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# **Decision Making in Engineering Planning and Design<sup>®</sup>**

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**1.0 Decision Making in Engineering Planning and Design Introduction**

This course illustrates the categories of decision making under conditions of Certainty, Uncertainty, and Risk and some tools as related to decision processes in engineering planning and design. Conditions of Certainty may be assumed as a reference or bench-mark for defining the best possible outcomes, but may also be used as a simplification often used to provide a first level of feasibility checking or for conditions with negligible risk. A first example of decision making under conditions of certainty is taken from the use of Linear Programming (LP) tools to optimize constrained resource allocation to optimize a monetary return. Both a graphical approach and the “Simplex Method” are introduced to illustrate how to identify an optimum. A second example is taken from the use of the Critical Path Method (CPM) as used in determining the planning of a project. Extensions of CPM to include elements of risk and the PERT Method in the decision making under conditions of Risk are deferred until that risk is discussed, but the CPM elements are used as a framework. A short discussion of decision making under conditions of Uncertainty is included with introduction of payoff tables and both optimistic and pessimistic approaches. Because the predominant form of decision making is under conditions of Risk, concepts of probability are discussed including conditional probability, Baye’s Theorem, expected values, the value of research and information, and the extensions of CPM to develop estimates in the PERT scheduling technique. Links are provided in several locations to free, open-source software tools associated with each topic. The tools and techniques are useful both during initial planning and as more information becomes available suggesting plan revision. The single most useful tool is a spreadsheet program and considerable patience to design custom solutions.

**2.0 Quantitative Decision Making in Engineering Design**

Engineering design may be defined as the Art of Applied Science. The term “Art” in Engineering brings qualitative elements of aesthetic principles and that which is appealing, beautiful, or of extraordinary significance. The use of “Science” refers to the systematically accumulated body of knowledge, principles, and laws based on hypotheses, experimentation, observation, and generalization that are the result of the scientific method as applied to the physical world. Much of that body of knowledge is encapsulated in the universal short-hand notation of Mathematics and/or the use of quantitative methods. Constraints are imposed on the design and the process by economic considerations. Quantitative methods are preferred in engineering, but qualitative decisions may also be needed in instances without appropriate quantitative measures. Still, we find it useful to convert qualitative considerations to quantitative considerations to make the decision process understandable and justifiable.



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The engineer begins a design process by defining the qualitative and quantitative initial objectives to be achieved as a result of the design, typically in the form of a set of objectives definitions suitable for initial review for approval as well as for completion comparison to indicate success. Design objectives are met using material and labor resources both in target product or service objectives as well as during the initial design process and the required effort. The design process is evaluated in its “effectiveness” regarding the measure of the design in meeting the objectives and the “efficiency” of the design regarding the resources needs in the target design product or services, and also resources needed to produce a design.

The evaluation of efficiency of the use of resources is a step in the design process both regarding the economics of the resulting outcome, as well as during the decision process of the design effort. The decision-making process can be quantified so that rational decisions are made prior to the expenditures and so that alternatives can be evaluated and optima identified prior to commitment. Much of the decision process is involved during the engineering planning phases but may also be required in stages during a project as information is produced by testing.

Each decision is expressed in terms of the “payoff” consequences (usually either a gain or loss) of the decision and the interaction of 1) the decision itself, with 2) the outcome or event that results from the decision.

### **3. Categories of Conditions of Decision Making**

Broadly, there are three categories of conditions of decision making defined as conditions of Certainty, Risk, and Uncertainty. In the following sections, the usage of all three categories are discussed and examples given.

Decision making under conditions of Certainty imply that factors of the decision are known in advance; outcomes can be quantified in terms of the use of resources and the decision is a comparison and selection made by applying optimization criteria (usually monetary costs or scheduled time expended) to the decision. Decisions made under implied conditions of Certainty are often used to determine the bounding costs associated with the decisions.

Decision making under conditions of Risk imply that the factors of the decision can be assigned a probability; all outcomes can be quantified in terms of the use of resources and the decision is a comparison and selection made by applying optimization criteria (usually “expected value”) to the decision. The assignment of probabilities is generally the most



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problematic issue in risk assessment, but experience can be used to make useful approximations and the quantitative process of using the probabilities is well understood.

Decision making under conditions of Uncertainty imply that few or none of the factors of the decision are known in advance; outcomes can be quantified in terms of the use of resources but the decision is a comparison and selection made by applying different optimization criteria (usually monetary costs or scheduled time expended) to the decision with respect to the expenditure bounds. Decisions made under implied conditions of Uncertainty are also often used to evaluate some of the bounding costs associated with the decisions.

**4. Decision making under conditions of Certainty**

Despite the possibilities of default by suppliers and personnel, contracted suppliers or services are often treated as if the outcomes are certain. For budgetary bounding purposes with appropriate contingency plans, such a treatment is reasonable. It may seem trivial to make decisions under conditions of certainty, but there are a few classical examples that demonstrate some complexities involved.

4.1 Linear Programming (LP) for Optimizing Combinations - For illustration, we introduce a simple hypothetical scenario involving components for assembly of two products. We define the quantity of the first product produced as X and Y as the quantity of the second product produced. We can make no more than 4,000 of X and 6,000 of Y due to absolute capacity limits. We also need a critical component that is constrained by a service that can produce as many as 36,000 units, but 6 are required for each X produced and 4 are required for each Y produced. We earn \$400 for each X produced and \$300 for each Y produced. We know all the relationships with certainty, but we do not know what assignment of resources to X and Y production provides the maximum return.

The constraints are:

$$X \leq 4,000 \quad [4.1]$$

$$Y \leq 6,000 \quad [4.2]$$

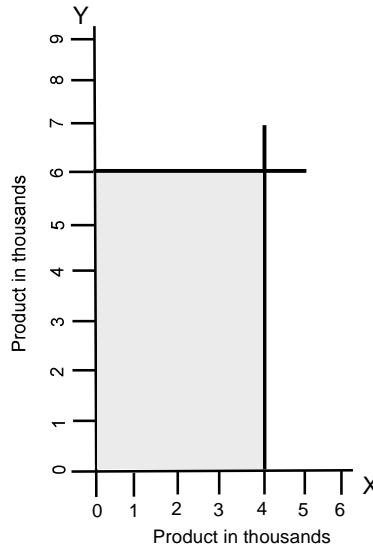
$$6 * X + 4 * Y \leq 36,000 \quad [4.3]$$

$$Return = \$400 * X + \$300 * Y \quad [4.4]$$



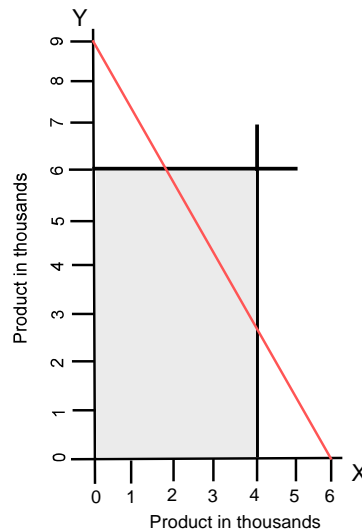
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We plot the relationships initially using the first two constraints to define a “feasible region” (grey) as follows:



**Plot 4.1 – Initial Bounds on a Feasible Region of Solution**

We see that we can produce any combination of X up to 4,000 units and Y up to 6,000 units and thus we have made two upper bounds on a feasible solution.



**Plot 4.2 – Initial Bounds plus Critical Component Feasible Region of Solution**



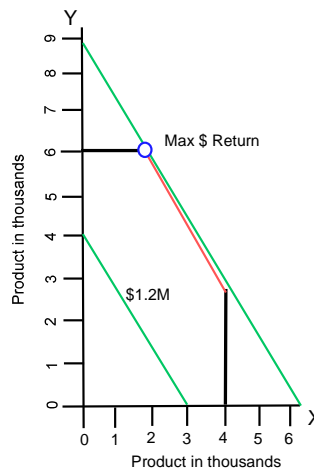
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We now show the upper bound imposed by the critical component. To achieve a formula to plot, we solve for Y in terms of X at the boundary as follows:

$$4 * Y = 36,000 - 6 * X \quad [4.5]$$

$$Y = 9,000 - 1.5 * X \quad [4.6]$$

We identify the two intercepts as X = 0, Y = 9,000 and X = 6,000, Y = 0 with the straight line of combinations between.



**Plot 4.3 –Feasible Region of Solution plus Plots of Returns of \$1.2M and Maximum**

We next plot the equation for return, showing a case with \$1,200 return from the intercepts at X = 0, Return = 4,000 \* \$300 = \$1.2million, and at Y = 0, Return = 3,000 \* \$400 = \$1.2million. We solve the return equation for the slope as follows:

$$\$300 * Y = Return - \$400 * X \quad [4.7]$$

$$Y = \frac{Return}{\$300} - \frac{\$400}{\$300} X = \frac{Return}{\$300} - 1.33 * X \quad [4.8]$$

We do not yet know the solution for the maximum return, but we see that a return equation with the slope of negative 1.33 appears to be nearly parallel with the slope of negative 1.5 for the critical component constraint. We can identify the intersection between the maximum constraint on Y and the constraint on the critical component, substitute for Y, solve for X, and determine the best combination of resources for maximum return as follows:



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$$Y_{optimum} = 9,000 - 1.5 * X_{optimum} = 6,000 \quad [4.9]$$

$$1.5 * X_{optimum} = 3,000 \quad [4.10]$$

$$X_{optimum} = 2,000 \quad [4.11]$$

$$Return_{optimum} = \$400 * X_{optimum} + \$300 * Y_{optimum} \quad [4.12]$$

$$Return_{optimum} = \$400 * 2,000 + \$300 * 6,000 \quad [4.13]$$

$$Return_{optimum} = \$800,000 + \$1,800,000 = \$2,600,000 \quad [4.14]$$

The graphical approach shown above for solving such problems is useful for illustrating the principles, but it is impractical for larger sets of constraints. We will employ the “Simplex Method” for larger problems.

We use the same constraints from the example above, but employ the Simplex method to solve. As a first step, we introduce two “slack” variables in the quantities of each product produced to reduce the inequalities to equalities as follows:

$$X + S_X = 4,000 \quad [4.15]$$

$$Y + S_Y = 6,000 \quad [4.16]$$

The result is that both X and Y inequality equations are augmented with the new unknown slack quantities,  $S_X$  and  $S_Y$  respectively to eliminate the inequalities. Likewise, we can modify the critical component constraint as follows:

$$6 * X + 4 * Y + S_C = 36,000 \quad [4.17]$$

This third “slack” variable  $S_C$  represents the amount of unused critical components.

We express the return without a slack variable as before and we wish to maximize return as follows:



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$$\text{Return} = \$400 * X + \$300 * Y \quad [4.18]$$

To illustrate the Simplex Method of solving Linear Programming problems, we construct an initial “tableau” from the linear equations including slack variables as follows:

<b>Tableau 1</b>	<b>X</b>	<b>Y</b>	<b>Sx</b>	<b>Sy</b>	<b>Sc</b>		<b>Quantity</b>	<b>Available</b>
<b>Max X</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>=</b>	<b>4000</b>	
<b>Max Y</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>=</b>	<b>6000</b>	
<b>Critical</b>	<b>6</b>	<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>=</b>	<b>36000</b>	
	<b>400</b>	<b>300</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>=</b>	<b>Return</b>	
<b>Coordinates</b>	<b>0</b>	<b>0</b>	<b>4000</b>	<b>6000</b>	<b>36000</b>			

Each constraint equation is listed, and additionally the return equation. Zero entries are included wherever there is no contribution from a slack variable. No return is realized from any slack variable, so they are all zero contribution to returns. In addition, we list the “coordinates” of the solution to aid a sequence.

The strategy of the Simplex method for maximization is following the successive evaluation of vertices of the feasible region. What makes the method efficient is that the search along vertices uses the marginal return of incremental improvements to direct the search in the direction of increasing returns.

For our example, we can evaluate the tableau starting from the (X = 0, Y = 0) vertex coordinates (corresponds to the lower left corner in the graph) and evaluate the simultaneous equations for Sx = 4,000, Sy = 6,000, and Sc = 36,000 with a result that the return is zero (again, slack variables do not contribute to return). What we see from tableau 2, is that each incremental unit of X contributes \$400 in returns, but each unit of Y only contributes \$300 and we are directed to increase X preferentially.

<b>Tableau 2</b>	<b>X</b>	<b>Y</b>	<b>Sx</b>	<b>Sy</b>	<b>Sc</b>		<b>Quantity</b>	<b>Available</b>
<b>Max X</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>=</b>	<b>4000</b>	<b>4000</b>
<b>Max Y</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>=</b>	<b>6000</b>	
<b>Critical</b>	<b>6</b>	<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>=</b>	<b>36000</b>	<b>6000</b>
	<b>400</b>	<b>300</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>=</b>	<b>Return</b>	
<b>Coordinates</b>	<b>0</b>	<b>0</b>	<b>4000</b>	<b>6000</b>	<b>36000</b>		<b>\$0</b>	





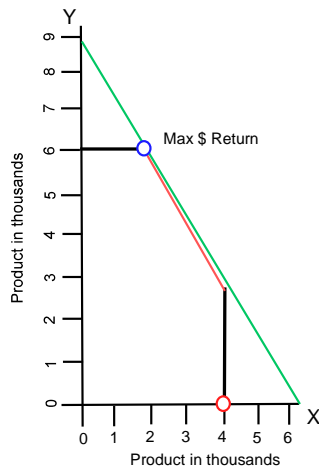
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Because incremental increases in X provide maximal increases in Return, we preferentially increase the coordinate X, but we do not yet know to what extent. To guide the effect of contribution, we add the “Available” units column to evaluate each row that increases with X, and the available number of units available. We see that the Max X row has 4000 units available and the Critical row has 6000 units available. For the Max X row, the relationship is obvious, but for the Critical row, we see that the column for X has a coefficient weighting of 6 and the total “Quantity” allows  $6 * 6,000 = 36,000$  total units.

We proceed to the new vertex as shown below:

<b>Tableau 3</b>	<b>X</b>	<b>Y</b>	<b>Sx</b>	<b>Sy</b>	<b>Sc</b>	<b>Quantity</b>	<b>Available</b>
<b>Max X</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>= 4000</b>	<b>0</b>
<b>Max Y</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>= 6000</b>	<b>6000</b>
<b>Critical</b>	<b>6</b>	<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>= 36000</b>	<b>3000</b>
	<b>400</b>	<b>300</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>= Return</b>	
<b>Coordinates</b>	<b>4000</b>	<b>0</b>	<b>0</b>	<b>6000</b>	<b>12000</b>	<b>\$1,600,000</b>	

For convenience, we show this vertex on the graph at the lower right marked by a red circle.



**Plot 4.4 –Feasible Region, Plot of Maximum, and Tableau 3 Simplex Vertex**

We have traversed from the origin to the lower right vertex by following the maximum rate of return.



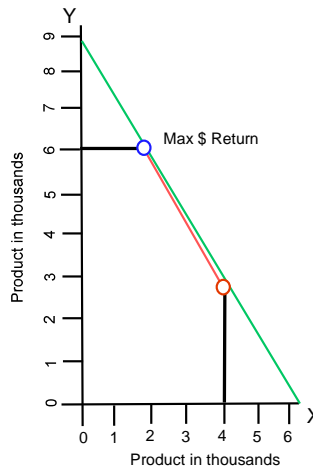
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In tableau 3, we have slack in both  $S_y$  and  $S_c$  with at least 3000 units of Y available limited by the critical component availability.

We proceed to the new vertex as shown below:

<b>Tableau 4</b>	<b>X</b>	<b>Y</b>	<b>S<sub>x</sub></b>	<b>S<sub>y</sub></b>	<b>S<sub>c</sub></b>	<b>Quantity</b>	<b>Available</b>
<b>Max X</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>= 4000</b>	<b>0</b>
<b>Max Y</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>= 6000</b>	<b>3000</b>
<b>Critical</b>	<b>6</b>	<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>= 36000</b>	<b>0</b>
	<b>400</b>	<b>300</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>= Return</b>	
<b>Coordinates</b>	<b>4000</b>	<b>3000</b>	<b>0</b>	<b>3000</b>	<b>0</b>	<b>\$2,500,000</b>	

For convenience, we show the vertex for tableau 4 on the graph marked by a new red circle.



**Plot 4.5 –Feasible Region, Plot of Maximum, and Tableau 4 Simplex Vertex**

We have done well, but there is still room for improvement. At the new ( $X = 4000$ ,  $Y = 3000$ ) vertex, there is still slack left in the  $S_y$  variable, but none in either  $S_x$  or  $S_c$ . We have 3000 units of Y available and the question arises as to whether we can improve the return. If we decrease the slack in Y, we must increase the slack in X. The question we face is the relative benefit of trading one form of slack for the other, and do we generate a marginal increase in return.

For this case, each unit of Y that we utilize (decreasing the available slack 3000 units of Y) generates an additional \$300 in revenue. Each unit of X that we decrease (increasing the



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slack 0 units of X) costs us \$400 in revenue. However, keeping the slack in critical components to zero, we decrease 6 slack units of Y for every 4 slack units of X that we increase. Therefore, we can trade decreasing 3000 Y slack units and increasing 2000 X units. The trade is  $\$300 * 3000 = \$900,000$  gain in Y returns for  $\$400 * 2000 = \$800,000$  cost in X returns and a net benefit of \$100,000 for the trade.

We proceed to the new vertex as shown below:

<b>Tableau 5</b>	<b>X</b>	<b>Y</b>	<b>Sx</b>	<b>Sy</b>	<b>Sc</b>	<b>Quantity</b>	<b>Available</b>
<b>Max X</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>= 4000</b>	<b>2000</b>
<b>Max Y</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>	<b>= 6000</b>	<b>0</b>
<b>Critical</b>	<b>6</b>	<b>4</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>= 36000</b>	<b>0</b>
	<b>400</b>	<b>300</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>= Return</b>	
<b>Coordinates</b>	<b>2000</b>	<b>6000</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>\$2,600,000</b>	

There exists a matrix-manipulation method called “pivoting” that automates the goal-directed search amongst the vertices but it is far too complicated to include here. We have explored the nature of the Simplex Method, its algorithmic basis, and followed its search to find the same optimum presented using the graphical method. The Simplex Method can be used for problems that are too large for the simple graphical approach. Free, Open-Source programs that utilize the Simplex Method are available and a link is provided here to obtain an example from the following website: <http://www.gnu.org/software/glpk/>

Another similar example of decisions under conditions of certainty includes the “Traveling Salesman” problem. With a number of intermediate destinations, the “salesman” is required to plan a route to the succession of destinations to cover each within scheduling constraints imposed but also to minimize travel costs. A mathematical method called “dynamic programming” is used to establish feasible solutions and the decision process is the selection of an optimum among alternative feasible solutions. The Simplex Method has been used also. In Computer Science, the problem is defined as NP-Hard because exact solutions are known but the computational effort grows quickly as the list of destinations increases.

4.2 Scheduling and CPM - Another classical example is the assignment of productive efforts to personnel and equipment to satisfy the desired outcomes and minimize costs. The solution is well understood both for the case of certainty as well as risk. There are two general methods and both employ “graph theory” to determine a solution.



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The first method, known as Critical Path Method (CPM), assumes that all tasks and durations are well known and the issue is purely one of scheduling. Each task is assigned a list of resources required and a set of precedent dependencies that must occur prior to that task initiation. A graph is prepared (or a data structure in a computer program) that determines the sequence of tasks from a beginning to an end. Beginning with tasks that have no precedent requirement, the graph is prepared between task beginning/end events from the start to finish with each task's beginning following completion of precedent task's completion. The graph is traversed twice, first beginning at the start and determining the earliest possible completion at each event between tasks, and second returning from the finish to determine the latest possible start at each event. Those events with the earliest completion and latest start are defined as being on the "Critical Path" through the graph. Tasks between events that are not on the "Critical Path" through the graph have a difference in earliest starts that defines the "slack" associated with scheduling that task. Other dependencies including the availability of resources are used to determine actual scheduled begin/end times for those tasks.

Variations of CPM are often used to determine assignment of personnel to adjust work loading amongst personnel and to evaluate alternative approaches. Typically, the those tasks on the "Critical Path" through the graph are closely evaluated for alternate approaches to shorten the overall project duration as may be required to achieve critical dates.

The similar Program Evaluation and Review Technique (PERT), provides for the assignment of probable minimum, maximum, and most-likely task durations and so makes decisions under conditions of Risk. In PERT, as well as CPM, the "Critical Path" through the graph is identified and managed most carefully because that sequence of tasks determines the completion date. We discuss PERT further later in decisions under conditions of Risk.

4.2.1 Events and Tasks/Activities Lists - Events are denoted on the graph as a circle and are associated with dates. Tasks, or activities, are actions that consume resources and are associated with duration to execute. The CPM chart is a collection of Events (circles) connected by directed line segments representing the tasks or activities.

For CPM, developed by DuPont in the 1950's, the duration of each activity was assumed to be known in advance. For describing many projects, certainty is a reasonable assumption. For instance, the time associated with the "curing" of chemical reactions is reasonably well known as to be assumed to be certain. A construction project that requires concrete to harden



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before forms are removed and/or a load is applied may be considered to require a certain minimum duration either from the specification on the concrete or applicable building codes. Likewise, brewing beer requires a certain minimum duration for fermentation reactions, and such examples are extensive. Projects with more or less uncertain durations may be estimated with a degree of risk and are better describe using PERT illustrated later.

We illustrate by making an extensible list of tasks/activities in Table 4.1 enumerating each and its duration from which the chart is constructed. In addition, we construct an initial list of events that are expected. We illustrate using a patio construction project example as follows:

Task	Title: Patio Project Tasks	Duration: Days
1	Site engineering planning	3
2	Materials purchase	4
3	Site preparation	2
4	Concrete pour, surface, & cure	28
5	BBQ construction & cure	4

**Table 4.1 – Initial List of Task/Activities for CPM Plan**

The following list of 9 events in Table 4.3 illustrates the iterative nature of the planning process because there must be at least eight tasks between nine distinguishable events, so we will construct the plan accordingly and modify as necessary.

Event	Title: Patio Project Events	Earliest Date	Latest Date
A	Customer design approval		
B	Plans/permit approved		
C	Site prepared		
D	Site inspection		
E	Slab concrete poured		
F	Slab finished		
G	Slab inspection		
H	BBQ constructed		
I	Final inspection		

**Table 4.2 – Initial List of Events for CPM Plan**



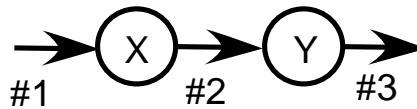
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4.4.2 Events located on the Graph – We place all the events on the graph as follows:



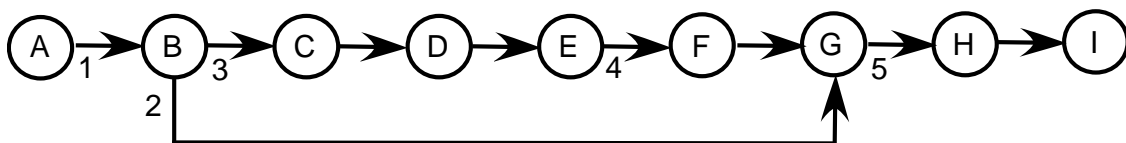
**Chart 4.1 – Initial CPM Chart with Arbitrary Event Placement**

4.2.3 Task/Activities located on the Graph – Using the list of tasks, we make a first attempt at connecting events with enumerated task/activities as directed line-segments between the events similar to the example of connectivity below:



**Chart 4.2 – CPM Chart Detail Showing Task/Activity for a Sequence**

Each task/activity starts with an event and ends with a different event. We map all the tasks to the graph of events with, in this case, insufficient task/activities to connect our understanding of the sequence. We remedy the need for additional task/activity designations by adding “dummy” task/activity arrows with neither title nor duration.



**Chart 4.3 – Example CPM Chart Detail Showing Task/Activity Dependency Network**

In this case, we re-visit the table of tasks, adding columns to indicate start event and end event, and adding rows for the “dummy” task/activities we found necessary to add to the graph to complete a path from the beginning event to the ending event. Note, at this stage of graph construction, it is just as common to require extra “dummy” events to be added to make a coherent graph.



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<b>Task</b>	<b>Title: Patio Project Tasks</b>	<b>Start</b>	<b>End</b>	<b>Time</b>
1	Site engineering planning	A	B	3
2	Materials purchase	B	G	4
3	Site preparation	B	C	2
4	Concrete pour, surface, & cure	E	F	28
5	BBQ construction & cure	G	H	4
6		C	D	
7		D	E	
8		F	G	
9		H	I	

**Table 4.3 – Updated List of Task/Activities for CPM Plan with Terminating Events**

We have added four new task/activities to complete the graph. The first added task/activity is required to join event C to D or the site prepared event with the site inspection. In some jurisdictions, this requires a same-day call for a site inspection task and will be entered as zero duration. The second added task/activity is required to join event D to E for the slab inspected. Again, a same-day, zero duration call for concrete task is entered. The third added task/activity is required to join event F to G for the slab finished with a slab inspection. Again, a same-day inspection is entered as zero duration. The fourth added task/activity is required to join event H to I or the BBQ constructed and the zero duration final inspection.

<b>Task</b>	<b>Title: Patio Project Tasks</b>	<b>Start</b>	<b>End</b>	<b>Time</b>
1	Site engineering planning	A	B	3
2	Materials purchase	B	G	4
3	Site preparation	B	C	2
4	Concrete pour, surface, & cure	E	F	28
5	BBQ construction & cure	G	H	4
6	Call for inspection	C	D	0
7	Call for concrete	D	E	0
8	Call for inspection	F	G	0
9	Call for inspection	H	I	0

**Table 4.4 – Updated List of Named Task/Activities for CPM Plan**

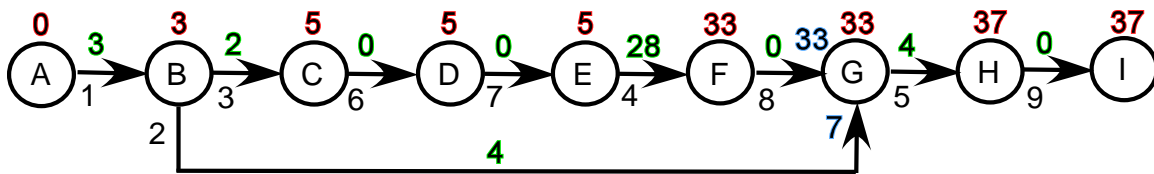


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It is worthwhile to note that we have added several task/activities including the duration of calls for inspection. We can treat these calls as reminders that we may need to schedule inspections in some jurisdictions and therefore alter the plan accordingly.

In addition, we note that we have designated task/activity 2 describing material purchase as 4-day duration with an early start at event B, but a completion that is not required until the BBQ needs the materials. This task illustrates parallel task/activity paths through the graph.

4.2.4 Earliest Start Analysis – We traverse the graph from start to finish twice. The first time through, we determine the earliest possible start date for each task/activity and denote the latest to the earliest time for an event.



**Chart 4.4 – CPM Chart with Task/Activity Durations and Earliest Event Times**

We traverse the graph from the beginning event through each task/activity to the final event. For convenience, we annotate the task/activity duration numerals in green above each. The first or starting event is given a time reference of zero and is annotated in red above the event. Beginning with task/activity 1, we expend 3 days to reach event B. Because there is no other way to reach event B, that is its earliest possible date. Using the Task/Activity table below, we determine each event from its predecessors and the task/activity duration separating them. From each Start-Event we get the earliest start date for each task/activity and we add the duration to get the Earliest End date. For those events with only one predecessor task/activity, the event date is the earliest end from that predecessor. For the case of event G, however, there are two predecessor events and the time associated with G is the latest of the possible earliest end dates marked in blue of the predecessors, 33 days in this case.





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Task	Title: Patio Project Tasks	Event		Time	Earliest		Latest	
		Start	End		Start	End	Start	End
1	Site engineering planning	A	B	3	0	3		
2	Materials purchase	B	G	4	3	7		
3	Site preparation	B	C	2	3	5		
4	Concrete pour, surface, & cure	E	F	28	5	33		
5	BBQ construction & cure	G	H	4	33	37		
6	Call for inspection	C	D	0	5	5		
7	Call for concrete	D	E	0	5	5		
8	Call for inspection	F	G	0	33	33		
9	Call for inspection	H	I	0	37	37		

**Table 4.5 –Named Task/Activities for CPM Plan with Start/End Dates**

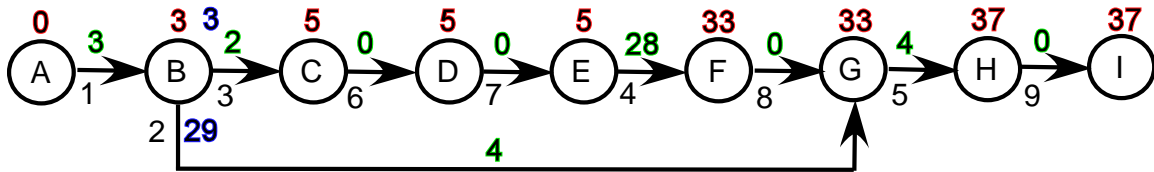
Event	Title: Patio Project Events	Earliest	Latest
		Date	Date
A	Customer design approval	0	
B	Plans/permit approved	3	
C	Site prepared	5	
D	Site inspection	5	
E	Slab concrete poured	5	
F	Slab finished	33	
G	Slab inspection	33	
H	BBQ constructed	37	
I	Final inspection	37	

**Table 4.6 –Events for CPM Plan with Earliest/Latest Dates**



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To complete the evaluation, we traverse the graph in the opposite direction:



**Chart 4.5 – CPM Chart with Task/Activity Durations and Latest Event Times**

In the traversal from the end to the beginning, we evaluate the latest end as the date of the event at the end of the task/activity and find the latest start by subtracting the duration.

Task	Title: Patio Project Tasks	Event		Time	Earliest		Latest	
		Start	End		Start	End	Start	End
1	Site engineering planning	A	B	3	0	3	0	3
2	Materials purchase	B	G	4	3	7	29	33
3	Site preparation	B	C	2	3	5	3	5
4	Concrete pour, surface, & cure	E	F	28	5	33	5	33
5	BBQ construction & cure	G	H	4	33	37	33	37
6	Call for inspection	C	D	0	5	5	5	5
7	Call for concrete	D	E	0	5	5	5	5
8	Call for inspection	F	G	0	33	33	33	33
9	Call for inspection	H	I	0	37	37	37	37

**Table 4.7 –Task/Activities for CPM Plan with Critical Path Start/End Dates in RED**



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Event	Title: Patio Project Events	Earliest Date	Latest Date
A	Customer design approval	0	0
B	Plans/permit approved	3	3
C	Site prepared	5	5
D	Site inspection	5	5
E	Slab concrete poured	5	5
F	Slab finished	33	33
G	Slab inspection	33	33
H	BBQ constructed	37	37
I	Final inspection	37	37

**Table 4.8–Events for CPM Plan with Critical Path Event Dates in RED**

Every event and task/activity with identical earliest and latest dates is designated as “critical.” All critical events and task/activities are on the “critical path” through the graph and define the ending date. Recall that all task/activities for the CPM method are considered to be known with certainty and therefore the planning decisions are made as decisions under conditions of certainty.

**5. Decision making under conditions of Uncertainty**

To develop the concepts of Decision making under conditions of Uncertainty, we introduce “Payoff Table 1” example for a product development decision.

		Event	
		Success	Failure
Decision	Accept	\$100,000	(\$30,000)
	Reject	(\$1,000)	(\$1,000)

**Table 5.1 - Payoff Table for Decision under Uncertainty**

The hypothetical decision involves accepting a project with a \$100,000 fee on successful completion, but a \$30,000 penalty on failure to complete. Regardless of the decision to submit the bid for the project, \$1,000 costs are required in the bid proposal.



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An optimist with no information would choose the decision to prepare the bid and assume that the payoff would be \$100,000 without reservation. Such an optimistic decision, choosing the decision with the largest payoff regardless of the event is defined as the “maximax” rule.

A true pessimist with no information would choose the decision to reject all such bids and fail to ever bid at all. Such a pessimistic decision, choosing the decision with the largest payoff with the worst-case event is defined as the “maximin” rule. The true pessimist under these conditions would never make any decision and thus eliminate even the cost of bid preparation.

The value of preparing a decision table under conditions of Uncertainty is useful because it presents a clear presentation of all possibilities and the importance of the decision process.

**6. Decision making under conditions of Risk**

In some sense, Decision making under conditions of Risk is the predominant form of decision making because the event outcomes are neither completely known nor completely unknown. What differentiates the process of Decision making under conditions of Risk is the assignment of probabilities to events and evaluation of outcomes associated with those decisions. Key to the process is the concept of probability and its applications. There are three classical approaches to defining probability, as well as an approach that is entirely intuitive. The categories are the classical, relative frequency, and axiomatic approaches.

6.1 Enumeration - One classical approach to probability is based on enumeration of possible outcomes with the implicit assumption that all outcomes are “equally likely.” For games of chance including dice games and some games with playing cards some reasonable estimates are made this way. For example, a pair of “fair” dice can each have one of six possible outcomes in a “fair” toss, shown with the associated sum of spots enumerated below:



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<b>Fair</b>						
<b>Dice</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

**Table 6.1 - Enumeration of Possible Sums of Events with Two Dice**

There are 36 possible outcomes with one outcome each having the values of 2 or 12 and the consequent probability assigned of  $1/36$  for either of those cases. There are two outcomes with the value of 3 or 11 and the consequent probability assigned of  $2/36$  for either of those cases. There are three outcomes with the value of 4 or 10 and the consequent probability assigned of  $3/36$  for either of those cases. There are four outcomes with the value of 5 or 9 and the consequent probability assigned of  $4/36$  for either of those cases. There are five outcomes with the value of 6 or 8 and the consequent probability assigned of  $5/36$  for either of those cases. Finally, there are six outcomes with the value of 7 and the consequent probability assigned of  $6/36$  for that case.

A similar exercise can be used to enumerate all the possible “hands” of a card game and define the probability of events such as improving the value of the hand by making appropriate decisions.

The primary objection by the mathematics community concerning this classical definition of probability is the relatively “circular” dependency of probability or likelihood on the assumption of equal-probability outcomes of each number on the face of each die. That objection is overcome by the later development of the axiomatic approach to the definition of probability.

6.2 Polling - In a similar fashion to enumeration, there is a classical concept of probability obtained by polling or voting amongst a community of participants. Such a community process is useful for reaching a consensus for group decisions and has shown surprisingly good results in numerous cases. One such refinement gives rise to the Delphi-Technique with



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several rounds of blind polling and distribution of intermediate results to all. The distribution of estimates tends to become more of a consensus value with a narrower range of estimation as successive polls are taken. The idea is that the participants who are less certain adjust their estimates closer to the mean in successive rounds. Underlying the process is the idea that the experience base of the group provides some sense of history with greater weight given to participants who are personally more certain, but avoids an element of argumentation to sway opinion.

This consensus approach including the Delphi Technique, however, has not had wide acceptance in engineering decision-making except in forensic examinations of engineering failures and as such is more associated with the polling of recognized experts in court cases. Use of this form of “probability” for defining outcomes gives a result that may be manipulated as if the decision were made under conditions of risk, but is truly more closely related to decision under uncertainty. The Delphi Technique however has value when associated with developing task probability estimates for the PERT Chart and building a useful consensus among team members responsible for estimating tasks.

6.3 Relative Frequency approach to definition of probability - The relative frequency approach to defining probability is experimentally based. It consists of running numerous trials and tabulating outcomes.

For instance, the “Coin Toss” or “flip of a coin” gives rise to the production of the two mutually-exclusive results of either “heads” or “tails.” Using the classical approach, the two outcomes would be assigned equal probabilities of  $\frac{1}{2}$  with no experimentation required. In an experiment, such an experiment might assign the “a-priori” (before the fact) estimate of exactly those probabilities, but clearly, the first trial can have only one result. Running multiple trials and accumulating results provides a history with accumulated occurrences of each result. As the number of trials increases, the distribution of results tends to approach the values of  $\frac{1}{2}$  for each outcome, but multiple trials are not expected to be duplicates.

The history of trial result outcomes defines a test “population” that can be described using well-defined descriptive “parameter” measures such as the mean, median, mode, standard-deviation, and others. A more precise definition would describe the parameters as describing some known collection of test instances and the term “statistic” would describe the same quantitative measures as applied to a sub-set of the population drawn as a sample from the population.



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The collection of measures, however, is termed “Descriptive Statistics” whether applied to the entire population or to only a sample. In instances of large numbers of independent samples drawn from a population, the “statistics” tend to approach the “parameters” in what is known as a “Law-of-Large-Numbers” fashion. Such is the case with the example of the “Coin Toss” with the result of outcome probability estimates approaching  $\frac{1}{2}$  over large sample sizes.

The relative frequency approach to defining probability is used extensively in engineering to find an “a-posteriori” or “after-the fact” estimate of probable outcomes involved in production testing. Design of a product with “a-priori” estimates of design parameters can be verified by comparing with “a-posteriori” measurements from test and measurement or quality-control activities.

6.4 Axiomatic definition of probability - A purely mathematical approach to the definition of probability, is made by defining three fundamental mathematical axioms and deriving all other relationships from those definitions.

The three fundamental axioms used to define probability are:

- I. The probability of an event is assigned a number between zero and one inclusive,
- II. The probability of occurrence of mutually exclusive events is the sum of individual probabilities, and
- III. The sum all possible mutually exclusive events is unity.

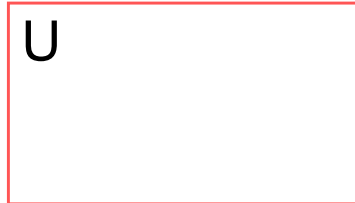
We re-iterate:

- i.  $0 < p(A) < 1$ , for event A
- ii.  $p(A \cup B) = p(A) + p(B)$
- ii.  $S = \sum(p(A_i)) = 1$ , for the exhaustive mutually exclusive set of events  $A_i$



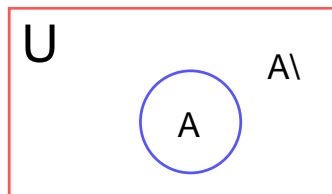
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6.4.1 Venn Diagram - The Venn Diagram is a useful tool for visualization of probability concepts as shown below:



**Diagram 6.1 – Venn Diagram of the Universal Set U**

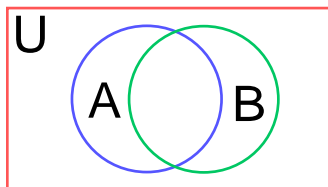
The rectangular area designated as **U** is used to represent the set of all possible outcomes. Because **U** represents the set of all possible outcomes, it is assigned  $p(\mathbf{U}) = 1$ .



**Diagram 6.2 – Venn Diagram of Set A and Complement A\ in the Universal Set U**

The circular area **A** within **U** is used to represent a sub-set of **U** for some possible set of outcomes. The remaining area **A\'** inside **U**, but outside **A** is used to represent exhaustive mutually-exclusive subsets. We note that:  $p(\mathbf{A}) + p(\mathbf{A}') = p(\mathbf{U}) = 1$ .

6.4.2 Intersection and Union of Sets -



**Diagram 6.2 – Venn Diagram of Intersecting Sets A and B in the Universal Set U**

The Venn Diagram can be used to illustrate more complicated relationships. We illustrate two possible sub-sets of **U** with outcomes **A** and **B** that are not mutually exclusive. We





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enumerate the Intersection of sets  $A \cap B$  as all events that are only in both  $A$  and  $B$ . We define the Union of sets  $A \cup B$  as all events that are in either  $A$  or  $B$ , but we do not count the elements in the Intersection twice. Much as we did with the enumeration of the values associated with the roll of two dice, we assign a probability to an event within a set of events.

6.4.3 Bayes' Theorem and Conditional Probability - An important tool for decision making under conditions of Risk is Bayes' Theorem. The theorem is developed using two possible sub-sets of  $U$  with outcomes  $A$  and  $B$  that are not mutually exclusive as in Diagram 6.2, and the  $A \cap B$  Intersection of the sets.

Bayes' Theorem is introduced using a sequence approach to the sub-sets  $A$  and  $B$  that are not mutually exclusive. We define "conditional probabilities" as either the probability of an event belonging to set  $B$  given that the event has already been identified as being in set  $A$ . In short-hand notation we define the probability of  $B$  given that  $A$  has already occurred. Each is evaluated as a ratio of the probability of the event space of Intersection of sets  $p(A \cap B)$  to either the probability of one of the event spaces. For the two sets, we define  $p(B|A)$ , probability of  $B$  given  $A$ , and  $p(A|B)$ , probability of  $A$  given  $B$ . We can also show the conditional probability relationships

$$p(B|A) = \frac{p(B \cap A)}{p(A)} \quad [6.1]$$

$$p(A|B) = \frac{p(B \cap A)}{p(B)} \quad [6.2]$$

From the two "conditional probabilities" and with some manipulation, we see:

$$p(B|A)p(A) = p(B \cap A) = p(A|B)p(B) \quad [6.3]$$

Thus, we can infer one conditional probability from the other and a ratio, as follows:

$$p(B|A) = p(A|B) \frac{p(B)}{p(A)} \quad [6.4]$$

Likewise, we can define several other conditional probabilities as follows:

$$p(B|A \setminus) = \frac{p(B \cap A \setminus)}{p(A \setminus)} \quad [6.5]$$



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$$p(B|A) = \frac{p(B \cap A)}{p(A)} \quad [6.6]$$

$$p(B|A^c) = \frac{p(B \cap A^c)}{p(A^c)} \quad [6.7]$$

$$p(A|B) = \frac{p(B \cap A)}{p(B)} \quad [6.8]$$

$$p(A|B^c) = \frac{p(B \cap A^c)}{p(B^c)} \quad [6.9]$$

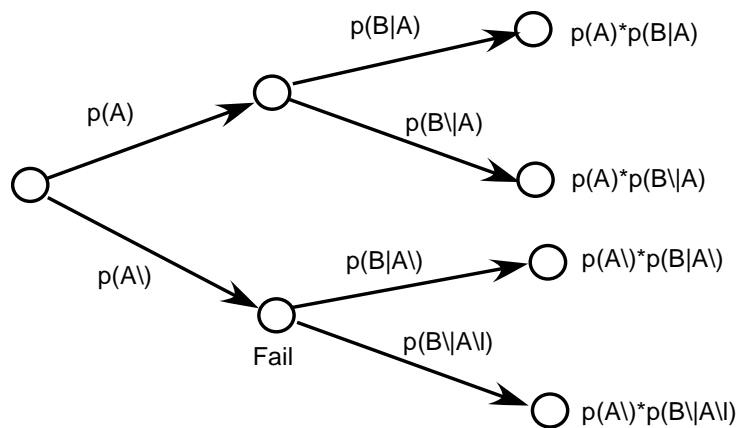
$$p(A|B^c) = \frac{p(B \cap A^c)}{p(B^c)} \quad [6.10]$$

We can also combine conditional probabilities to derive other probabilities that are not known directly. For example:

$$p(B) = p(B \cap A) + p(B \cap A^c) = p(B|A)p(A) + p(B|A^c)p(A^c) \quad [6.11]$$

$$p(A) = p(B \cap A) + p(B^c \cap A) = p(A|B)p(B) + p(A|B^c)p(B^c) \quad [6.12]$$

We can also derive the forms for the complements, but it is usually simpler to derive the complement as a sum with unity.



**Diagram 6.3 – Conditional Probability Outcome Tree Diagram**



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To illustrate the use of Bayes' Theorem, we introduce a medical example and assume that we have a stable population in a city with some occurrences of the disease tuberculosis. We choose a person (possibly ourselves) from the population for testing. We have available a test for the disease but it is imperfect showing both false positives and false negatives. We assume extensive testing with the result from prior testing that a person with tuberculosis will have a positive test with a probability of 0.98, but also the probability is 0.05 that a person without the disease will test positive.

Let us also assume that the probability that a randomly selected person in the city has the disease is 0.01 from the ongoing medical records of diagnoses for the infection rate. We assign  $p(\mathbf{A})$  to the probability of the person tested actually having tuberculosis.

$$p(A) = 0.01 \quad [6.13]$$

$$p(A\setminus) = 0.99 \quad [6.14]$$

We ask the question concerning a personal test. If an individual tests positive for the disease, what is the probability that they actually are infected, or are disease free? What are the probabilities that if the person tests negative for the disease that they actually are infected, or are disease free?

We assign  $p(\mathbf{B})$  to the probability of any random test being positive for tuberculosis.

We note that we already know  $p(\mathbf{A})$  from the infection rate in the city. However, we do not know  $p(\mathbf{B})$ , which increases as tests are administered in populations with higher rates of infection, but we do know from the prior testing that true positives will be:

$$p(B|A) = 0.98 \quad [6.15]$$

$$p(B\setminus|A) = 0.02 \quad [6.16]$$

And, it follows that the probability of false positives will be:

$$p(B|A\setminus) = 0.05 \quad [6.17]$$



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$$p(B|A) = 0.95 \quad [6.18]$$

As for  $p(B)$ , we can calculate that:

$$p(B) = p(B \cap A) + p(B \cap A^c) = p(B|A)p(A) + p(B|A^c)p(A^c) \quad [6.19]$$

$$p(B) = 0.98 * 0.01 + 0.05 * 0.99 = 0.0593 \quad [6.20]$$

So we can substitute:

$$p(A|B) = \frac{p(B \cap A)}{p(B)} = \frac{p(B|A)p(A)}{p(B|A)p(A) + p(B|A^c)p(A^c)} \quad [6.21]$$

We evaluate the expression to obtain:

$$p(A|B) = \frac{p(B|A)p(A)}{p(B|A)p(A) + p(B|A^c)p(A^c)} = \frac{0.98 * 0.01}{0.0593} = 0.165 \quad [6.22]$$

The conclusion we make about the result of a positive test is that the probability of infection is 16.5% and far greater than the infection rate of 1%, but it does not indicate with certainty the presence of tuberculosis. Those managing the spread of the disease would probably re-test the individual, and preferably with a different, possibly more expensive and more accurate, test to confirm the diagnosis.

We can further utilize the example to evaluate the probability of the absence of the disease despite the positive test result as follows:

$$p(A^c|B) = \frac{p(B|A^c)p(A^c)}{p(B|A)p(A) + p(B|A^c)p(A^c)} = \frac{0.05 * 0.99}{0.0593} = 0.835 \quad [6.23]$$

From this result, the individual can take hope because the probability that the disease is not present is 83.5% due to the history of false positives. However, this is less than the 99% probability in the general population. Again, further testing would probably be recommended.

We might wonder about the sense of security the individual may feel in case of negative test results. To determine the probability that the disease is present despite a negative result, we evaluate:



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$$p(B^c) = 1 - p(B) = 0.9407 \quad [6.24]$$

$$p(A|B^c) = \frac{p(B^c|A)p(A)}{p(B^c|A)p(A) + p(B|A)p(A)} = \frac{0.02 \cdot 0.01}{0.9407} = 0.0002 \quad [6.25]$$

Finally, to be complete, we evaluate the probability that the disease is not present given a negative test result as:

$$p(A^c|B^c) = \frac{p(B^c|A^c)p(A^c)}{p(B^c|A^c)p(A^c) + p(B|A^c)p(A^c)} = \frac{0.95 \cdot 0.99}{0.9407} = .9998 \quad [6.26]$$

By calculation, we have verified the intuitive result that the conditional probabilities can be paired as complement for either positive or negative results, but the actual probabilities for one of the pair must be calculated. We will return to the use of Bayes' theorem and its applications in an engineering setting after we introduce a discussion of probability and expected values for outcomes.

## 7. Expected Value

In decision making under conditions of Risk, the values associated with the decisions are not certain, nor are they entirely uncertain. Instead, we adopt a discount associated with the risk. In this way, a monetary value associated with an outcome having a probability of 100% is taken at full value, an outcome with probability of 0% is taken at no value, and an outcome with probability  $p\%$  is taken at  $p\%$  value. This discounted value is defined as the expected value.

$$Value = \$outcome * p(outcome) \quad [7.1]$$

Human nature, however, does not always present behavior that exhibits this proportional relationship. A person buying a lottery ticket for \$1 with a payoff of \$1million but a probability of winning of 1 in 20 million is not acting purely rationally concerning the expected value. The expected value of the lottery ticket is:

$$Value = \frac{\$1,000,000}{20,000,000} = \$0.05 \quad [7.2]$$

This person is trading \$1 for a ticket with \$0.05 expected value. This non-rational behavior can be "explained" using the concept that the value to the player of \$1 is much less than the expected "nickel" value of \$1million payoff. Psychologists assume that another proportionality factor related to utility of the amounts is included in this expectation of value.



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That is, the utility of a “dollar-in-hand” is perceived as having much less utility than the \$1million payoff, should it occur. In that sense, behavior is similar to an engineering start-up that offers a risky funding opportunity to investors. For this discussion, however, we ignore this psychological behavior and ignore the “utility” of the monetary amounts and decide on expected values using monetary values and outcome probabilities alone.

Let us return to the example presented in the prior Table 5.1 example, but assign probabilities to the outcomes and define Table 7.1, as follows:

		Event	
		Success	Failure
Probability		p(S)	p(F)
Decision	Accept	\$100,000	(\$30,000)
	Reject	(\$1,000)	(\$1,000)

**Table 7.1 - Payoff Table for Decision under Risk**

We evaluate the expected value of an outcome using the expected value of the decision as follows:

		Event		Expected Value
		Success	Failure	
Probability		p(S)	p(F)	
Decision	Accept	\$100,000	(\$30,000)	$p(S)*\$100k-p(F)*\$30k$
	Reject	(\$1,000)	(\$1,000)	$-p(S)*\$1k-p(F)*\$1k$

**Table 7.2 - Payoff Table for Decision under Risk with Expected Values**

The expected value of a decision is the payoff of a success weighted by the probability of success plus the payoff of failure weighted by probability of failure. The expected value of a decision to accept the project is given by:

$$p(S) * \$100k - p(F) * \$30k = EV(Accept) \quad [7.3]$$

Because the success/failure alternatives are mutually exclusive and exhaustive, we express both in terms of the probability of success:



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$$p(F) + p(S) = 1 \quad [7.4]$$

$$p(F) = 1 - p(S) \quad [7.5]$$

$$EV(Accept) = p(S) * \$100k - (1 - p(S)) * \$30k \quad [7.6]$$

$$EV(Accept) = p(S) * \$70k - \$30k \quad [7.7]$$

Similarly, the expected value of a decision to reject the project is given by:

$$p(S) * (-\$1k) + p(F) * (-\$1k) = EV(Reject) = -\$1k \quad [7.8]$$

A pertinent question to ask is: “What must p(S) be to make the outcomes equal?”  
So we solve for p(S):

$$EV(Accept) = p(S) * \$70k - \$30k = EV(Reject) = -\$1k \quad [7.9]$$

$$p(S) * \$70k = \$29k \quad [7.10]$$

$$p(S) = \frac{\$29k}{\$70k} = .414 = 41.4\% \quad [7.11]$$

There are indications from historical reports that only 5% of industrial products are a market success, but the reports are over 50 years old and suspect in today’s marketplace. However, if we use the 5% p(S) estimate as a benchmark, we would never engage in such projects.

If instead, we insert that probability of success and assume the same costs to prepare for the decision, as well as the same cost of failure, we can ask the question: “What would the payoff need to be for a p(S) = 5% to break-even?”

$$EV = .05 * Payoff - .95 * \$30k = -\$1k \quad [7.12]$$

$$Payoff = \frac{.95 * \$30k - \$1k}{.05} = \$550k \quad [7.13]$$



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This example illustrates that, with a historical probability of success of 5% of projects, and the costs & penalties shown, no project should be undertaken for less than \$550k to ensure a favorable expected value.

Although these examples are hypothetical, they illustrate the use of expected values in Decision making under conditions of Risk.

**8.0 Applying Bayes’ Theorem to Decision making under conditions of Risk –**

Bayes’ Theorem has wide application in problems of decision making under conditions of Risk. We show a few of the applications to encapsulate the concepts of conditional probability as it modifies the decision process, as well as derivative concepts related to the value of information.

8.1 Research and Testing in Project Decision Evaluation - In the example project that we have been developing in Table 7.1 and its derivations, let us assume that there exists some test or research effort that could be expended to provide revised probability estimates much like the test for tuberculosis in the prior example. For the sake of convenience, let us assume that the probabilities associated with the test are those used in the prior example, We assume that the test shows a 5% false-positive rate, as well as a 2% false-negative rate. We use these rates to be similar to the prior medical example. For the sake of discussion, let us assume that the test costs \$0.5k to administer in the Table 8.1 project selection example.

First, let us revisit Table 1 with the presumption that the probability of success p(S) is the 5% from historical experience:

Table 1 - P(S)=5%		Event		Expected Value
		Success	Failure	
Probability		p(S)=.05	p(F)=.95	
Decision	Accept	\$100,000	(\$30,000)	.05*\$100k-.95*\$30k=-\$23.5k (loss)
	Reject	(\$1,000)	(\$1,000)	-.05*\$1k-.95*\$1k=-\$1k (loss)

**Table 8.1 - Payoff Table for Decision under Risk with p(Success) = 5%**





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We see that both decision outcomes provide a loss but the decision to “Reject” the project is less and would lead us to always reject these proposals. Let us now examine what the decision might be if we have available the test introduced above.

We define  $p(F)$  as  $p(S)$  so that we can use the prior formulations. We also introduce the test as  $p(T)$  and its complement  $p(T^c)$  to describe the four conditional probabilities. From the prior testing experience, we define:

$$p(T|S) = 0.98 \quad [8.1]$$

$$p(T^c|S) = 0.02 \quad [8.2]$$

So we assume the test is 98% correct in identifying projects that will be a success, but has a 2% false-negative indication.

Likewise, from the history of the test’s indication of probability of false positives we have:

$$p(T|S^c) = 0.05 \quad [8.3]$$

$$p(T^c|S^c) = 0.95 \quad [8.4]$$

Therefore, we can evaluate the conditional probabilities of testing for projects with a 5% historical proportion of success without testing as:

$$p(S|T) = \frac{p(T|S)p(S)}{p(T|S)p(S) + p(T|S^c)p(S^c)} = \frac{.98 * .05}{.98 * .05 + .05 * .95} = .508 \quad [8.5]$$

We will find it useful to identify the denominator in the equation above as:

$$p(T|S)p(S) + p(T|S^c)p(S^c) = p(T) = .98 * .05 + .05 * .95 = .0965 \quad [8.6]$$

And we can also derive the conditional probability complement:

$$p(S^c|T) = 1 - .508 = .492 \quad [8.7]$$



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In the event of a negative test result, we derive:

$$p(S|T\backslash) = \frac{p(T\backslash|S)p(S)}{p(T\backslash|S)p(S)+p(T\backslash|S\backslash)p(S\backslash)} = \frac{0.02*0.05}{0.02*0.05+0.95*0.95} = 0.0011 \quad [8.8]$$

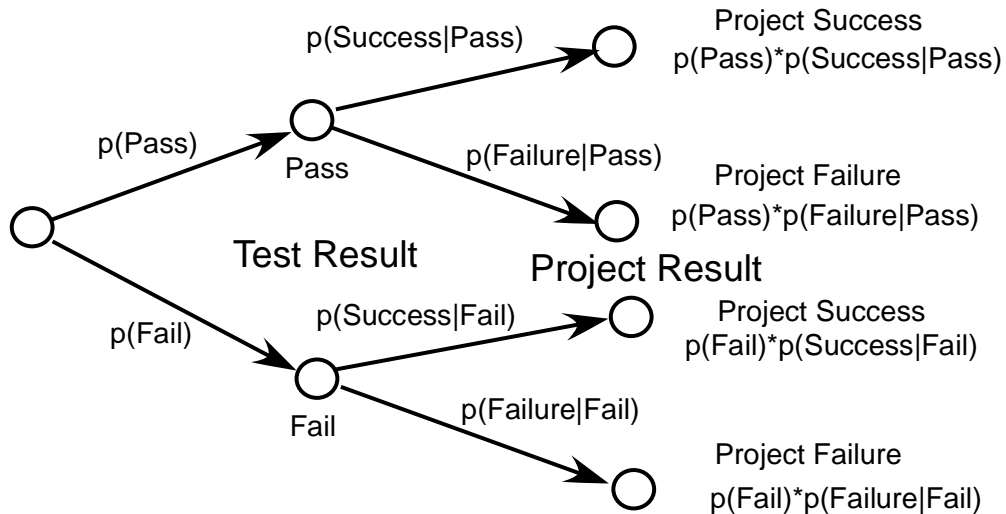
We will find it useful to identify the denominator in the equation above as:

$$p(T\backslash|S)p(S) + p(T\backslash|S\backslash)p(S\backslash) = p(T\backslash) = .02 * .05 + .95 * .95 = .9035 \quad [8.9]$$

And we can derive as the conditional probability complement:

$$p(S\backslash|T\backslash) = 1 - .0011 = .999 \quad [8.10]$$

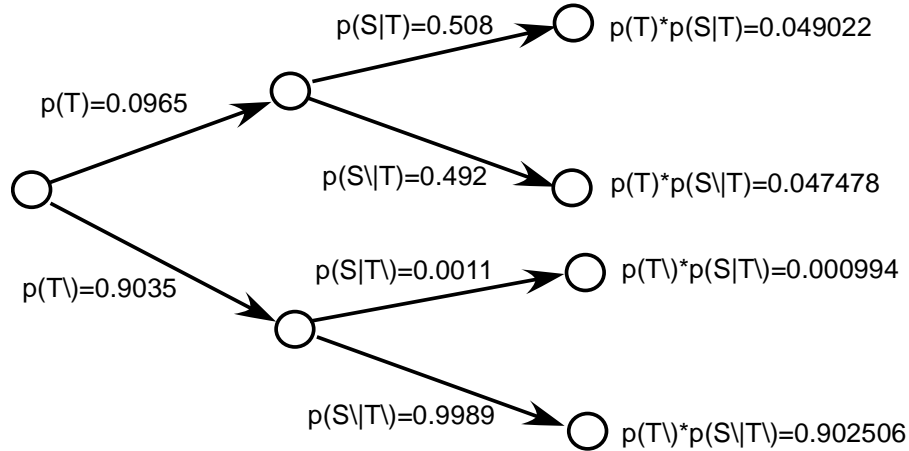
We are now prepared to evaluate expected values using the test prior to making a decision about pursuing the project.



**Diagram 8.1 – Conditional Probability Project Outcome Tree Diagram**



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**Diagram 8.2 – Project Conditional Probability Tree Diagram**

If we administer the test and the result is positive, we have the expected payoff table:

		Event		Expected Value
		Success	Failure	
Probability	Accept	\$100,000	(\$30,000)	.508*\$100k-.492*\$30k=\$36.4k (gain)
	Decision	Reject	(\$1,000)	(\$1,000)

**Table 8.2 - Payoff Table for Decision under Risk after Positive Test**

Therefore, we would choose to accept the project with the expected value of \$36.4k.

If we administer the test and the result is negative, we have the expected payoff table:

		Event		Expected Value
		Success	Failure	
Probability	Accept	\$100,000	(\$30,000)	.001*\$100k-.999*\$30k=\$29.9k (loss)
	Decision	Reject	(\$1,000)	(\$1,000)

**Table 8.3 - Payoff Table for Decision under Risk after Negative Test**



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Therefore, we would choose to reject the project with the resulting cost of \$1k, clearly a loss, but the minimum loss.

Finally, we have calculated that the test is expected to be positive in only 9.65% of instances and negative in 90.35% of instances, so that from the expected values, we will result in:

$$EV(Project) = .0965 * \$36.4k - .9035 * \$1k = \$2.6091k \quad [8.11]$$

We incur the cost of \$0.5k to administer the test for all cases and thus we reduce the expected value to \$2.1081k a far better result than with no test information.

We see from the result above that we can make use of testing to revise the probability of success sufficiently that we will decide to accept 9.65% of such projects and generate an average of over \$2k/project for a collection of projects. Note, that we are still generating the \$100k payoff but not on every project accepted. Further, the project requires an investment in testing of \$0.5k prior to making a decision to proceed. This becomes an interesting problem to attempt to represent on a Critical Path Chart.

8.2 The Value of Information - We will calculate expected values of outcomes under differing assumptions; perfect information, no information, and with the testing providing conditional probabilities. We then compare to the expected values to establish the values of the information.

8.2.1 The Value of a Project with Perfect Information - If we assume in Table I we are able to always identify a project's success or failure, we would always accept projects that will be a success and always reject projects that will fail.

The expected value of the payoff of \$100k on 5% of projects that will be a success and paying the cost of \$1k on the remaining 95% of projects that will fail provides the expected value of:

$$EV\left(\text{Project} \frac{w}{\text{Perfect}} \text{Information}\right) = .05 * \$100k - .95 * \$1k \quad [8.12]$$

$$EV(\text{Project } w/\text{Perfect Information}) = \$4.05k \quad [8.13]$$



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On average, this is the best that we can do in this regard and is the expected value of a project with perfect information.

8.2.2 The Value of a Project with No Information - If we assume in Table 7.1 we are not able to identify a project's success or failure, we would always reject projects but that may cost \$1k regardless.

$$EV \left( \text{Project} \begin{matrix} w \\ No \end{matrix} \text{Information} \right) = -.05 * \$1k - .95 * \$1k = -\$1k \quad [8.14]$$

This seems ridiculous to spend the money knowing we will always reject all projects, but in many cases the resulting expected value does not always provide a clear indication that we should avoid making any decision. We use these results to make a point with the expected values but we could still argue that we would avoid making any decision and thus incur zero costs.

8.2.3 The Value of a Project with Test Information – We have already investigated the expected value of a project using testing. The expected value before we incur the cost of the testing is \$2.6091k, from which we reduce the expected value by the costs of the test incurred. In this regard, the \$2.1091k expected value of a project with the available testing represents the value of the project with the imperfect information.

The difference between the expected value of the outcome with test information and the prior expected value of the outcome with the imperfect information provided by testing is the “value of perfect information.”

$$V(\text{Perfect Information}) = \$4.05k - \$2.11k = \$1.94k \quad [8.15]$$

The value of perfect information is a bit misleading in comparison to the case we have outlined because we have already paid the cost of testing for the example. What is left is the incremental value of perfect information remaining to the decision maker.

8.2.4 The Value of Test Information – We see that if we exclude the results of testing, returning instead to the original premises, the expected value of a project is actually always a loss of \$1k but we would never accept any projects.



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With reference to the case with no information, the expected value of perfect information would be:

$$V(\text{Perfect Information}) = \$4.05k - (-\$1k) = \$5.05k \quad [8.16]$$

We have chosen to spend \$0.5k for an imperfect test reducing the value of perfect information from \$5.05k to \$1.94k and thus gaining some imperfect information in the process. The reduction in the value of the perfect information remaining is the gain we have produced by the testing.

$$V(\text{Test Information}) = \$5.05k - \$1.94k = \$3.11k \quad [8.17]$$

### **9. PERT as CPM with Task/Activity Risk Estimates**

In projects with estimates for task/activity duration rather than assuming values with certainty, the PERT Technique has provided a tool for using the estimates to provide an estimate for the project duration as well as a measure of the uncertainty of that estimate.

9.1 Positive Task/Activity Durations - Each task/activity can be any positive duration, including zero, but they cannot provide negative time durations. Probability density functions that have this characteristic are considered “one-sided” and PERT uses a class belong to the of “Beta” distributions.

9.2 Beta distribution parameters – For use in the PERT charts, each task/activity requires three parameter estimates for:

- 1) “a,” the minimum duration estimate,
- 2) “c,” the maximum duration estimate, and
- 3) “b,” the most-likely duration estimate.

From the three estimated parameters, two pertinent statistics are derived, the mean and the standard deviation of the task/activity duration estimate as follows.

The mean is calculated as:

$$\mu(\text{duration}) = \frac{a+4b+c}{6} \quad [9.1]$$



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The standard deviation is calculated as:

$$\sigma(\text{duration}) = \frac{c-a}{6} \quad [9.2]$$

The Critical Path in PERT is evaluated the same way that it is in the CPM Method outlined in section 4 above, but the task/activity calculated mean values are used instead of the assumed or even most-likely values. In addition, the standard deviation of the entire project duration is calculated using the task/activity standard deviation values.

We illustrate using the same project as in section 4 above, but with entirely new time estimates.

Task	Title: Patio Project Tasks	Start	End	Time				
				Min	Mode	Max	Mean	$\sigma$
1	Site engineering planning	A	B	2	3	5	3.167	0.5
2	Materials purchase	B	G	1	4	5	3.667	0.667
3	Site preparation	B	C	1	2	4	2.167	0.5
4	Concrete pour, surface, & cure	E	F	27	28	29	28	0.333
5	BBQ construction & cure	G	H	3	4	5	4	0.333
6	Call for inspection	C	D	0	1	2	1	0.333
7	Call for concrete	D	E	0	1	2	1	0.333
8	Call for inspection	F	G	0	1	2	1	0.333
9	Call for inspection	H	I	0	1	2	1	0.333

**Table 9.4 –List of Task/Activities for PERT Plan**

9.3 Completion Date Using PERT - We calculate the estimated completion date as the sum of task/activity mean values on the critical path. We have already established that the critical path for this project includes all task/activities except the materials purchase of 3.667 days. The summation of mean values evaluates to 41.3 days, but we note that we have introduced a one-day effort for four of the formerly zero-effort tasks. The CPM method provided a 37 day total with four zero-effort tasks, so we are not actually very different. The primary reason that we are nearly the same is that both are dominated by task/activity 2 and that is nearly deterministic based on curing concrete.



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9.4 Completion Date Standard Deviation Using PERT – From each task/activity standard deviation, we calculate its variance as the square of the standard deviation. The variances are summed and the sum is the variance of the task completion date. For our example, the sum-of-squares of the standard deviations is 1.1667 and the square-root of that variance 1.08 is the standard deviation of the critical path.

It should be mentioned that the combination of the variances from multiple independent beta distributions (as is the case with essentially any set of independent distributions) is nearly a Gaussian or “Standard Normal” distribution. This result is a consequence of the “Central Limit Theorem” and proves quite useful in this case.

We can construct bounds on the estimated completion date knowing that in 95% of cases, the actual completion will fall at the estimate  $\pm 2$  standard deviations, or essentially  $\pm 2$  days for this project.

### **10.0 Summary and Conclusions**

This course illustrates the categories of decision making under conditions of Certainty, Uncertainty, and Risk and some tools as related to decision processes in engineering planning and design. Conditions of Certainty were assumed as a reference or bench-mark for defining the best possible outcomes, and also were used as a simplification to provide a first level of feasibility checking. A first example of decision making under conditions of certainty was shown using Linear Programming (LP) tools to optimize constrained resource allocation to optimize a monetary return. Both a graphical approach and the “Simplex Method” were introduced to illustrate how to identify an optimum. A second example of decision making under conditions of certainty was introduced using the Critical Path Method (CPM) to plan a project. A short discussion of decision making under conditions of Uncertainty was included to introduce payoff tables and both optimistic and pessimistic extremes. Because the predominant form of decision making is under conditions of Risk, concepts of probability were discussed in depth including conditional probability, Bayes’ Theorem, expected values, the value of research and information, and the PERT scheduling technique. Links are provided in several locations to free, open-source software tools associated with each topic. The tools and techniques are useful both during initial planning and as more information becomes available to suggest plan revision. The single most useful tool is a spreadsheet program and considerable patience to design custom solutions.