A SunCam online continuing education course

Open Channel Flow Measurement – Weirs and Flumes

by

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1. Introduction

Measuring the flow rate of water in an open channel typically involves some type of obstruction in the path of flow. The two major categories of open channel flow measurement devices discussed in this course are the weir and the flume. A weir is a vertical obstruction that the water must flow over. The measured height of water above the top of the weir (the weir crest) can be used to calculate the flow rate. A flume consists of a constriction in the cross-sectional area of flow. The measured height of water entering the flume and the height in the constricted area (the throat) can be used to calculate the water flow rate. This course included descriptions, equations and example calculations for sharp crested (V-notch and rectangular weirs), broad crested weirs, and Parshall flumes. A spreadsheet to assist with rectangular weir calculations, using either U.S. or S.I. units, is included with the course.

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Image Credit: Nigel Davies, Weir at Llygad Llwchwr

2. Learning Objectives

At the conclusion of this course, the student will

- Be familiar with standard terminology used in connection with sharp-crested weirs for open channel flow measurement

- Be able to use the Kindsvater-Carter equation to calculate the flow rate over a suppressed rectangular weir for given head over the weir and weir dimensions

- Be able to use the Kindsvater-Carter equation to calculate the flow rate over a contracted rectangular weir for given head over the weir and weir dimensions

- Know the conditions required in order to use a simpler equation instead of the Kindsvater-Carter equation to calculate the flow rate over a suppressed rectangular weir and over a contracted rectangular weir for given head over the weir and weir dimensions

- Know the conditions required in order to use the equation, $Q = 2.49\ H^{2.48}$, to calculate the flow rate over a V-notch weir for given head over the weir and weir dimensions

- Be able to use the Kindsvater-Carter equation to calculate the flow rate over a V-notch weir for notch angles other than 90^{o}, given head over the weir and weir dimensions

- Be familiar with installation and use guidelines for sharp-crested weirs for open channel flow measurement

- Be able to calculate the flow rate over a broad crested weir for specified channel width and head over the weir crest when the flow over the weir crest is critical flow

- Be able to determine the minimum weir height needed to ensure critical flow over a broad crested weir for specified flow rate, channel width, and approach depth.

- Be familiar with the general configuration of a Parshall flume and how it is used to measure open channel flow rate

- Be able to determine whether the flow through a Parshall flume is free flow or submerged flow if given the throat width, H_a and H_b for the flow

- Be able to calculate the flow rate through a Parshall flume for given throat width and H_a if it is free flow.

- Be able to calculate the flow rate through a Parshall flume for given throat width, H_a, and H_b if it is submerged flow

3. Topics Covered in This Course

I. Introduction (2 min)

II. Learning Objectives for the Course (5 min)

III. Sharp Crested Weirs

A. Introduction to Sharp Crested Weirs (10 min)
B. V-Notch Weirs (20 min)
C. Suppressed Rectangular Weirs (30 min)
D. Contracted Rectangular Weirs (25 min)
E. Installation and Use Guidelines (20 min)

4. Sharp Crested Weirs

A. Introduction to Sharp Crested Weirs

A sharp crested weir is simply a flat vertical plate placed in a channel that causes the water to flow over it. The top of the weir will have a sharp edge and is called the weir crest. The sectional view in Figure 1 shows flow over a sharp crested weir along with some terminology and parameters commonly used in connection with sharp crested weirs.

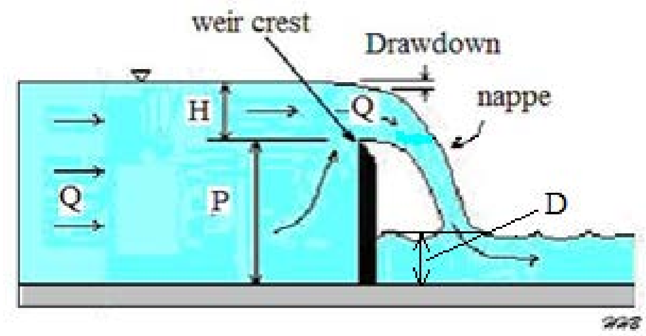

Figure 1. Parameters for Flow Over a Sharp Crested Weir

The parameters shown in Figure 1 are as follows:

- **Q** = volumetric flow rate over the weir (cfs – U.S., m^3/s – S.I. units)
- **H** = the head over the weir, that is the height of water above the weir crest, measured upstream of the drawdown effect (ft – U.S., m - S.I. units)
- **P** = the height of the weir crest above the bottom of the channel (ft – U.S., m – S.I. units)
- **D** = the depth of water downstream of the weir (ft – U.S., m = S.I. units)

The meaning of some terminology is also illustrated in the diagram.

- The **weir crest** is the upper edge of the weir.
- The **drawdown** is the reduction in elevation of water directly over the weir crest because of the acceleration of the water as it approaches the weir. (The head, H, should be measured upstream of the drawdown effect.)
- The term **nappe** refers to the sheet of water that flowing down, just downstream of the weir.

Some other terms are also commonly used in connection with flow over sharp crested weirs as follows:

- The **Velocity of Approach** is calculated as the volumetric flow rate through the channel, Q, divided by the cross-sectional area of flow at the upstream location where the head over the weir is measured.
- **Free flow** refers to the flow condition in which there is free access of air under the **nappe**, as shown in Figure 1
- **Submerged flow** refers to the flow condition in which the downstream water level is above the **weir crest** elevation.

The equations for sharp crested weirs that will be presented and discussed in this course all apply only for **free flow** conditions. If the weir crest is submerged (**submerged flow**), then the head over the weir can't be measured accurately.

Three types of sharp crested weirs will be discussed in this course. These three types of weirs (V notch, suppressed rectangular, and contracted rectangular) are shown in Figure 2.

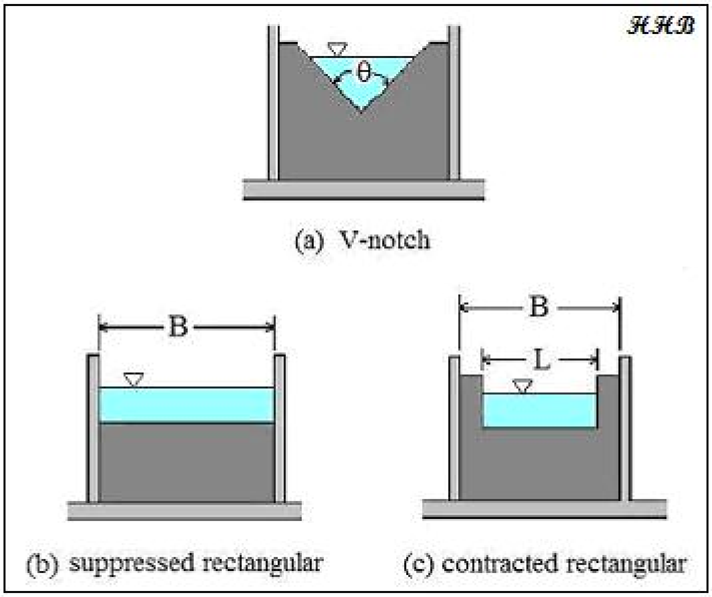

Figure 2. Common Types of Sharp Crested Weirs

B. V Notch Weirs

The V notch weir works well for measuring low water flow rates, because the decreasing surface width and flow area with decreasing head over the weir allow accurate measurement of the head even at low flow rates. The easiest V notch weir configuration for calculation of flow rate is a fully contracted, 90^o V notch weir. Equation (1), is recommended for this configuration in reference #1, (U.S. Bureau of Reclamation, *Water Measurement Manual*). The conditions needed for a fully contracted V notch weir are given below Equation (1). Figure 3, below the equation, known as the Cone Equation, shows the meaning of the parameters P, S, and H.

$$\mathbf{Q = 2.49\ H^{2.48}} \qquad (1)$$

Subject to: $\mathbf{P \geq 2\ H_{max}}$, $\mathbf{S \geq 2\ H_{max}}$, where:

- P = the height of the V notch vertex above the channel bottom
- S = the distance from the V notch opening at the top of the water overflow to the channel wall.
- H_{max} = the maximum expected head over the weir.

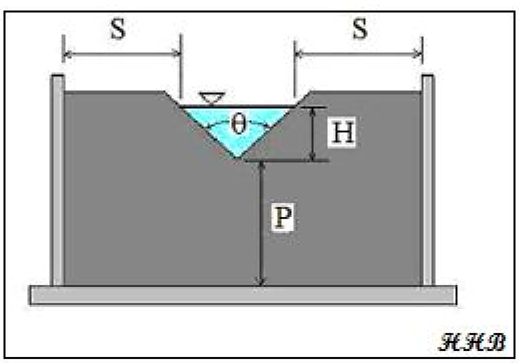

Figure 3. V Notch Weir Parameters

The constants in Equation (1) are for U.S. units, with Q in cfs and H, P & S in ft. For **S.I. units** the Cone Equation becomes:

$$\mathbf{Q = 1.36\ H^{2.48}} \qquad (2)$$

Subject to: $\mathbf{P \geq 2\ H_{max}}$, $\mathbf{S \geq 2\ H_{max}}$,

In this S.I. version of the equation, Q must be in m^3/s with H, P & S in m.

The recommended useable range for H, the head over a V notch weir in reference #1 is:

$\mathbf{0.2\ ft \leq H \leq 1.25\ ft}$ or in S.I. units: $\mathbf{0.06\ m \leq H \leq 0.38\ m}$

A general requirement for any sharp-crested weir is that the weir crest must be at least 0.2 ft (0.06 m) above the water surface immediately downstream of the weir, that is: $\mathbf{P - D \geq 0.2\ ft}$ (U.S. units) or $\mathbf{P - D \geq 0.06\ m}$ (S.I. units).

Example #1: Calculate the flow rate for the minimum recommended head (0.2 ft) over a fully contracted V notch weir and the flow rate for the maximum recommended head (1.25 ft) over a fully contracted V notch weir.

Solution: Substituting H = 0.2 ft and H = 1.25 ft into Equation (1) gives the following results:

$$Q_{min} = (2.49)(0.2^{2.48}) = \underline{\mathbf{0.046\ cfs}} = \mathbf{Q_{min}}$$

$$Q_{max} = (2.49)(1.25^{2.48}) = \underline{\mathbf{4.33\ cfs}} = \mathbf{Q_{max}}$$

Note that for the V notch weir to be fully contracted, we must have $P \geq 2\ H_{max}$ or $P \geq 2.5$ ft.

Notch angles other than 90^o lead to a bit more complicated (but still manageable) calculations. The Bureau of Reclamation, in reference #1, recommends the use of Equation (3) below (known as the Kindsvater-Carter equation) for V notch weirs with a notch angle other than 90^o.

$$\mathbf{Q = 4.28\, C_e \, Tan\, (\theta/2)\, (H + k)^{5/2}} \qquad (3)$$

The parameters in this equation and their required units are as follows:

- Q = the discharge over the weir in cfs
- H = the head over the weir in ft
- θ = the angle of the V notch
- C_e = the effective discharge coefficient, which is dimensionless (An equation for C_e as a function of θ is given below.)
- k is a head correction factor in ft (An equation for k as a function of θ is given below.)

Equations (4) and (5), shown below, which are expressions for C_e and k in terms of the notch angle, θ were derived by Excel curve fitting using the graphs shown below for Ce vs θ and k vs θ, prepared from curves in the USBR Water Measurement Manual. Note that the notch angle must be expressed in degrees for Equation (4) and Equation (5).

$$C_e = 0.6028 - (0.0007364)\theta + (5.179 \times 10^{-6})\theta^2 \qquad (4)$$

$$k = 0.01456 - (0.0003401)\theta + (3.286 \times 10^{-6})\theta^2 - (1.042 \times 10^{-8})\theta^3 \text{ ft} \qquad (5)$$

Figure 4 below shows the curve used to derive Equation (4), and Figure 5 below shows the curve used to derive Equation (5). The tables of values came from similar curves in the U.S. Bureau of Reclamation, *Water Measurement Manual*, reference #1. These curves show the pattern of variation of C_e and k due to variation in θ.

θ, degrees	Ce
20	0.59
40	0.582
60	0.577
80	0.577
100	0.581

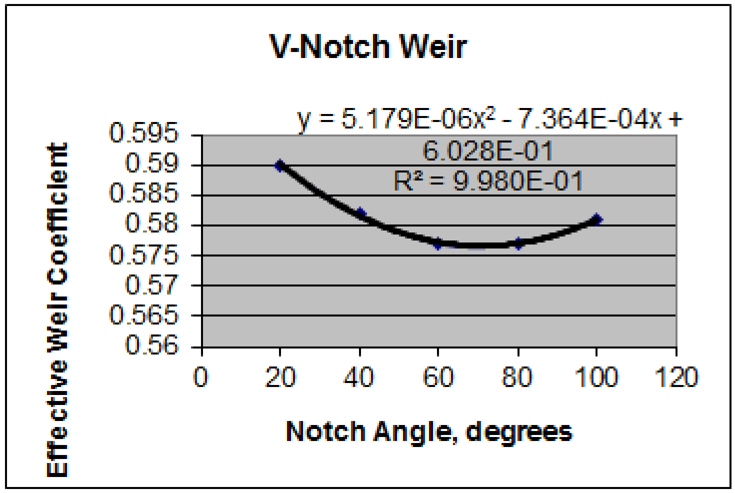

Figure 4. V Notch Weir – Effective Discharge Coefficient, C_e

θ, degrees	k
20	0.009
40	0.0055
60	0.0038
80	0.003
100	0.003

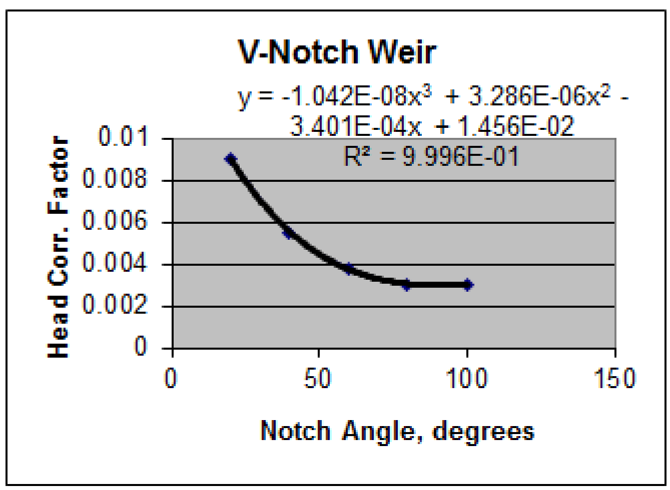

Figure 5. V Notch Weir – Head Correction Factor, k

Example #2: Estimate the flow rate through a fully contracted V notch weir with a head over the weir of 1.25 feet for a) a notch angle of 30°, b) a notch angle of 70°, and c) at notch angle of 90°. Use the Kindsvater-Carter equation [Equation (3)] for all three notch angles.

Solution: These calculations consist of substituting the value of θ into Equation (4) to calculate C_e, and into Equation (5) to calculate k. Then values of C_e, k, θ, and H are substituted into Equation (3) to calculate the flow rate, Q. The solution for the three notch angles is summarized in the table below.

H, ft	θ, degrees	Ce	k, ft	Q, cfs
1.25	30	0.585	0.0070	1.19
1.25	70	0.577	0.0033	3.04
1.25	90	0.578	0.0030	4.35

Note that the flow rate calculated for the 90° notch angle is essentially the same here as the value calculated with Equation (1) in Example #1.

For use with **S.I. units**, the Kindsvater-Carter equation changes slightly to become:

$$\mathbf{Q = 3.26\, C_e \, Tan\, (\theta/2)\, (H + k)^{5/2}} \qquad (6)$$

Where Q is in m^3/s and H& k are in m. Equation (4) for C_e remains the same for calculations in S.I. units, because C_e is dimensionless. The right hand side of Equation (5) must be multiplied by 0.3048, in order to convert k from ft to m. Thus, for S.I. units:

$$k = 0.3048[\, 0.01456 - (0.0003401)\theta + (3.286 \times 10^{-6})\theta^2 - (1.042 \times 10^{-8})\theta^3\,] \quad m \qquad (7)$$

C. Suppressed Rectangular Weirs

Figure 6 shows a suppressed rectangular weir. Its significant feature is that the weir goes across the entire channel. The name suppressed weir comes from the fact that the end contractions (see figure 2 (c)) are not present (are suppressed).

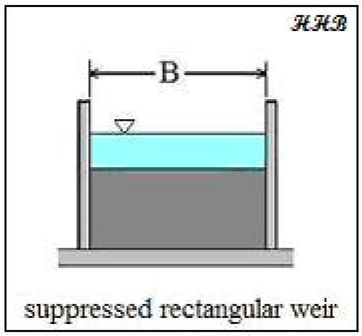

Figure 6. A Suppressed Rectangular Weir

The Kindsvater-Carter method, which will be discussed here, was originally presented by C.E. Kindsvater and R.W. Carter (reference #3) in 1959. It is recommended by the Bureau of Reclamation (reference #1) for use with sharp crested rectangular weirs.

The Kindsvater-Carter equation for either a suppressed or a contracted, sharp crested, rectangular weir is:

$$\mathbf{Q = C_e L_e H_e^{3/2} = C_e(L + k_b)(H + k_h)^{3/2}} \quad (8)$$

In this equation:

- Q = the volumetric flow rate over the weir (in cfs for U.S. or m^3/s for S.I. units)
- C_e = the effective discharge coefficient of the weir (dimensionless)
- L_e = the effective length of the weir = $L + k_b$, (in ft for U.S. or m for S.I. units)
- H_e = the effective head over the weir = $H + k_h$ (in ft for U.S. or m for S.I. units)

- k_b = a correction factor to obtain effective length (k_b = - 0.003 ft = - 0.001 m for a suppressed rectangular weir.)
- L = the length of the weir crest (in ft for U.S. or m for S.I. units)
- k_h = a correction factor for the measured head (k_h = 0.003 ft = 0.001 m for a suppressed rectangular weir.)
- H = The measured head over the weir (in ft for U.S or m for S.I. units)

For a suppressed rectangular weir, C_e, can be calculated from the following equation (from reference #1): (P is the height of the weir crest above the channel bottom, as shown in Figure (1)).

$$\mathbf{C_e = 0.4000(H/P) + 3.220} \qquad (9)$$

Substituting this expression for C_e, together with k_b = - 0.003 and k_h = 0.003 into Equation (8) gives the following general form of the Kindsvater-Carter equation for the flow rate over a *suppressed, rectangular, sharp crested weir*:

(U.S. units): $$\mathbf{Q = [0.4000(H/P) + 3.220](L - 0.003)(H + 0.003)^{3/2}} \qquad (10)$$

This is the form of the equation for U.S. units, in which Q is in cfs, and H, L, & P are in ft. A more general form of the equation for C_e (from reference #2) is as follows:

$$\mathbf{C_e = [0.075(H/P) + 0.602](2/3)[(2g)^{1/2}]} \qquad (11)$$

Equation (11) can be used for either U.S. units or S.I. units. For U.S. units, with g = 32.17 ft/sec^2, Equation (11) reduces to Equation (9). For S.I. units, Equation (11) should be used to calculate Ce, with g = 9.81 m/s^2. Thus **for S.I. units**, the Kindsvater-Carter equation for a *suppressed, rectangular, sharp crested weir* is:

$$\mathbf{Q = [0.075(H/P) + 0.602](2/3)[(2g)^{1/2}](L - 0.001)(H + 0.001)^{3/2}} \qquad (12)$$

(for S.I. units – Q in m^3/s, H, L & P in m, and g = 9.81 m/s^2):

Use of the Kindsvater-Carter equation is subject to the following conditions:

L $\geq$ 0.5 ft (or **L $\geq$ 0.15 m**); **P $\geq$ 0.33 ft** (or **P $\geq$ 0.1 m**); and **H/P $\leq$ 2.4**

Also the general requirements for any sharp-crested weir that were noted above in the V-Notch weir discussion must be met as follows:

P – D $\geq$ 0.2 ft (U.S. units) or **P – D $\geq$ 0.06 m** (S.I. units)

and **H $\geq$ 0.2 ft** (or **H $\geq$ 0.06 m**)

A simplified version of this equation, called the Francis Equation, can be used for many cases of flow over a suppressed rectangular weir. The Francis equation below is given in both reference #1 and reference #2, to be used only if the specified conditions are met.

(U.S. units) $$Q = 3.33\, B\, H^{3/2} \qquad (13)$$

Subject to the conditions that: **H/P $\leq$ 0.33** and **H/B $\leq$ 0.33**

This equation is for use with the following U.S. units: Q in cfs; B, P, & H in ft.

For S.I. units, Equation (12) becomes:

(S.I. units) $$Q = 1.84\, B\, H^{3/2} \qquad (14)$$

Subject to the conditions that: **H/P $\leq$ 0.33** and **H/B $\leq$ 0.33**

This equation is for use with the following S.I. units: Q in m^3/s; B, P, & H in m.

Example #3: Calculate the flow rate over a suppressed rectangular weir that is in a 4 ft wide rectangular channel with the weir crest 1.5 ft above the channel bottom, for a) H = 0.25 ft and b) H = 0.5 ft.

Solution: From the given information, P = 1.5 ft and B = L = 4 ft.

a) H/P = 0.25/1.5 = 0.167 and H/B = 0.25/4 = 0.0625 ; Since both these ratios are less than 0.33, Equation (13) can be used.

$$Q = 3.33\,(4)(0.25^{3/2}) = \underline{\mathbf{1.67\ cfs}}$$

b) H/P = 0.5/1.5 = 0.33 and H/B = 0.5/4 = 0.125, so:

$$Q = 3.33\,(4)(0.5^{3/2}) = \underline{\mathbf{4.71\ cfs}}$$

Example #4: Repeat Example #3, using the more general form of the Kindsvater-Carter equation (Equation (10)) instead of Equation (13).

Solution: Substituting given values of P, L, and H into Equation (10):

a) $$Q = [0.4000(0.25/1.5) + 3.220](4 - 0.003)(0.25 + 0.003)^{3/2} = \underline{\mathbf{1.67\ cfs}}$$

b) $$Q = [0.4000(0.5/1.5) + 3.220](4 - 0.003)(0.5 + 0.003)^{3/2} = \underline{\mathbf{4.78\ cfs}}$$

These calculations can be done conveniently with Worksheet 2 in the Rectangular Weir Spreadsheet that was provided with this course. The screenshot on the next page, from the suppressed rectangular weir-U.S. tab, shows the solution to Example #4 for h = 0.5 ft.

Note that the calculated flow rates from the two equations are close, but not quite the same. H = 0.5 is on the borderline of meeting the H/P requirement for use of Equation (13), which leads to the variation between the two results.

Calculation of Flow Rate Over a Rectangular Weir - U.S. Units					
1. Sharp-crested, Suppressed					
Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*					
Inputs			**Calculations**		
Height of weir crest above channel invert, **P** =	1.5	ft	H/B =	0.13	
Width of channel, **B** (= weir length, **L**) =	4	ft	H/P =	0.33	
Measured head over the weir, **H** =	0.5	ft	P - D =	1.00	ft
Depth of water downstream of weir, D =	0.5	ft			
Flow rate calculated with the Francis equation:			Flow Rate, **Q** =	**4.71**	**cfs**
H/P>0.33, requirements for Francis Eqn are not met.					
See the results using the general Kindsvater-Carter equation below.					
Flow rate calculated with the general Kindsvater-Carter equation:					
			Flow Rate, Q =	**4.78**	**cfs**
Requirements for Kindsvater-Carter Equation are met. See results above.					

Figure 7. Screenshot of Suppressed Rectangular Weir Spreadsheet

Example #5: Calculate the flow rate over a suppressed rectangular weir in the same channel described in Examples #3 & #4, using both Equation (10) and Equation (13) for each of the following values of H, and calculate the % difference between the two results for each value of H.

a) H = 0.6 ft; b) H = 0.8 ft; c) H = 1.5 ft

Solution: As in the previous two examples, P = 1.5 and B = L = 4. Substituting these values, along with the appropriate value of H for each part of the solution, leads to the results summarized in the table below. All of these calculations can be done with the Rectangular Weir Spreadsheet that was provided with the course.

	H, ft	H/B	H/P	Q(10)	Q(13)	% diff.
a)	0.6	0.15	0.40	**6.33**	**6.19**	**2.21 %**
b)	0.8	0.20	0.53	**9.87**	**9.53**	**3.44 %**
c)	1.5	0.38	1.0	**26.66**	**24.47**	**6.67 %**

Note the increasing % difference as the ratios H/P and/or H/P become increasingly larger. This emphasizes the importance of using the more comprehensive Equation (10) when H/P and/or H/B is greater than 0.33.

Example #6: Calculate the flow rate over a suppressed rectangular weir in a 2 ft wide rectangular channel with P = 2.5 ft, for H = 0.2 ft and for H = 1.25 ft. Compare the results with the results calculated in Example #1 for a V notch weir with the same two values of H.

Solution: For H = 0.2 ft, H/P and H/B are both less than 0.33, so the simpler Equation (13) can be used, giving:

$$Q = 3.33\,(2)(0.2^{3/2}) = \underline{\mathbf{0.596\ cfs}}$$

For H = 1.25 ft, H/P and H/B are both greater than 0.33, so the more complicated Equation (10) must be used, giving:

$$Q = [0.4000(1.25/2.5) + 3.220](2 - 0.003)(1.25 + 0.003)^{3/2} = \underline{\mathbf{9.58\ cfs}}$$

A comparison with the V-notch weir results from Example #1 is summarized in the table below:

Head over the weir, ft	Q for V notch Weir, cfs	Q for 2 ft Suppressed Rectangular Weir, cfs
0.2	0.046	0.596
1.25	4.55	9.58

Note that the V notch weir can measure a much lower flow rate at the minimum head over the weir (0.2 ft), while the 2 ft suppressed rectangular weir can measure a larger flow rate at the maximum head over the weir (1.25 ft)

D. Contracted Rectangular Weirs

Figure 8 below shows a contracted rectangular weir. As shown in that diagram, the length of the weir, L, is less than the channel width, B for this type of rectangular weir.

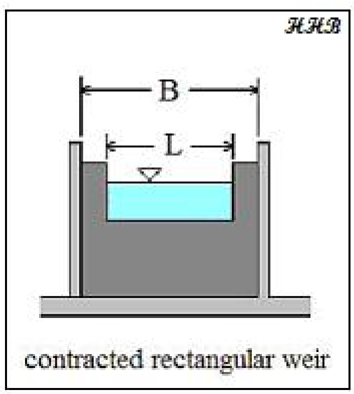

Figure 8. A Contracted Rectangular Weir

The simplified version of the Kinsdvater-Carter Equation, Equation (14) below, known as the Francis Equation, may be used only if the three conditions given below the equation are met.

(U.S. units): $\mathbf{Q = 3.33(L - 0.2\,H)(H^{3/2})}$ (15)

Subject to the condition that:

i) $\mathbf{B - L \geq 4\ H_{max}}$; ii) $\mathbf{P \geq 2\ H_{max}}$; and iii) $\mathbf{H/L \leq 0.33}$

In Equation (15): L and H must be in ft and Q will be in cfs.

Converting to S.I.units gives Equation (16) below, in which H and L are in m and Q will be in m^3/s (The required conditions for use of Equation (16) are the same as those just given above for Equation (15):

$$\text{(S.I. units):} \quad \mathbf{Q = 1.84(L - 0.2\ H)(H^{3/2})} \tag{16}$$

If any of the three conditions needed for use of Equations (15) or (16) are not met, then the general Kindsvater-Carter equation for a rectangular weir must be used. The starting point for setting up this equation for a contracted rectangular weir is Equation (8) from page 12, which is shown again below.

$$\mathbf{Q = C_e\, L_e\, H_e^{3/2} = C_e(L + k_b)(H + k_h)^{3/2}} \tag{8}$$

For a contracted rectangular weir, k_h is still equal to 0.003 ft or 0.001 m, as with the suppressed rectangular weir, however the value of k_b is not constant. Its value depends upon the value of L/B, and can be calculated from the following equations, which were derived by Excel curve fitting using data from a graph of k_b vs L/B similar to Figure 9 below, from the USBR *Water Measurement Manual* (Ref #2):

For $0 \leq L/B \leq 0.5$:

$$\mathbf{k_b = 0.0139(L/B)^3 + 0.000476\,(L/B)^2 + 0.000885(L/B) + 0.00770\ ft} \tag{17}$$

For $0.5 \leq L/B \leq 1.00$:

$$\mathbf{k_b = -\,8.0889(L/B)^5 + 28.186(L/B)^4 - 39.056(L/B)^3 + 26.84(L/B)^2 - 9.1139(L/B) + 1.2302\ ft} \tag{18}$$

For a contracted rectangular weir, C_e depends upon both H/P & L/B, and can be calculated from the following equations:

$$\mathbf{C_e = C_1(H/P) + C_2} \tag{19}$$

Where C1 and C2 are functions of L/B as shown in the table below, which is table 1 in the USBR *Water Measurement Manual* (Ref #1).

L/B	C_1	C_2
0.2	-0.0087	3.152
0.4	0.0137	3.164
0.5	0.0612	3.173
0.6	0.0995	3.178
0.7	0.1602	3.182
0.8	0.2376	3.189
0.9	0.3447	3.205
1.0	0.4000	3.220

The equations for k_b and C_e were adapted from information in reference #4. Graphs showing k_b as a function of L/B and C_e as a function of H/P and L/B are available from various sources, including reference #1. The equations above are more convenient for calculating values of k_b and C_e, but Figures (9) and (10) are helpful in showing the pattern of the relationships.

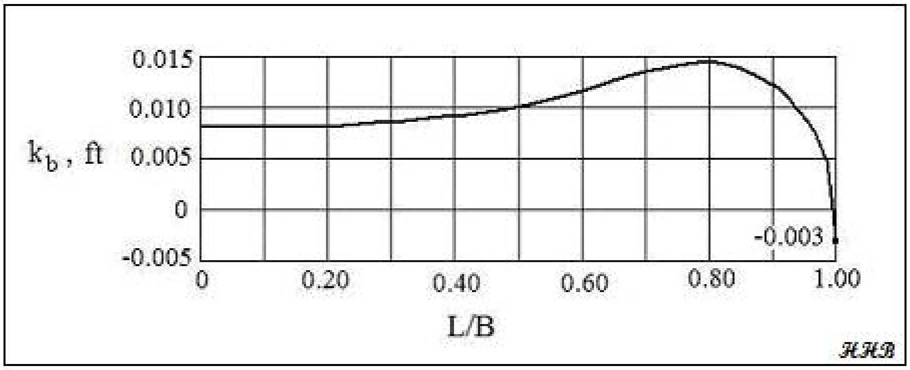

Figure 9. k_b as a function of L/B

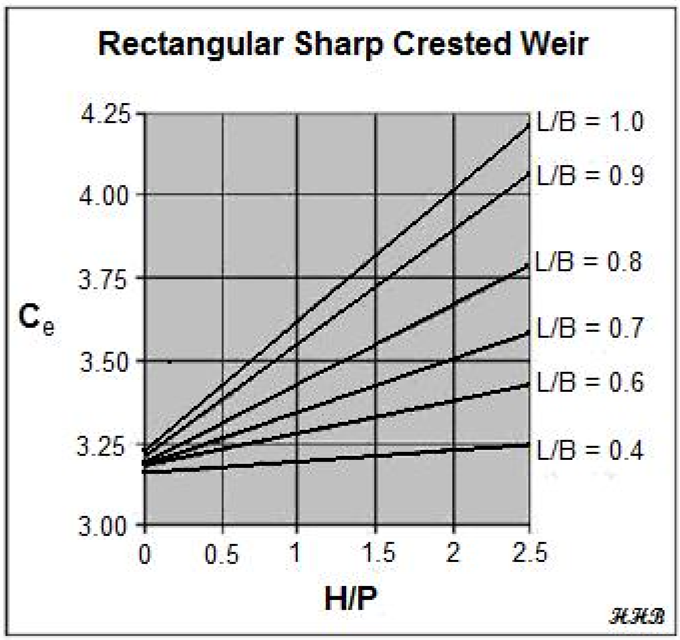

Figure 10. C_e as a Function of L/B and H/P

Example #7: Calculate the volumetric flow rate over a contracted rectangular weir using both Equation (15) (the Francis Equation) and Equation (8) (the Kindsvater-Carter Equation) [together with Equations (17) through Equation (19)] for each of the following sets of channel parameters. Also check on whether the three conditions required for use of the Kindsvater-Carter Equation are met for each set of channel parameters.

	B, ft	L, ft	P, ft	H, ft
a)	2	1	0.5	0.2
b)	2	1	0.5	0.4
c)	4	2	1	0.4
d)	4	2	1	0.8
e)	4	2	1	1.5

Solution: The calculations for this example can be done with the rectangular weir spreadsheet. The results are summarized in the table below along with a calculation of the % difference based on the Kindsvater-Carter result. A screenshot showing the spreadsheet solution for part (d) is shown as Figure 11 below.

The three conditions for use of the Kindsvater-Carter Equation can be written as:

$$(B - L)/4H \geq 1 \quad ; \quad P/2H \geq 1 \quad ; \quad H/L \leq 0.33$$

RESULTS:

	(B – L)/4H	P/2H	H/L	Francis Eqn Q(15), cfs	K-C Eqn Q(8), cfs	% diff
a)	1.25	1.25	0.20	0.29	0.30	3.3%
b)	0.63	0.63	0.40	0.78	0.83	6.0%
c)	1.25	1.25	0.20	1.62	1.65	1.8%
d)	0.63	0.63	0.40	4.38	4.63	5.4%
e)	0.33	0.33	0.75	10.40	11.87	12.4%

Note that the two equation agree reasonably well for part (a) and part (c), the two configurations that meet all three of the requirements for use of the Francis Equation. Also, the greater the divergence from the required conditions, the greater the difference between the results for the two equations.

This provides evidence of the importance of using the more complicated Kindsvater-Carter equation for contracted rectangular weir calculations, if the specified conditions needed for use of the simpler Equation (15) are not met. It also indicates the suitability of using the simpler Equation (15) in those cases where the required conditions, $(B – L)/4H \geq 1$; $P/2H \geq 1$; and $H/L \leq 0.33$, are all met.

Calculation of Flow Rate Over a Rectangular Weir - U.S. Units
2. Sharp-crested, Contracted

Instructions: *Enter values in blue boxes. Spreadsheet calculates values in yellow boxes*

Inputs			**Calculations**		
Height of weir crest above channel invert, **P** =	1	ft	**(B - L)/4H =**	0.63	
Width of channel, **B** =	4	ft	**P/2H =**	0.63	
Length of weir, **L** =	2	ft	**H/L =**	0.40	
Measured head over the weir, **H** =	0.8	ft	**P - D =**	0.50	ft
Depth of water downstream of weir, **D** =	0.5	ft			

Flow rate calculated with the Francis equation: **Flow Rate, Q** = **4.38** cfs

(B - L)/4H < 1, requirements for Francis Eqn are not met.
See the results using the Kindsvater-Carter equation below.

Flow rate calculated with the Kindsvater-Carter equation:

(This calculation is only made if the requirements for **Flow Rate, Q** = **4.63** cfs
the Francis equation are not met and the conditions for the Kindsvater-Carter equation are met.)

The values calculated for $\mathbf{C_e}$ and $\mathbf{k_b}$ and used to calculate **Q** are shown below.

Requirements for Kindsvater-Carter Equation are met. See results above.

H/P =	0.8		**L/B** =	0.500
$\mathbf{k_b}$ =	**0.01000**	ft	Next lower L/B value from table below:	0.5
			Next higher L/B value from table below:	0.6

Ce for L/B =	0.5	equals	3.20360
Ce for L/B =	0.6	equals	3.23770
Ce for L/B =	0.500	equals	**3.20360**

L/B	C_1	C_2
0.2	-0.0087	3.152
0.4	0.0137	3.164
0.5	0.0612	3.173
0.6	0.0995	3.178
0.7	0.1602	3.182
0.8	0.2376	3.189
0.9	0.3447	3.205
1.0	0.4000	3.220
0.50	0.0612	3.173
0.60	0.0995	3.178

Figure 11. Screenshot of Contracted Rectangular Weir Spreadsheet

E. Installation and Use Guidelines

This section contains installation and measurement guidelines for sharp crested weirs, based on information in the U.S. Bureau of Reclamation, *Water Measurement Manual* (reference #1).

i) <u>All Sharp Crested Wiers:</u>

a) The upstream face of any weir plate or bulkhead must be plumb, smooth, and perpendicular to the channel axis.

b) The thickness of the crest and side plates at the top must be between 0.03 and 0.08 inch.

c) The upstream edges of weir openings should be straight and sharp. Use machining or filing perpendicular to the upstream face to remove burrs or scratches. Abrasive cloth or paper shouldn't be used to smooth off these edges. Knife edges are a safety hazard and they damage easily, so they should be avoided.

d) The nappe or overflow sheet shouldn't touch the downstream face of the crest and side plates.

e) The highest downstream water level should be at least 0.2 ft below the crest elevation.

f) For a very small head over the weir, use frequent observations to be sure that the Nappe is continuously ventilated.

g) The head over the weir should be measured as the difference between the elevation of the weir crest and the elevation of the water surface at a point that is upstream of the weir at least four times the maximum head over the weir.

h) The channel upstream of the weir must be kept free of sediment deposits.

i) Ten average approach channel widths of straight, unobstructed approach are needed if the weir crest is more than 50% of the channel width.

j) Twenty average approach channel widths of straight, unobstructed approach are needed if the weir crest is less than 50% of the channel width.

k) If the upstream flow is supercritical, a hydraulic jump should be forced to occur upstream far enough to give 30 measuring heads of straight, unobstructed approach after the jump.

l) The head over the weir should be at least 0.2 ft.

<u>For V Notch Weirs:</u>

In addition to the general conditions, a) through l) above:

a) The bisector of the V notch angle should be plumb.

b) The maximum measured head for a fully contracted V notch weir should be 1.25 ft.

c) H/B should b less than 0.2 for a fully contacted V notch weir.

d) For a fully contracted V notch wier, the average width of the approach channel should be at least 3 ft.

e) The vertex of the V notch should be at least 1.5 ft above the bottom of the channel for a fully contracted V notch weir.

<u>For Rectangular, Sharp Crested Weirs:</u>

In addition to the general conditions, a) through l) above:

a) The entire crest should be level.

b) The crest length should be at least 6 inches.

c) The crest height should be at least 4 inches.

d) H/P should be less than 2.4.

5. Broad Crested Weirs

A. General Configuration

Broad crested weirs are used primarily for flow measurement and regulation of water depth in canals, rivers, and other natural open channels, because the broad crested weir is more robust with regard to variations in upstream conditions than the sharp crested weir. The diagram below shows the general configuration of a broad crested weir, which is basically a flat-topped obstruction extending across the entire channel.

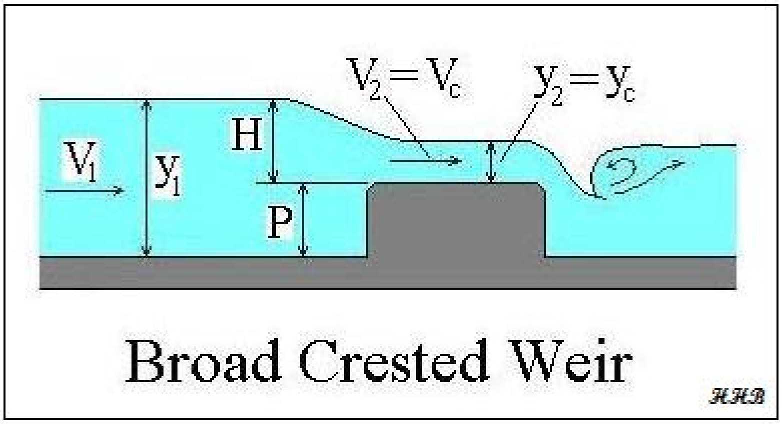

Figure 12. Parameters for a Broad Crested Weir

The parameters shown in Figure 12 are the head over the weir, **H**; the height of the weir, **P**; the approach velocity, $\mathbf{V_1}$; the approach depth, $\mathbf{y_1}$; the velocity over the weir crest, $\mathbf{V_2}$; and the water depth over the weir crest, $\mathbf{y_2}$. It is also noted that the flow over the weir crest should be critical flow, making $\mathbf{V_2 = V_c}$ and $\mathbf{y_2 = y_c}$.

B. Equations for Flow Over a Broad Crested Weir

When the flow over the weir crest is critical flow, the equation for calculating the flow rate over the weir is quite straightforward, with the following general equation:

$$\mathbf{Q = C\,L\,H^{1.5}} \qquad \text{(for critical flow over the weir)} \qquad (23)$$

Where for U.S. units:

- **Q** is the open channel flow rate (flow rate over the weir) in cfs.
- **L** is the length of the weir crest in ft. (the width of the opening perpendicular to the flow direction)
- **H** is the head over the weir (as shown in Figure 12) in ft.
- **C** is the discharge coefficient

For S.I. units:

- **Q** is the open channel flow rate (flow rate over the weir) in m^3/s.
- **L** is the length of the weir crest in m. (the width of the opening perpendicular to the flow direction)
- **H** is the head over the weir (as shown in Figure 12) in m.
- **C** is the discharge coefficient

The theoretical value for the discharge coefficient, **C**, for critical flow over the weir is $(g^{0.5})(2/3)^{1.5}$. For the U.S. units shown above, with g = 32.2 ft/sec^2, the theoretical value is **C = 3.09**. For the S.I. units shown above, with g = 9.81 m/sec^2, the theoretical value is **C = 1.70**. In the 2012 edition of the FHWA publication, “Hydraulics of Safe Bridges” (Reference #9 at the end of this course), it is stated that the discharge coefficient for a broad-crested weir is typically in the range from **2.6 to 3.05** for U.S. units or **1.44 to 1.68** for S.I. units.

Note that the length of the weir crest perpendicular to the flow direction is typically called the weir crest length or weir length, while the length of the weir crest parallel to flow is typically called the weir crest breadth or weir breadth.

Also, note that the typical **C** values given above are for free flow over the weir. Pages 3-27 and 3-28 in the 2012 edition of FHWA Hydraulic Design of Safe Bridges (Reference #9 for this course) provides discussion of submerged flow over a broad-crested weir and curves for determining a value for the discharge coefficient, **C**, for submerged or partially submerged flow.

Also, you may find equations or tables for determining the discharge coefficient for a broad-crested weir based on various broad-crested weir parameters. Coverage of those tables and/or equations is beyond the scope of this course.

Example #8: Calculate the flow rate in a 4 ft wide open channel in which the head over a broad crested weir, **H**, is measured to be 2.5 ft. Assume that there is free flow over the weir, that the weir goes across the entire channel, that it was designed to provide critical flow over the crest for all anticipated heads over the weir and that the value of 2.9 can be used for the discharge coefficient.

Solution: For the specified condition the flow rate can be calculated from Equation (23) with **H** = 2.5, **L** = 4, and **C** = 2.9. Thus:

$$Q = 2.9\,L\,H^{1.5} = 2.9(4)(2.5^{1.5}) = \underline{\mathbf{45.8\ cfs = Q}}$$

C. Determining Weir Height Needed for Critical Flow Over the Weir

The presence of a broad crested weir obstruction in an open channel will cause the water velocity to increase as it passes over the weir crest, in comparison with the velocity in the approach channel. Increasing the height of the weir will increase the velocity over the weir crest, up to the point at which the flow over the weir crest becomes critical flow. In order to use the rather simple equation (23) or (24) above to calculate the flow over the weir, the weir height must be great enough to cause critical flow over the weir crest. The following three fundamental fluid mechanics equations can be used to calculate the minimum weir height, P_{min}, needed to cause critical flow over the weir.

1. The energy equation [with y_1, V_1, y_2, and V_2 as shown in Figure (12)] :

$$y_1 + V_1^2/2g = P + y_2 + V_2^2/2g$$

2. The definition of average velocity in an open channel, assuming an approximately rectangular channel cross-section:

$$V = Q/yB$$

3. The equation based on minimum specific energy at critical flow conditions in a rectangular channel:

$$y_c = (Q^2/gB^2)^{1/3} \qquad (24)$$

Writing the energy equation and the average velocity definition equation for the case in which the flow over the weir crest is critical flow gives:

$$y_1 + V_1^2/2g = P_{min} + y_c + V_c^2/2g \qquad (25)$$

$$V_1 = Q/y_1B \qquad (26)$$

$$V_c = Q/y_cB \qquad (27)$$

If The flow rate, Q, through a channel of width, B, has flow depth, y_1, in the approach channel, then the four equations above [Equations (25), (26), (27), & (28)] can be used to calculate the minimum weir height, P_{min}, needed to give critical flow over the weir, as demonstrated in Example #9.

Example #9: Calculate the minimum weir height needed to give critical flow over a broad crested weir in a 15 ft wide channel with a maximum flow rate of 250 cfs at an approach depth of 5 ft.

Solution: Using Equations (24), (25), (26), & (27), with Q = 250 cfs, y_1 = 5 ft, and B = 15 ft, the calculations proceed as follows:

Equation (27): $V_1 = Q/y_1B = 250/(5)(15) = 3.33$ ft/sec

Equation (25): $y_c = (Q^2/gB^2)^{1/3} = [\,(250^2)/(32.17)(15^2)\,]^{1/3} = 2.05$ ft

Equation (28): $V_c = Q/y_cB = 250/(2.05)(15) = 8.12$ ft/sec

Equation (26): $P_{min} = y_1 + V_1^2/2g - y_c - V_c^2/2g$

$= 5 + (3.33^2/64.34) - 2.05 - (8.12^2/64.34) =$ **2.10 ft = P_{min}**

6. The Parshall Flume

A. Introduction to the Parshall Flume

The Parshall flume is used for open channel flow measurement in a variety of applications, especially for flows that contain suspended solids, as in a wastewater treatment plant. The diagrams in Figure 13 show a plan and elevation view of a Parshall flume.

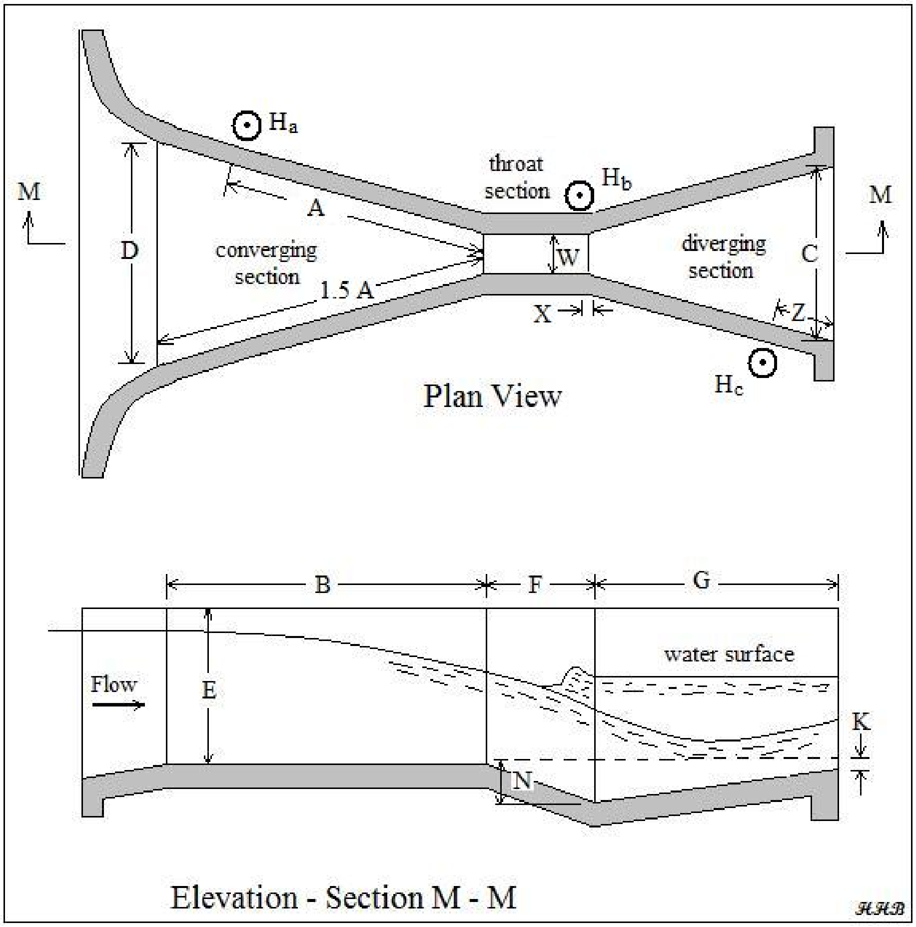

Figure 13. Plan and Elevation View of a Parshall Flume

The plan view of the Parshall flume shows a converging section, a throat, and a diverging section. The bottom of a Parshall flume also has prescribed variations as shown in the elevation view. Figure 13 was prepared from information in ref. #1.

The size of a Parshall flume is typically specified by its throat width. Thus a 2 ft Parshall flume means one with a 2 ft throat width. The table below gives some of the required Parshall flume dimensions for several throat widths ranging from one foot to 50 feet. Figure 13 above shows the dimension represented by each of the letters in the table. This information came from reference #1, which contains a more complete table.

Dimensions for Various Size Parshall Flumes

W, ft	A, ft-in	C, ft-in	D, ft-in	E, ft-in	F, ft-in	G, ft-in	N, ft-in	X, ft-in
1	3-0	2-0	2-9 ¼	3-0	2-0	3-0	0-9	0-2
3	3-8	4-0	5-1 7/8	3-0	2-0	3-0	0-9	0-2
5	4-4	6-0	7-6 3/8	3-0	2-0	3-0	0-9	0-2
8	5-4	9-0	11-1 ¾	3-0	2-0	3-0	0-9	0-2
15	7-8	18-4	25-0	6-0	4-0	10-0	1-6	0-9
30	12-8	34-8	40-4 ¾	7-0	6-0	14-0	2-3	0-9
50	19-4	56-8	60-9 ½	7-0	6-0	20-0	2-3	0-9

B. Free Flow and Submerged Flow

Free flow refers to the flow condition in which the downstream channel conditions have no effect on the flow rate through the throat of the flume. For free flow in a Parshall flume, there will be a hydraulic jump visible in the flume throat. For flow conditions in which the downstream channel conditions cause the flow to ‘back up’ into the throat, the flow is referred to as “submerged flow,” and no hydraulic jump is visible in the throat.

The locations, H_a and H_b on Figure 13 are used to measure the head, H_a and the head H_b. The ratio of these two heads, H_b/H_a Is used as a measure for whether the flow can be considered free flow or if it is submerged flow.

For 1” $\leq$ W $\leq$ 3” : $H_b/H_a \leq 0.5$ means free flow and $H_b/H_a > 0.5$ means submerged flow.

For 6” $\leq$ W $\leq$ 9” : $H_b/H_a \leq 0.6$ means free flow and $H_b/H_a > 0.6$ means submerged flow.

For 1’ $\leq$ W $\leq$ 8’ : $H_b/H_a \leq 0.7$ means free flow and $H_b/H_a > 0.7$ means submerged flow.

For 8’ $\leq$ W $\leq$ 50’ : $H_b/H_a \leq 0.8$ means free flow and $H_b/H_a > 0.8$ means submerged flow.

C. Equations for Free Flow through a Parshall Flume

The equation for calculation of flow rate through a Parshall Flume under free flow conditions is

$$\mathbf{Q_{free} = C\ H_a^{\,n}} \qquad (28)$$

Where:

- Q_{free} is the flow rate through a Parshall flume with free flow conditions, in cfs for U.S. or m^3/s for S.I. units.
- H_a is the head measured at the point in the converging section of the Parshall flume that is identified in Figure 12, in ft for U.S. or m for S.I. units.
- C and n are constants for a specified Parshall flume throat width, W.

Values for C and n for a range of throat widths are given in the table on the next page for both U.S. units and S.I. units.

Example #10: Determine the flow rate through a Parshall flume with 1 ft throat width, when H_a = 1.5 ft and H_b = 0.6 ft.

Solution: $H_b/H_a = 0.6/1.5 = 0.4$. Since this ratio is less than 0.7, this is free flow. From the table on the next page, C = 4.00 and n = 1.522, so

$$Q = Q_{free} = 4.00\ H_a^{1.522} = 4.00\ (1.5^{1.522}) = \underline{\mathbf{7.41\ cfs\ =\ Q}}$$

Constants for Parshall Flume Free Flow* Equation, $Q_{free} = CH_a^n$				
Throat Width, W		n	C, U.S. units*	C, S.I. Units**
1 in	2.5 cm	1.55	0.338	0.060
2 in	5.1 cm	1.55	0.676	0.121
3 in	7.6 cm	1.55	0.992	0.177
6 in	15.2 cm	1.58	2.06	0.381
9 in	22.9 cm	1.53	3.07	0.535
12 in	30.5 cm	1.522	4.00	0.691
18 in	45.7 cm	1.538	6.00	1.056
2 ft	.610 m	1.550	8.00	1.429
3 ft	.914 m	1.566	12.00	2.184
4 ft	1.219 m	1.578	16.00	2.954
5 ft	1.524 m	1.587	20.00	3.732
6 ft	1.829 m	1.595	24.00	4.518
7 ft	2.134 m	1.601	28.00	5.313
8 ft	2.438 m	1.607	32.00	6.115
10 ft	3.048 m	1.6	39.38	7.463
12 ft	3.658 m	1.6	46.75	8.859
15 ft	4.572 m	1.6	57.81	10.96
20 ft	6.096 m	1.6	76.25	14.45
25 ft	7.620 m	1.6	94.69	17.94
30 ft	9.144 m	1.6	113.13	21.44
40 ft	12.19 m	1.6	150.00	28.43
50 ft	15.24 m	1.6	186.88	35.41
* for U.S. units, H_a is in ft and Q_{free} is in cfs				
** for S.I. units, H_a is in m and Q_{free} is in m^3s				

D. Equations for Submerged Flow through a Parshall Flume

The general equation used to calculate the flow rate through a Parshall flume that is flowing under submerged flow conditions is:

$$\mathbf{Q_{subm} = Q_{free} - Q_{corr}} \qquad (29)$$

Where:

- Q_{subm} is the flow rate through a Parshall flume that is under submerged flow conditions, in cfs for U.S. or in m^3/s for S.I. units.
- Q_{free} is the flow rate calculated using the equation, $Q_{free} = C\ H_a^n$, as described in section D, in cfs for U.S. or in m^3/s for S.I. units.
- Q_{corr} is a flow correction factor, which is calculated with the appropriate equation for the throat width of the Parshall flume from the equations below, in cfs for U.S. or in m^3/s for S.I. units.

The flow correction factor, Q_{corr}, can be calculated from the appropriate equation below, based on the throat width of the Parshall flume.

1. For 1 ft $\leq$ W $\leq$ 8 ft: $\mathbf{Q_{corr} = 0.000132\ M\ H_a^{2.123}\ e^{9.284(S)}}$ (30)

Where:

- $S = H_b/H_a$
- The value of M is selected from the table below, based on the throat width.

W, ft	1	1.5	2	3	4	5	6	7	8
M	1	1.4	1.8	2.4	3.1	3.7	4.3	4.9	5.4

2. For 8 ft $\leq$ W $\leq$ 50 ft: $\mathbf{Q_{corr} = (W/10)\,(3.364 + 20.19\,S^2\,\ln S)^2}$ (31)

Where:

- $S = H_b/H_a$

Example #11: Determine the flow rate through a Parshall flume with 1 ft throat width, when H_a = 1.5 ft and H_b = 1.25 ft.

Solution: H_b/H_a = 1.25/1.5 = 0.8333. Since this ratio is greater than 0.7, this is submerged flow.

$$Q = Q_{subm} = Q_{free} - Q_{corr}$$

From the table on page 37, C = 4.00 and n = 1.522, so

$$Q_{free} = 4.00\,H_a^{1.522} = 4.00\,(1.5^{1.522}) = 7.41\text{ cfs}$$

For W = 1 ft: $Q_{corr} = 0.000132\,M\,H_a^{2.123}\,e^{9.284(S)} = 0.000132\,(1)(1.5^{2.123})e^{9.284(0.8333)}$

$Q = Q_{free} - Q_{corr} = 7.41 - 0.715 =$ **<u>6.70 cfs = Q</u>**

E. Installation and Use Guidelines

This section contains installation and measurement guidelines for Parshall flumes, based on information in the U.S. Bureau of Reclamation, *Water Measurement Manual* (reference #1).

1. Flumes shouldn't be installed close to turbulent flow, surging flow, or unbalanced flow.

2. The flow in the approach channel should be tranquil flow, that is, it should be fully developed flow in a long straight channel with a mild slope, free of curves, projections, and waves.

3. If the throat width is more than half of the approach channel width, then 10 throat widths of straight unobstructed approach channel are needed.

4. If the throat width is less than half of the approach channel width, then 20 throat widths of straight unobstructed approach channel are needed.

5. If there is supercritical flow upstream of the Parshall flume, then a hydraulic jump should be forced to occur, and 30 measuring heads of straight unobstructed approach channel is needed after the jump.

6. When baffles are used to smooth and correct the approach flow, then 10 measuring heads of channel should be provided between the baffles and the measuring station.

7. If feasible, the approach velocity should be greater than 1 ft/sec.

8. The Froude number in the approach channel should be less than 0.5 for all flow conditions for a distance of at least 30 measuring heads upstream of the Parshall flume.

7. Summary

Sharp-crested weirs, broad crested weirs and flumes are commonly used for flow rate measurement in open channels. Three types of sharp-crested weirs: V-notch, suppressed rectangular, and contracted rectangular, are covered in detail in this course. Emphasis is on calculation of flow rate over a weir for given head over the weir and weir/channel dimensions. For each of the three types of sharp-crested weir, a general equation with a wide range of applicability is presented and discussed along with equations and/or graphs as needed for use with the main equation. Also, for each of the three types of sharp-crested weir, a simpler equation is presented along with a set of conditions under which the simpler equation can be used. Similarly equations are given for broad crested weir and Parshall flume calculations, including flow rate calculation. Several worked examples are included covering all three types of sharp crested weirs, for broad crested weirs, and for Parshall flumes. Practical installation and use guidelines for sharp-crested weirs and for Parshall flumes are presented.

8. References

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