A SunCam online continuing education course

Introduction to Compound Channel Flow Analysis for Floodplains

by

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Course Summary

Natural and restored waterways do not usually posses the typical cross-sectional areas undergraduate engineering students spend the majority of their time studying in hydraulics courses. Rather, these waterways are generally compound channels consisting of a main channel and floodplains of varying geometry and roughness. This course covers multiple compound channel analysis methods including the Colebatch method, Cox method, Horton method, Krishnamurthy and Christensen method, Lotter method, Pavlovskii method, segmented conveyance method, and Yen methods. The differences in these methods, results, and typical application of these methods are discussed in the course material. Additionally, alternative methods used by some hydraulic software programs are introduced.

Learning Objectives

After completing this course participants should be able to:

1. Recall basic hydraulic definitions required for further discussion of composite and compound channel analysis.

2. Understand the differences between various methods for subdividing an open channel cross-section.

3. Apply eight different composite and compound channel analysis methods to an open channel of their choosing.

4. Understand the range of solutions different composite and compound channel analysis methods typically provide.

5. Recognize why many hydraulic software programs use additional methods more complex than the methods presented in this course.

Introduction

Most natural and restored waterways are not comprised of simple cross-sectional areas such as rectangles, trapezoids, triangles, or simple curved shapes. Most natural waterways, at a minimum, are composed of a main channel and one or more adjacent flood plains. Such channels are referred to as compound channels since they are made up of more than one of these basic geometric shapes. During low and normal flows water remains entirely in the main channel (Figure 1A). During high flows water fills the entire main channel and proceeds to spill over into the flood plain, or overbank, areas (Figure 1B).

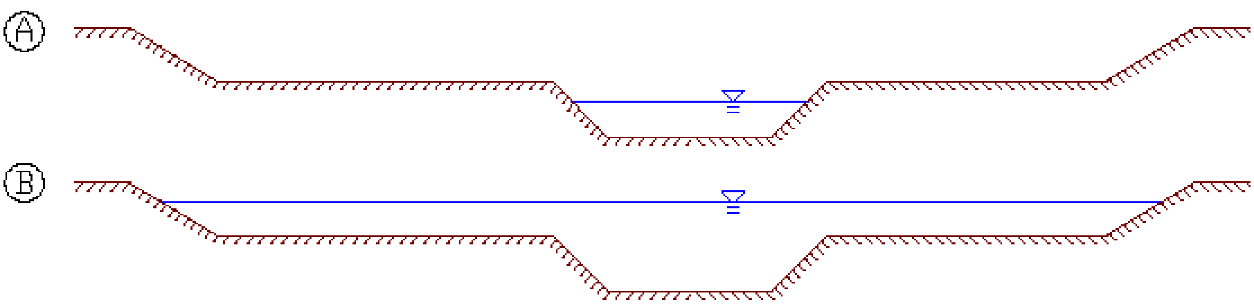


Figure 1: Idealized Cross-Section of a Natural Waterway during (A) Low/Normal Flow and (B) High Flow

In most instances the surface of the main channel provides less frictional resistance than the surfaces of flood plains since often main channels consist of bare soil or rock and most flood plain surfaces consist of grass, brush, and/or trees. A channel that has varying frictional surface resistance in a given cross-section is said to be a composite channel. In nature many channels are composite compound channels.

Basic Hydraulic Review

Before jumping into a discussion about the analysis of compound channels a brief overview of open channel flow is probably warranted. At the very least this will provide clear definitions of terms used later in the course. At most it may briefly summarize terms and equations you have not used or thought about in years.

Open Channel Flow Classification

Flow in open channels may be categorized by relating the flow to either time or space. There are two classifications of flow related to time. Steady flow is flow where the discharge (sometimes referred to as flow rate) and water depth in the section of interest does not change over time (during the time period of interest). Unsteady flow is where the discharge or water depth in the section of interest changes over time (again during the time period of interest). Additionally there are two classifications of flow related to space. Uniform flow occurs when the discharge and water depth are the same over the entire length of interest. Varied flow occurs when the discharge and/or water depth are different over the entire length of interest.

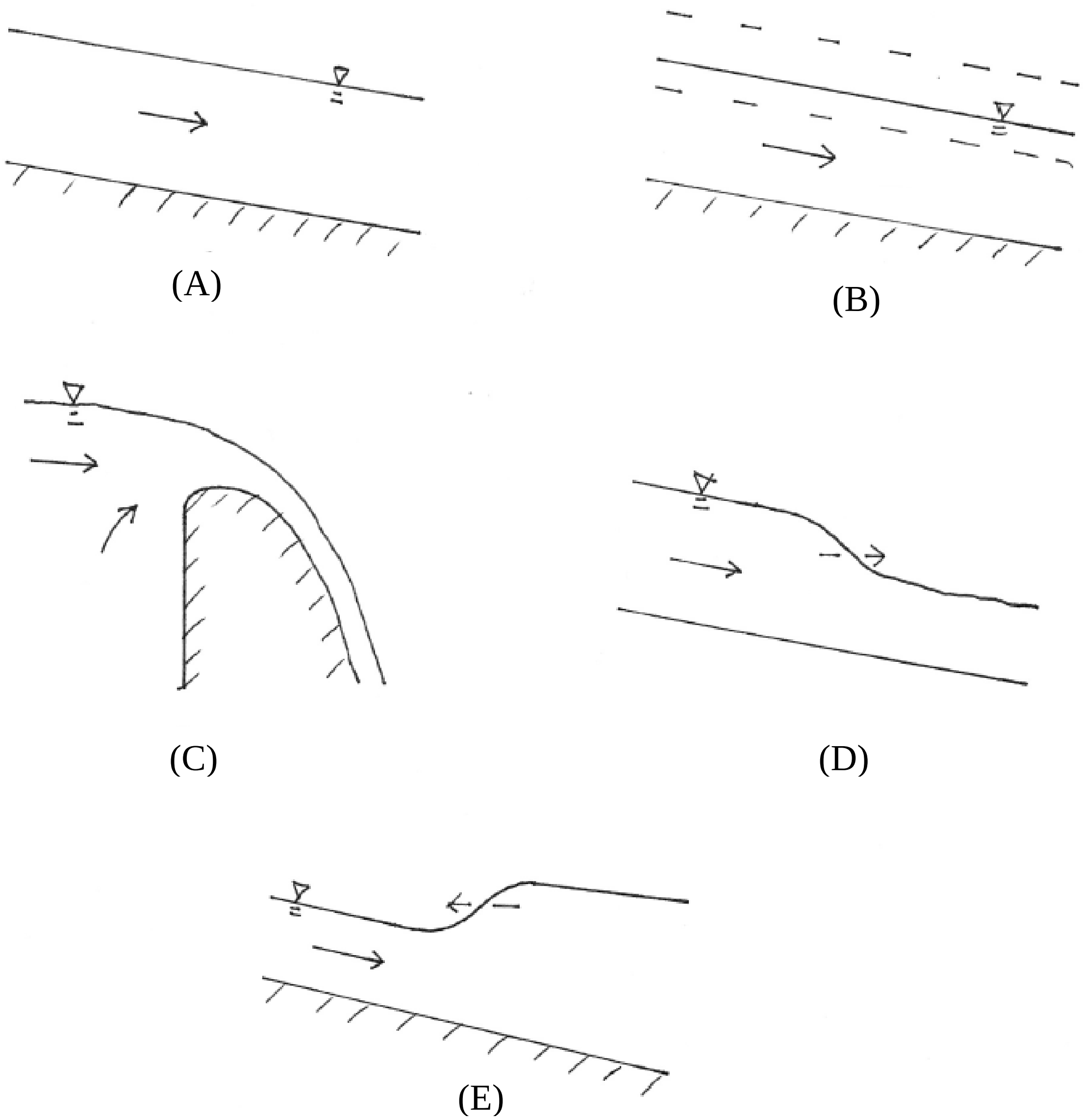


Figure 2: Open Channel Flow Classification: (A) Uniform Flow, (B) Unsteady Flow, Uniform Flow (C) Steady, Varied Flow, (D) Unsteady, Varied Flow, and (E) Unsteady, Varied Flow.

Grade Lines

In open channel flow the hydraulic grade line (HGL) and the water surface elevation are one and the same. The HGL is the sum of the elevation head and water depth. The energy grade line (EGL), again for the specific condition on open channel flow, is the sum of the elevation head, water depth, and velocity head.

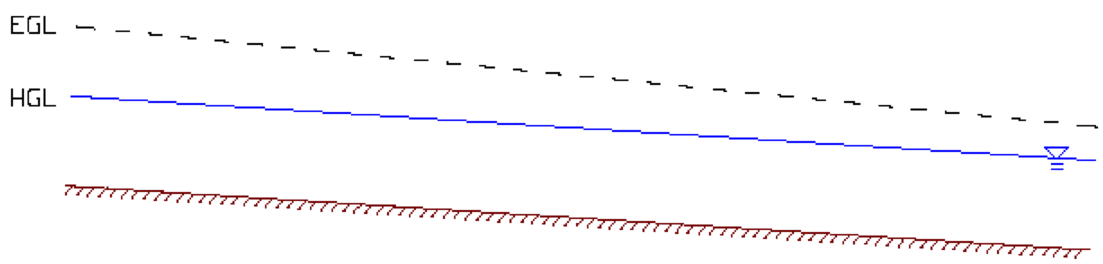


Figure 3: HGL and EGL for an Open Channel

Uniform Flow

More precisely, uniform flow occurs in a channel or channel section when both of the following conditions are met:

1. The water depth, flow area, discharge/flow rate, and velocity distribution remain constant throughout the entire channel section of interest.

2. The EGL (friction slope or S_f), HGL (water surface or S_{ws}), and channel bottom (S) are all parallel to each other.

Unfortunately uniform flow rarely occurs in a natural setting. As a result other analyses methods have been developed and are discussed later in his course, however often uniform flow is assumed for ease of design in certain situations.

Hydraulic Radius

The hydraulic radius is defined as the water cross-sectional, *A*, area divided by the wetted perimeter, *P*. The water cross-sectional area is simply the area of a given cross-section comprised of water and the wetted perimeter is the distance over which the water cross-section is in contact with a surface other than air.

The equation for calculating the hydraulic radius is given below.

$$R = \frac{A}{P} \quad (Equation\ 1)$$

Where: R = Hydraulic radius
A = Water cross-sectional area
P = Wetted perimeter

For example in Figure 4 below the water cross-sectional area is equal to 20 ft^2 and the wetted perimeter is 14 ft. As a result the hydraulic radius can be calculated as 1.43 ft.

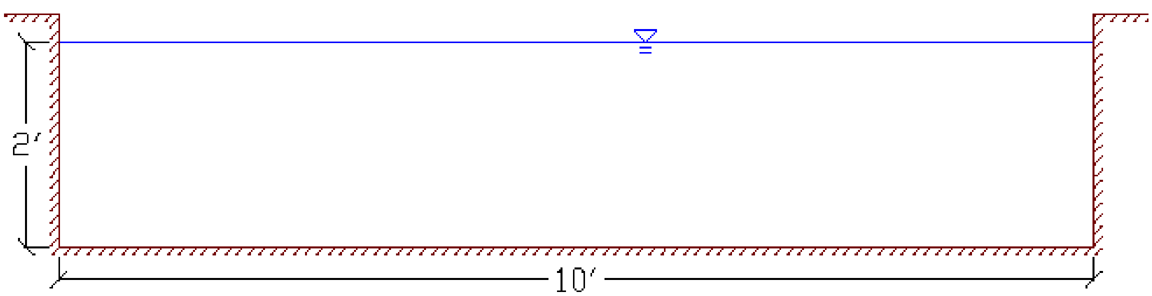


Figure 4: Rectangular Open Channel Cross-Section

The applicable equations for calculating the water cross-sectional areas, wetted perimeters, and top widths of different common channel shapes are provided in Figure 5.

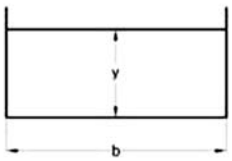
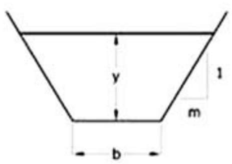
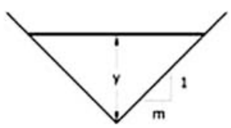
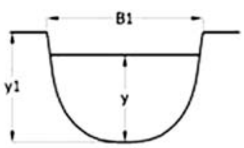
Section	Area, A	Wetted Perimeter, P	Top Width, B
Rectangular 	by	$b + 2y$	b
Trapezoidal 	$y(b + my)$	$b + 2y(1 + m^2)^{1/2}$	$b + 2my$
Triangular 	my^2	$2y(1 + m^2)^{1/2}$	$2my$
Circular  $\theta = 2\cos^{-1}\left[1 - 2\left(\frac{y}{d}\right)\right]$	$\frac{(\theta - sin\theta)d^2}{8}$	$\frac{\theta d}{2}$	$d \sin\left(\frac{\theta}{2}\right)$
Parabolic  $x = \frac{4y}{B}$	$\frac{2By}{3}$	$\left(\frac{B}{2}\right)\left[(1 + x^2)^{1/2} + \left(\frac{1}{x}\right)\ln\left(x + (1 + x^2)^{1/2}\right)\right]$	$B_1\left(\frac{y}{y_1}\right)^{1/2}$

Figure 5: Area, Wetted Perimeter, and Top Width for Common Channel Shapes

Mean Velocity

For many hydraulic computations the mean velocity for a given cross-section of water is assumed to be the discharge divided by the water cross-sectional area.

$$V = \frac{Q}{A} \quad (Equation\ 2)$$

Where: V = Mean velocity
Q = Discharge
A = Water cross-sectional area

Shear Velocity

Shear velocity is a way to describe the shear stress between the bottom of a channel and the fluid moving over it in terms of velocity units (distance over time). For uniform flow it is commonly calculated using the following equation.

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{\gamma RS}{\rho}} = \sqrt{gRS} \quad (Equation\ 3)$$

Where: u_* = Shear velocity
τ_0 = Shear stress at the bottom of the channel
ρ = Fluid density
γ = Fluid specific weight
R = Hydraulic radius
S = Bed slope
g = Gravitational acceleration

Manning Formula

Many different friction formulas have been developed for uniform open channel flow over the years. Common formulas include the Manning formula, Kutter formula, Hazen-Williams formula, and multiple variations of the Darcy-Weisbach formula. In the United States the Manning formula is most widely used so it will be used exclusively throughout this course. The formula is appropriate for use in uniform flow conditions and is given below.

$$V = \frac{K_n}{n} R^{2/3} S^{1/2} \qquad (Equation\ 4)$$

Where: V = Mean velocity, in ft/s [m/s]

K_n = Coefficient equal to 1.49 when R is in ft and V is in ft/s and equal to 1.0 when R is in m and V is in m/s

n = Manning's Roughness Coefficient, a dimensionless coefficient generally obtained from a table

R = Hydraulic radius, in ft [m]

S = Bed slope, in ft/ft [m/m]

EXAMPLE:

Calculate the mean velocity, in ft/s, for uniform flow in the channel shown in Figure 4, a bed slope of 0.01 ft/ft, and a Manning's Roughness Coefficient of 0.013.

SOLUTION:

$$V = \frac{K_n}{n} R^{2/3} S^{1/2} = \frac{1.49}{0.013}\left(\frac{20}{14}\right)^{2/3} (0.01)^{1/2} = 14.5\ ft/s$$

Table 1 provides minimum, typical, and maximum values of Manning's Roughness Coefficient, n, for a variety of channel and stream types. Inspection of the table shows that n values are not always constant. Weathering, vegetation changes, the time of year (primarily due to vegetation changes), flow depth, etc can influence and subsequently change the value. For natural channels experience in conjunction with a comprehensive table of n values and ranges is probably the best method for estimating a value for n since roughness coefficients often vary between cross-sections and field studies are often cost prohibitive except for during very largest projects.

Channel/Stream Description	Minimum	Typical	Maximum
Lined or Built-Up Channels			
Concrete, trowel finish	0.011	0.013	0.015
Concrete, float finish	0.013	0.015	0.016
Concrete, unfinished	0.014	0.017	0.020
Masonry, cemented rubble	0.017	0.025	0.030
Masonry, dry rubble	0.023	0.032	0.035
Excavated or Dredged Channels			
Earth, straight and uniform			
Clean, recently completed	0.016	0.018	0.020
Clean, after weathering	0.018	0.022	0.025
With short grass, few weeds	0.022	0.027	0.033
Earth, winding and sluggish			
No vegetation	0.023	0.025	0.030
Grass, some weeds	0.025	0.030	0.033
Dense weeds or aquatic plants in deep channels	0.030	0.035	0.040
Earth bottom and rubble sides	0.028	0.030	0.035
Stony bottom and weedy banks	0.025	0.035	0.040
Cobble bottom and clean sides	0.030	0.040	0.050
Channels not maintained, weeds and brush uncut			
Dense weeds, high as flow depth	0.050	0.080	0.120
Clean bottom brush on sides	0.040	0.050	0.080
Natural Streams			
Minor streams (top width at flood stage < 100 ft)			
Clean, straight, full stage, no rifts or deep pools	0.025	0.030	0.330
Same as above, but more stones and weeds	0.030	0.035	0.040
Clean, winding, some pools and shoals	0.033	0.040	0.045
Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
Flood Plains			
Pasture, no brush, short grass	0.025	0.030	0.035
Pasture, no brush, high grass	0.030	0.035	0.050
Scattered brush, heavy weeds	0.035	0.050	0.070
Light brush and trees, in winter	0.035	0.050	0.060
Light brush and trees, in summer	0.040	0.060	0.080
Medium to dense brush, in winter	0.045	0.070	0.110
Medium to dense brush, in summer	0.070	0.100	0.160
Cleared land with tree stummps, no sprouts	0.030	0.040	0.050
Heavy timber stand, few down trees, little undergrowth, flood stage below branches	0.080	0.100	0.120
Major streams (top width at flood stage > 100 ft)	Less than for similar minor streams		

Table 1: Values of Manning's Roughness Coefficient *n* (selected values from Sturm Table 4-1)

It should be noted that *n* can also change value over time due to the formation and subsequent disappearance of bed forms (irregularities in the bottom of the channel). The presence and change in size of bed forms is the result of sediment movement caused by flow changes. In this course we will assume *n* remains constant.

Pictures of natural channels and flood plains along with their estimated or calculated *n* values can be a useful tool to help engineers estimate the *n* value(s) of a composite channel or compound channel subareas being analyzed, especially if the engineer does not have much previous field experience. Some government agencies, most notably the Unites States Geological Survey (USGS) and Federal Highway Administration (FHA), post example pictures and *n* values on their websites or provide publications on the topic. A number of web links and sample pictures are included below.

Channels

USGS (for Western United States):
http://wwwrcamnl.wr.usgs.gov/sws/fieldmethods/Indirects/nvalues/index.htm

USGS Water Supply Paper 1849 *Roughness Characteristics of Natural Streams*:
http://pubs.usgs.gov/wsp/wsp_1849/

Flood Plains

FHWA-TS-84-204/USGS Water Supply Paper 2339 *Guide for Selecting Manning's Roughness Coefficients for Natural Channels and Flood Plains*:
http://www.fhwa.dot.gov/bridge/wsp2339.pdf

Figure 6: West Fork Bitterroot River near Connor, Montana. n = 0.036, mean depth = 4.7 ft, mean velocity = 7.70 ft/s. Channel Description – Bed composed of gravel and boulders with d_{50} = 172 mm and d_{84} = 265 mm, left bank is lined with brush and right bank is lined with tress (courtesy of the U.S. Geological Survey).

Figure 7: Mission Creek near Cashmere, Washington. n = 0.057, mean depth = 1.53 ft, mean velocity = 3.92 ft/s. Channel Description – Bed composed of angular-shaped boulders as great as 1 ft in diameter, banks are lined on both sides with brush (courtesy of the U.S. Geological Survey).

Figure 8: Bayou de Lourte near Farmerville, Louisiana. Flood plain n = 0.11. Flood plain description – Large, tall trees including oak, gum, ironwood, and pine; sparse ground cover and undergrowth (courtesy of the U.S. Geological Survey).

When personal experience is lacking in a given situation a step-by-step procedure may be used for either a channel or flood plain. This procedure estimates an n value using the following equation.

$$n = (n_b + n_1 + n_2 + n_3 + n_4)m \quad (Equation\ 5)$$

Where: n = Manning's Roughness Coefficient
n_b = The base value for a straight, uniform, smooth channel
n_1 = Correction for surface irregularities
n_2 = Correction for variations in the shape and size of the channel cross-section
n_3 = Correction for obstructions
n_4 = Correction for vegetation and flow conditions
m = Correction factor for channel meandering

The base value and correction factors can be obtained from the FHA/USGS publication *Guide for Selecting Manning's Roughness Coefficients for Natural Channels and Flood Plains* mentioned previously.

EXAMPLE:

Estimate the *n* value for the Middle Oconee River, near Athens Georgia on April 17, 1959. The bed is sand and gravel (1.0 mm median diameter). The plan view and four cross-sections of the river reach are shown below. The picture provided was taken at location 1175 in the upstream direction.

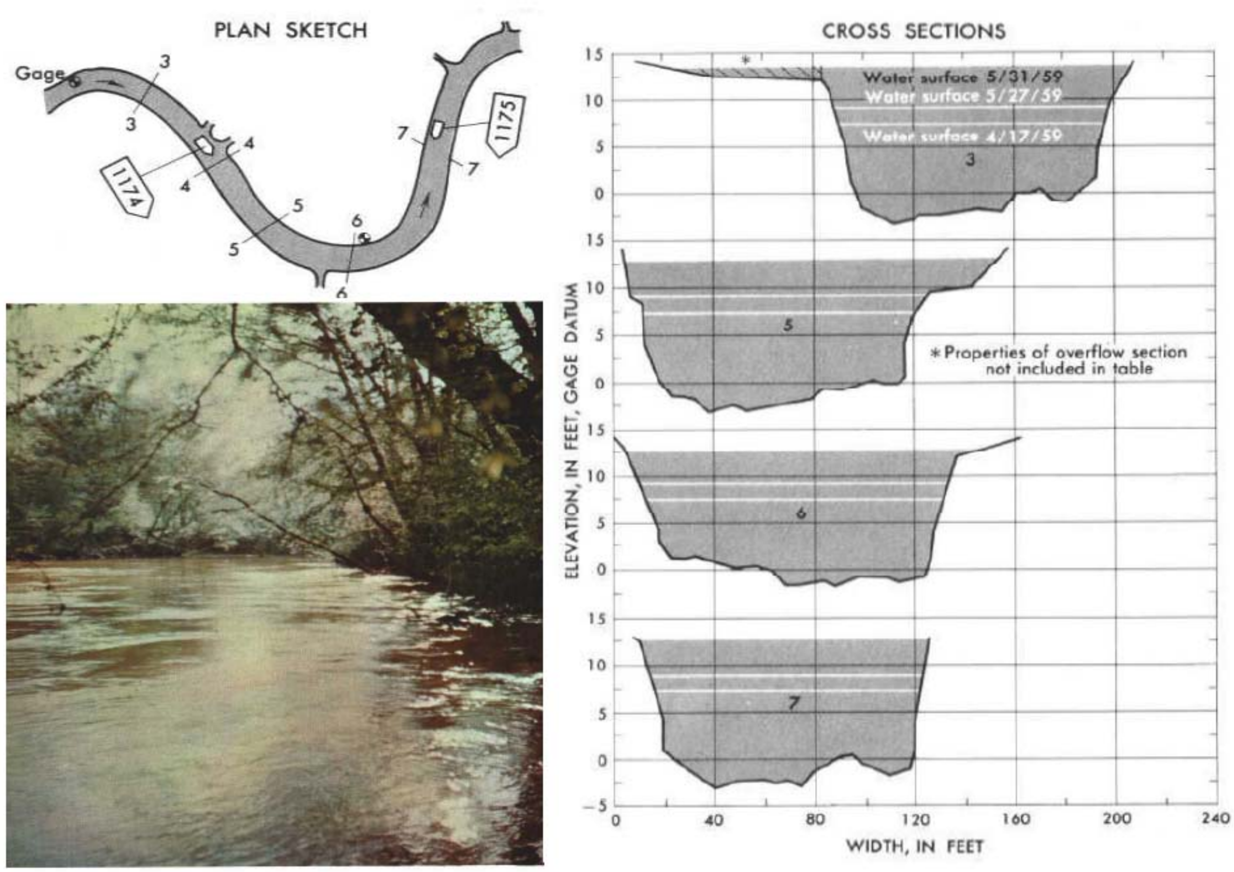


Figure 9: Middle Oconee River Plan, Cross-Sections, and Photo (courtesy of the U.S. Geological Survey).

SOLUTION:

Using the tables in the FHA/USGS publication *Guide for Selecting Manning's Roughness Coefficients for Natural Channels and Flood Plains* determine the values of n_b, n_1, n_2, n_3, n_4, and m.

n_b = 0.026 (from Table 1)
n_1 = 0 (select "Smooth" from Table 2 since there is no indication of substantial bank erosion)
n_2 = 0.010 (select "Alternating frequently" from Table 2 since cross-sections indicate the main flow channel alternates from side-to-side)
n_3 = 0 (select "Negligible" from Table 2 since there is little evidence in the photo to indicate substantial obstructions, though this is difficult to verify without field observations)
n_4 = 0 (since no indication of vegetation over the majority of the wetted perimeter can be observed and since the depth of flow is high relative to any vegetation that may be present)
m = 1.30 (select "Severe" from Table 2, since the ratio of channel length, the straight blue line in Figure 10, to valley length, the curved blue line in Figure 10, is greater than 1.5)

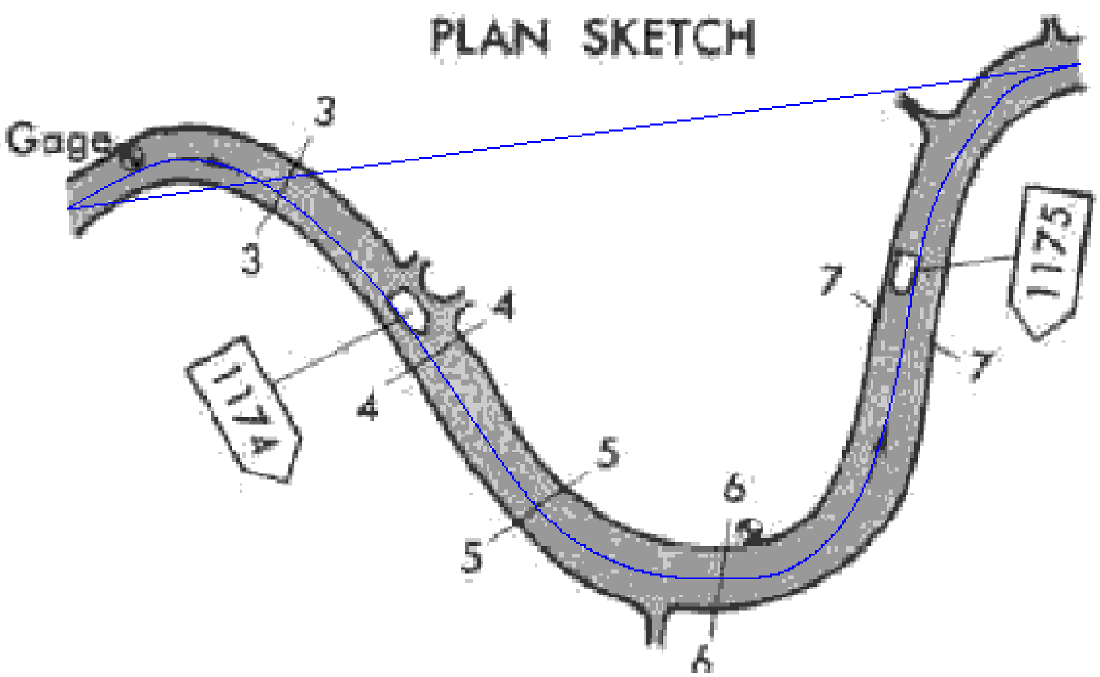


Figure 10: Channel Length and Valley Length Lines (courtesy of the U.S. Geological Survey).

Then plug these values into Equation 5.

$$n = (n_b + n_1 + n_2 + n_3 + n_4)m = (0.026 + 0 + 0.010 + 0 + 0)(1.30) = 0.047$$

The n value calculated by the USGS (using much more information than available to us) was 0.044. Thus our estimation is within seven percent of their estimated value.

Conveyance

Conveyance, which is a measure of the carrying capacity of a channel or channel subarea, is often a convenient term to use in open channel flow calculations using the Manning formula. It combines the channel geometry and roughness into a single parameter. Conveyance is defined in Equation 6.

$$K = \frac{K_n}{n} A R^{2/3} \quad (Equation\ 6)$$

Where: K = Conveyance, in English units [S.I. units]
K_n = Coefficient equal to 1.49 when R is in ft and V is in ft/s and equal to 1.0 when R is in m and V is in m/s
n = Manning's Roughness Coefficient [dimensionless]
R = Hydraulic radius, in ft [m]
A = Water cross-sectional area, in ft^2 [m^2]

Discharge can then be calculated using Equation 7.

$$Q = K S_f^{1/2} \quad (Equation\ 7)$$

Where: Q = Discharge, in ft^3/s [m^3/s]
K = Conveyance, in English units [S.I. units]
S_f = Friction slope, which is the slope of the EGL, in ft/ft [m/m]

Open Channel Cross-Section Subdivision

Various experts have suggested many different methods for subdividing composite and/or compound channels into subareas. Generally subdivision is recommended if either of the following is true.

1. The ratio of overbank width to depth is greater than 5 or

2. The ratio of main channel depth to overbank depth is greater than 2

Samples of these various proposed subdivision methods include:

1. Extending vertical lines from some or all geometric break point to the water surface and extending vertical lines from every location where surface roughness changes to the water surface (Figure 11A).

2. Extending main channel geometry lines to the water surface, which in many cases is the extension of a diagonal line. This attempts to make the dividing lines as near as possible to the zero-shear surfaces (Figure 11B).

3. Extending the horizontal line of the flood plain across the main channel to create a main lower main channel and upper flood channel (Figure 11C).

4. Bisecting some or all angles at geometric break points and all locations of surface roughness change (Figure 11D).

5. Various combinations of the above methods.

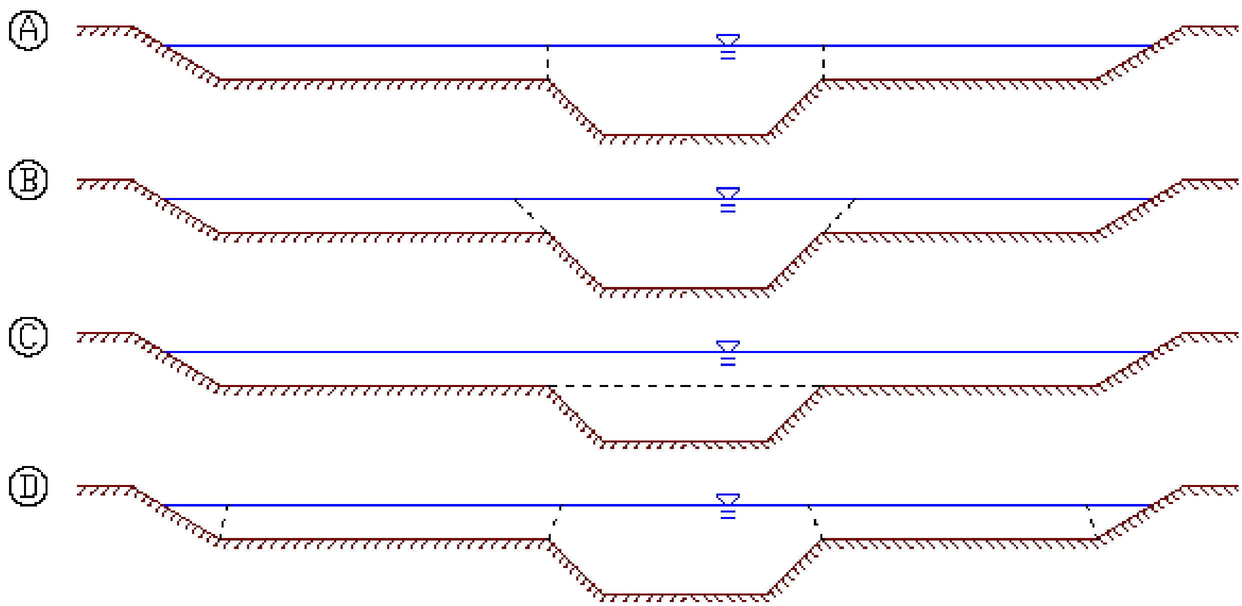


Figure 11: Typical Channel Cross-Section Subdivision Methods (Note: This figure assumes the entire main channel and the entire area of each flood plain each have a single surface roughness value.)

Usually internal water interfaces between subareas are not considered part of the wetted perimeter for any of the subareas. This approach suggests internal shear stresses at the interfaces are zero, though this is only true, or nearly true, in certain circumstances. Additionally the main channel is often not a common geometric shape and must be further broken into subareas such as in Figure 12.

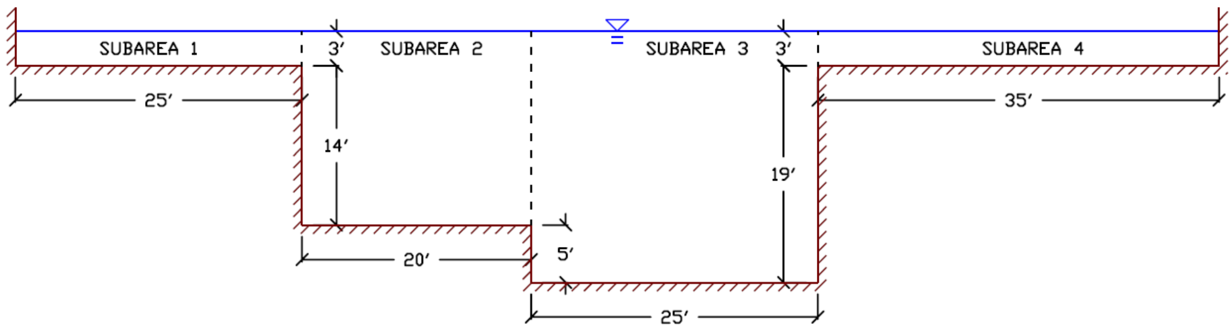


Figure 12: Example Subdivision of Main Channel

The wetted perimeters in Figure 12 would therefore be 28 ft, 34 ft, 49 ft, and 38 ft for subareas 1, 2, 3, and 4 respectively.

Later in the course we will compare the results of using different subdivision methods on the same compound channel. There is no universally accepted opinion about what subdivision method is best, however a number of separate studies have suggested diagonal subdivision may result in the smallest error. As is often the case, engineering judgment must be used on a case by case basis.

Compound and Composite Channel Analysis Methods

Over time many different formulas have been developed for analyzing composite and compound channels. Yen lists 17 different equations for calculating a composite roughness coefficient in his 2002 paper entitled "Open Channel Flow Resistance". The variety and magnitude of these different methods is due to varying assumptions regarding the relationship between discharges, velocities, shear stresses, and forces between the subareas. This course covers seven different methods for calculating a composite roughness coefficient. The methods include the Pavlovskii method, Lotter method, Horton method, Colebatch method, Krishnamurthy and Christensen method, Cox method, and Yen methods.

Some educators and practitioners believe these composite roughness coefficient methods should primarily be used for non-compound channels where the channel beds and sides are made of dissimilar materials or for main channels with oddly shaped geometry and differing roughness coefficients. Additionally they suggest a composite roughness may be used for compound channels where there are not significant differences in roughness between the main channel and flood plains. In most compound channel instances these engineers instead suggest it is more appropriate to use the segmented conveyance method. As a result this course will also present this method.

Each method discussed will show an example calculation for the exact same channel. Later results for a different channel will be provided to help show the variability and differences between the methods. The channel geometry used for the example calculations is shown in Figure 13. The bed slope for the channel reach of interest is 0.001 ft/ft, the main channel is a clean, straight natural stream with no rifts or deep pools, and the floodplains are covered with scattered brush and heavy weeds.

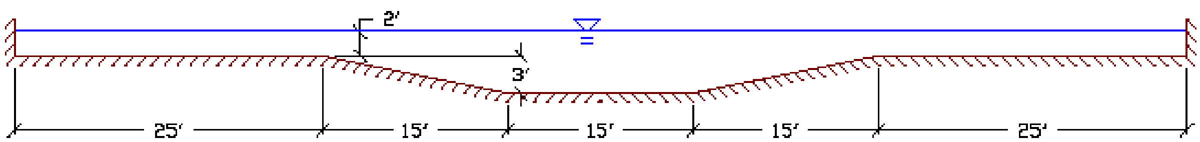


Figure 13: Channel Geometry for Compound Channel Analysis Example

Pavlovskii Method

The Pavlovskii method was developed in the 1930s by Nikolai Nikolaevich Pavlovskii, a Russian engineer and professor. Its derivation was based on the assumption that the total channel resistance force to flow is equal to the sum of subarea resistance forces. Unlike some later methods, no weighting scheme is used. The method may be used for both irregularly shaped open channels and irregularly shaped closed channels.

The method calculates a composite Manning's Roughness Coefficient for a compound channel using the following formula.

$$n_c = \sqrt{\frac{\sum_1^N (P_N n_N^2)}{P}} = \sqrt{\frac{P_1 n_1^2 + P_2 n_2^2 + \cdots + P_N n_N^2}{P}} \qquad (Equation\ 8)$$

Where: n_c = Composite Manning's Roughness Coefficient
P = Wetted perimeter
n = Manning's Roughness Coefficient
N = Subscripts denoting individual subareas of the entire compound channel section

EXAMPLE SOLUTION:

Using "typical" values from Table 1 select Manning's Roughness Coefficients for the main channel and floodplains. Choose values of 0.030 and 0.050 respectively. Next we must decide how to subdivide, or break, the cross-section up into subareas. For this example we will use two vertical lines 25 ft inside the outer edge of each floodplain resulting in a main channel with a wetted perimeter of 45.59 ft and two floodplains each with a wetted perimeter of 27 ft. The overall compound channel wetted perimeter is 99.59 ft. The floodplain on the left will be considered subarea 1, the main channel will be subarea 2, and the floodplain on the right will be subarea 3. The resulting composite Manning's Roughness Coefficient is calculated as follows:

$$n_c = \sqrt{\frac{\sum_1^N (P_N n_N^2)}{P}} = \sqrt{\frac{(27)(0.050)^2 + (45.59)(0.030)^2 + (27)(0.050)^2}{99.59}} = 0.042$$

Next the mean velocity is calculated using the Manning formula.

$$V = \frac{K_n}{n_c} R^{2/3} S^{1/2} = \frac{1.49}{0.042}\left(\frac{280}{99.59}\right)^{2/3} (0.001)^{1/2} = 2.23\ ft/s$$

Finally the discharge is calculated using Equation 2 and solving for Q.

$$Q = AV = (280)(2.23) = 624\ ft^3/s$$

Note: Rounding was carried out on each step shown above for ease of hand calculation. Similar rounding was used for all examples in the course. Therefore use of a spreadsheet or hydraulic software program generally results in slightly different values than shown.

Lotter Method

The Lotter method was developed in the 1930s by G. K. Lotter. Its derivation was based on the concept that total channel discharge equals the sum of subarea discharges. The method is normally used for irregularly shaped open channels such as natural floodplains.

$$n_c = \frac{PR^{5/3}}{\sum_1^N \left(\frac{P_N R_N^{5/3}}{n_N}\right)} = \frac{PR^{5/3}}{\frac{P_1 R_1^{5/3}}{n_1} + \frac{P_2 R_2^{5/3}}{n_2} + \cdots + \frac{P_N R_N^{5/3}}{n_N}} \qquad (Equation\ 9)$$

Where:
- n_c = Composite Manning's Roughness Coefficient
- P = Wetted perimeter
- R = Hydraulic radius
- n = Manning's Roughness Coefficient
- N = Subscripts denoting individual subareas of the entire compound channel section

EXAMPLE SOLUTION:

Using the same Manning's Roughness Coefficients and channel subdivision as previously discussed for this example results in a main channel water cross-sectional area of 180 ft^2 and two floodplains each with water cross-sectional areas of 50 ft^2. The overall compound channel water cross-sectional area is 280 ft^2. The water cross-sectional areas and wetted perimeters can be used to calculate the hydraulic radius for each subarea and the entire channel. The resulting composite Manning's Roughness Coefficient, mean velocity, and discharge are calculated as follows:

$$n_c = \frac{PR^{5/3}}{\sum_1^N \left(\frac{P_N R_N^{5/3}}{n_N}\right)} = \frac{(99.59)(\frac{280}{99.59})^{5/3}}{\frac{(27)(\frac{50}{27})^{5/3}}{(0.050)} + \frac{(45.59)(\frac{180}{45.59})^{5/3}}{(0.030)} + \frac{(27)(\frac{50}{27})^{5/3}}{(0.050)}} = 0.031$$

$$V = \frac{K_n}{n_c} R^{2/3} S^{1/2} = \frac{1.49}{0.031}\left(\frac{280}{99.59}\right)^{2/3} (0.001)^{1/2} = 3.03\ ft/s$$

$$Q = AV = (280)(3.03) = 848\ ft^3/s$$

Horton Method

The Horton method was developed in the 1930s by Robert Horton, a noted American hydrologist, soil scientist, and ecologist. He is the namesake of what is today commonly referred to as Horton overland flow. The Horton Method derivation was based on the assumption that the total cross-sectional mean velocity is equal to each and every of the subarea cross-sectional mean velocities. This method was also independently proposed by Hans Albert Einstein, Albert Einstein's son and a highly regarded researcher and professor, around the same time. The method is usually used for irregularly shaped closed channels such as those with arched covers; however it is sometimes used in open channels in instances of steep banks or extremely wide, flat floodplains. Since for many compound channels the assumption that velocities in the main channel and flood plains are equal would be very false, this method should be used carefully.

$$n_c = \left(\frac{\sum_1^N (P_N n_N^{1.5})}{P}\right)^{2/3} = \frac{(P_1 n_1^{1.5} + P_2 n_2^{1.5} + \cdots + P_N n_N^{1.5})^{2/3}}{P^{2/3}} \qquad (Equation\ 10)$$

Where: n_c = Composite Manning's Roughness Coefficient
P = Wetted perimeter
n = Manning's Roughness Coefficient
N = Subscripts denoting individual subareas of the entire compound channel section

EXAMPLE SOLUTION:

Using the same Manning's Roughness Coefficients and channel subdivision as previously discussed for this example the resulting composite Manning's Roughness Coefficient, mean velocity, and discharge are calculated as follows:

$$n_c = \left(\frac{\sum_1^N (P_N n_N^{1.5})}{P}\right)^{2/3} = \frac{((27)(0.05)^{1.5} + (45.59)(0.03)^{1.5} + (27)(0.05)^{1.5})^{2/3}}{(99.59)^{2/3}} = 0.041$$

$$V = \frac{K_n}{n_c} R^{2/3} S^{1/2} = \frac{1.49}{0.041}\left(\frac{280}{99.59}\right)^{2/3} (0.001)^{1/2} = 2.29\ ft/s$$

$$Q = AV = (280)(2.29) = 641\ ft^3/s$$

Colebatch Method

The Colebatch method was developed in the 1940s by G. T. Colebatch. It is the same as the Horton method except water cross-sectional area is used instead of wetted perimeter in the calculation of a composite Manning's Roughness Coefficient. The method is normally used for irregularly shaped open channels such as natural floodplains.

$$n_c = \left(\frac{\sum_1^N (A_N n_N^{1.5})}{A}\right)^{2/3} = \frac{(A_1 n_1^{1.5} + A_2 n_2^{1.5} + \cdots + A_N n_N^{1.5})^{2/3}}{A^{2/3}} \quad (Equation\ 11)$$

Where: n_c = Composite Manning's Roughness Coefficient
A = Water cross-sectional area
n = Manning's Roughness Coefficient
N = Subscripts denoting individual subareas of the entire compound channel section

EXAMPLE SOLUTION:

Using the same Manning's Roughness Coefficients and channel subdivision as previously discussed for this example the resulting composite Manning's Roughness Coefficient, mean velocity, and discharge are calculated as follows:

$$n_c = \left(\frac{\sum_1^N (A_N n_N^{1.5})}{A}\right)^{2/3} = \frac{\left((50)(0.050)^{1.5} + (180)(0.030)^{1.5} + (50)(0.050)^{1.5}\right)^{2/3}}{(280)^{2/3}} = 0.038$$

$$V = \frac{K_n}{n_c} R^{2/3} S^{1/2} = \frac{1.49}{0.038}\left(\frac{280}{99.59}\right)^{2/3} (0.001)^{1/2} = 2.47\ ft/s$$

$$Q = AV = (280)(2.47) = 692\ ft^3/s$$

Krishnamurthy and Christensen Method

Muthusamy Krishnamurthy, a graduate research assistant, and Professor Bent Christensen at the University of Florida proposed their method in the early 1970s. Their method attempts to take into account the logarithmic velocity distribution in the vertical water column of a channel (i.e. velocity at the water's surface is greatest and decreases logarithmically between the water surface and bottom of the channel).

$$n_c = \exp\left[\frac{\sum_1^N (P_N h_N^{1.5} \ln n_N)}{\sum_1^N (P_N h_N^{1.5})}\right] \quad (Equation\ 12)$$

$$n_c = \exp\left[\frac{P_1 h_1^{1.5} \ln n_1 + P_2 h_2^{1.5} \ln n_2 + \cdots + P_N h_N^{1.5} \ln n_N}{P_1 h_1^{1.5} + P_2 h_2^{1.5} + \cdots + P_N h_N^{1.5}}\right] \quad (Equation\ 12)$$

Where: n_c = Composite Manning's Roughness Coefficient
P = Wetted perimeter
h = Water cross-sectional depth
n = Manning's Roughness Coefficient
N = Subscripts denoting individual subareas of the entire compound channel section

EXAMPLE SOLUTION:

Using the same Manning's Roughness Coefficients and channel subdivision as previously discussed for this example the resulting composite Manning's Roughness Coefficient, mean velocity, and discharge are calculated as follows:

$$n_c = \exp\left[\frac{(27)(2)^{1.5}\ln(0.050) + (45.59)(5)^{1.5}\ln(0.030) + (27)(2)^{1.5}\ln(0.050)}{(27)(2)^{1.5} + (45.59)(5)^{1.5} + (27)(2)^{1.5}}\right] = 0.034$$

$$V = \frac{K_n}{n_c} R^{2/3} S^{1/2} = \frac{1.49}{0.034}\left(\frac{280}{99.59}\right)^{2/3} (0.001)^{1/2} = 2.76\ ft/s$$

$$Q = AV = (280)(2.76) = 773\ ft^3/s$$

Cox Method

The Cox method, also known as the U.S. Army Corps of Engineers Los Angeles District Method, was developed in the 1970s by R. G. Cox for the United States Army Corps of Engineers. Its derivation was based on the idea that total shear velocity should be equal to a weighted sum of subarea shear velocities and in equation form it sums the roughness coefficients of subareas weighted by the corresponding subarea water cross-sectional area. The method is typically used for irregularly shaped open channels such as natural floodplains.

$$n_c = \frac{\sum_1^N (A_N n_N)}{A} = \frac{A_1 n_1 + A_2 n_2 + \cdots + A_N n_N}{A} \quad (Equation\ 13)$$

Where: n_c = Composite Manning's Roughness Coefficient
A = Water cross-sectional area
n = Manning's Roughness Coefficient
N = Subscripts denoting individual subareas of the entire compound channel section

EXAMPLE SOLUTION:

Using the same Manning's Roughness Coefficients and channel subdivision as previously discussed for this example the resulting composite Manning's Roughness Coefficient, mean velocity, and discharge are calculated as follows:

$$n_c = \frac{\sum_1^N (A_N n_N)}{A} = \frac{(50)(0.050) + (180)(0.030) + (50)(0.050)}{280} = 0.037$$

$$V = \frac{K_n}{n_c} R^{2/3} S^{1/2} = \frac{1.49}{0.037}\left(\frac{280}{99.59}\right)^{2/3} (0.001)^{1/2} = 2.54\ ft/s$$

$$Q = AV = (280)(2.54) = 711\ ft^3/s$$

Yen Methods

Ben Chie Yen, a professor at the University of Illinois at Urbana-Champaign, proposed a number of different methods in the early 1990s. A number of the method derivations were based on the premise that total shear velocity should be equal to a weighted sum of subarea shear velocities. Different weighting factors were given to the same basic equation based on various assumptions regarding the relationships between velocities and hydraulic radii of the subdivided areas. This resulted in the hydraulic radii terms being raised to the different powers. Only the equation with hydraulic radii terms raised to the 1/6 power is presented here.

$$n_c = \frac{\sum_1^N \left(\frac{P_N n_N}{R_N^{1/6}}\right)}{\frac{P}{R^{1/6}}} = \frac{\frac{P_1 n_1}{R_1^{1/6}} + \frac{P_2 n_2}{R_2^{1/6}} + \cdots + \frac{P_N n_N}{R_N^{1/6}}}{\frac{P}{R^{1/6}}} \quad (Equation\ 14)$$

Where: n_c = Composite Manning's Roughness Coefficient
P = Wetted perimeter
n = Manning's Roughness Coefficient
R = Hydraulic radius
N = Subscripts denoting individual subareas of the entire compound channel section

EXAMPLE SOLUTION:

Using the same Manning's Roughness Coefficients and channel subdivision as previously discussed for this example the resulting composite Manning's Roughness Coefficient, mean velocity, and discharge are calculated as follows:

$$n_c = \frac{\sum_1^N \left(\frac{P_N n_N}{R_N^{1/6}}\right)}{\frac{P}{R^{1/6}}} = \frac{\frac{(27)(0.050)}{(\frac{50}{27})^{1/6}} + \frac{(45.59)(0.030)}{(\frac{180}{45.59})^{1/6}} + \frac{(27)(0.050)}{(\frac{50}{27})^{1/6}}}{\frac{(99.59)}{(\frac{280}{99.59})^{1/6}}} = 0.042$$

$$V = \frac{K_n}{n_c} R^{2/3} S^{1/2} = \frac{1.49}{0.042}\left(\frac{280}{99.59}\right)^{2/3} (0.001)^{1/2} = 2.23\ ft/s$$

$$Q = AV = (280)(2.23) = 624\ ft^3/s$$

Segmented Conveyance Method

The segmented conveyance method, which is also referred to as the divided-channel method, does not utilize a composite roughness coefficient. When using the Manning formula for compound channel analysis this method is often the recommendation method. It assumes the EGL slope is the same in all cross-sectional subareas. Since continuity requires the discharges of the subareas to equal the total discharge it logically follows that the conveyance of the subareas should equal the total conveyance.

$$K_{channel} = \sum_{1}^{N} \frac{K_n}{n_N} A_N {R_N}^{2/3} = \frac{K_n}{n_1} A_1 {R_1}^{2/3} + \frac{K_n}{n_2} A_2 {R_2}^{2/3} + \cdots + \frac{K_n}{n_N} A_N {R_N}^{2/3} \quad (Equation\ 15)$$

Where: $K_{channel}$ = Conveyance, in English units [S.I. units]
K_n = Coefficient equal to 1.49 when R is in ft and equal to 1.0 when R is in m
n = Manning's Roughness Coefficient [dimensionless]
R = Hydraulic radius, in ft [m]
A = Water cross-sectional area, in ft^2 [m^2]
N = Subscripts denoting individual subareas of the entire compound channel section

Discharge can then be calculated for uniform flow using Equation 16.

$$Q = K_{channel} S^{1/2} \quad (Equation\ 16)$$

Where: Q = Discharge, in ft^3/s [m^3/s]
$K_{channel}$ = Conveyance, in English units [S.I. units]
S = Bed slope, in ft/ft [m/m]

EXAMPLE SOLUTION:

Using the same Manning's Roughness Coefficients and channel subdivision as previously discussed for this example the conveyance and discharge are calculated as follows:

$$K_{channel} = \sum_{1}^{N} \frac{K_n}{n} A R^{2/3}$$

$$K_{channel} = \frac{(1.49)}{(.050)}(50)\left(\frac{50}{27}\right)^{2/3} + \frac{(1.49)}{(.030)}(180)\left(\frac{180}{45.59}\right)^{2/3} + \frac{(1.49)}{(.050)}(50)\left(\frac{50}{27}\right)^{2/3} = 26{,}826$$

$$Q = K_{channel} S^{1/2} = (26{,}826)\ (0.001)^{1/2} = \ 848\ ft^3/s$$

The segmented conveyance method and Lotter method are based on the same principles and in fact their expressions for discharge can be shown to be mathematically identical. The two methods always produce the same discharge value. They are both presented because their forms of input, how they are described conceptually at intermediate steps, and their intermediate results are different.

Method Differences and Application

Table 2 displays the discharge results just calculated and ranks the discharge values from the various methods. Rank 1 denotes the largest discharge and rank 7 the smallest discharge. In this case there was a tie for both the 1st and 7th ranks.

Method	Q [ft^3/s]	Rank
Pavlovskii Method	624	7
Cox Method	711	4
Lotter Method	848	1
Horton Method	641	6
Colebatch Method	692	5
Krishnamurthy and Christensen Method	773	3
Yen Method (with R raised to the 1/6 power)	624	7
Segmented Conveyance Method	848	1

Table 2: Summary of Example Results

The mean discharge for these eight methods is approximately 720 ft^3/s, but the differences in discharge between the methods are substantial. The largest discharge (Lotter method and segmented conveyance method) is over 35 percent larger than smallest discharge (Pavlovskii method and Yen method). One of the first questions this may raise is whether the rank of a given method remains constant. For example does the Lotter method always produce the largest discharge?

One way to answer this question is to calculate the discharges for another composite compound channel using all the methods. The first iteration of these calculations will use the same subdivision method as the example problem (vertical line extension as shown in Figure 11A). A second and third iteration will use two other subdivision methods mentioned previous (diagonal line extension as shown in Figure 11B and horizontal line extension as shown in Figure 11C respectively).

The channel geometry used for these calculations is shown in Figure 14. The bed slope for the channel reach of interest is 0.001 ft/ft. The main channel is a clean, straight natural stream with no rifts or deep pools ($n = 0.030$). The floodplains are covered with a heavy timber stand with few down trees, little undergrowth, and flood stage below the branches ($n = 0.10$). The results of these calculations are shown in Table 3.

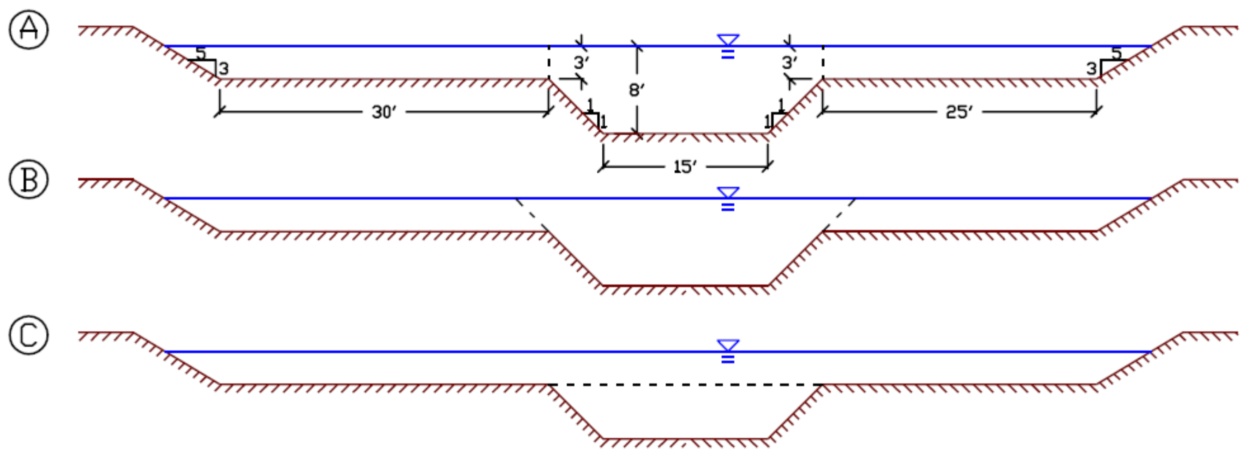


Figure 14: Channel Subdivision for Table 3 Calculation Results

	Subdivision A		Subdivision B		Subdivision C	
Method	**Q [ft^3/s]**	**Rank**	**Q [ft^3/s]**	**Rank**	**Q [ft^3/s]**	**Rank**
Pavlovskii Method	471.0	8	471.0	8	471.0	8
Cox Method	611.6	4	628.6	4	498.9	5
Lotter Method	1,072.6	1	1,138.3	1	651.2	2
Horton Method	487.0	7	487.0	6	487.0	6
Colebatch Method	570.7	5	584.8	5	479.0	7
Krishnamurthy and Christensen Method	882.0	3	882.0	3	717.9	1
Yen Method (with $R^{1/6}$)	489.8	6	486.5	7	510.5	4
Segmented Conveyance Method	1,072.6	1	1,138.3	1	651.2	2

Table 3: Summary of Results for Different Subdivision Methods

We can see that for a different compound channel using the same subdivision method that there are some differences in ranking. The first five rankings remain the same, but the Yen method moves up from 7th to 6th, the Horton method moves down from 6th to 7th, and the Pavlovskii method moves down from 7th to 8th.

Next you may notice that some methods (the Pavlovskii and Horton methods) give the same discharge result regardless of the method of subdivision. For this specific compound channel one method has little variability (the Yen method) and some methods exhibit great variability (the Cox, Lotter, Colebatch, and Krishnamurthy and Christensen methods). In fact for this channel the Lotter Method has a difference of about 75 percent between subdivision methods C and B. In many instances however, the discharge values from the named methodologies produce greater differences than the method of subdivision.

Finally we can see that the method of subdivision seems to make a large difference in the rankings. For example when using subdivision method C the Lotter method no longer produces the largest discharge, the Yen method's ranking ranges from 4 to 7, and only the Pavlovskii keeps the same ranking with all three subdivisions methods.

While results vary depending on the geometry and roughness coefficients some general trends can be seen. The Lotter, segmented conveyance, and Krishnamurthy and Christensen methods often produce larger discharges and the Pavlovskii method often produces smaller discharges.

Just as with different subdivision methods, there is no "best" method for calculating a composite n value for every different channel. If possible engineers should compare the depth and discharge measurements from a stream gage during a recent flooding event to the various methods to try and determine the most accurate method for the channel. Of course that does not mean the same method will be the best method in all reaches of the channel. The USGS does have a large system of gages, but often no gage exists in the desired area. In such cases other indicators, like high water marks or personal observations, along with engineering judgment must be used.

In 1980 Motayed and Krishnamurthy used data from 36 gauged streams in four states to test four different composite n methods, three of which were presented in this course (the Horton method, Lotter method, and Krishnamurthy and Christensen method). They found the mean error between the computed n_c and measured n_c was smallest for the Lotter method. In 2007 Yang, Cao, and Liu studied a single artificial composite compound channel at 50 different discharge rates. They tested five different composite n methods, three of which were presented here (the Cox method, Lotter method, and Krishnamurthy and Christensen method). They also found the mean relative error for n_c was smallest for the Lotter method.

Commercially available software programs are available which can solve for the composite roughness coefficient and conveyance methods. Many of the programs also allow the user to draw irregular channel cross-sections which saves much time compared to manually calculating channel and subdivided channel subarea properties such as hydraulic radii. One such program commonly used is Bentley's FlowMaster. One drawback of these software programs is they often do not allow you options in how to subdivide the channel.

Methods Used in Hydraulic Software Programs

As mentioned previously, uniform flow is not all that common in nature. As a result using the Manning formula, or any other uniform flow formula, is not adequate in many situations. Since natural channels often change cross-section on a continual basis, even occurrences of uniform flow in short sections would require analysis of each separate section. More often than not natural channels exhibit varied flow. While a software program like FlowMaster has the ability to handle varied flow in a composite compound channel (a moderately complicated spreadsheet could also be used) the number of consecutive segments needed to analyze the scope of many engineering studies lends itself to specifically designed hydraulic software.

Most hydraulic software packages currently used for flood modeling are either one-dimensional or two-dimensional models. One-dimensional models can only directly model flow in the longitudinal direction. This means flow movement vertically and side-to-side in the channel is not considered. Geometrically the model consists of cross-sections spread out along the river length. One-dimensional models generally use the concepts of conservation of mass and conservation of momentum to calculate a mean velocity and water depth at each cross-section. Two-dimensional models directly model flow in both the longitudinal and lateral (side-to-side) directions. Geometrically they consist of a continuous surface composed of many finite elements. In theory two-dimensional models are more accurate since they eliminate some of the major simplifying assumptions of one-dimensional models; however the data collection and parameter input effort is much more intensive. A few of the commonly used hydraulic software packages include HEC-RAS, MIKE 11, MIKE 21, and FLO-2D. Since two-dimensional models use small scale finite elements they do not require the use of composite n values. As a result only a brief discussion of HEC-RAS and MIKE 11 are included in this course since when using these, or other programs, it is important to recognize what methods are being used and/or if users are allowed to pick their method of preference.

HEC-RAS

The Army Corps of Engineers' HEC-RAS (Hydrologic Engineering Center River Analysis System) program can be used for one-dimensional steady and unsteady flow hydraulic analysis. Complementing program capabilities include sediment transport and water temperature modeling.

For gradually varied steady flow analysis the energy equation is used to determine the water surface profile. First the cross-section is subdivided in a manner similar to Figure 15. The default method of channel subdivision uses vertical subdivision lines. The only alternate subdivision method also uses vertical subdivision lines but in greater number, thus creating more subareas.

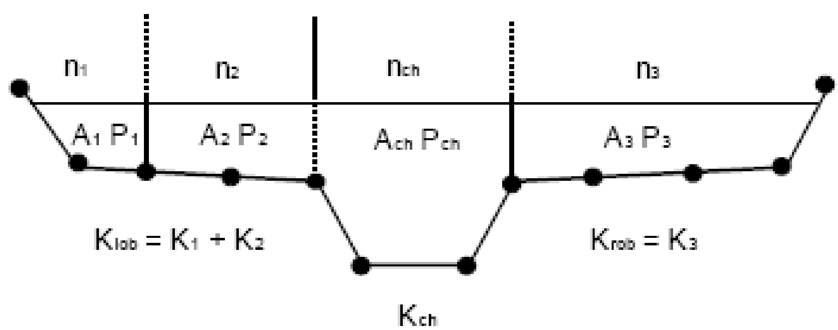


Figure 15: HEC-RAS Default Subdivision Method (courtesy of the U.S. Army Corps of Engineers)

The *K* term in Figure 15 stands for conveyance, which was previously defined. The composite *n* value of the main channel, if required, is calculated using the Horton method. A composite *n* value is not calculated for the entire cross-section prior to calculating the total discharge. Rather the conveyance for the right and left overbank areas is calculated by summing the conveyance of each overbank subarea. Then each overbank, or flood plain, conveyance is added to the conveyance of the main channel to determine the total conveyance. This total conveyance is then used to calculate total discharge. This is an example of the segmented conveyance method described earlier in the course.

For unsteady flow HEC-RAS still uses the concepts of conservation of mass and conservation of momentum, but in the form of partial differential equations. Consequently an implicit finite difference solution method is utilized.

MIKE 11

MIKE 11 is a one-dimensional steady and unsteady flow hydraulic program developed by DHI Water & Environmental. Additional software capabilities include sediment transport and water quality modeling.

MIKE 11 does not have a true steady flow solution methodology. Instead it uses a quasi-steady solution method when needed and otherwise uses an unsteady implicit finite difference methodology. The program also has a quasi-two-dimensional steady flow methodology that computes a composite *n* value using a method not covered in this course.

Conclusion

In recent years more and more sophisticated modeling software has been developed for general river modeling and flood analysis. Likewise the need for data collection and input to feed these models has increased as well. While these models will likely result in continued improvement in our ability to accurately predict flooding events it seems likely there will still be the need for engineers to use some of the basic methodologies developed for composite compound channels either due to lack of input data, lack of time, or lack of funds.

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