is the inductor voltage

V_S is the source voltage

t is the time after the switch closing

τ is the discharge time constant

After 5 time constants, or 20 seconds, the voltage is approximately the steady state value. For an inductor, the time constant is:

$$\tau_L = \frac{L}{R}$$

Where:

τ is the time constant for the inductor

L is the inductance

R is the resistance in the inductor discharge path

Current through the inductor has a similar exponential decay behavior. The inductor is acting as the source for the discharge circuit. As the source voltage decays, the current through the circuit also decays exponentially to zero. Assume that the total current is 6A and $\tau = 4s$.



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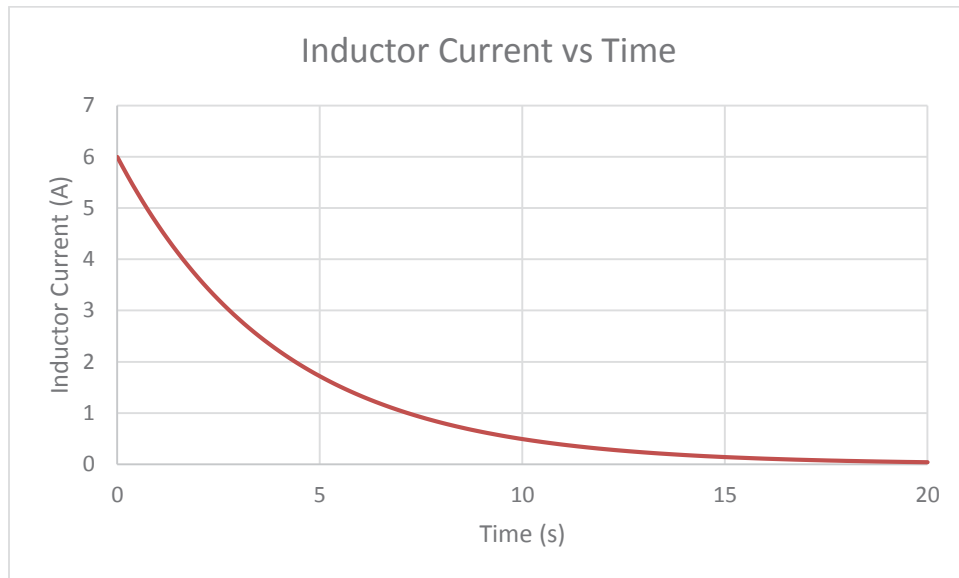


Figure 35 Inductor Current vs Time for a Discharging Inductor

Inductor current over time can be determined using the following equation:

$$I_L = I_{Max} e^{-\frac{t}{\tau}}$$

Where:

I_L is the inductor current

I_{Max} is the maximum current

t is the time after the switch opening

τ is the time constant

After 5 time constants, the current is approximately the steady state value of zero.

Imaginary Numbers

In order to describe impedance, we need to cover imaginary numbers. The imaginary number j is used commonly in electrical engineering, where $j = \sqrt{-1}$. To simplify things as far as impedance is concerned, we will say that resistors are real and capacitors and inductors are imaginary. Figure 36 shows the Real-Imaginary plane.



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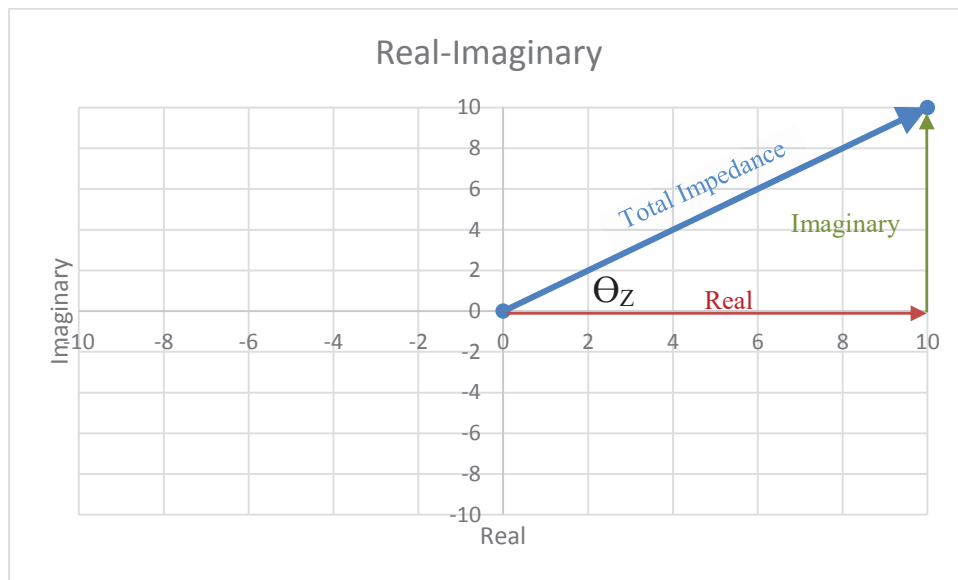


Figure 36 Real-Imaginary Plane

Figure 36 represents the total impedance of a circuit as shown by the blue line. The red line on the x-axis represents the real component of the impedance. The green line represents the imaginary component of the impedance. The angle between the real and imaginary components (Θ_z) is called the impedance angle.

Impedance

Just as resistance is the opposition to current flow in a DC circuit, impedance is the opposition to current flow in an AC circuit. There are some additional factors to consider for AC circuits. Capacitors and inductors create phase shifts in the current which reduces the effective current flow. This effect is called reactance. Impedance is the algebraic sum of resistance, capacitive reactance, and inductive reactance taking into account their respective real and imaginary components.

Resistance in an AC circuit is the same as a DC circuit.

To calculate the total resistance of a series circuit, simply add all of the resistors.

$$\text{Total Resistance } (R_T) = \sum_{x=1}^n R_x = R_1 + R_2 + \dots + R_n$$



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To calculate the resistance of a parallel circuit, the resistors add as inverses.

$$\text{Total Resistance } (R_T) = \frac{1}{\sum_{x=1}^n \frac{1}{R_x}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

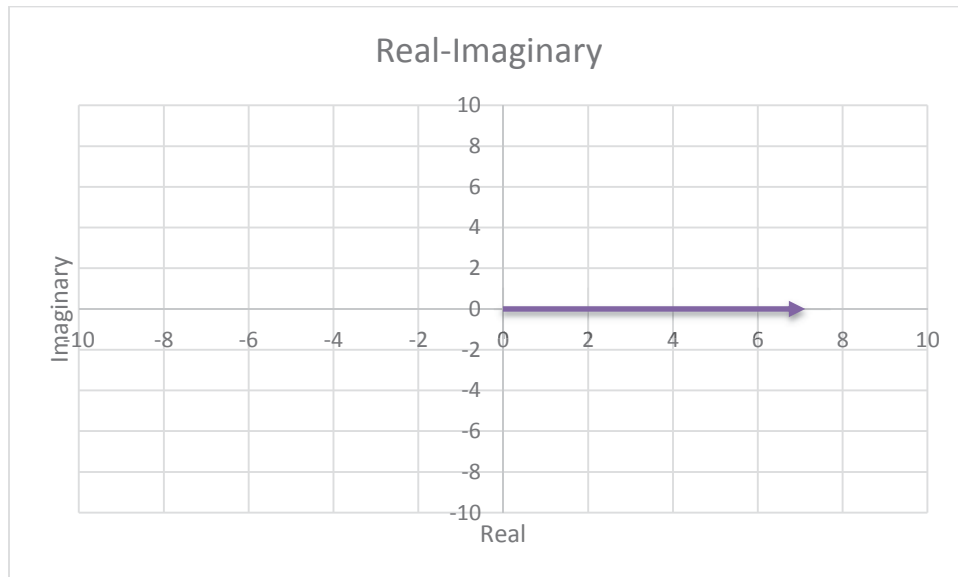


Figure 37 Purely Resistive Load

Figure 37 shows that a purely resistive load has only a real element. There is no imaginary contribution.

Capacitive reactance (X_C)

$$X_C = \frac{1}{\omega C}$$

Where:

ω is the angular frequency in radians

$$\omega = 2\pi f$$

Therefore:

$$X_C = \frac{1}{2\pi f C}$$



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Capacitive impedance (Z_C)

$$Z_C = \frac{1}{j\omega C}$$
$$Z_C = \frac{X_C}{j}$$
$$Z_C = \frac{X_C}{j} * \frac{j}{j}$$
$$Z_C = -jX_C$$

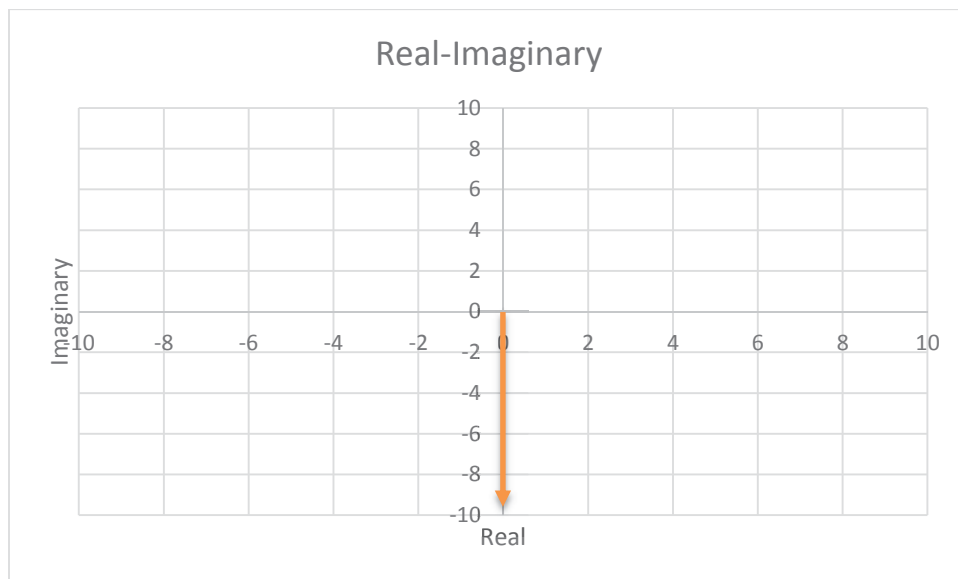


Figure 38 Purely Capacitive Load

Figure 38 shows that a purely capacitive load has only an imaginary magnitude and direction. The capacitive impedance ($-jX_C$) tells us that capacitors will be in the negative imaginary direction ($-j$).

Capacitors add the opposite of resistors. To add the capacitors of a series circuit, add the inverses.

$$Total\ Impedance\ (Z_T) = \frac{1}{\sum_{x=1}^n \frac{1}{Z_{C_x}}} = \frac{1}{\frac{1}{Z_{C1}} + \frac{1}{Z_{C2}} + \dots + \frac{1}{Z_{Cn}}}$$



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To calculate the impedance of a parallel circuit of capacitors, the capacitors values are summed.

$$\text{Total Impedance } (Z_T) = \sum_{x=1}^n Z_{C_x} = Z_{C1} + Z_{C2} + \dots + Z_{Cn}$$

Inductive reactance (X_L)

$$X_L = \omega L$$

Where:

ω is the angular frequency in radians

$$\omega = 2\pi f$$

Therefore:

$$X_L = 2\pi fL$$

Inductive impedance (Z_L)

$$Z_L = j\omega L$$

$$Z_L = jX_L$$

Inductors add the same as resistors. For a series circuit:

$$\text{Total Impedance } (Z_T) = \sum_{x=1}^n Z_{L_x} = Z_{L1} + Z_{L2} + \dots + Z_{Ln}$$

To calculate the impedance of a parallel circuit of inductors, the inductors add as inverses.

$$\text{Total Impedance } (Z_T) = \frac{1}{\sum_{x=1}^n \frac{1}{Z_{L_x}}} = \frac{1}{\frac{1}{Z_{L1}} + \frac{1}{Z_{L2}} + \dots + \frac{1}{Z_{Ln}}}$$



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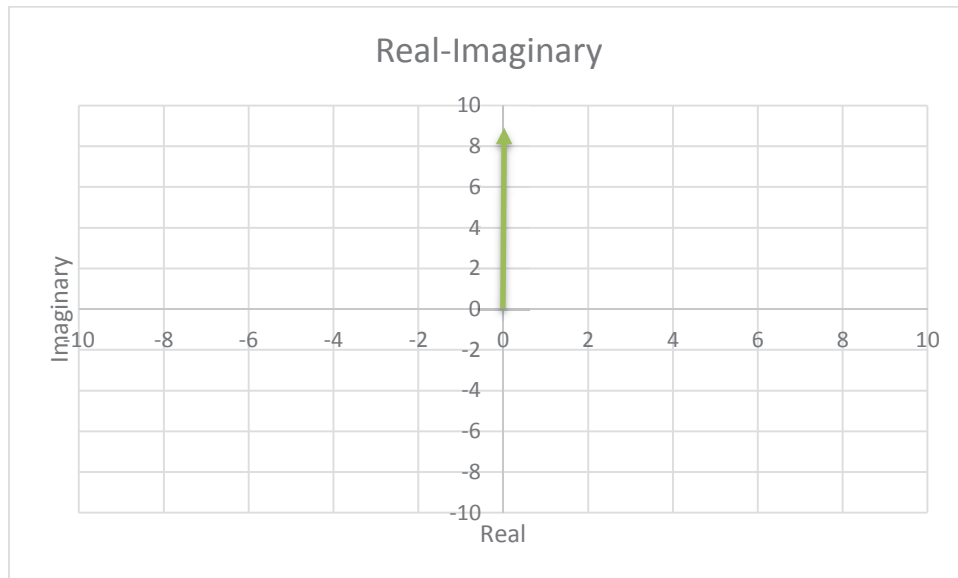


Figure 39 Purely Inductive Load

Figure 39 shows that a purely inductive load has only an imaginary magnitude and direction. The inductive impedance (jX_L) tells us that inductors will be in the positive imaginary direction ($+j$).

Total Impedance Example

Example: Given the following. Frequency = 60Hz, $R_1 = 15\Omega$, $C_1 = 530.5\mu\text{F}$, and $L_1 = 23.88\text{mH}$. Determine the total impedance of the circuit.

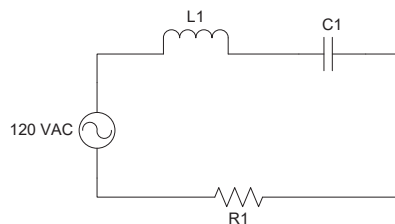


Figure 40 RLC Circuit

1. Calculate the capacitive reactance.

$$X_C = \frac{1}{\omega C}$$



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$$X_C = \frac{1}{2\pi fC}$$

$$X_C = \frac{1}{2\pi(60\text{Hz})(530.5\mu\text{F})}$$

$$X_C = 5\Omega$$

2. Calculate the inductive reactance.

$$X_L = \omega L$$

$$X_L = 2\pi fL$$

$$X_L = 2\pi(60\text{Hz})(23.88\text{mH})$$

$$X_L = 9\Omega$$

3. Calculate the total impedance.

$$Z_T = R_1 + Z_{C1} + Z_{L1}$$

$$Z_T = 15\Omega + (-j5\Omega) + j9\Omega$$

$$Z_T = 15\Omega + j4\Omega$$

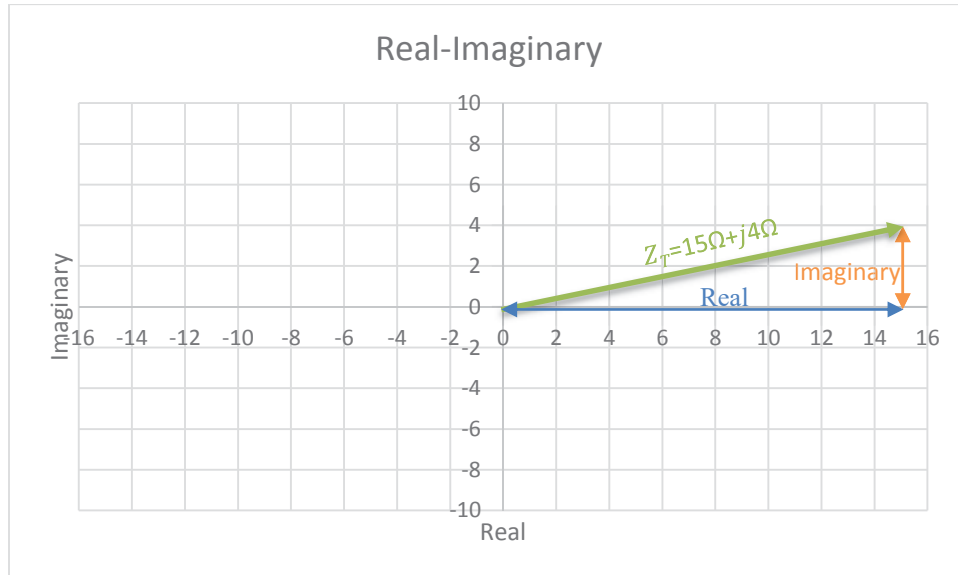


Figure 41 Total Impedance



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Applying Ohm's Law to Complex Circuits

As you recall, Ohm's law provides the relationship between Voltage, Current, and Resistance. Ohm's law can be used for circuits containing inductors and capacitors as well.

Assume the voltage source is at 0 degrees phase shift, so $\theta_v = 0$.

Expressing voltage as a sine wave:

$$V_{RMS}(t) = \frac{V_{peak}}{\sqrt{2}} * \sin(\omega t + \theta_v)$$

If $\theta_v = 0$, then

$$V(t) = \frac{V_{peak}}{\sqrt{2}} * \sin(\omega t)$$

Expressing voltage as a phasor:

$$V_{RMS}(t) = \frac{V_{peak}}{\sqrt{2}} * e^{j(\omega t + \theta_v)}$$

or

$$\bar{V} = \frac{V_{peak}}{\sqrt{2}} \angle \theta_v$$

Expressing current as a sine wave:

$$I_{RMS}(t) = \frac{I_{peak}}{\sqrt{2}} * \sin(\omega t + \theta_i)$$

Expressing voltage as a phasor:

$$I_{RMS}(t) = \frac{I_{peak}}{\sqrt{2}} * e^{j(\omega t + \theta_i)}$$

or

$$\bar{I} = \frac{I_{peak}}{\sqrt{2}} \angle \theta_i$$



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Expressing impedance by combining voltage and current phasors:

$$V = I * Z$$

$$Z = \frac{\bar{V}}{\bar{I}}$$

$$Z = \frac{\frac{V_{peak}}{\sqrt{2}} < \theta_v}{\frac{I_{peak}}{\sqrt{2}} < \theta_i}$$

$$Z = \frac{V_{peak} < \theta_v}{I_{peak} < \theta_i}$$

$$Z = \frac{V_{peak}}{I_{peak}} < (\theta_v - \theta_i)$$

$$Z = \frac{V_{peak}}{I_{peak}} < \theta_z$$

If you have impedance in rectangular form, you can convert it to the polar form above.

$$Z = a + jb$$

$$|Z| = \sqrt{a^2 + b^2}$$

$$\theta_z = \tan^{-1} \frac{b}{a}$$

$$Z = |Z| < \theta_z$$

Let's use the impedance from the previous example, $Z_T = 15\Omega + j4\Omega$.

$$Z = 15\Omega + j4\Omega$$

$$|Z| = \sqrt{15^2 + 4^2}$$

$$|Z| = 15.52\Omega$$

$$\theta_z = \tan^{-1} \frac{4}{15}$$

$$\theta_z = 14.93^\circ$$

$$Z = 15.52\Omega < 14.93^\circ$$

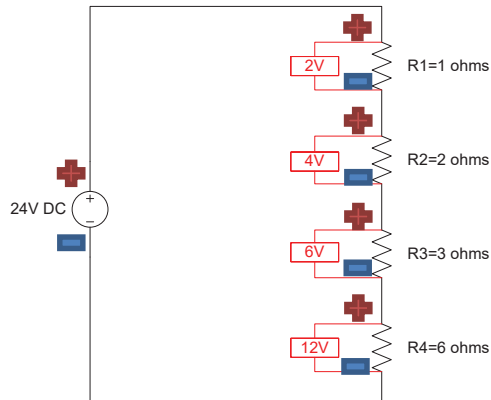


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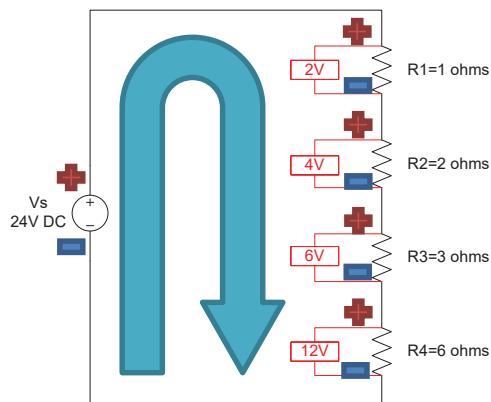
Kirchoff's Laws

Kirchoff's Laws are used in circuit analysis to determine the voltage and current at a specific point in a circuit.

1. Kirchoff's Voltage Law - The algebraic sum of the voltage in any closed loop is equal to zero.
 - a. Assign polarities to all of the circuit elements. The source is assigned a positive value where the current leaves the source. The other elements are assigned a positive where current flows in.



- b. Pick a point on a wire. Starting from the tail of the blue arrow, move around the loop.



$$-V_S + V_{R1} + V_{R2} + V_{R3} + V_{R4} = 0$$



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c. Using Ohm's Law, substitute the current through the circuit.

$$\begin{aligned}
 -V_S + I * R_1 + I * R_2 + I * R_3 + I * R_4 &= 0 \\
 -24V + I * 1\Omega + I * 2\Omega + I * 3\Omega + I * 6\Omega &= 0 \\
 I * (1\Omega + 2\Omega + 3\Omega + 6\Omega) &= 24V \\
 I &= \frac{24V}{12\Omega} \\
 I &= 2A
 \end{aligned}$$

d. Plug the current value in to find: V_{R1} , V_{R2} , V_{R3} , V_{R4} .

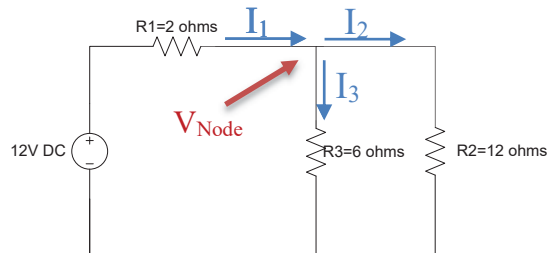
$V_{R1} = I * R_1$	$V_{R2} = I * R_2$	$V_{R3} = I * R_3$	$V_{R4} = I * R_4$
$V_{R1} = 2A * 1\Omega$	$V_{R2} = 2A * 2\Omega$	$V_{R3} = 2A * 3\Omega$	$V_{R4} = 2A * 6\Omega$
$V_{R1} = 2V$	$V_{R2} = 4V$	$V_{R3} = 6V$	$V_{R4} = 12V$

e. Verify Kirchoff's Voltage Law:

$$\begin{aligned}
 -V_S + V_{R1} + V_{R2} + V_{R3} + V_{R4} &= 0 \\
 -24V + 2V + 4V + 6V + 12V &= 0 \\
 -24V + 24V &= 0
 \end{aligned}$$

2. Kirchoff's Current Law - The algebraic sum of the currents entering and leaving a node is equal to zero.

a. Assume a direction for current flow at a node.



b. Write an equation based on the direction of the arrows:

Sum of currents entering node = Sum of currents leaving node

$$I_1 = I_2 + I_3$$

c. Pick the node to be a voltage called V_{Node} .

d. Determine each of the currents using Ohm's law.

$$I_1 = \frac{V}{R}$$



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$$I_1 = \frac{\text{Voltage on left side of } R_1 - \text{voltage on right side of } R_1}{R_1}$$

$$I_1 = \frac{12V - V_{Node}}{R_1}$$

$$I_2 = \frac{V}{R}$$

$$I_2 = \frac{\text{Voltage on top side of } R_2 - \text{voltage on bottom side of } R_2}{R_2}$$

$$I_2 = \frac{V_{Node} - 0}{R_2}$$

$$I_2 = \frac{V_{Node}}{R_2}$$

$$I_3 = \frac{V}{R}$$

$$I_3 = \frac{\text{Voltage on top side of } R_3 - \text{voltage on bottom side of } R_3}{R_3}$$

$$I_3 = \frac{V_{Node} - 0}{R_3}$$

$$I_3 = \frac{V_{Node}}{R_3}$$

- e. Plug in the current equations into Kirchoff's Current Law to solve for the node voltage.

$$I_1 = I_2 + I_3$$

$$\frac{12V - V_{Node}}{R_1} = \frac{V_{Node}}{R_2} + \frac{V_{Node}}{R_3}$$

$$\frac{12V - V_{Node}}{2\Omega} = \frac{V_{Node}}{12\Omega} + \frac{V_{Node}}{6\Omega}$$

$$12V - V_{Node} = 2\Omega \left(\frac{V_{Node}}{12\Omega} + \frac{V_{Node}}{6\Omega} \right)$$

$$12V = V_{Node} + 2\Omega \left(\frac{V_{Node}}{4\Omega} \right)$$

$$12V = V_{Node} + \left(\frac{V_{Node}}{2} \right)$$



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$$12V = \frac{3}{2}V_{Node}$$
$$V_{Node} = 8V$$

- f. Plug the node voltage back into the original Ohm's law equations to find each current.

$$I_1 = \frac{12V - V_{Node}}{R_1}$$
$$I_1 = \frac{12V - 8V}{2\Omega}$$
$$I_1 = 2A$$

$$I_2 = \frac{V_{Node}}{R_2}$$
$$I_2 = \frac{8V}{12\Omega}$$
$$I_2 = \frac{2}{3}A$$

$$I_3 = \frac{V_{Node}}{R_3}$$
$$I_3 = \frac{8V}{6\Omega}$$
$$I_3 = \frac{4}{3}A$$

- g. Verify Kirchoff's Current Law

$$I_1 = I_2 + I_3$$
$$2A = \frac{2}{3}A + \frac{4}{3}A$$
$$2A = \left(\frac{2}{3} + \frac{4}{3}\right)A$$
$$2A = \left(\frac{6}{3}\right)A$$
$$2A = 2A$$



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Summary

Kirchoff's Laws and Ohm's Law can be used to determine the voltages and currents at any point in a circuit. An understanding of the basic circuit elements and the principles of AC and DC sources provides the basis for understanding electrical theory.

If you have any recommendations for this course or would like to see a specific electrical course, please contact me.